Quasi-resonance excitation of stationary disturbances in compressible boundary layers

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***Abstract* —** In the paper the resonance theory was used for an explanation of generation reasons of the longitudinal structures in the compressible boundary layer by external vorticities. Researches are conducted as in case of subsonic Mach numbers, and in case of a supersonic flow. As a result of researches Eigen values of a uniform boundary problem have been obtained and the corresponding Eigen functions are constructed. By researches of other authors it has been established that two-dimensional stationary disturbances in a subsonic boundary layer on a flat plate are damped on the longitudinal coordinate by a power law, the exponent is Eigen value of a boundary problem. The present results coincide with their data completely. The researches of three-dimensional disturbances which were conducted for the first time have shown that their fading rate down a stream depends on wave numbers in the lateral direction poorly. However, there are the optimal values of the wave number in the lateral direction, in which perturbations are damped down a stream the most poorly. If in case of subsonic speeds decrements of perturbations of the first mode doesn't depend neither on a Reynolds number, nor on value of a lateral wave number, then in case of M=2 the nature of a perturbations reduction on longitudinal coordinate depends both on a wave number, and on a Reynolds number.

Intensive generation of longitudinal structures takes place under a condition when parameters of external waves are close to parameters of Eigen stationary perturbations of a boundary layer. Data of the resonant theory are coordinated with direct calculations of an interaction of external disturbances with a boundary layer satisfactorily.

***Keywords* —** boundary-layer, longitudinal structures, vorticities, supersonic flow, laminar-turbulent transition

I. Introduction

THIS paper is an extended version of a theoretical investigation originally presented at the 13th International Conference on Applied and Theoretical Mechanics (Venice, Italy, April 26-28, 2017) and published in [1].

Present studies are concerned with the transition of a laminar flow into turbulence one in the flat plate boundary layer. The transition of an attached boundary layer from a laminar to a turbulent state is usually classified as either being through a natural mode 1) or a bypass mode 2).

1) Natural transition is the dominant mode for flows, where the freestream turbulence level is low enough. In this case scenario of transition is following. Interaction of external disturbances with a boundary layer or small perturbations imposed directly inside the boundary layer generate characteristic nonstationary fluctuations which amplify at rather long distances from a leading edge of a plate. At an achievement of larger amplitudes the nonlinear interaction takes place, which ends by the boundary layer transition in a turbulent state.

2) Bypass transition takes place at high levels of freestream disturbances and is characterized by the formation of stationary longitudinal structures. Longitudinal structures can be observed both in a result of non-linear interaction of oblique waves, usually in an unstable region [2], and in a stable area because of the linear interaction of an external turbulence with a boundary layer.

The first study [3] of the flow in the boundary layer in the presence of an external turbulence showed that in the boundary layer near the front edge of the plate the low-frequency oscillations take place. Similar studies were continued about two decades later in [4], where the streaky structures were observed. The later these structures were called as the Klebanoff's modes. These structures is clearly visible in Fig. 1, which are taken from [5].

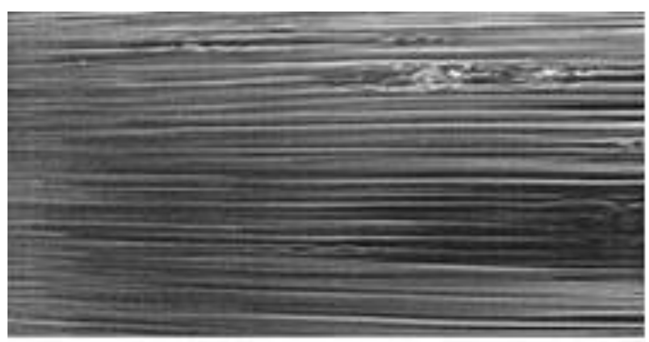

Fig.1 Flow visualization of streaky structures in boundary layers affected by free-stream turbulence [5]

The first theoretical works on the development of small two-dimensional stationary disturbances in the incompressible boundary layer on a flat plate were [6-8]. In them it was shown that perturbations of flow parameters in the boundary layer fade down on a flow under the degree law. They had shown that the parameters disturbances of the flow q_i in the boundary layer damped down the flow of power law. Exponents were Eigen numbers of the formulated problem

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on Eigen values. Interest in a study of longitudinal structures in many respects was defined by [2] where they were found experimentally. As it was mentioned above, their nature could be associated both with a nonlinear interaction of perturbations, and linear interaction of an external turbulence with boundary layer. An interaction of an external turbulence with a subsonic boundary layer on a flat plate was researched experimentally in [4, 5, 9, 10] and in some other papers which review can be found in [5]. It was noted in all these works that in the interaction result of an external turbulence with boundary layer in stable region relatively small perturbations in the subsonic boundary layer longitudinal structures were observed. Longitudinal velocity profile of stationary disturbances excited by an external turbulence, at least in the low frequency range, has a bell-shaped type, Fig.2, maximum which is located at a distance from the wall y/δ ≈2.5, where $\delta=\sqrt{x\nu_\infty/u_\infty}$, x — the distance from the leading edge plate, u_∞,ν_∞ - speed and kinematical viscosity of a ram airflow.

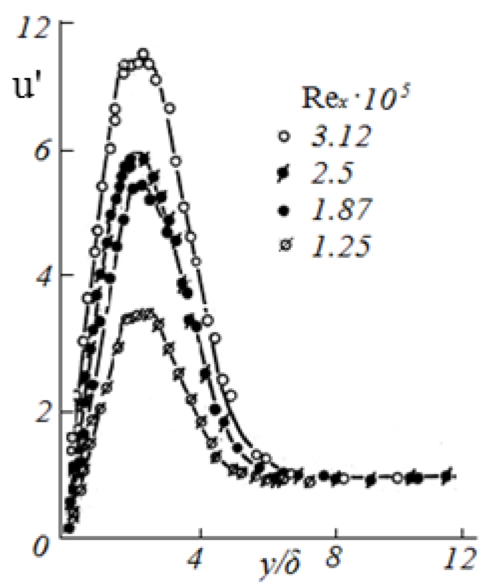

Fig.2 Distribution of the low-frequency longitudinal velocity on the boundary layer for different Reynolds numbers, Re_x [10] $\mathrm{Re}_x = xu_\infty/\nu_\infty$

For the first time theoretically an interaction of an external longitudinal vorticity with a subsonic boundary layer is explored in [11]. There it was set that under the influence of the periodic external vorticity in the lateral direction the amplitude of longitudinal structure increased linearly down a flow and was inversely proportional to its period.

It completely matched with the dependence of the perturbation amplitude on the boundary layer thickness given in [8]. At the same time the dependence of the perturbation amplitude of the longitudinal velocity on the normal coordinate was coincided with data [6,7] which were obtained for two-dimensional perturbations.

It should be noted that the theory [11] is applicable only in case of enough large periods of an external vorticity. More exact results, using parabolized stability equations, were obtained in [12]. The calculations revealed that the form of longitudinal velocity perturbations profiles does not depend on the structure period practically. Its amplitude increased in proportion to the thickness of the boundary layer.

In the subsequent theoretical and numerical study of the stationary disturbances generation in the supersonic boundary layer by external waves were continued in [13-23].

To some extent the external turbulence interaction with a boundary layer can be described by means of a continuous spectrum of the stability task. For the first time a connection between the continuous spectrum and the task of an interaction of external disturbances (acoustic) with a parallel flow in the boundary layer was specified in [24]. Perhaps the most actively the possibility of the description of vortex perturbations interaction of an external flow with a boundary layer by means of a continuous spectrum began to be used after appearance of papers [25, 26].

Another explanation for a generation of the intense perturbations in the boundary layer by external waves is a qusi-resonant interaction of external disturbances with their Eigen waves. Apparently in [27] for the first time an attention was paid to quasi-resonant generation of oscillations within the considered flow area under the influence of harmonic perturbations in the time. There the system of the differential equations of first order with harmonic in time the right part had been considered. Homogeneous system has set of Eigen frequency ω_i. Therefore disturbance amplitude will significantly exceed values of the right part if its frequency ω differs a little from Eigen frequency ω_i.

The aim of this work was to study the development of internal stationary perturbations in the compressible boundary layer on a flat plate and to describe theirs on the basis of resonance theory.

II. Problem Statement and Basic Equations

The linear statement is considered. The flow of a compressible gas in a boundary layer on a flat plate is taken as an initial undisturbed flow. Disturbances in a boundary layer we shall consider in orthogonal coordinate system (ξ,ψ,z) [19] connected with stream-surfaces of basic flow and look like $\tilde{a}(\xi,\psi)\exp(i\alpha\xi+i\beta z-i\omega t)$. Here ψ - flow function; for a plate $\xi = x+O(\mathrm{Re}^{-2})$; $\mathrm{Re}=\sqrt{u_\infty x/\nu_\infty}$; x, y, z - longitudinal, normal to a wall and transversal co-ordinates of the Cartesian system with the beginning on an edge of a plate. Gas is perfect with a constant Prandtl number, Pr. Resulting a set of Navier-Stokes equations to a linear view, using estimations on the whole degrees of a Reynolds number, Re, rejecting the members order Re^{-2} respect to the main ones, the properties of a critical layer [19] and neglecting by a deformation of a perturbations distribution with changing of coordinate x it is possible to receive the dimensionless equations:

$$\tilde{v}' = \rho T' \tilde{v} - T\left(f_0 u'\right)\tilde{u} - \tilde{u}_w - i_c T\,\tilde{r} -$$
$$-\left(f_2 \rho T'\right)\tilde{T} - f_1 T\,\tilde{u}' + f_2 \tilde{T}',$$
$$\tilde{p}' = -\left(i_c + r_h u\right)\tilde{v} + i_x \tilde{\tau}_{12} + i_z \tilde{\tau}_{23} - 2\mu_r \tilde{u}_w',$$
$$\tilde{\tau}_{12}' = i_x \tilde{p} + \left(i_c + f_1 u' + u\partial\right)\tilde{u} +$$
$$f_2 u' \tilde{r} - \tilde{i}_T + f_2 \tilde{u}' + \rho u' \tilde{v}$$
$$\tilde{u}' = -i_x \tilde{v} - u' \mu_t \tilde{T} + \tilde{\tau}_{12} / \mu_r,,$$
$$\tilde{\tau}_{23}' = i_z \tilde{p} + \left(i_c - \mu_a\right)\tilde{w} - i_z \mu_r \tilde{u}_w + f_2 \tilde{w}', \qquad (1)$$
$$\tilde{w}' = -i_z \tilde{v} + \tau_{23} / \mu_r,,$$
$$\tilde{q}' = i\omega RT\,\tilde{p} + \rho H' \tilde{v} + f_2 H' \tilde{r} - u\tilde{i}_t + f_2 u\tilde{u}'$$
$$+\left(i_c u + f_1 H' + f_2 u'\right)\tilde{u} + f_2 \tilde{h}' + \left(i_c - \mu_a/\mathrm{Pr}\right)\tilde{h},$$
$$\tilde{h}' = -\mathrm{Pr}\,u'\tilde{u} - h'\mu_t \tilde{T} + \mathrm{Pr}\left(\tilde{q} - u\tilde{\tau}_{12}\right)/\mu_R,$$

where the stroke means a derivative on Y; $dY = d\psi / u\,\mathrm{Re}$;

$\tilde{p}, \tilde{v}, \tilde{u}, \tilde{w}, \tilde{h}$ - amplitudes of pressure; normal to a surface of a plate, longitudinal and transversal speeds; enthalpy disturbances. The expressions for $\tilde{\tau}_{12}, \tilde{\tau}_{23}, \tilde{q}$ can be found in [19].

Additional members of the system are of the form:

$$\tilde{u}_w = i_x \tilde{u} + i_z \tilde{w};\; i_t = i_x \mu_r \tilde{u}_w + \mu_a \tilde{u};$$
$$\tilde{r} = \tilde{\rho}/\rho = g_m \tilde{p} - \rho\tilde{T};\; i_c = \mathrm{Re}\,u_c = i\,\mathrm{Re}(u\alpha - \omega);$$
$$i_x = i\alpha\,\mathrm{Re}\,T;\; i_z = i\beta\,\mathrm{Re}\,T;$$
$$\mu_a = (i_x^2 + i_z^2)\mu_r;\; r_h = \mathrm{Re}\,h_1 = f_0 u' + f_1 \rho T';$$
$$\mu_T = d\ln\mu / dT;\; f_1 = -\psi\big/\left(2\,\mathrm{Re}^2 u\right);\; \mathrm{Re} = \sqrt{\xi}$$

The system (1) was normalized with the help of following scales: ν_∞ / u_∞ - length, ν_∞ / u_∞^2 - time, μ_∞ - viscosity and flow function, u_∞ - velocity and its disturbances, T_∞ - temperature, ρ_∞ - density , u_∞^2 - enthalpy, $\rho_\infty u_\infty^2$ - pressure and disturbances of viscous stresses, $\rho_\infty u_\infty^3$ - value $\tilde{q}$, u_∞^2 / T_∞ - specific heat (the index ∞ corresponds to values in the incident airflow). In this case: $g_m = \gamma \mathrm{M}^2$, $g_{m1} = (\gamma - 1)\mathrm{M}^2$, where $\gamma = c_p / c_V$ - relation of heat capacities, M– Mach number.

The system (1) can be represented in the form:

$$\mathbf{Z}' = A(\eta, \mathrm{Re}, M, \omega, \beta, \alpha)\mathbf{Z}. \qquad (2)$$

$\mathbf{Z} = \left(\tilde{p}, \tilde{v}, \tilde{u}, \tilde{w}, \tilde{h}, \tilde{\tau}_{12}, \tilde{\tau}_{23}, \tilde{q}\right)$, A—quadratic matrix of given functions of Re and Y.

In the absence of external perturbations the system of equations (2) is solved with the following boundary conditions. The disturbances of speeds and a temperature on a surface and in infinity are equals to zero:

$$z_2(0) = z_3(0 = z_4(0) = z_5(0) = 0,\; (z_5'(0) = 0)$$
$$z_2(\infty),\; z_3(\infty),\; z_4(\infty),\; z_5(\infty) = 0 \qquad (3)$$

Thereby the task on own values (the task of stability) is formulated. For example, at given values of Re, M, ω and β, the value of a wave number α is searched. In case of positive values of an imaginary part of a wave number α_i the flow is stability and vice versa.

In the presence of external perturbations of a type of $\tilde{a}_0(\xi, \psi)\exp\left(i\alpha_0 \xi + i\beta z - i\omega t\right)$ in a matrix of A it is necessary to replace α on α_0. Boundary conditions take the form:

$$\tilde{v}(0) = \tilde{u}(0 = \tilde{w}(0) = \tilde{T}(0) = 0,\; (\tilde{T}'(0) = 0);\;\; \mathbf{Z}(\infty) = \mathbf{Z}_0; \qquad (4)$$

III. Results

A. Egen stationary disturbance

In this paper the case of stationary perturbations is researched, which parameters differ from parameters of nonstationary perturbations negligible in case $\omega = 2\pi f \nu_\infty / u_\infty^2 < 10^{-6}$, f — the frequency in Hertz. All distributions given below were normalized on maximum values of the longitudinal velocity which located in the range: 2.5<Y<3.5.

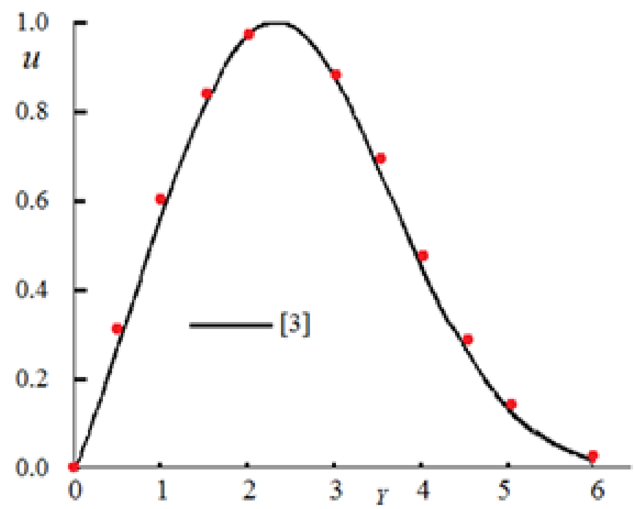

Fig. 3 Comparison of calculation results (M=0.2) of the longitudinal velocity distribution with data of [7] (M=0)

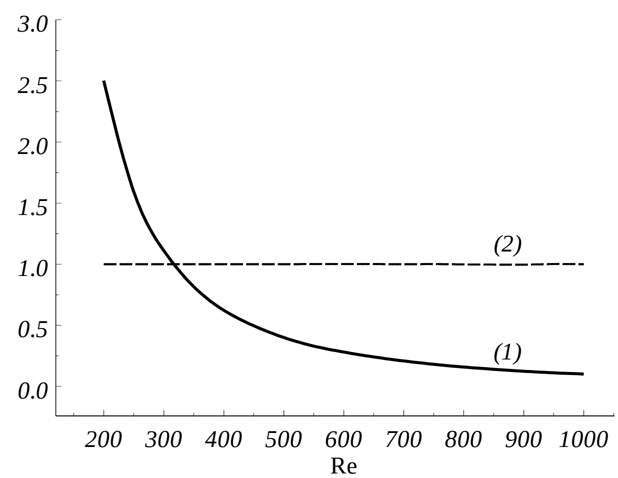

Fig. 4 Dependence of a rate amplification αi (1) and (2) on a Reynolds number, M=0.2

For the greater confidence in calculations, comparison of our results with data of [3], received at M=0 and β = 0, was carried out. There within of boundary layer equations, the stationary and nonstationary perturbations are researched. There it was shown, that longitudinal logarithmic derivative of on longitudinal coordinate has the form $\frac{\partial \ln(\bar{u}_k)}{\partial x} = i(\alpha_r + i\alpha_i) = -\lambda_k / x$.

λ_k - an infinite set of numbers. The first four of them with a precision of three digit are equal: 1.0, 1.89, 2.81, 3.76.

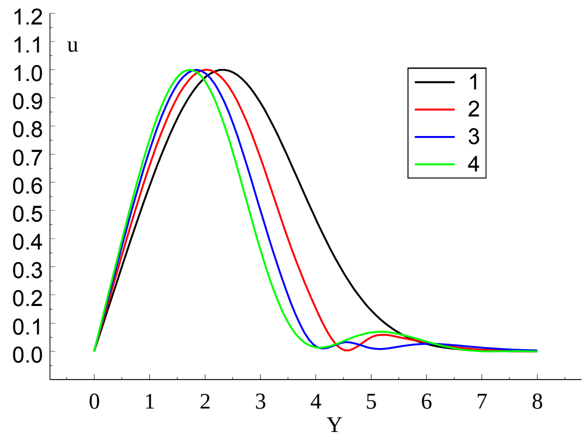

Fig.5 Eigenfunctions for λ_i=*1.0*(1), *1.89*(2), *2.81*(3), *3.76*(4). Re=200, *β=0, M=0.1*

The distribution of the longitudinal velocity perturbation in a boundary layer is given in Fig.3: present calculations at M=*0.1, β=0,* Re= *200,* $\alpha_i = 2.50\cdot10^{-5}$ (symbol) and data of [7] at M=*0*, (solid line). Dependence of a rate amplification α_i and $\lambda_1 = \alpha_i \operatorname{Re}^2$ on a Reynolds number is given in Fig. 4. Data show that $\alpha_i \sim 1/\operatorname{Re}^2$, and $\lambda_1 = 1$ (in the full accordance with the results of [7]). Eigen functions for different λ_i are shown in Fig.5, which agree completely with data of [6].

Along with two-dimensional perturbations in subsonic boundary layer (M=*0.1*) calculations carried out for three dimensional stationary disturbance, $\beta \neq 0$, in subsonic and supersonic boundary layer, unlike researches of [6, 7] (M=*0*, $\beta = 0$).

Distributions of longitudinal, normal and lateral velocities perturbations (u, v, w) on a boundary layer are shown in Fig. 6 and Fig. 7, when Re=200 and 10^3 (M=*0.2*, $\beta=0.7\cdot10^{-4}$). In addition distributions of phases, *ph*, of longitudinal velocities are shown in Fig.7. If in case of low Reynolds numbers (Re=200) there is no influence of a wave number *β* on the velocities perturbations distribution, then with increase in Re it not so. Especially it is visible on phase shift. If in case of Re=200 phase shift on a boundary layer is equal to zero, as well as in a case *β =0*, then in case of Re=10^3 it reaches about one radian, (about 60°). At the same time the phase decreases with an increase of the coordinate *Y*. It is connected with the fact that a Reynolds number (coordinate *x*), boundary layer thickness and an effective wave number *β*Re increase. It carries to more strong influence of a lateral velocity on longitudinal velocity through a continuity equation.

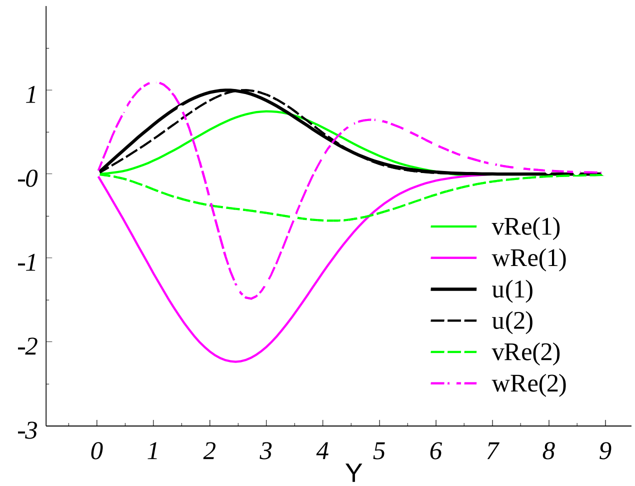

Fig. 6 Distributions of longitudinal, normal and lateral velocities perturbations and a phase shift on a boundary layer at Reynolds numbers *200*(1) и 10^3 (2), (M=*0.2*, $\beta=7\cdot10^{-4}$)

From presented data (Fig. 6) it is visible that normal and lateral velocities have an order 1/Re.

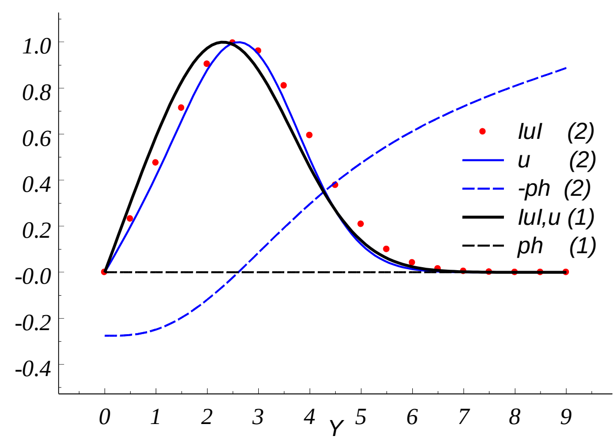

Fig. 7 Distributions of maximum, */u/*, and actual, *u*, values of velocity perturbations and a phase shift on a boundary layer at Re =*200(1)* and 10^3*(2)* (M=*0.2*, $\beta=7\cdot10^{-4}$)

In case of supersonic speeds along with velocities perturbations disturbances of the mass flow play an important role. Moreover, at hot-wire anemometer using in experiments the mass flow perturbations are measured as a rule. Therefore in Fig. 8 the mass flow perturbation m distribution in a boundary layer is shown in case of M=2 not only velocity perturbations. Symbols show the distribution of the velocity perturbation for M = 0.1. It is interesting to note that for the small Reynolds number (Re=200) velocity perturbation distribution does not depend on the Mach number.

Fig. 7 and 8 show that deformation of a longitudinal velocity profile in a supersonic boundary layer in case of a Reynolds number change is similar to its change in a case of a subsonic boundary layer. However full phase shifts of velocity and mass flow perturbations on a boundary layer in case of

M=2 are approximately equal π=3.14 and they are positive while in case of a subsonic boundary layer they are much less and negative (Fig. 6). At small Reynolds numbers the influence parameter β is insignificantly, the phase shift on boundary layer is absent.

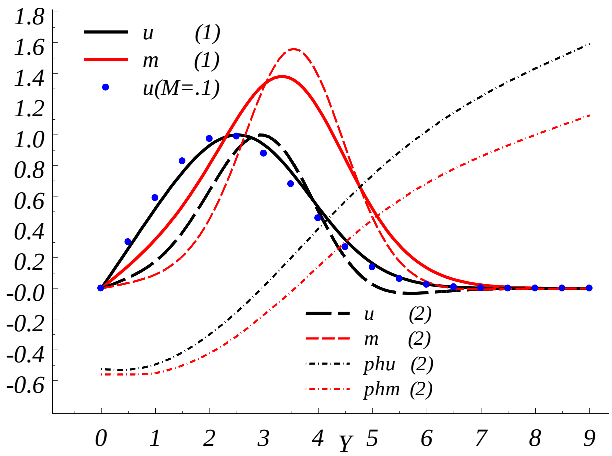

Fig. 8 Distributions *u*, *m* и *ph* on a boundary layer at M=2. Re=*200 (1)*, 10^3 *(2)*; $\beta=7\cdot10^{-4}$

Dependences of decrements and their product on Re^2 for two-dimensional (*(01), (02)*), and also three-dimensional (*(b1), (b2)*) perturbations on the Reynolds number are shown in Fig. 9 (M=*2*). Here, for comparing, dependence $\alpha_i\mathrm{Re}^2$ on the Reynolds number is given for a case of subsonic speeds (M=*0.2*) at $\beta=7\cdot10^{-4}$. These data show that the spatial decrements weakly dependent on the Mach number. However, it is possible to observe the small increase of the decrement with an increase of the Mach number at Re>450. .

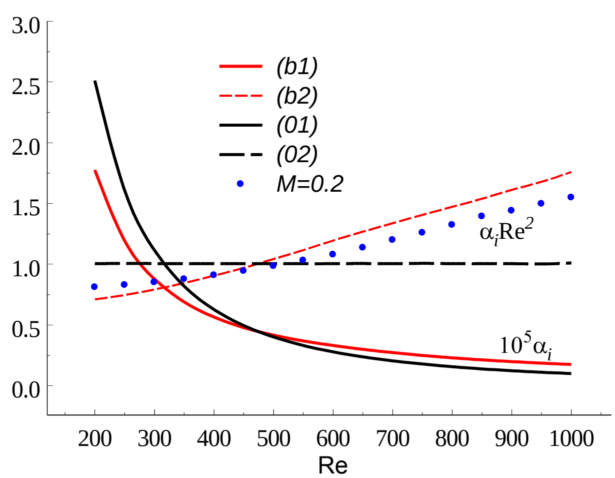

Fig. 9 Decrements dependences, α_i, and product $\alpha_i\mathrm{Re}^2$ for $\beta=0$ (*(01),(02)*) and $\beta=7\cdot10^{-4}$ (*(b1),(b2)*) on Reynolds number at M=*2*

At Re<400 three-dimensional perturbations decrements ($\beta=7\cdot10^{-4}$) are smaller than the two-dimensional perturbations decrements. At Re>400 on the contrary, decrements of three-dimensional perturbations are greater than two-dimensional perturbations decrements. In case of Re=1000 they are about 1.5 times more than decrements of two-dimensional perturbations.

In Fig. 10 dependences of decrements on a wave number β are shown. It is possible to note that in the given dependences there are minima at $\beta=\beta^*$.

Calculations show that if M=2 then β^*Re=0.1 at all Reynolds numbers. At subsonic speeds (M=0.2) value $\beta^*\mathrm{Re} \approx 0.08$.

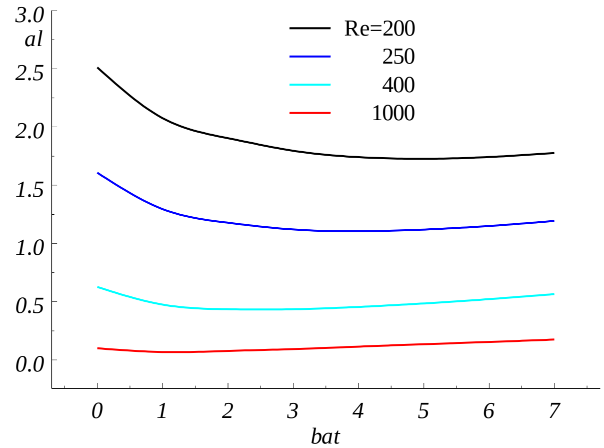

Fig. 10 Decrements dependences on a lateral wave number at M=*2* (*al* = $\alpha_i\cdot10^5$, *bet* =$\beta\cdot10^4$).

B. Resonance theory of the interaction of external perturbations with boundary layer

In the presence of external perturbations, problem (2), (4) can be reduced to solving of inhomogeneous differential equations with zero boundary conditions similar to the conditions (3). Indeed, let's introduce a vector-function **W**= **Z**-φ(*Y*)$\mathbf{Z}_0$. The function φ(Y) must satisfy to conditions: φ(0)=0, φ(∞)=1. Then instead of the system (2) we have a non-uniform system of equations

$$\mathbf{W}' - A(Y,\mathrm{Re},M,\omega,\beta,\alpha_0)\mathbf{W} \equiv L_0\mathbf{W} = \varphi(Y)A\mathbf{Z}_0 - \varphi'(Y)\mathbf{Z}_0 \quad (5)$$

with boundary conditions similar to (3):

$$w_2(0) = w_3(0 = w_4(0) = w_5(0) = 0,\ (w_5'(0) = 0)$$
$$w_2(\infty),\ w_3(\infty),\ w_4(\infty),\ w_5(\infty) = 0 \quad (6)$$

It is possible to show that in case of small values $|\alpha-\alpha_0|$ amplitudes any component of a vector **W** and components of a vector **Z** will be proportional to $1/|\alpha-\alpha_0|$. For example normalized longitudinal speed perturbations can be taken as $u_n = (u/u^0)|\alpha^0-\alpha_0|/|\alpha-\alpha_0|$

Results of calculations on interactions of external perturbations of the form $\tilde{a}_0\exp(i\beta z)$ with a subsonic boundary layer are given below. Similar investigations were done in papers [11, 12, 19, 20].

Dependences of amplitude peaks of normalized longitudinal velocities on a Reynolds number in case of M=*0.2* and different values of a wave number β are given in Fig. 11. Here u^0 is the amplitude peak of a longitudinal velocity at Re=*200.* From the presented data it is visible that in case of small wave numbers (*bet =0; 1*) amplitudes of stationary perturbations, grows proportionally to longitudinal coordinate x ~Re_x = Re^2 as it was predicted in [8, 11]. In case of large wave numbers (*bet=7*) it grows proportionally to $\mathrm{Re} = \sqrt{\mathrm{Re}_x}$.

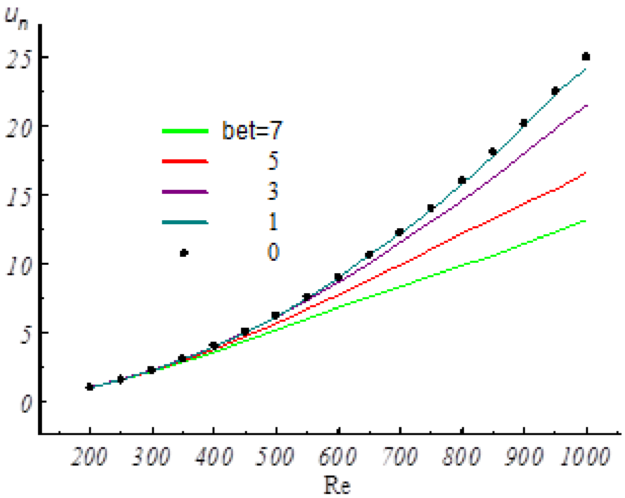

Fig. 11. Dependences of normalized velocity perturbations amplitudes on the Reynolds number at M=*0.2* and different values $bet=\beta \cdot 10^4$

Dependences of the amplitude peaks of a normalized longitudinal velocity on wave number $bet= \beta \cdot 10^{-4}$ at different Reynolds numbers are shown in Fig. 12. Here u^0 is the amplitude peak of a longitudinal velocity at $\beta=0$. These results show that perturbations amplitude of a longitudinal velocity rises with β linearly in the area of small values of wave number that are also consistent with the findings of [8, 11].

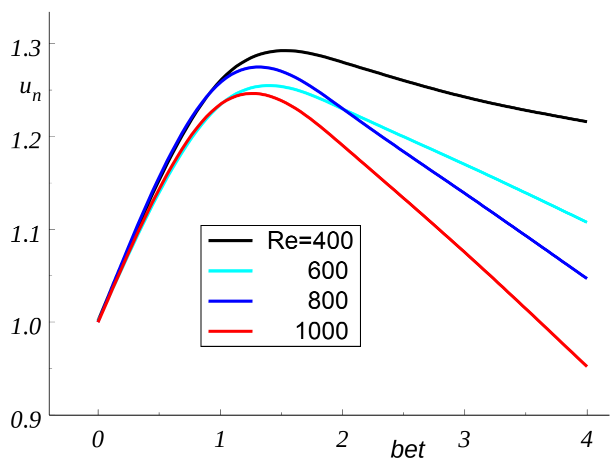

Fig. 12. Dependences of normalized velocity perturbations amplitudes on wave number at M=*0.2* and different values Re

From fig. 12 it is possible to conclude that in case of the fixing Reynolds number the maximum of perturbations are achieved approximately at $\beta^*\mathrm{Re}=0.1$. Calculations of paper [12] showed that the amplitude of excited stationary perturbations, which were proportional to $\exp\left(i(\beta z + \beta y / 3)\right)$, reached its maximum also when $\beta^*\mathrm{Re}\approx 0.1$.

Due to the fact that for the data, presented on Fig. 12, the value of u_n changes slightly in the range $1 < \mathrm{bet} < 1.5$, for dependences of the disturbances amplitude excited by external waves on the Reynolds number it is possible to use the curve of Fig. 11, corresponding to *bet=1*. In papers [19, 20] it was obtained that the maximum values of the excited velocities amplitudes are observed at $\beta^*\mathrm{Re}\approx 0.5$. With regard to the dependencies shown in Fig. 11 it is different from the amplitude dependencies of disturbances on Reynolds number presented in [12, 19, 20], where there are peaks. One reason of such discrepancy is related to the exponential dependence of a amplitude of an external wave on $\beta^2 x$, $\exp(-\beta^2 \mathrm{Re}^2)$. Taking into account such attenuation dependences $u=u_n\exp(-\beta^2\mathrm{Re}^2)$ on a Reynolds number at M=*0.2* for three values of a wave number β is shown in Fig. 13. Taking into account such correction it is possible to notice that $\beta^*\mathrm{Re}\approx 0.8$ what is coordinated with data [20].

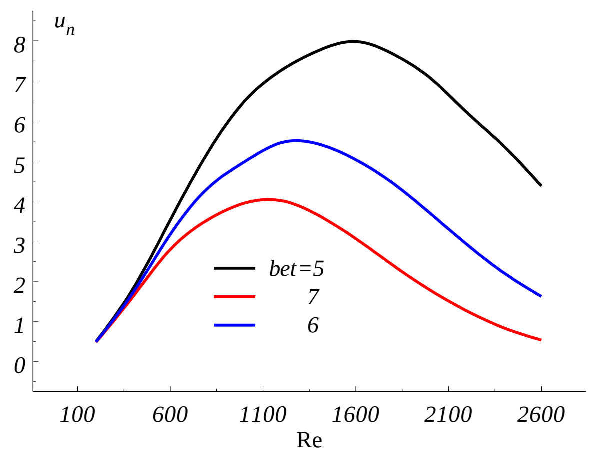

Fig. 13 Dependences of normalized velocity perturbations amplitudes, *u*, on the Reynolds number at M=*0.2* and different values *bet*

Similar dependencies of normalized velocity perturbations amplitudes on the Reynolds number at M=2.*0* are shown in Fig.14. From these data it is possible to see that the disturbance amplitude of longitudinal velocity under the influence of an external vorticity decreases in case of a supersonic boundary layer.

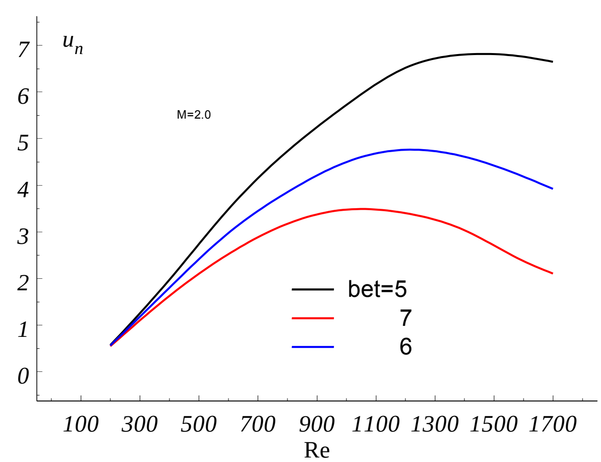

Fig. 14 Dependences of velocity perturbations amplitudes, *u*, on the Reynolds number at M=*2* and different values $bet=\beta \cdot 10^4$

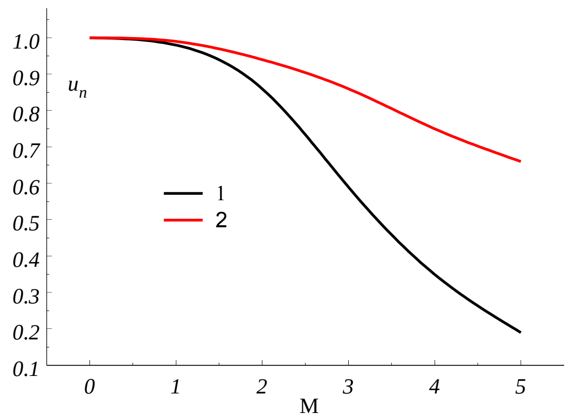

Fig. 15 Dependences of velocity perturbations amplitudes, *u*, on the Reynolds number at M=*2* and different values *bet*.

The effect of the Mach number on amplitude perturbations of normalized longitudinal velocity is shown in Fig. 15. As the basic velocity, $u_{0,}$ was accepted velocity at the Mach number M=0. Despite distinction data the resonant theory, 2, and results of [17], 1, the tendency of an influence of the Mach number on the amplitude of excited disturbances in a boundary by an external vorticity in the both cases is identical. Distinction of these dependences is caused by influence by a flow nonparallelism on distributions of disturbances parameters in a boundary layer which was considered in [17] and couldn't be considered in the resonant theory, in the principle.

IV. Conclusions

As a result of the conducted researches it was received:

1. Calculations results of Eigen two-dimensional stationary disturbances under the classical theory completely coincide with data [6, 7]. Longitudinal velocity profile of stationary disturbances excited by an external turbulence has a bell-shaped type, maximum which is located inside of the boundary layer. For the exponential dependence of the maximum amplitude on Re_x, $u_{max}= Re_x^{-n}$, the maximum position drifts to the wall with increase of n.

2.The direct connection of an excitation of internal perturbations of the boundary layer by external waves with resonant theory of their interaction with the boundary layer is established. Obtained on its basis amplitude dependences of internal perturbations excited by an external vorticity, on a wave number and a Reynolds number are coordinated with direct calculations which are available in the known literature.

3. The first time data on the eigenvalue problem of three-dimensional stationary disturbances as for subsonic and supersonic boundary layers are received. In case of low Reynolds numbers there is no influence of a wave number β on the velocities perturbations distribution. With increasing of Reynolds number the longitudinal velocity phase shift on the boundary layer appears. Decrements of three-dimensional perturbations decrease in inverse proportion to longitudinal coordinate at small Reynolds numbers and in inverse proportion to thickness of a boundary layer at large numbers of Reynolds. It was established that amplitudes of excited disturbances in a boundary decrease with Mach number. There is a characteristic wavenumber β*Re, at which the amplitude of streamwise velocity perturbations inside the boundary layer attains its maximum. This result explains the experimentally observed emergence of laterally periodic streaky structures.

References

[1] S.A.Gaponov, Resonance theory of stationary longitudinal structures in the boundary layer, *WSEAS Transactions on Fluid Mechanics*, vol. 12, pp. 58-64, 2017 .

[2] P. S. Klebanoff, K. D. Tidstrom, Evolution of amplified waves leading to transition in a boundary layer with zero pressure gradient, *Nat. Aero. and Space Adm.*, Tech. Note D-195, 1959.

[3] H. L. Dryden, Air flow in the boundary layer near a plate. *NACA Tech. Rep. 562*, 1937.

[4] P. S. Klebanoff, Effect of free stream turbulence on a laminar boundary layer. *Bull. Am. Phys.Soc.*, vol. 16, 203–216, 1971.

[5] M. Matsubara, P. H. Alfredsson, Disturbance growth in boundary layers subjected to free-stream turbulence, *J. Fluid Mech.*, vol. 430, pp. 149-168, Mar. 2001.

[6] P.A Libby, H. Fox, Some perturbation solutions in laminar boundary-layer theory. Part 1. The momentum equation, *J. Fluid Mech.*, vol. 17, no. 3, pp. 433-449, Nov. 1963.

[7] C.E. Grosch, T.L Jackson., A.K. Kapila, Non-separable eigenmodes the incompressible boundary layer, In *Instability, Transition and Turbulence*, Springer-Veriag, 1992, pp. 127-136.

[8] P. Bradshaw The effect of wind-tunnel screens on nominally two-dimensional boundary layers, *J. Fluid Mech*., vol. 22, no. 4, pp. 679-687, Aug. 1965.

[9] J. M. Kendall, Boundary-layer receptivity to freestream turbulence, *AIAA Paper 90-1504*, 1990.

[10] V. S. Kosorygin, N. F. Polyakov, T. T. Suprun, and E. Ya. Epik, Development of disturbances in the laminar boundary layer on a plate at high levels of external flow turbulence, in: *Instability of Subsonic and Supersonic Flows*, ITAM, SB USSR AS, Novosibirsk 1982, pp. 85-92 (in Russian).

[11] S. C. Crow, The spanwise perturbation of two-dimensional boundary layers, *J. Fluid Mech*, vol. 24, no. 1, pp. 153-104, Jan. 1966.

[12] F.P. Bertolotti, Response of the Blasius boundary layer to free-stream vorticity. Physics of fluids, Vol.9, No 8, 1997, pp. 2286-2299.

[13] A. N. Gulyaev, V. E. Kozlov, V. P. Kuznetsov, B. I. Mineev, and A. N. Sekundov, "Interaction of a laminar boundary layer with outer turbulence, *Fluid Dynamics*, vol. 24, no. 5, pp. 700-710, Sep.–Oct. 1989.

[14] M. V. Ustinov, Receptivity of the flat-plate boundary layer to free-stream turbulence. *Fluid Dynamics*, vol. 38, no. 3, pp. 397–408, Sep. 2003.

[15] T. A. Zaki, P. A. Durbin, Mode interaction and the bypass route to transition, *J. Fluid Mech*., vol. 531, pp. 85–111, May 2005.

[16] P. Ricco, The pre-transitional Klebanoff modes and other boundary-layer disturbances induced by small-wavelength free-stream vorticity, *J. Fluid Mech*., vol. 638, pp. 267–303, Nov. 2009.

[17] M. W. Johnson, Bypass transition receptivity modes, *International Journal of Heat and Fluid Flow*, Vol. 32, no. 2, pp. 392–401, Apr. 2011.

[18] M. E. Goldstein, Effect of free-stream turbulence on boundary layer transition, *Phil. Trans. R. Soc. A*, vol. 372, no. 2020, June 2014, DOI: 10.1098/rsta.2013.0354.

[19] S. A. Gaponov, A. V. Yudin, Interaction of hydrodynamic external disturbances with the boundary layer, *Journal of Applied Mechanics and Technical Physics*, vol. 43, no. 1, pp. 83–89, Jan. 2002.

[20] S. A. Gaponov, Interaction of external vortical and thermal disturbances with boundary layer, *International journal of mechanics*, vol.1, no.1, pp. 15-20, 2007.

[21] J. Joo, P. A. Durbin, Continuous Mode Transition, in High-speed Boundary layers, *Flow Turbulence Combust*, vol. 88, no.3, pp. 407–430, Apr. 2012.

[22] F. Qin, X. Wu, Response and receptivity of the hypersonic boundary layer past a wedge to free-stream acoustic, vortical and entropy disturbances, *J. Fluid Mech*., vol. 797, pp. 874-915, June 2016.

[23] X. Wu, M. Dong, Entrainment of short-wavelength free-stream vortical disturbances in compressible and incompressible boundary layers, *J. Fluid Mech.*, vol. 797, pp 683- 728, June 2016.
[24] S. A. Gaponov, Interaction between a supersonic boundary layer and acoustic disturbances, *Fluid Dynamics*, vol.12, no 6, pp. 858-862, Nov.–Dec. 1977.
[25] C. E. Grosch, H. Salwen, The continuous spectrum of the Orr-Sommerfeld equation, Part 1, The spectrum and the eigenfunctions, *J. Fluid Mech.*, vol. 87, no. 1, pp. 33-54, Jul 1978.
[26] C. E. Grosch, H. Salwen, The continuous spectrum of the Orr-Sommerfeld equation, Part 2, Eigenfunction expansions, *J. Fluid Mech*., Vol. 104, pp. 445-465, Mar. 1981.
[27] A. E. Trefethen, S. C. Reddy, T.A. Driscoll, Hydrodynamic stability without eigenvalues, *Science*, vol. 261, no. 5121, pp. 578-584, July 1983.