

Buckling and free vibration analysis of sandwich plate

E. Kormanikova and K. Kotrasova

Abstract—The paper presents buckling and free vibration analysis of sandwich plate with laminate facings. The sandwich plate is subjected to the Tsai-Wu criterion. Then buckling analysis of the sandwich plate is performed in two steps. First, a finite element method is used to determine the overall buckling load of the sandwich plate. The second part of the analysis is free vibration analysis. The investigated sandwich plate is simply supported at all boundaries and loaded by an uniaxial uniform load.

Keywords—Buckling Analysis, Free Vibration Analysis, Sandwich Plate, Tsai-Wu Criterion.

I. INTRODUCTION

THE typical sandwich structure consists of three layers. The outer layers are made of high strength material such as steel, fibre reinforced laminates etc., which can transfer axial forces and bending moments, while the core is made of lightweight materials such as foam, alder wood etc. The material used in sandwich core must be resistant to compression and capable of transmitting shear [1, 2].

A fibre reinforced laminates consist of stack of composite layers. A composite material of laminate layer can be defined as a heterogeneous mixture of two or more homogeneous phases, with their different physical properties, which have been bonded together. Properties of composite material are clearly distinct from the properties of its components [3]. The most important aspect of composite materials, in which the reinforcement are fibres, is the anisotropy caused by the fibre orientation. It is necessary to give special attention to this fundamental characteristic of fibre reinforced composites and the possibility to influence the anisotropy by material design for a desired quality [4]. The sandwich panels are one of the types of composite materials that are used in civil structures.

The finite element method (FEM) is the effective method for the numerical solution of problem formulated in partial differential equations [5-8]. Using FEM formulation, buckling and free vibration analysis can be solved numerically [9-11].

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A sensitivity of a composite plate is important analysis for design of structures. In the sensitivity analysis the design variables are changes between their lower and upper bounds in a specified number of steps [12-16].

II. STATIC ANALYSIS OF SANDWICH PLATES

A sandwich can be defined as a special laminate with three layers. The thin cover sheets, i.e. the layers 1 and 3, have the thicknesses h_1 for the lower skin and h_3 for the upper skin (Fig. 1). The thickness of the core is h_2 . In a general case h_1 does not have to be equal to h_3 , but in the most important practical case of symmetric sandwiches $h_1 = h_3$.

To formulate the governing differential equations for sandwich plates we draw the conclusion from the similarity of the elastic behaviour between laminates and sandwiches in the first order shear deformation theory and all results derived for laminates can be applied to sandwich plates. We restrict our considerations to symmetric sandwich plates with thin cover sheets. There are differences in the expressions for the flexural stiffness, coupling stiffness and the transverse shear stiffness of laminates and sandwiches. Furthermore there are essential differences in the stress distributions.

The assumptions about deformation are:

a) For the sandwich thin cover sheets gilt Kirchhoff's assumptions about deformation. In-plane stress-strain state is accrued in the sandwich thin cover sheets.

b) The sandwich core with the thickness h_2 transfers only shear stresses perpendicular to the mid-plane of the cover sheets. The material characteristics is the shear modulus G_2 .

c) All points in the normal line have the equal deflections $w_1 = w_2 = w_3 = w$.

d) All layers are perfectly bonded.

We can write the shear deformations

$$\gamma_{xz2} = \left(\frac{u_{12} - u_{32}}{h_2} + \frac{\partial w}{\partial x} \right) = \left(\frac{u_1 - u_3}{h_2} + \frac{d}{h_2} \frac{\partial w}{\partial x} \right) \quad (1)$$

$$\gamma_{yz2} = \left(\frac{v_{12} - v_{32}}{h_2} + \frac{\partial w}{\partial y} \right) = \left(\frac{v_1 - v_3}{h_2} + \frac{d}{h_2} \frac{\partial w}{\partial y} \right) \quad (2)$$

where d is the distance of sheets mid-planes.

$$d = h_2 + \frac{h_1 + h_3}{2}. \quad (3)$$

Most sandwich structures can be modelled and analyzed using the shear deformation theory for laminate plates. The

components of vector of internal forces N , M , V at the sandwich element in the (x, z) plane are shown in the Figure 2.

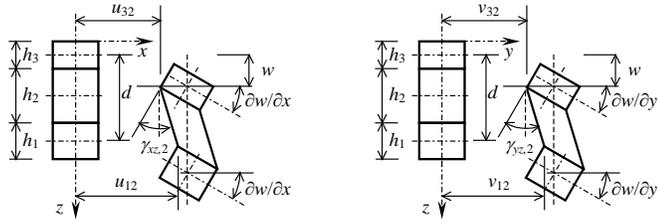


Fig. 1 geometry of deformation

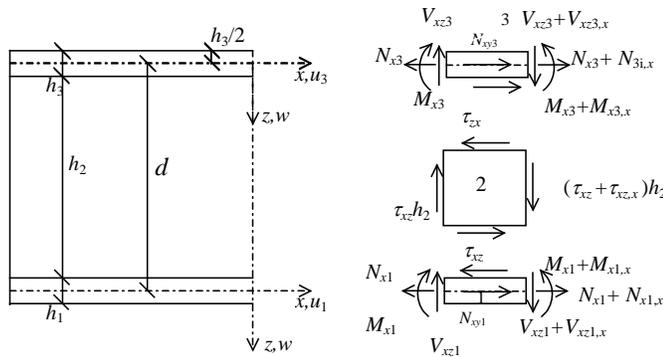


Fig. 2 internal forces at the sandwich element in the (x, z) plane

There are the normal forces in cover sheets $i = 1, 3$

$$N_{xi} = D_{Ni} \left(\frac{\partial u_i}{\partial x} + \nu_i \frac{\partial v_i}{\partial y} \right), \quad N_{yi} = D_{Ni} \left(\nu_i \frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} \right),$$

$$N_{xyi} = \frac{D_{Ni}(1-\nu_i)}{2} \left(\frac{\partial u_i}{\partial y} + \frac{\partial v_i}{\partial x} \right), \quad (4)$$

where

$$D_{Ni} = E_i h_i / (1-\nu_i^2). \quad (5)$$

The bending moments and the shear forces in cover sheets we can write as

$$M_{xi} = -D_{Mi} \left(\frac{\partial^2 w}{\partial x^2} + \nu_i \frac{\partial^2 w}{\partial y^2} \right), \quad M_{yi} = -D_{Mi} \left(\nu_i \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right),$$

$$M_{xyi} = -D_{Mi}(1-\nu_i) \frac{\partial^2 w}{\partial x \partial y}, \quad (6)$$

$$V_{xzi} = -D_{Mi} \left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2} \right), \quad V_{yzi} = -D_{Mi} \left(\frac{\partial^3 w}{\partial y^3} + \frac{\partial^3 w}{\partial x^2 \partial y} \right), \quad (7)$$

where

$$D_{Mi} = E_i h_i^3 / 12(1-\nu_i^2). \quad (8)$$

The shear stresses in the core are written

$$\tau_{xz} = G_2 \gamma_{xz2} = \frac{G_2}{h_2} \left(u_1 - u_3 + d \frac{\partial w}{\partial x} \right),$$

$$\tau_{yz} = G_2 \gamma_{yz2} = \frac{G_2}{h_2} \left(v_1 - v_3 + d \frac{\partial w}{\partial y} \right). \quad (9)$$

The equilibrium equations for internal forces are following

$$\frac{\partial N_{xi}}{\partial x} + \frac{\partial N_{yxi}}{\partial y} + \frac{\partial V_{xzi}}{\partial z} = 0, \quad \frac{\partial N_{xyi}}{\partial x} + \frac{\partial N_{yxi}}{\partial y} + \frac{\partial V_{yzi}}{\partial z} = 0, \quad i=1,3$$

$$\frac{\partial V_{xz}}{\partial x} + \frac{\partial V_{yz}}{\partial y} + p = 0, \quad (10)$$

where

$$\frac{\partial V_{xz1}}{\partial z} = -\tau_{xz}, \quad \frac{\partial V_{xz3}}{\partial z} = \tau_{xz}, \quad (11)$$

$$V_{xz} = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} + \frac{\partial V_{xz}}{\partial z}, \quad V_{yz} = \frac{\partial M_{yx}}{\partial x} + \frac{\partial M_y}{\partial y} + \frac{\partial V_{yz}}{\partial z},$$

$$\frac{\partial V_{xz}}{\partial z} = \tau_{xz} h_2, \quad \frac{\partial V_{yz}}{\partial z} = \tau_{yz} h_2. \quad (12)$$

The solving of unknown functions $u_1(x,y)$, $u_3(x,y)$, $v_1(x,y)$, $v_3(x,y)$, $w(x,y)$ have to perform the boundary conditions for each boundary.

III. FREE VIBRATION AND BUCKLING ANALYSIS OF SANDWICH PLATE

The equations to determine the natural frequencies of symmetric sandwich panel are following

$$D_{11} \frac{\partial^2 \bar{\alpha}}{\partial x^2} + D_{66} \frac{\partial^2 \bar{\alpha}}{\partial y^2} + (D_{12} + D_{66}) \frac{\partial^2 \bar{\beta}}{\partial x \partial y} -$$

$$-k^s A_{55} \left(\bar{\alpha} + \frac{\partial w}{\partial x} \right) - I \frac{\partial^2 \bar{\alpha}}{\partial t^2} = 0, \quad (13)$$

$$(D_{12} + D_{66}) \frac{\partial^2 \bar{\alpha}}{\partial x \partial y} + D_{66} \frac{\partial^2 \bar{\beta}}{\partial x^2} + D_{22} \frac{\partial^2 \bar{\beta}}{\partial y^2} -$$

$$-k^s A_{44} \left(\bar{\beta} + \frac{\partial w}{\partial y} \right) - I \frac{\partial^2 \bar{\beta}}{\partial t^2} = 0, \quad (14)$$

$$k^s A_{55} \left(\frac{\partial \bar{\alpha}}{\partial x} + \frac{\partial^2 w}{\partial x^2} \right) + k^s A_{44} \left(\frac{\partial \bar{\beta}}{\partial y} + \frac{\partial^2 w}{\partial y^2} \right) -$$

$$-\rho_m h \frac{\partial^2 w}{\partial t^2} = 0, \quad (15)$$

$$\rho_m = \frac{1}{h} \sum_{k=1}^N \rho_k (z^{(k)} - z^{(k-1)}), \quad (16)$$

$$I = \frac{\rho_m h^3}{12} \frac{1}{3} \sum_{k=1}^N \rho_k (z^{(k)})^3 - (z^{(k-1)})^3,$$

where

k^s is the transverse shear deformation factor given by value 5/6,

ρ_k is the mass density of the k^{th} layer.

For the simply supported plate let

$$w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} \sin \left(\frac{m\pi x}{a} \right) \sin \left(\frac{n\pi y}{b} \right) e^{i\omega_{mn} t}, \quad (17)$$

$$\bar{\alpha}(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A'_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{i\omega_{mn}t}, \quad (18)$$

$$\bar{\beta}(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B'_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{i\omega_{mn}t}, \quad (19)$$

where

m, n – are integers only,

a, b – are the panel dimensions in x, y axis direction respectively,

ω_{mn} – is natural angular velocity.

A set of homogeneous equations is used to solve the natural frequencies of vibration

$$\begin{pmatrix} \dot{L}'_{11} & \dot{L}'_{12} & \dot{L}'_{13} \\ \dot{L}'_{12} & \dot{L}'_{22} & \dot{L}'_{23} \\ \dot{L}'_{13} & \dot{L}'_{23} & \dot{L}'_{33} \end{pmatrix} \begin{pmatrix} A'_{mn} \\ B'_{mn} \\ C'_{mn} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \quad (20)$$

Matrix elements are given by the formulas

$$\begin{aligned} \dot{L}'_{11} &= L_{11} - \frac{\rho_m h^3}{12} \omega_{mn}^2, & \dot{L}'_{22} &= L_{22} - \frac{\rho_m h^3}{12 \omega_{mn}^2}, \\ \dot{L}'_{33} &= L_{33} - \rho_m h \omega_{mn}^2, \end{aligned} \quad (21)$$

where

$$\begin{aligned} L_{11} &= D_{11} \lambda_m^2 + D_{66} \lambda_n^2 + k^s A_{55}, \\ L_{12} &= (D_{12} + D_{66}) \lambda_m \lambda_n, \\ L_{13} &= k^s A_{55} \lambda_m, \\ L_{22} &= D_{66} \lambda_m^2 + D_{22} \lambda_n^2 + k^s A_{44}, \\ L_{23} &= k^s A_{44} \lambda_n, & L_{33} &= k^s A_{55} \lambda_m^2 + \lambda_n^2, \end{aligned} \quad (22)$$

$$\lambda_m = \frac{m\pi}{a}, \quad \lambda_n = \frac{n\pi}{b}. \quad (23)$$

If the rotatory inertia terms are neglected then $\dot{L}'_{11} = L_{11}$, $\dot{L}'_{22} = L_{22}$, and we get

$$\omega_{mn}^2 = \frac{(Q L_{33} + 2 L_{12} L_{23} L_{13} - L_{22} L_{13}^2 - L_{11} L_{23}^2)}{\rho_m h Q}, \quad (24)$$

$$Q = L_{11} L_{22} - L_{12}^2.$$

Also applies

$$\begin{aligned} A'_{mn} &= \frac{L_{12} L_{23} - L_{22} L_{13}}{Q} C'_{mn}, \\ B'_{mn} &= \frac{L_{12} L_{13} - L_{11} L_{23}}{Q} C'_{mn}. \end{aligned} \quad (25)$$

In a similar way the governing equations for buckling problems can be derived. In the matrix equations (20) only the differential operator \dot{L}'_{33} is substituted by

$$L'_{33} = L_{33} - \left(N_1 \frac{\partial^2}{\partial x^2} + 2N_6 \frac{\partial^2}{\partial x \partial y} + N_2 \frac{\partial^2}{\partial y^2} \right). \quad (26)$$

IV. TSAI-WU CRITERION

We can distinguish the failure between fiber failure (FF) and inter fiber failure (IFF). In the case of plane stress, the IFF criteria discriminates three different modes. The IFF mode A is when perpendicular transversal cracks appear in the lamina under transverse tensile stress with or without in-plane shear stress. The IFF mode B denotes perpendicular transversal

cracks, but in this case they appear under in-plane shear stress with small transverse compression stress. The IFF mode C indicates the onset of oblique cracks when the material is under significant transversal compression.

Strength of a composite layer in any other direction is evaluated based on various failure criteria. The basic premise in predicting the failure of fibre-reinforced layers using maximum stress and maximum strain criteria is the same as for isotropic material. Failure is predicted when the maximum stress along the fibre or transverse to the fibre directions exceed the strength of the tension or compression.

Tsai-Wu criterion is the general form of the failure criterion for orthotropic materials under plane stress. The assumption is expressed as

$$F_{01} \sigma_1 + F_{11} \sigma_1^2 + 2F_{12} \sigma_1 \sigma_2 + F_{02} \sigma_2 + F_{22} \sigma_2^2 + F_{44} \tau_{12}^2 < 1, \quad (27)$$

where

$$\begin{aligned} F_{01} &= \frac{1}{X_t} - \frac{1}{X_c}, & F_{11} &= \frac{1}{X_t X_c}, & F_{02} &= \frac{1}{Y_t} - \frac{1}{Y_c}, \\ F_{22} &= \frac{1}{Y_t Y_c}, & F_{12} &= -\frac{1}{2 \sqrt{X_t X_c Y_t Y_c}}, & F_{44} &= \frac{1}{S^2}. \end{aligned} \quad (28)$$

The failure criterion for orthotropic material under strain assumption is expressed as

$$G_{01} \varepsilon_1 + G_{11} \varepsilon_1^2 + G_{12} \varepsilon_1 \varepsilon_2 + G_{02} \varepsilon_2 + G_{22} \varepsilon_2^2 + G_{44} \gamma_{12}^2 < 1, \quad (29)$$

where

$$\begin{aligned} G_{01} &= F_{01} E_{11} + F_{02} E_{12}, & G_{02} &= F_{02} E_{22} + F_{01} E_{12}, \\ G_{11} &= F_{11} E_{11}^2 + F_{22} E_{12}^2 + F_{12} E_{11} E_{12}, \\ G_{22} &= F_{22} E_{22}^2 + F_{11} E_{12}^2 + F_{12} E_{22} E_{12}, \\ G_{12} &= 2E_{12} (F_{11} E_{11} + F_{22} E_{22}) + 2F_{12} (E_{12}^2 + E_{11} E_{22}), \\ G_{44} &= F_{44} E_{44}^2. \end{aligned} \quad (30)$$

When $F_{12} = \frac{-1}{2X_t X_c}$, the Tsai-Wu criterion is reduced to Tsai-

Hill criterion, and when $F_{12} = \frac{-1}{2X_t X_c}$ the Tsai-Wu criterion

is reduced to Hoffman criterion [3].

These failure criteria are used to calculate a failure index (F.I.) from the computed stresses and user-supplied material strengths. A failure index denotes the onset of failure, and a value less than 1 denotes no failure. The failure index according to this theory is computed using the following equation

$$I_F = F_{01} \sigma_1 + F_{11} \sigma_1^2 + 2F_{12} \sigma_1 \sigma_2 + F_{02} \sigma_2 + F_{22} \sigma_2^2 + F_{44} \tau_{12}^2. \quad (31)$$

Failure load factor is inverse value to the failure index.

V. SOLUTION AND RESULTS

The sandwich plate (Fig. 3) is made of a 6-layer Boron-Epoxy laminate facings $[\theta/\theta-60/\theta+60]_s$ and polystyrene core. The thickness h of the laminate is 0.001m. The material properties for laminate layers are given as:

$$E_1 = 194 \text{ GPa}, \quad E_2 = 8.7 \text{ GPa}, \quad G_{12} = 3.2 \text{ GPa}, \quad \nu_{12} = 0.33,$$

$$\rho = 2100 \text{ kg/m}^3$$

$$X_t = 1300 \text{ MPa}, \quad X_c = 2000 \text{ MPa}, \quad Y_t = 140 \text{ MPa}, \quad Y_c = 300 \text{ MPa}, \quad S = 90 \text{ MPa}.$$

The material properties for sandwich core are given as:

$E = 42\text{MPa}$, $\nu = 0.3$, $\sigma_u = 1\text{MPa}$, $\rho = 150\text{kg/m}^3$.

The plate is simply supported at all boundaries and loaded by an uniaxial uniform load (Fig. 3). Thickness h is for the facings and 8^*h is for the core (Fig. 4).

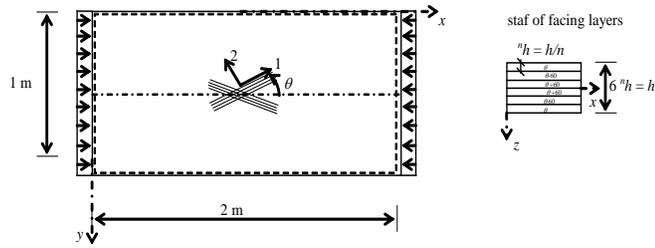


Fig. 3. geometry of the sandwich plate

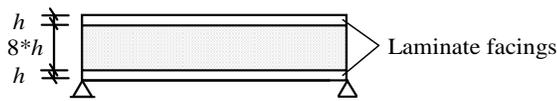


Fig. 4. cross-section of sandwich plate

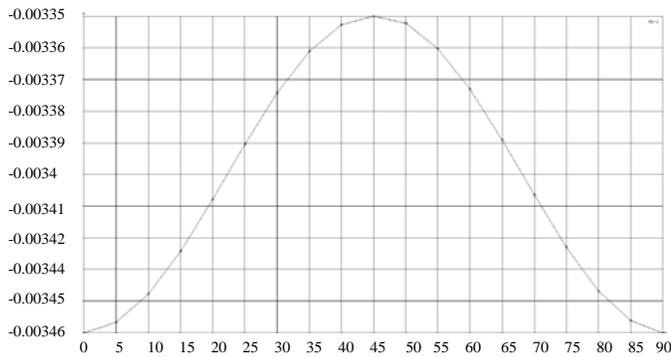


Fig. 5. maximum F.I for changed angle orientation $\theta - 90^\circ$

TABLE I
FIRST 10 BUCKLING LOAD FACTORS

Eigen Value	Buckling Load Factor
1	12.79965
2	15.44049
3	19.73424
4	21.25276
5	29.79095
6	41.38276
7	54.31924
8	56.70496
9	57.96148
10	61.83806

TABLE II
FIRST 10 FREQUENCIES
IN BUCKLING ANALYSIS

Frequency	Frequency [Hz]
1	60.1931
2	72.1556
3	93.6620
4	98.7033
5	137.297
6	166.673
7	245.163
8	255.635
9	163.688
10	274.553

TABLE III
FIRST 10 NATURAL FREQUENCIES
IN FREE VIBRATION ANALYSIS

Frequency	Frequency [Hz]
1	47.9450
2	77.1406
3	126.157
4	163.474
5	176.255
6	194.218
7	195.647
8	245.632
9	286.601
10	318.363

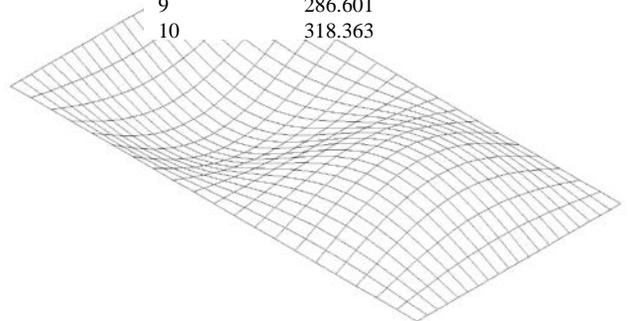


Fig. 6. first natural mode shape in buckling analysis

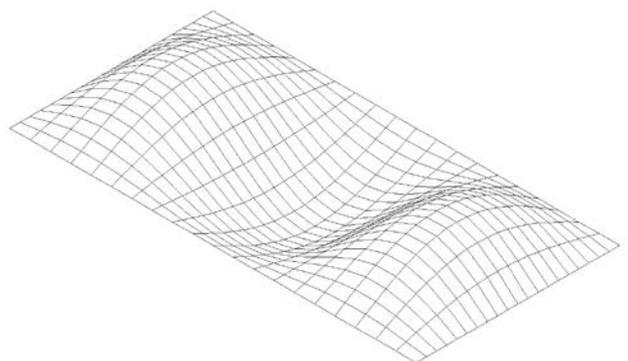


Fig. 7. second natural mode shape in buckling analysis

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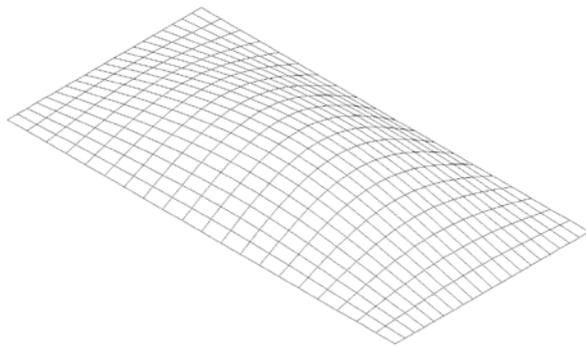


Fig. 8. third natural mode shape in buckling analysis

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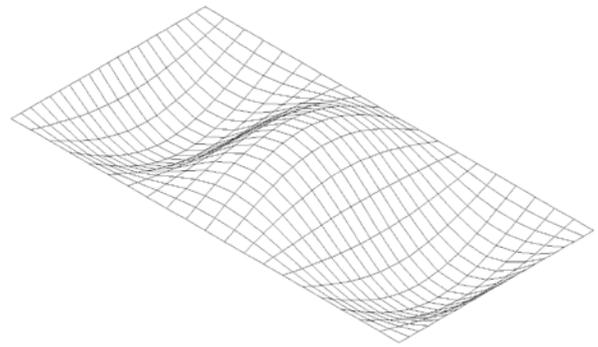


Fig. 12. third natural mode shape in free vibration analysis

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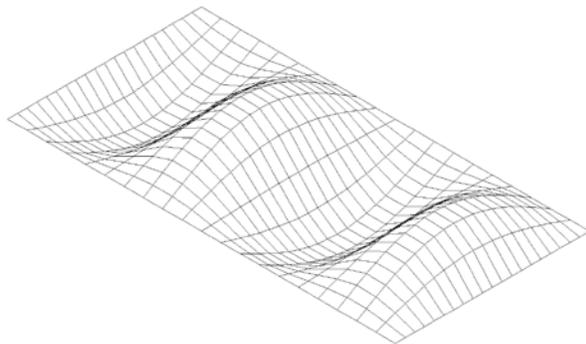


Fig. 9. fourth natural mode shape in buckling analysis

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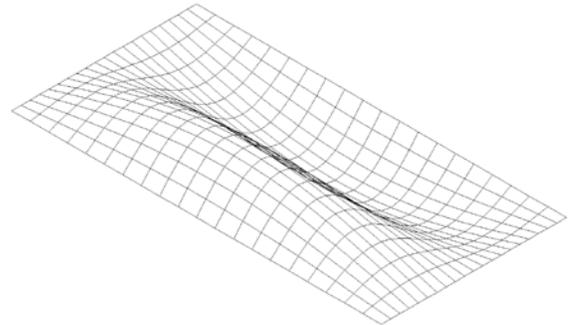


Fig. 13. fourth natural mode shape in free vibration analysis

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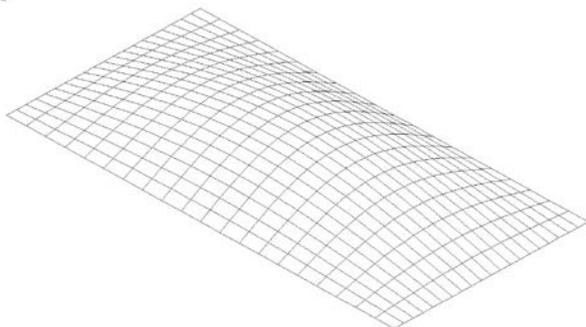


Fig. 10. first natural mode shape in free vibration analysis

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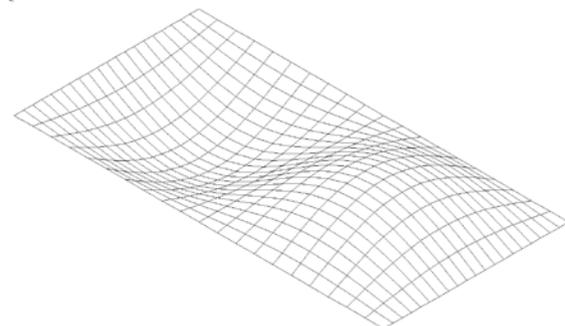


Fig. 11. second natural mode shape in free vibration analysis

VI. DISCUSSION AND CONCLUSION

The paper deals with a modeling of buckling and free vibration analysis of sandwich plates. To predict the inception of buckling, in-plane resultant forces must be included.

From sensitivity analysis (Fig. 5) one can see, that angle orientation has minor influence on the maximum failure index. The reason is the quasi-isotropic character of the laminate facings. Tsai-Wu criterion is violated, than failure load factor is 289. The results for the buckling factors are shown in Table 1. The first buckling load factor is 22.5 times less than maximum failure load factor.

For the fibre angle $\theta = 45^\circ$, the buckling and free vibration analysis was done. The first 10 frequencies in buckling and frequency optimization analysis can be seen in Table 2 and 3, respectively. The first ten frequencies in buckling analysis are higher than in free vibration analysis.

For symmetric laminates the buckling modes for $\theta = 0^\circ - 30^\circ$ are nearly the same. For fibre angles $\theta = 30^\circ, 45^\circ, 60^\circ, 90^\circ$ the buckling modes have different shapes. The buckling mode shapes are symmetric to the symmetric axis in loading direction (Figs. 6-9). Natural mode shapes in buckling analysis are different than in free vibration analysis (Figs. 10-13).

Buckling and free vibration analyses play very important role in the investigation of sandwich plates. Natural mode shapes in buckling analysis depend on fibre angle orientation and have different shapes then for isotropic homogeneous plates. There are significant differences between behaviour of homogeneous and heterogeneous materials.

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