Influence of non-linear behavior on seismic response parameters

Rita Greco, Alessandra Fiore and Ivo Vanzi

Abstract**—** This paper deals with the effects of post-yielding stiffness on the response of non-linear structural systems under seismic actions. In detail, two hysteretic models are considered: a bilinear plastic model, representing structures that exhibit strain hardening characteristics, and an elastic-perfectly-plastic model, widely adopted in design codes. For these models, a parametrical study is carried out to assess the influence of post-yielding stiffness ratio on some non-linear response quantities, such as the response modification factor, the ductility demand and the damage index. The analysis is developed on a SDOF system subject to El Centro earthquake and demonstrates that the elastic-plastic constitutive law is excessively conservative to evaluate structural deformations. Finally the study provides useful information also from an energetic point of view.

Keywords **-** Post-yield stiffness ratio, Inelastic Displacement Ratio, Strength Reduction Factor, Dissipated energy, Damage Functional.

I. Introduction

A correct evaluation of strength and peak lateral deformation demand on structures is fundamental to limit structural damage, especially when structures are subjected to severe ground motions. In current design processes, inelastic behaviour is taken into account through strength reduction factors that allow structures to be designed for lateral forces smaller than those required to remain elastic during severe earthquakes [1-3]. For this reason, inelastic design spectra are generally obtained from elastic spectra scaling-down the latter by strength reduction factors (*q* and *R* for Eurocode8 [4] and US FEMA [5] respectively). These factors account for ductility demand and vibration period. Likewise, the approximate estimation of inelastic displacement in structures is based on the elastic system response, amplified by the "displacement modification factor *C*", which depends on ductility demand and structural period. Based on these empirical coefficients, several studies have shown that inelastic response can significantly differ from the response obtained by time-history analyses [6].

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Hence, the need to improve the currently recommended procedures, for instances by evaluating in a proper and more realistic way the used hysteretic models. A comparison between elastic perfectly plastic -EPP- model and other models in strength modification factor assessment can be found for example in [7, 8]. However, at the present time, in design codes the definition of inelastic response spectra is limited to the elastic-perfectly-plastic (EPP) systems. This is, indeed, the easiest way to model inelastic force-resistance; moreover, it is the basis for early relationships between seismic action and response modification factors, but few structural characteristics are included. First, an EPP model is not suited to describe the global behaviour of steel and reinforced concrete structures subjected to earthquakes after the yielding. Indeed, it does not take into account the strain hardening effect and post-elastic stiffness values different from zero. Few researchers have investigated the post-elastic stiffness effects on strength and displacement demand of structures. Most of them does not provide a general result, but discuss the local influence of the post-yielding stiffness ratio on particular parameters as part of other sensitivity analyses. For example, Garcia and Miranda [9] refer to post-yielding stiffness ratio α_k effects on the maximum inelastic displacement demand (for α_k equal to 3, 5 and 10%). They show that the maximum inelastic deformation of the bilinear system becomes smaller, with respect to the one of the elasto-plastic system, as the strength ratio increases. For periods of vibration larger than about 1,0 s, the ratio between maximum deformations remains approximately constant. Instead, for periods smaller than about 0,5s, the maximum deformation of the system with positive post-yield stiffness can be significantly smaller than that of elasto-plastic systems. The study, then, establishes the short period range in which the inelastic deformation ratio *C* is sensitive to strength and stiffness. Other researchers have demonstrated that *C* is lower in the acceleration-sensitive spectral region due to post-yielding stiffness [10], so that a big difference would be expected in evaluating the seismic response of structures with short period. Moreover, the comparison with the response data with expectations based on the well known “equal displacement” rule, allows saying that the Newmark rule concept is valid only for EPP systems. Other authors have pointed out that the post-yielding stiffness influence on ductility demand is not significant for constant strength system, especially over long periods [11]. Therefore, the response of constant-ductility systems can be conservatively estimated by using the EPP model [12]. The difference obtained in the analysis of the response is of the order 10-20%, but it is not predominant on inelastic strength demand [13]. A review on post-yielding stiffness effects was provided in [14]. In this report the authors conclude that for the same ductility factor, post-yielding stiffness limits the deformations of bilinear elasto-plastic (BP) systems compared to EPP ones, for periods smaller than the characteristic ground motion period. This observation is valid for α_k variable between 0 and 10%. Furthermore, the percentage reduction of deformation is roughly independent from period. A larger reduction is achieved for bigger values of ductility factor. The study concludes that the reduction of deformation due to post-yielding stiffness effects is slight for realistic values of α_k and ductility μ (for instance, for μ=4 and α_k=3%, the reduction of deformation is less than 15% over the observed period range). These results support the theory of Riddell and Newmark [15] and Riddell et al. [12] according to whom, EPP model provides a useful conservative estimate of deformation for small ductility systems. The analysis also shows that post-yielding stiffness effect reduces inelastic deformation in the acceleration sensitive region, more successfully for constant values of *R* (C_R) than for constant ductility μ (C_μ). As concerns the yield-strength reduction factor *R*, the same report shows that *R* is weakly affected by α_k, being *R* slightly larger for BP systems [13]. An evaluation of the FEMA-273 procedure to estimate displacements [16] demonstrates that the ratio between inelastic and elastic displacement at constant strength ratio C_R, analysed for different values of post-yielding stiffness (5% and 25%), tends to be substantially smaller than the one corresponding to the EPP case. Therefore, neglecting α_k effects in the deformation analysis is too conservative for seismic evaluation of structures characterized by period in the acceleration sensitive region. This is not considered relevant for real structures, which are usually strong enough to remain elastic for most ground motions [14].

However, at present, the studies to evaluate response spectrum and strength reduction factor *R* are mainly focused on EPP models. In [17] the authors studied the inelastic time-history analysis of numerous SDOF and MDOF systems to investigate the influence of post-yielding stiffness on the inelastic seismic response. The analytical results showed that, for SDOF systems, the larger positive post-yielding stiffness would result in smaller maximum displacement and especially residual displacement; for MDOF systems, the larger positive post-yielding stiffness results in more uniform distribution of hysteresis energy dissipation and smaller variation of the maximum inelastic story drift. Therefore, a larger post-yielding stiffness will result in a better control of structural performance under earthquake, so that the performance based design can be easily implemented. Similarly in [18] inelastic response spectra are investigated through the ductility demand, the yield strength reduction factor and the inelastic deformation ratio. In [19] the same topic is further enhanced by proposing a rational approach able to estimate the inelastic deformation ratio for SDOF bilinear systems by rigorous nonlinear analysis.

Some recent studies on the influence of inelastic behaviour on inelastic demand concern the effects of smooth hysteretic behavior [20, 21], which is more representative of the actual structural behavior than piece-wise linear hysteretic models. By considering the effect of smooth hysteretic behavior on the inelastic deformation ratio *C*, the accuracy of the inelastic displacement demand calculated from elastic displacement demand results increased than existing formulas without smooth effects. In [22] the median ductility demand ratio for 80 ground motions was presented for different levels of normalized yield strength, defined as the yield strength coefficient divided by the peak ground acceleration (PGA). The influence of the post-yielding stiffness on the ductility demand was investigated. It was found that the post-to-pre-yielding stiffness ratio has no effect on the median ductility demand for systems with a normalized yield strength greater than one. For systems with normalized yield strength smaller than one, the post-to-pre-yielding stiffness ratio reduces the ductility demand only for periods longer than 0.2 sec and has essentially little effect on the ductility demand for longer periods. Results showed that ignoring post-to-pre-yielding stiffness ratio in estimating the ductility demand is too conservative for seismic evaluation of structures with periods in the acceleration-sensitive region.

In [23] the inelastic displacement ratio is calculated including also a damage measure for SDOF systems subjected to a set of ground motions. The influences of post-yield stiffness and other factor is investigated. To study the effect of post-yield stiffness on inelastic displacement ratio, the inelastic deformation ratio *C* of bilinear systems with post-yield stiffness ratio (ratio between post-yield stiffness and initial stiffness) equal to α_k = 0.05 and α_k=0.10 are computed for all ground motions. Then, ratios between inelastic deformation ratio C of bilinear systems and inelastic deformation ratio *C* of EPP systems are calculated for each ground motion and each period of vibration. Results show that the mean ratios of *C* are smaller than 1.0 in the whole period region and increase slightly with the increase of period of vibration, which means that SDOF systems with positive post-yield stiffness ratio would lead to smaller C than those computed from EPP systems.

Other recent works on the above topics deal with base-isolated structures. Inelastic displacement ratio of base-isolated structures is studied in [24] by employing a two-degree-of-freedom model taking into account inelastic behavior of both isolators and superstructure. In [25], seismic reliability-based relationships between the strength reduction factors and the displacement ductility demand of nonlinear structural systems equipped with friction pendulum isolators, depending on the structural properties, are proposed.

Within this framework, the main objective of this study is to provide more information about the effects of post-elastic stiffness ratio α_k on inelastic structural response, by comparing the EPP model and the bilinear plastic (BP) one. Special emphasis is laid on the α_k effects on strength reduction factor *R*, ductility demand μ and inelastic displacement factor *C*. More in detail, the study is aimed at:

1. assessing the spectral region where the post-yielding stiffness effects on response modification factors and ductility demand cannot be neglected;
2. evaluating the influence of post-yielding stiffness on structural damage by the Park-Ang damage index;
3. understanding the energetic processes that cause the observed trends, trying to interpret the general system behaviour from an energetic viewpoint.

II. Modification Factors

The focus of this study is to perform a comparison among the most commonly adopted non-linear seismic response coefficients, such as the strength reduction factor, the inelastic displacement ratio and the damage index, adopting the BP model and the EPP one. In the next part of this section, a brief summary of these factors is reported.

A. Strength Reduction Factor

It is the ratio between the maximum elastic strength and the yield strength (Fig.1).

$$R = \frac{f_{\max}^{el}}{f_y} \tag{1}$$

It is used to derive the nonlinear strength response from the linear elastic strength demand. Moreover, it is adopted in standard design applications, since inelastic spectra can be obtained reducing the elastic ones by the factor *R*. This procedure represents a very simple way for accounting energy absorption process in inelastic systems. Strength Reduction Factor can be also written as the ratio between the acceleration of the linear system and the inelastic one:

$$R = \frac{S_a^{el}}{S_a^{in}} \tag{2}$$

The factor *R* has been widely studied, and various relationships $R-\mu-T$ have been developed.

The behaviour factor *q* is a strength modification factor conceptually equivalent to *R*. This latter is used in Eurocode 8 [26] and assumes different values depending on structural types, materials, class ductility and vibration period. Furthermore, in all analytical expressions of *R* and *q*, the influence of post yielding-stiffness ratio has not been taken into account (except for the Krawinkler and Nassar's formulation that includes the hardening parameter of the employed hysteretic model, see Table 1). Strength reduction due to the inelastic behaviour of systems can be also expressed by means of the mutual of *R*, the so-called *strength ratio*. It is defined as follows:

$$\alpha_f = \frac{f_y}{f_{\max}^{el}} \tag{3}$$

This formula allows limiting the parameter range into the [0,1] interval. The strength ratio measures the inelastic lateral strength demand compared to the elastic one (in elastic conditions $\alpha_f = 1$). In Eq. (3), f_y is the yield strength value and $f_{\max}^{el}$ represents the maximum elastic force experienced by the system without crossing the yield strength.

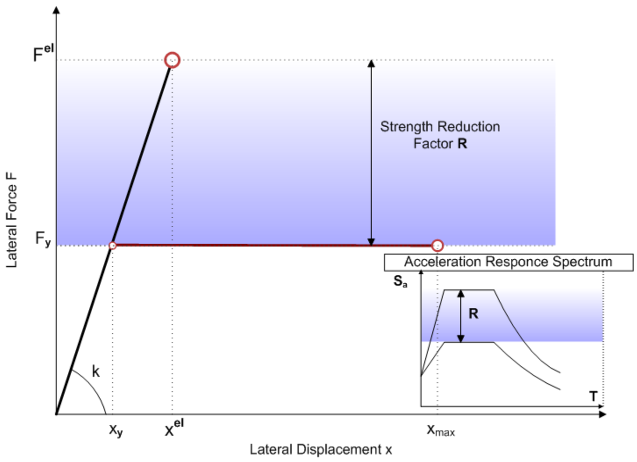

Fig. 1 Strength reduction factor.

B. Inelastic Displacement Ratio

It is the ratio between the maximum inelastic displacement demand and the maximum elastic displacement of a system with the same mass and initial stiffness and subject to the same earthquake:

$$C = \frac{x_{\max}^{inel}}{x_{\max}^{el}} \tag{4}$$

By introducing this factor, the maximum deformation of a system is given by the maximum deformation of the elastic system multiplied for *C*. Veletsos and Newmark [27] were the first to study how to obtain the inelastic response from the elastic one. They laid the basis of the well-known "equal energy" and "equal displacement" rules to define inelastic spectra. Afterwards, they derived a correlation between displacement amplification factor *C*, spectral period *T* and available ductility μ (Newmark and Hall, 1982). Both Newmark and Hall [28] and Miranda [29] proposed limit values for C ($C=\mu$ as $T\rightarrow 0$ and $C=1$ as $T\rightarrow\infty$). They also proved that C depends on lateral strength and vibration period [8]. C can also be estimated as follows [30]:

$$C = \frac{x^{i}_{\max}}{x^{el}_{\max}} = \frac{\mu}{R} \tag{5}$$

where μ is the ductility demand given by:

$$\mu = \frac{x^{i}_{\max}}{x_y} \tag{6}$$

III. Structural Model

In order to assess the influence on the response of post-yielding stiffness ratio, a simple SDOF system subjected to the El Centro ground motion [31] and characterized by an initial stiffness k_i, a viscous damping c and a hysteretic behavior (Fig. 2b) is considered. In detail, two models are compared: the EPP model (Fig. 2a, line 3) and the BP model (Fig. 2a, line 2). The EPP model represents the idealization of nonlinear behaviour [32, 33]. It is commonly employed to reproduce load history and to model the simplest form of hysteretic deformation cycles, by taking into account dissipated energy and post-yield excursions [34-41]. Due to its simplicity, EPP model is frequently used to investigate seismic inelastic response of structures. An EPP model presents a constant loading stiffness up to yielding, which occurs at yielding strength and displacement (f_y, x_y). Within the linearly elastic range, the system has a natural vibration period defined as follows:

$$T = 2\pi\sqrt{\frac{m}{k_i}} = 2\pi\sqrt{\frac{x_y}{f_y}} \tag{7}$$

An EPP model is generally used to represent structures without hardening effects; in fact, after the yielding branch, post-elastic stiffness is zero. Then, unloading occurs with stiffness equal to the linear-elastic one. The EPP model can be considered as a particular case of the BP one. In this latter, a finite slope is assigned to the post-yielding stiffness in order to simulate the strain hardening characteristics of steel and reinforced concrete. The BP model shows a hysteretic cycle similar to the one of an EPP model, but post-elastic stiffness assumes a generic value $\alpha_k k_i$ different from zero. It has to be emphasized that, neither the EPP model nor the BP one represent the behavior of real structures. Indeed, during reloading, members soften according to the "Bauschinger effect" [42]. Moreover, stiffness and strength degradation with inelastic deformation are not considered in these models. Nevertheless, the adoption of the BP model allows including the above-mentioned hardening characteristic, so that it is adequate for the goals of this study.

The motion equation for the BP model is:

$$m\ddot{x}(t) + c\dot{x}(t) + \alpha_k k_i x(t) + \left(1-\alpha_k\right)\ k_i z(t) = m\ddot{x}_g(t) \tag{8}$$

where:

$$\alpha_k = \frac{k_p}{k_i} \tag{9}$$

is the post-yielding stiffness ratio, defined as post-yielding stiffness k_p over initial elastic stiffness k_i. It depends on the inelastic system behavior. In this study, it has been assumed positive in order to simulate hardening effects ($\alpha_k > 0$).

For the EPP model, the motion equation becomes:

$$m\ddot{x}(t) + c\dot{x}(t) + k_i z(t) = m\ddot{x}_g(t) \tag{10}$$

being $k_p = \alpha_k k_i$. The expression of the internal hysteretic variable Z can be found in [43]. Herein it is assumed that the unloading and reloading of the hysteretic system occur without any deterioration of stiffness and strength.

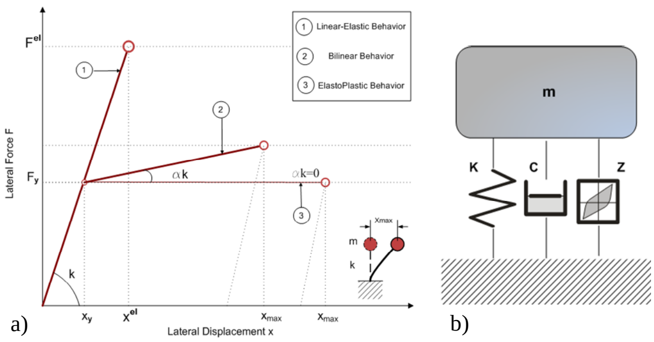

Fig. 2 EPP and BP behaviour (a) in a SDOF system (b).

IV. Description of the Analysis and Discussion of the Results

Only few relationships of modification factor proposed in literature to estimate nonlinear response, allow to account for the effects of post-yielding stiffness ratio.

In the present study, a sensitivity analysis is carried out with the aim of comparing the nonlinear response of an EPP system with a BP one, characterised by a post-yielding stiffness ratio α_k. A MATLAB algorithm has been developed to perform the nonlinear dynamic time-history analysis of the above-mentioned SDOF models. A numerical incremental step-by-step integration has been used to solve the ordinary differential equation of motion. The analysis has been carried out by varying the most significant parameters controlling the SDOF response. More in detail the following data have been considered:

1. The El Centro ground motion;
2. 9 different values of Strength Ratio α_f, varying in the interval [0.2 - 1;
3. 7 different values of Post-Yielding Stiffness Ratio α_k, varying between 0 (EPP case) and 30%;
4. A structural period T in the interval [0.1 - 2.0] sec.

A. Influence of Post-Yielding Stiffness Ratio on Acceleration Response

In order to explore the influence of post-yielding stiffness ratio, the acceleration response spectrum is evaluated for six different values of α_k, including also the EPP condition.

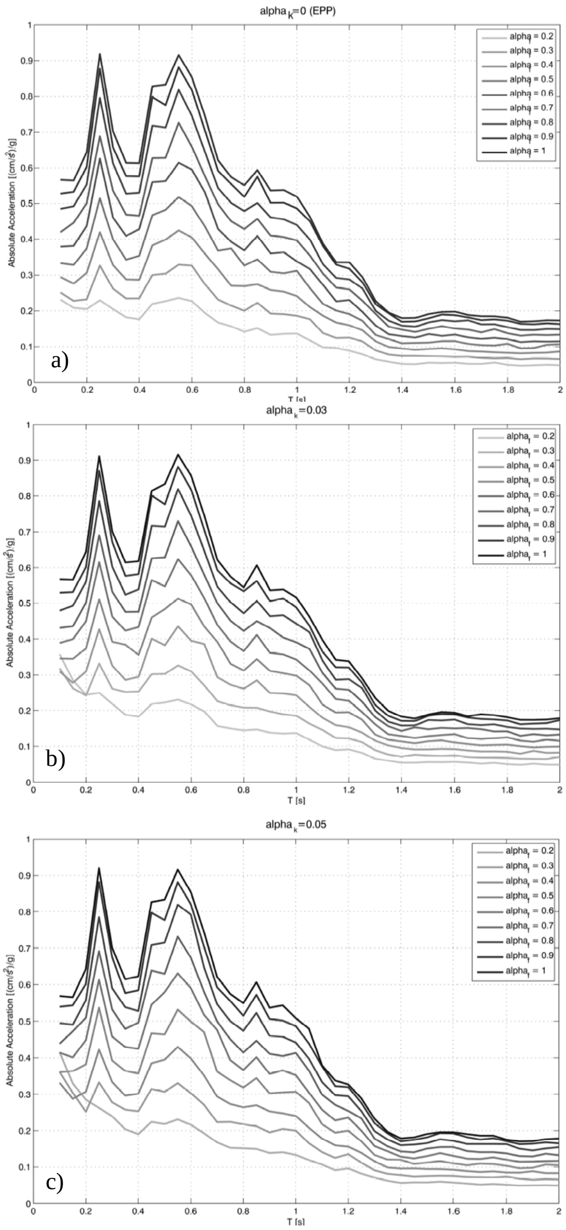

Fig. 3 Effects of the post-yielding stiffness ratio on the acceleration spectrum for α_k = 0 (a), 0.03 (b), 0.05 (c).

Results are shown in Figs. 3 and 4. It can be observed that only for α_k=0 (EPP model) (Fig.3.a) all inelastic acceleration spectral curves are reduced proportionally to the inelastic property of the system, if compared to the elastic spectrum. On the contrary, for a BP model (with variable values of α_k) it can be noted that, in a short period range and for small values of α_f (typical of systems with strong inelastic characteristics), the curves overlap in several points.

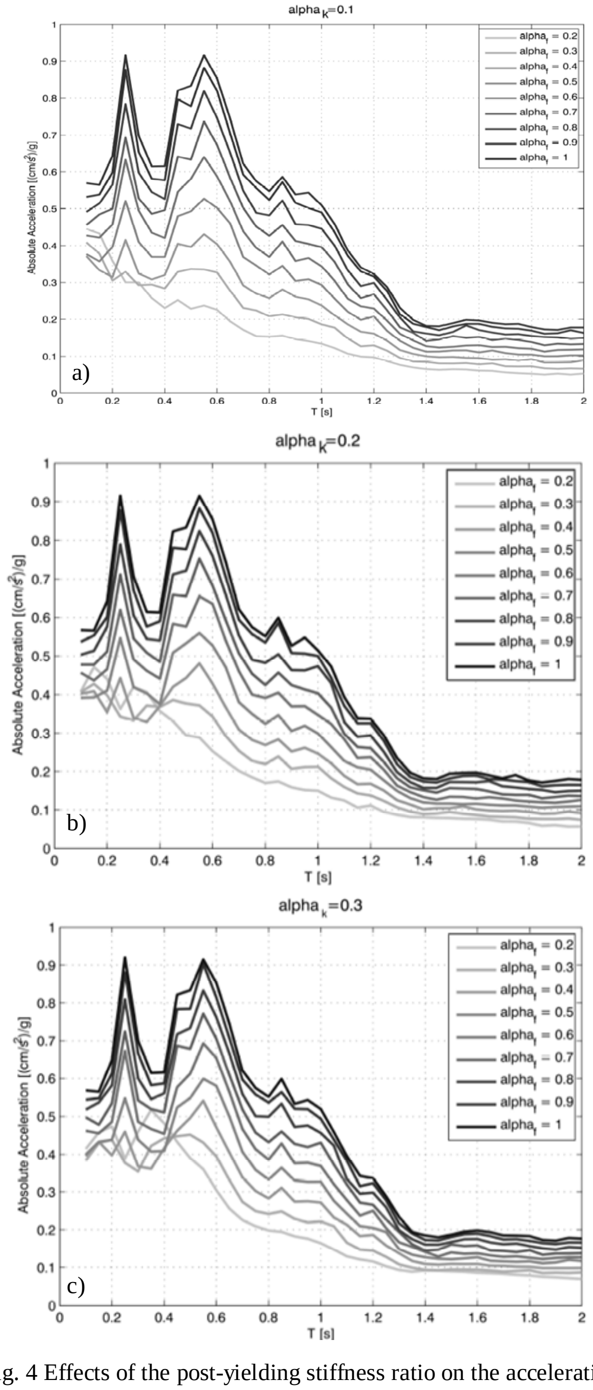

Fig. 4 Effects of the post-yielding stiffness ratio on the acceleration spectrum for α_k = 0.1 (a), 0.2 (b), 0.3 (c).

As α_k increases from 3% to 30% (Figs. 3 b-c and 4 a-c), the number of intersections increases as well, including more curves in a larger sensitivity range of period internal to $[0.1 \div 0.5]$ s.

Observing these results for a BP model, it can be concluded that the application of a strength reduction factor to the elastic system may be non-conservative. Indeed, as Figs. 3-4 show, an inelastic system with a fixed α_f can be more accelerated than a system characterized by larger α_f during an earthquake event; this happens in the critical period range $[0.1 \div 0.5]$ s.

For the reasons previously explained, the force modification factor R employed in seismic design should include post-yielding stiffness effects in inelastic strength demand of structures with hardening characteristics.

B. Influence of Post-Yielding Stiffness Ratio on Ductility Demand

In this section, the results of the investigation developed on ductility demand are shown. The sensitivity analysis furnishes results that agree with those presented in [44].

Figure 5 shows the ductility demand variability of the system with different values of α_k. As α_k increases, ductility globally tends to decrease and this trend is more relevant for short periods (i.e. approximately within the range $[0.1 \div 0.5]$s).

For example, a BP system with α_f=0.2 and α_k=10% has a ductility demand less than 37% with respect to the EPP one for a vibration period T=0.1s, and less than 10% for T=1s. As α_k increases, this trend becomes more evident, so that a system with post-elastic stiffness ratio equal to 30% and period 0.1s shows a reduction of ductility demand less than about 78%.

For a period equal to 1s, the reduction is about 22%; the trend for longer periods (and different α_k values) is more stable. It can be stated that the effects of post-yielding stiffness ratio are important in short periods, while for longer periods, the sensitivity to α_k is lower. Therefore, neglecting these effects causes too conservative assumptions.

Figure 6 shows the results of the investigation on the ductility demand. The ratio D between the ductility demand of a BP system characterized by a α_k ratio and the same quantity for a EPP model is introduced:

$$D = \frac{\mu_{BP(\alpha_k=\alpha_k\%)}}{\mu_{EPP(\alpha_k=0)}} \qquad (11)$$

It can be stated that for low values of α_f (Fig. 6a and 6b), and mainly for short periods, this ratio can assume values higher than 1 (in particular for high value of α_k). This means that the ductility demand of a BP system can be greater than the corresponding demand of the EPP model. Nevertheless, as α_f grows, the excursions above the unit tend to reduce (Fig. 6c and 6d), so that the global trend is less relevant, since D remains under the unit value.

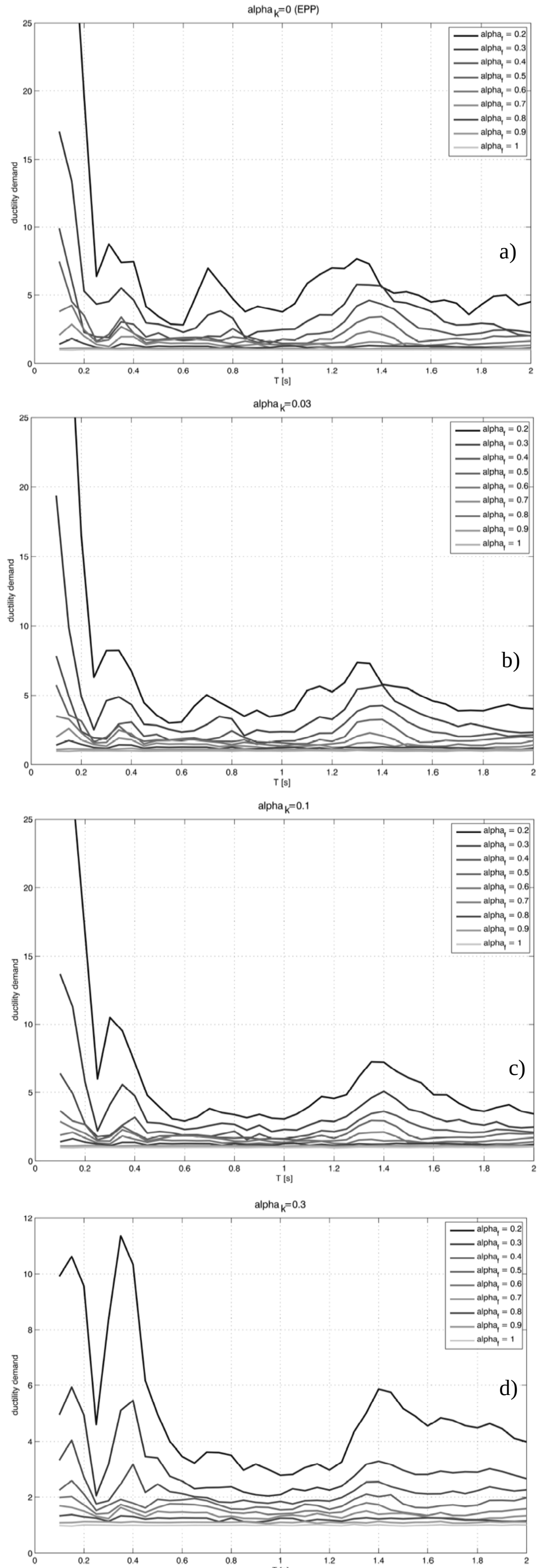

Fig. 5 Effects of post-yielding stiffness ratio on Ductility Demand for α_k = 0 (a), 0.03 (b), 0.1 (c), 0.3 (d).

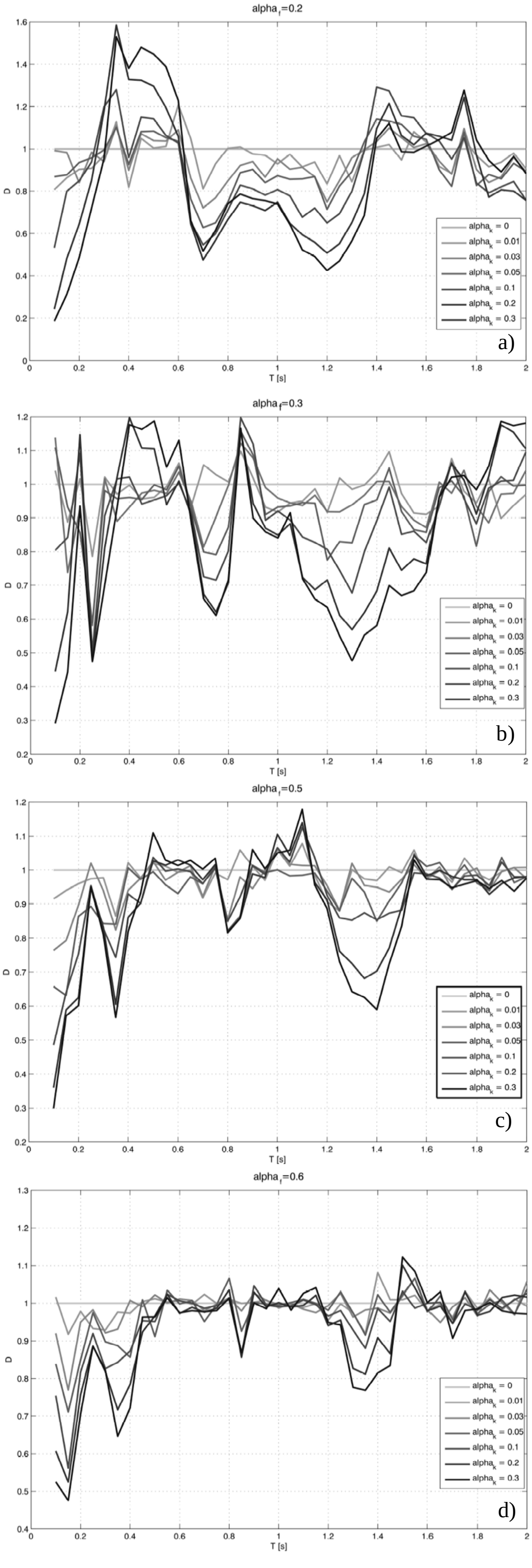

Fig. 6 Ductility ratio for α_f = 0.2 (a), 0.3 (b), 0.5 (c), 0.6 (d).

C. *Influence of Post-Yield Stiffness Ratio on Inelastic Displacement Ratio*

Figures 7 and 8 show the results of the analysis developed on the inelastic displacement ratio. Observing the plots, one can deduce that the variability of *C* for different values of α_k is similar to the one observed for ductility demand: *C* tends to decrease as α_k increases.

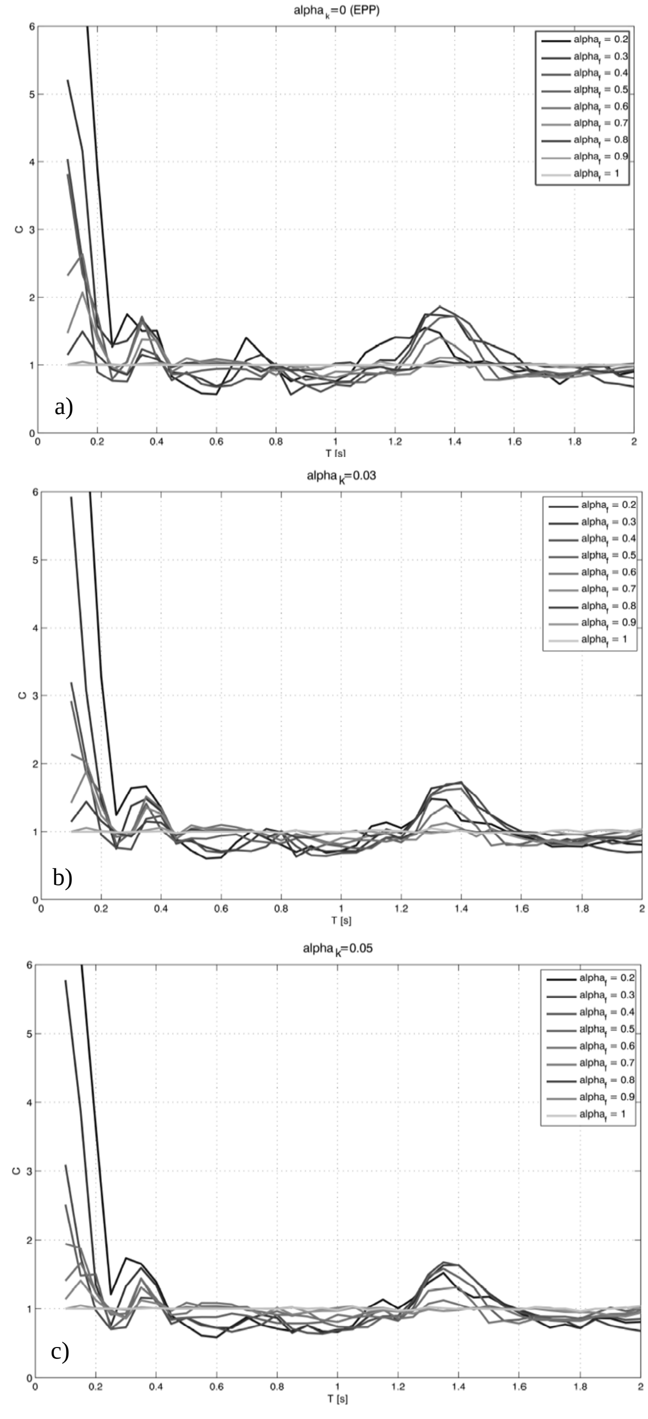

Fig. 7 . Effects of post-yielding stiffness ratio on C for α_k = 0 (a), 0.03 (b), 0.05 (c).

It has to be underlined that in many cases the most significant effects occur for short periods. The analytical formulation of *C* given in [9] and [28] is rigorously valid only for EPP systems. Indeed, the assumption of the "equal displacement" rule (which is considered a good approximation in case of a EPP system) for high values of α_k is excessively conservative, as Figs. 7.a, b and c (corresponding to $\alpha_k \geq 10\%$) show.

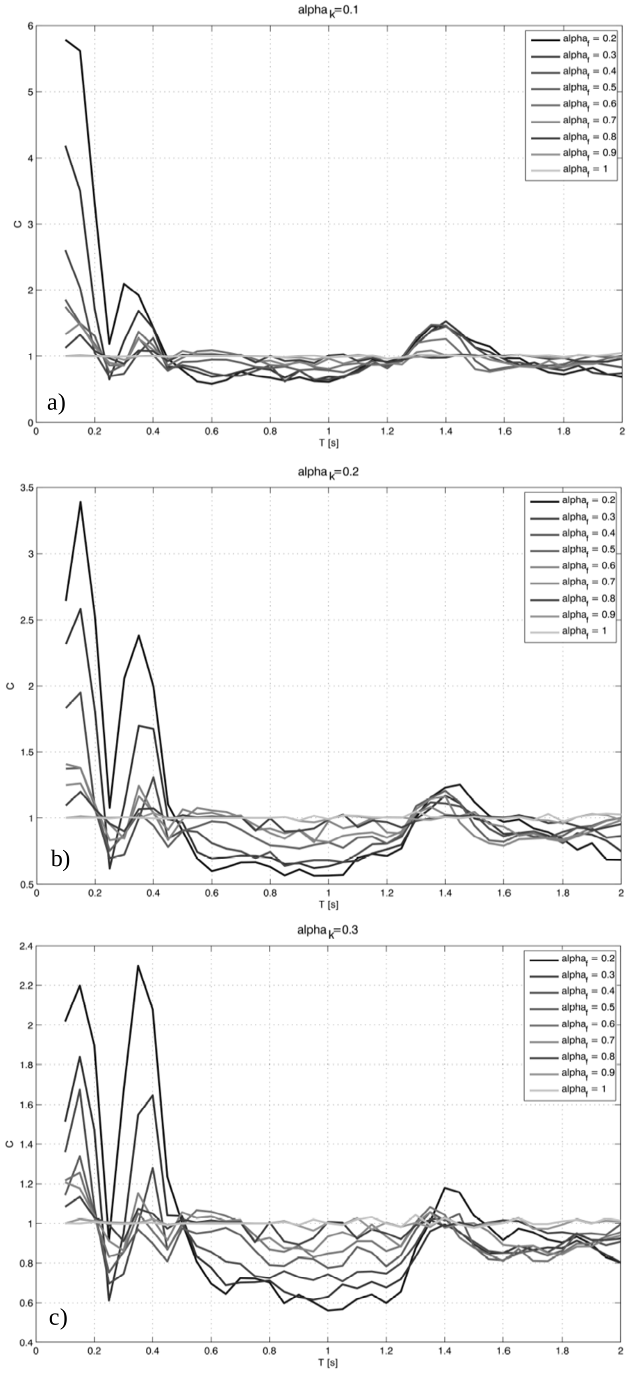

Fig. 8 . Effects of post-yielding stiffness ratio on C for α_k = 0.1 (a), 0.2 (b), 0.3 (c).

V. Energetic Interpretation of Results

In this section, the authors propose an energetic interpretation of some results before showed. As illustrated in the previous section, the effects of post-yielding stiffness ratio are more appreciable for systems with short vibration periods. A post-yielding stiffness sensitive region exists in the period range $[0.1 \div 0.5]$s. Hence, some energetic considerations arise. Firstly, one should consider that different dissipative capacities occur for systems with the same frequency but different values of α_f. In fact, an increase of α_f corresponds to a longer permanence of the system in the elastic domain, and then a lower amount of dissipated energy. Furthermore, the variation of α_k is related to the shape of the hysteretic cycle (and consequently to the amount of dissipated energy) and to the effective frequency, due to the stiffness reduction after the yielding event.

To correlate the results obtained from the above-mentioned numerical examples with energetic considerations, a parametric analysis on the dissipated energy has been carried out. For the generic BP model, the energy balance equation can be written as follows [31]:

$$\int_0^t \ddot{x}(t)\dot{x}(t)dt + 2\int_0^t \xi\omega \cdot \dot{x}^2(t)dt + \int_0^t \alpha_k \omega^2 \dot{x}(t)dt + \\ + \int_0^t (1-\alpha_k)\omega^2 z(t)\dot{x}(t)dt = \int_0^t \ddot{x}_g(t)\dot{x}(t)dt \quad (12)$$

where:

$$e_h(t) = \int_0^t (1-\alpha_k)\omega^2 z(t)\dot{x}(t)dt \quad (13)$$

is the hysteretic energy for unit mass.

A comparison between the maximum dissipated hysteretic energy e_h and the inelastic displacement ratio C is shown in Figs. 9 -12. Both e_h and *C* are plotted as function of α_k and α_f in a contour graph representation. Different figures correspond to various value of the period T.

Firstly, one can notice that the trend changes with the period range. For short periods, as α_f grows, *C* and e_h approximately monotonically decrease. More precisely, for *T* < 0.5 s (Figs. 9 a-c) it can be seen that contour lines of *C* plots quickly change with α_k and α_f. On the contrary, as the period increases (i.e. T > 0.5s) *C* surface shows a *minimum point* that can be well observed in the contour graphs (Figs. 11 a-c). A similar trend can also be observed in the dissipated energy plots (Figs. 10 a-c): these plots show a significant variability of e_h with α_k and α_f for short periods (*T* < 0.5 s). For larger vibration periods (Figs. 12 a-c), the energy shows a *maximum point* which corresponds to the point of minimum of *C*. Therefore, the study concludes that the dissipated hysteretic energy is sensitive to the stiffness ratio, particularly in short period range. Moreover, the existence of extreme points for lower natural frequency values demonstrate an inversion of behaviour according to α_k variation.

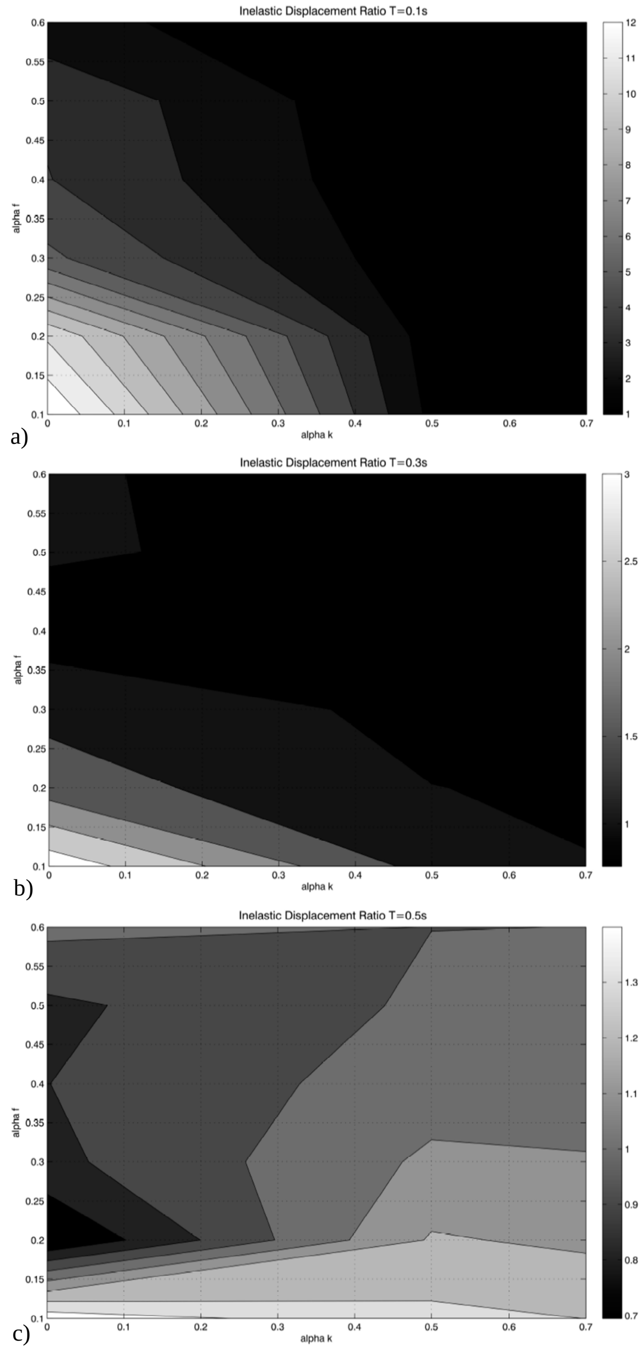

Fig. 9 . Effects of post-yielding stiffness and strength ratios on *C* for T = 0.1 (a), 0.3 (b), 0.5s (c)

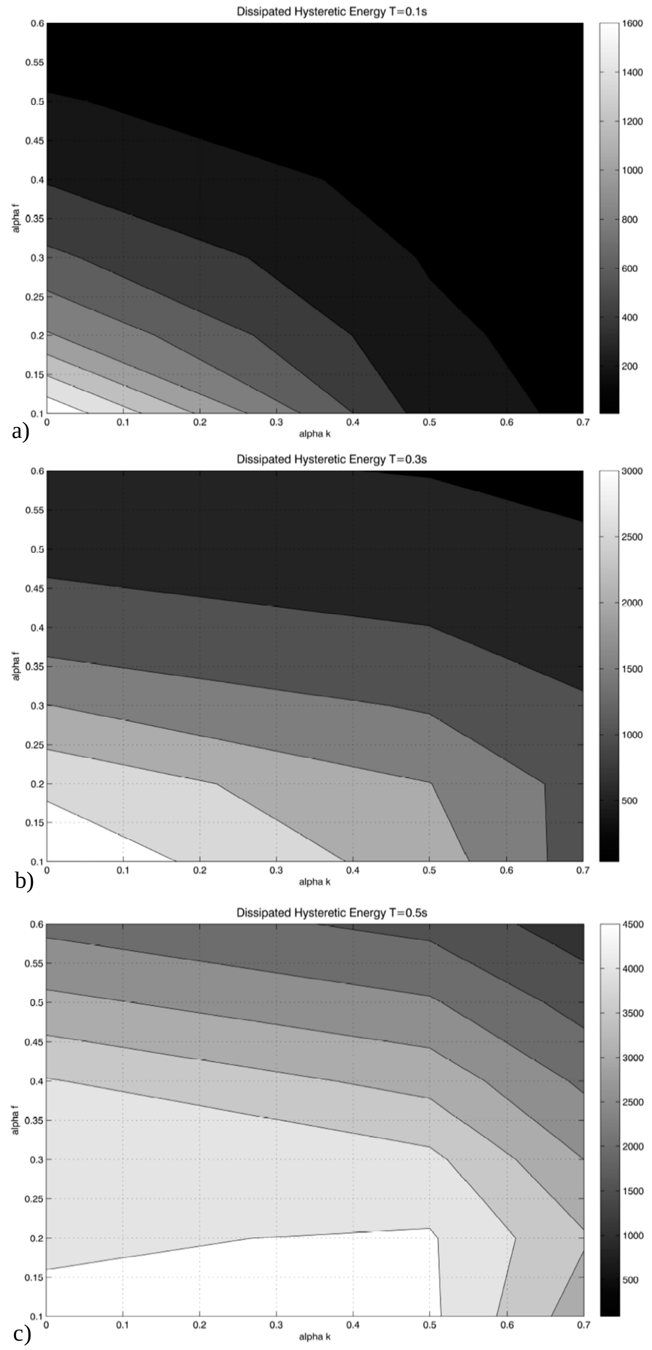

Fig. 10 . Effects of post-yielding stiffness and strength ratios on e_h for T = 0.1 (a), 0.3 (b), 0.5s (c)

There are two *dual counteracting effects* related to α_k and α_f: the first is of deformative nature and the second one of dissipative kind. For systems characterized by small periods, the deformative effect predominates. This is clearly visible, due to the big values of the inelastic displacement ratio *C*.

This behaviour may occur due to the relevant influence of the abrupt reduction of stiffness in plastic phase, after the yielding event.

For larger periods, the dissipative capacity of the inelastic system may prevail; this mitigates the stiffness reduction effects and compensates for the consequent plastic deformation. In short:

- *for low values of the period T (*approximately between 0.1 and 0.5 s), when the post-elastic stiffness of the BP system reduces, the energy dissipation increases. This shows a monotonically trend as α_k and α_f ratios decrease. Moreover, inelastic displacement grows up when the stiffness ratio decreases. This means that the stiffness reduction is prevalent over energy dissipation in this period range.

- *for high values of T* (approximately >0.5), *C* initially decreases with α_k and α_f. Once a minimum point is achieved, an increase occurs. The dissipated energy shows the same but inverse trend as demonstrated by the existence of a maximum point. Afterwards, the trend becomes monotonically increasing as α_k and α_f ratios decrease.

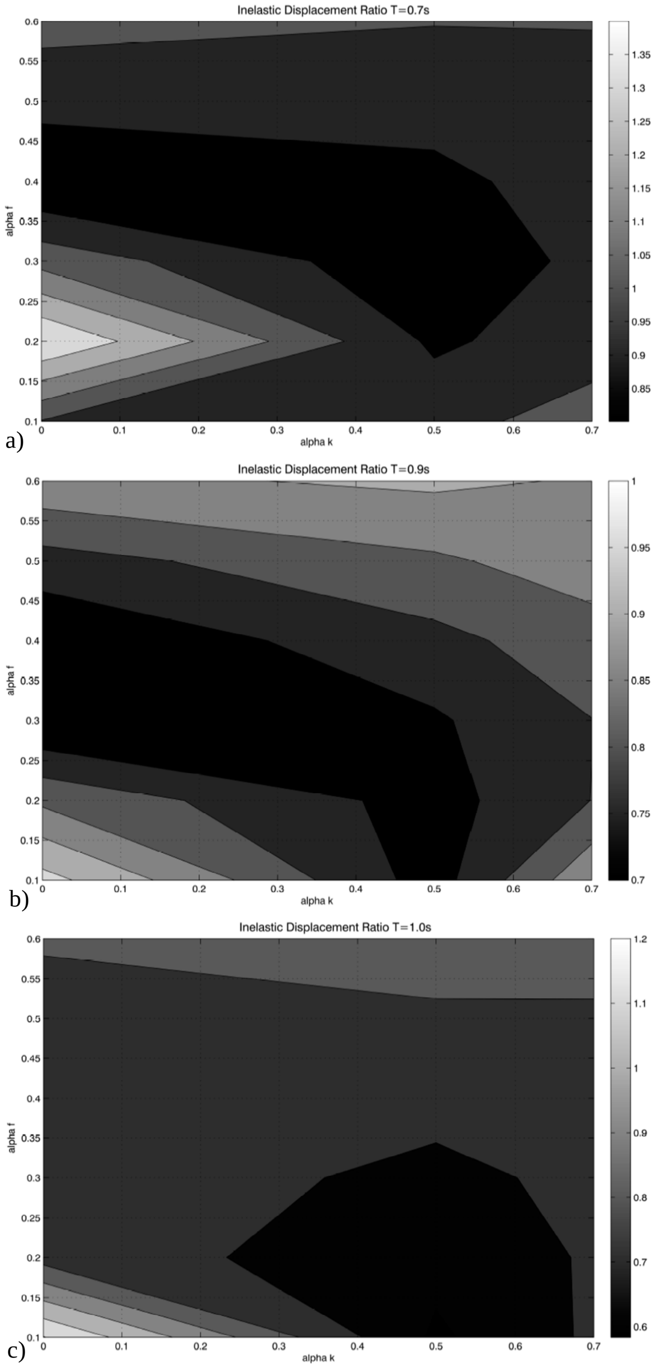

Fig. 11 . Effects of post-yielding stiffness and strength ratios on *C* for T = 0.7 (a), 0.9 (b), 1.0s (c)

This involves a reduction of loads on the structure.

Now, energy dissipation has a more relevant effect if compared with the stiffness reduction. The two mentioned effects are counteracting if related to the effect on inelastic displacement. Table I summarizes the above-mentioned effects.

The analysis of the hysteretic behavior and the recognition of extreme points, provide another important evidence that can be useful in design procedures.

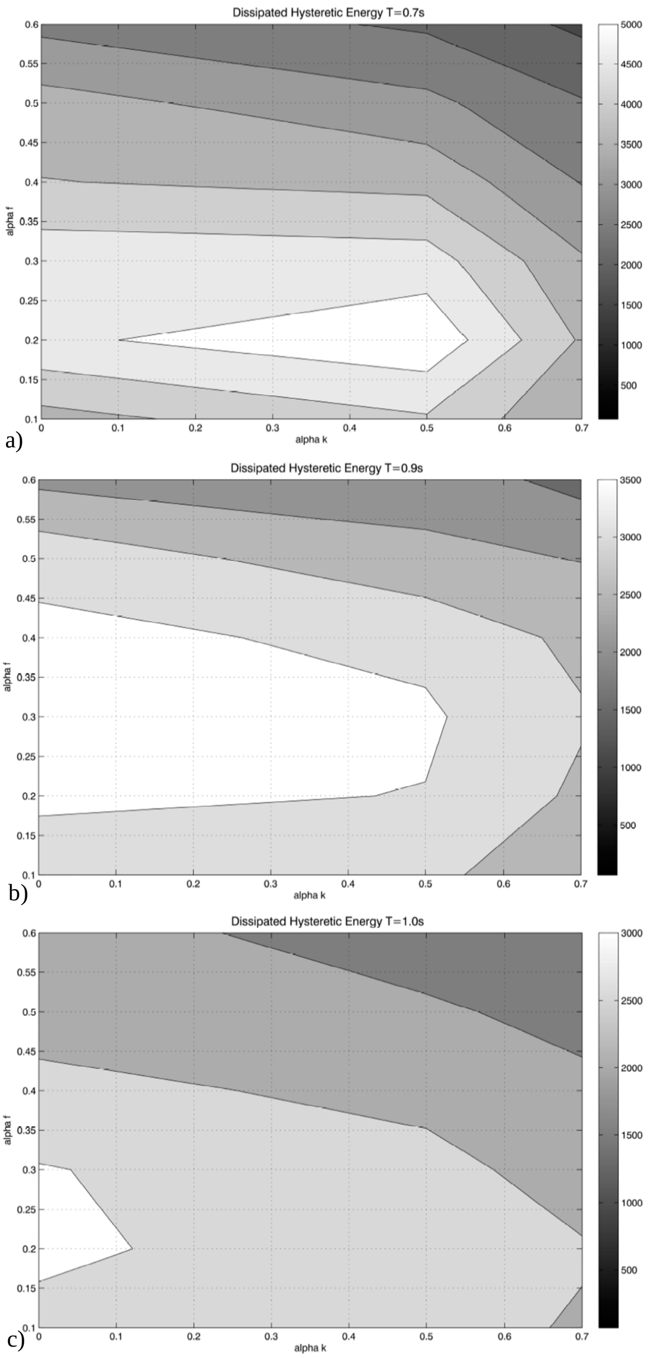

Fig. 12 . Effects of post-yielding stiffness and strength ratios on e_h for *T* = 0.7 (a), 0.9 (b), 1.0s (c)

As well known, the energy dissipation plays a fundamental role to limit seismic loads and damage level.

Indeed, as hysteretic dissipated energy increases, seismic performance improves due to the reduction of the kinetic and elastic energy introduced into the system during the earthquake event.

In the range of high natural periods, energy dissipation grows up when stiffness ratio decreases, and it reaches a maximum value. The maximum seismic performance of the system occurs therefore when its lateral displacement under dynamic actions is minimized and the dissipated energy is maximized.

Considering the dependence of the trend of these quantities from the stiffness and strength ratios, it can be concluded that a proper choice of α_k and α_f (for example assuming those values for which the minimum of C occurs) allows to minimize plastic deformation, through a higher amount of dissipated energy, thus improving the structure performance in seismic response.

Table I Synthesis about the dual effects on hysteretic behaviour as α_k decreases.

Vibration Period	*Prevalent Effect*	*Energy trend*	*Displacement trend*
Short	Stiffness Reduction	Decreasing	Increasing
Intermediate	Both effects	Maximum point	Minimum point
High	Energy Dissipation	Increasing	Decreasing

VI. Influence of post-yield stiffness ratio on global structural damage

Several studies in literature illustrate the importance of post-yielding stiffness in global damage evaluation [45]. For example, damage indices have been directly correlated to the natural frequency reduction of the entire structure [46] or to the ratio between the equivalent period of the structure and the initial one [47].

In this section, analogously with the investigation concerning the influence of α_k on the inelastic displacement ratio C and the dissipated energy, similar analyses are carried out with reference to the damage. To perform this investigation, the Park-Ang damage functional [48, 49] has been selected, due to its ability in reproducing the effects related to deformation and dissipated energy:

$$D_{P.A.} = \frac{x_{\max}^{in}}{x_{mon}^{u}} + \beta \frac{E_{hd}}{F_y x_{mon}^{u}} \tag{14}$$

In Eq. (14), x_{mon}^{u} is the limit displacement of the system under monotonic test and β is a regression coefficient, here assumed equal to 0.15, which corresponds to the average value among those proposed in literature for reinforced concrete structures [50]. The results of the sensitivity analysis carried out varying the structural period T, the post elastic stiffness ratio α_k and strength ratio α_k , are shown in Fig. 13. Firstly, one can observe that the Park-Ang damage index changes significantly according to the post-yielding stiffness reduction in small period range. For larger periods, the effect disappears (contour lines tend to become horizontal in Fig.13).

VII. Summary and Conclusions

The effects of the post-yielding stiffness ratio on non-linear seismic response parameters have been investigated by numerical time-history analyses. The study has been developed on simple SDOF systems subject to the El Centro ground motion record. A generic BP model has been then compared with the traditional EPP model. Several examples have been carried out to demonstrate that the reduction of stiffness in post-elastic phase must be necessarily taken into account, particularly in short period range. Acceleration, ductility demand and inelastic displacement ratio have been elaborated. The following considerations can be drawn up:

1. *Acceleration response spectra* suggest that drawing the inelastic acceleration response of a BP system from the elastic one reduced by a R factor, without incorporating the post-yielding stiffness influence, can be not conservative.
2. *Ductility demand* for systems with a post-yielding stiffness ratio different from zero is generally smaller than the corresponding EPP model demand. The numerical results show that a stiffness ratio cautiously assumed equal to zero can be excessively conservative. Moreover, the adoption of the EPP model may not be conservative if the system is characterized by strong inelastic properties (i.e. low values of strength ratio) in short period range.
3. *Inelastic Displacement Ratio* spectral analysis shows that for short period the inelastic response of the BP model is smaller than the corresponding EPP response. For long period the “equal displacement” rule is no longer valid, because inelastic response differs significantly from the elastic one.

The results obtained in this study allow affirming that the most significant effects of α_k on the inelastic response take place for values of α_k ratio greater than 10% and for short period range, with a limit approximately of 0.5 s.

For larger periods, the influence of post-yielding stiffness can be neglected, so that it is possible to locate "a stiffness ratio sensitive period range" in which the assumption of a simplified EPP model excessively overestimates inelastic response. This behaviour can be interpreted in the light of energetic considerations.

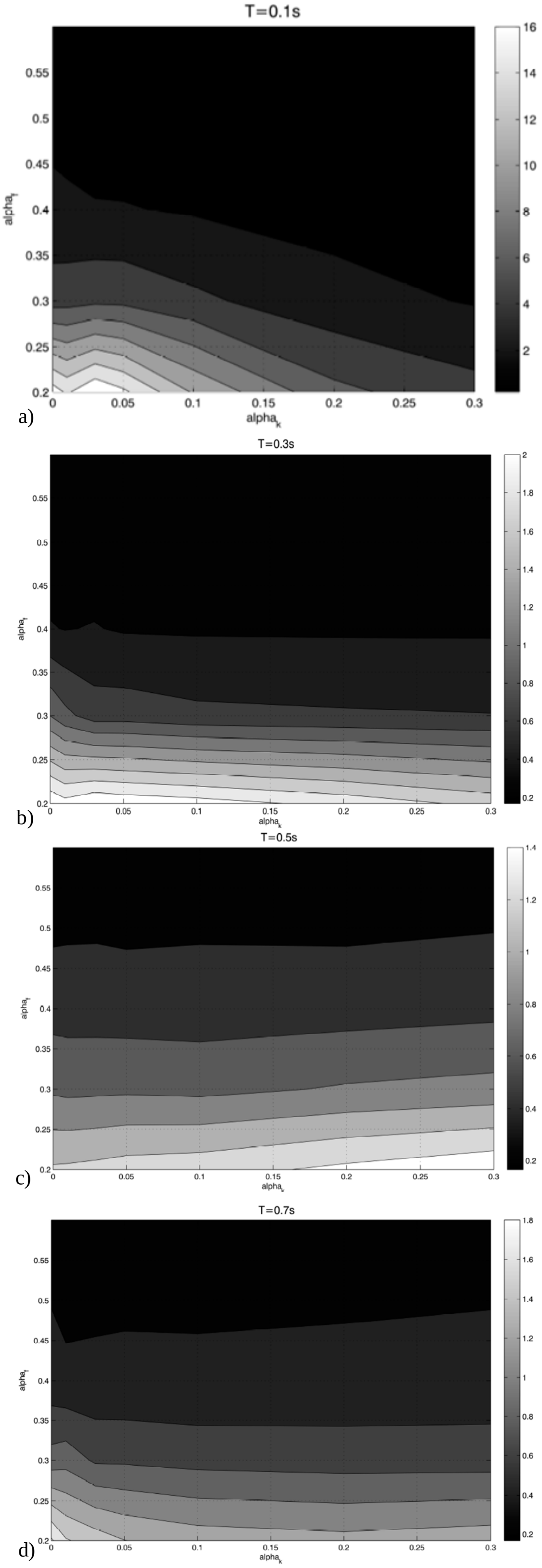

Fig. 13 . Effects of post-yielding stiffness ratio on Park – Ang Damage Index for T =0.1 (a), 0.3 (b), 0.5 (c) and 0.7 (d) s

Dissipated energy and displacement trends suggest that dual effects act on system with respect to the strength and the post-elastic stiffness. For intermediate and long vibration periods, the increase of the dissipated energy mitigates the undesirable effect of high lateral deformability due to the sudden reduction of stiffness after yielding.

According to these considerations, the level of seismic design performance can be also related to the stiffness ratio, because the reduction of plastic deformation is achieved in a particular range of stiffness and strength ratio and different structural conditions.

Finally, the Park-Ang functional damage is also sensitive to the post-yielding stiffness in short period range, confirming that global damage evaluation cannot be limited to the EPP case. This study strongly suggests to improve the currently recommended design procedures, since they do not consider the influence of the stiffness ratio in short period range.

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