

Probabilistic modelling of fatigue crack - Some observations about conditional probability

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Abstract — The article deals with time-dependent analysis of the failure probability of a load bearing element of a steel bridge. The theoretical model of fatigue crack propagation is based on linear fracture mechanics. The Latin Hypercube Sampling statistical method is used to calculate the unconditional and conditional probabilities. The conditional probability is calculated by excluding those random observations that have a detectable crack length observed during the inspection. The effect of included and excluded observations of input random quantities on the conditional failure probability is mapped and shown graphically. It is proven that the number of stress peaks per year can be considered as one random variable for any time interval of the life span of the bridge.

Keywords — Fatigue, steel, bridge, probability, stochastic, fracture mechanics, random sampling.

I. INTRODUCTION

Steel bridges in operation are subjected to repeated loads of considerable intensity and fatigue is an important consideration [1, 2]. Repeated application of live load may lead to failure of material even when the load level is lower than that for the ultimate limit states [3]. The process begins at the initiation site of initial cracks. Fatigue crack initiation often occurs at an inclusion, impurity or surface flaw, which acts as a local stress raiser and results in small scale plastic deformation.

Fatigue crack propagation is a process of the cycle-by-cycle accumulation of damage in a material undergoing fluctuating stresses and strains. All factors that affect the fatigue life of metal structures can be grouped into four categories: material, structure, loading, and environment [4]. The safety of steel bridges is ensured mainly by structural elements bearing their self-weight and traffic loads. Failure of the load carrying structure leads to a state of disrepair, degradation of the transport network and threatens traffic safety.

The state of the art review of metal fatigue has been published, for e.g. in [4-6]. Numerous methods have been proposed for the evaluation of the remaining fatigue life of load carrying steel structures and steel bridges [7-9], some of which are based on probabilistic approaches [10-14]. The state

of the art review of the analysis of the life cycle of steel bridges including reliability is presented in [15, 16].

The lifetime of steel bridges under real service conditions can be studied using methods based on linear fracture mechanics in combination with stochastic approaches [17, 18]. Stochastic methods can be divided into sensitivity [19-25], statistical [27, 28] and probability analyses [29, 30]. Structural performance can be measured in terms of reliability [31]. Common probabilistic approaches for assessing the reliability and safety of load bearing structural members are based on the calculation of the probability of failure, load effect, structural resistance, computational model, reliability function, probability of failure and reliability indicators [32-34].

Probabilistic structural design is a decision problem added to probabilistic structural analysis [31]. The current trend in these works is not the use of one, but of multiple MCDM methods in problems of ranking and selection [35-37].

The subject of this article is the research into probabilistic assessment of the fatigue life of existing steel bridges. The resistance of structural elements with cracks is studied based on the concept of linear fracture mechanics.

Fracture mechanical methods assume that the considered structural member contains a crack. The main focus is not on crack initiation but rather on the material behaviour at the crack tip once the member is loaded. The assessment of fatigue life requires that the random character of production, crack growth, applied load and subsequent failure due to cracks are duly taken into account [16]. The fatigue behaviour of steel is specific and therefore an important part of fatigue studies is information based on experimental research [38].

II. FATIGUE LIFE ASSESSMENT OF STEEL STRUCTURES

Existing bridges are assessed to provide evidence demonstrating whether it will safely function over the duration of a residual life [16]. It is based mainly on the results of assessing hazards, anticipated future load effects, material properties, geometry and on the structural state of the bridge.

A. Linear Fracture Mechanics

Linear fracture mechanics, particularly in the field of mechanical engineering, has been the subject of intensive research. It has gradually been modified and applied to the design of load bearing building structures. Linear elastic fracture mechanics analyses the propagation of an initial crack of magnitude a in dependence on the number of fatigue cycles

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N. Fatigue crack growth is generally described by Paris’s rule, which is expressed by Paris and Erdogan [39].

$$\frac{da}{dN} = C \cdot (\Delta K)^m, \quad (1)$$

where *m* and *C* are material-related parameters. Parameter *C* can be expressed as

$$\log(C) = c_1 + m \cdot c_2, \quad (2)$$

where *c*₁, *c*₂ can be considered for the steel grade S235 as *c*₁ = -11.141, *c*₂ = -0.507 [40]. The range of stress intensity factor ΔK can be determined by Broek [41].

$$\Delta K = \Delta\sigma_E \cdot \sqrt{\pi a} \cdot f(a), \quad (3)$$

where $\Delta\sigma_E$ is the quasi-constant stress range and *f(a)* is the calibration function obtained from experimental research [38] for pure bending in the form:

$$f(a) = 1.114 - 1.898a_{rel} + (1.659a_{rel})^2 + (1.04229a_{rel})^3, \quad (4)$$

where *a*_{rel} is dimensionless (relative) crack length calculated as *a*_{rel} = *a*/*W*, where *W* is the specimen width in the direction of crack propagation. The domain of equation (4) is [0.01, 0.5].

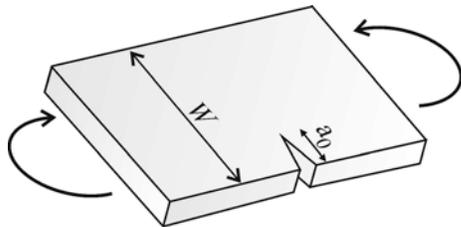


Fig. 1 Crack propagation in the structural detail with initial crack.

We can express the resistance from equation (1) as the accumulation of damage related to the growth from initial crack *a*₀ into a critical crack *a*_{cr}.

$$R_{cr} = \int_{a_0}^{a_{cr}} \frac{da}{[f(a) \cdot \sqrt{\pi \cdot a}]^m}, \quad (5)$$

As will be shown in the next paragraph, the calculation of the conditional failure probability is based on the resistance calculated for the detectable crack size *a*_d. Resistance for crack growth from initial crack *a*₀ into a detectable crack size *a*_d can be written analogously as:

$$R_d = \int_{a_0}^{a_d} \frac{da}{[f(a) \sqrt{\pi \cdot a}]^m}, \quad (6)$$

Fatigue limit state (failure) occurs when *R*_{cr} is equal to the

load effects *A*_t,

$$A_t = \Delta\sigma_E^m \cdot C \cdot N, \quad (7)$$

where *N* is the total number of cycles reached at the time the crack reaches its critical length *a*_{cr}, where *a*_{cr}/*W* ≤ 0.5. The value *A*_t (action) is a function of time, because the number of cycles *N* (cumulative effect of traffic) is a non-decreasing function in time. The reliability function can be written in a differential form based on previous equations as:

$$G = R_{cr} - A_t, \quad (8)$$

B. Probabilistic Analysis

Failure (brittle fracture) occurs if *G* < 0. The occurrence of failure is generally random because the initial crack *a*₀ and the other variables described in the preceding paragraph are random in nature. The input random variables are listed in the Table 1.

TABLE I
INPUT RANDOM VARIABLES

Characteristic	Pdf	Mean value	St. deviation
Initial crack size <i>a</i> ₀	lognormal	0.2 mm	0.06 mm
Detectable crack size <i>a</i> _d	Gauss	5 mm	0.6 mm
Parameter <i>m</i>	Gauss	3	0.03
Stress peaks range $\Delta\sigma_E$	Gauss	28 MPa	6 MPa
Specimen width <i>W</i>	Gauss	340 mm	17 mm
Stress peaks per year <i>N</i> _t	Gauss	10 ⁶	10 ⁵

Initial crack *a*₀ has a lognormal probability density function (pdf), the other random variables have Gauss pdf. The application of lognormal pdf for *a*₀ has been discussed within the logical context of probabilistic models of fatigue resistance [42]. Pdf of *a*₀ must have zero probability density close to zero lengths of *a*₀ and at the same time decrease very rapidly when observing higher values. The lognormal pdf of *a*₀ fulfils these conditions very well, but it is not the only applicable pdf [42].

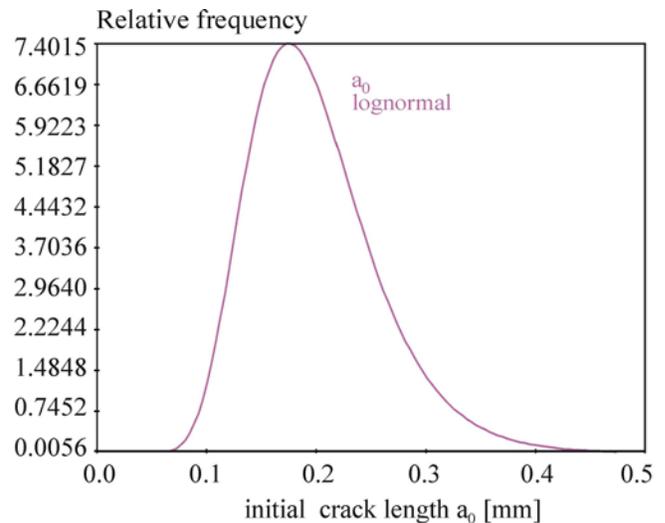


Fig. 2 Lognormal pdf of initial crack length *a*₀

The Latin Hypercube Sampling (LHS) method [43, 44] is used to analyze the probability of failure

$$P_f = P(G < 0), \tag{9}$$

The probability of failure is analyzed using 200 thousand runs of the LHS method. For such a high number of runs, the numerical precision of the LHS method approaches the Monte Carlo method (MC). It is still advantageous to use the LHS method, for e.g. due to the deterministic value of the minimum (0.0521 mm) and maximum (0.7019 mm) observations, see Fig. 3.

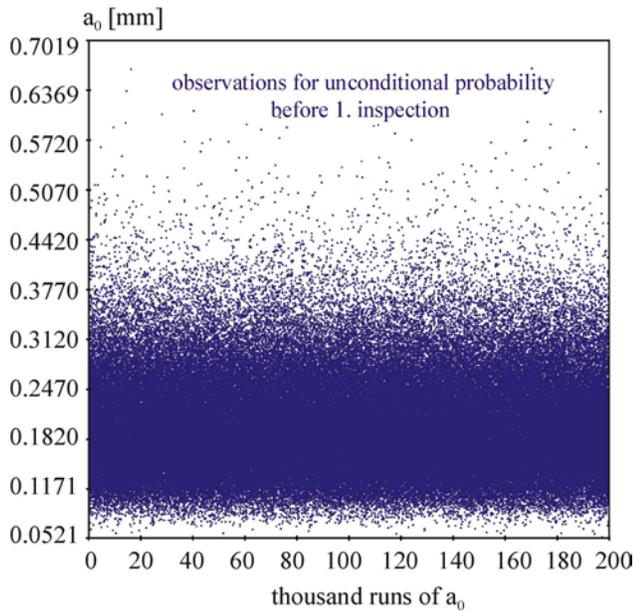


Fig. 3 LHS sampling of initial crack a_0

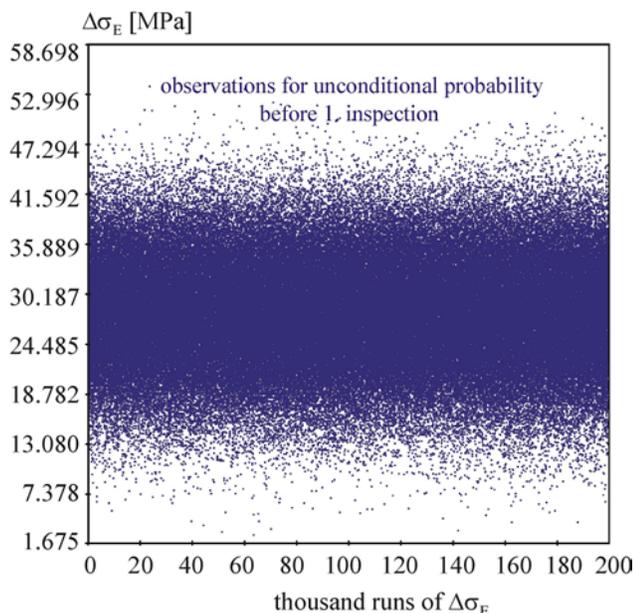


Fig. 4 LHS sampling of initial crack a_0

The minimum and maximum values of random realizations of the other random variables are also given deterministically. The minimum and maximum values of $\Delta\sigma_E$ are 1.675 MPa and 58.698 MPa, therefore, the use of Gauss pdf does not lead, for 200 thousand runs, to negative random realizations of $\Delta\sigma_E$, see Fig. 4. This is a big advantage of the LHS method in comparison to the MC method, where extreme values would have to be controlled or truncated histograms of inputs would have to be used.

Analysis of the probability of failure is performed in the time interval of zero to hundred and twenty years. The number of stress peaks per year N_i is considered for each individual year. One hundred and twenty statistically independent random variables N_i are needed in total. The number of stress peaks per y years is then calculated as the sum $N_1+N_2+\dots+N_y$.

C. Conditional probability

During the analysis of the probability of failure of bridge structures, a situation may arise where we are studying the probability of failure of a bridge in operation (phenomenon A) for which no crack was detected until time t_i (condition B). We are seeking the probability $G \leq 0$ of phenomenon A under condition B (when it is known that phenomenon B occurred), which can be written as $P(A|B)$. The conditional probability can be described using the MC method, see Fig. 5. If phenomenon B occurred, then we are only in the set B in Fig. 5, the rest of set A can be excluded.

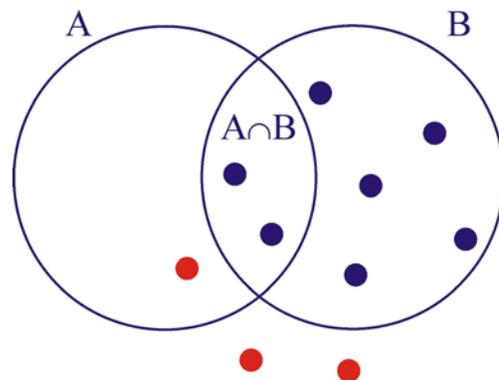


Fig. 5 The conditional probability described using the MC method

Let us consider ten random realizations of one bridge with the same traffic load. The bridges are put into operation at time $t_0=0$ without detectable cracks, but have random very small initial cracks a_0 and other random initial imperfections. All bridges are inspected at time t_1 . A crack of detectable size $a > a_d$ is found in three bridges. These observations are identified using condition (10), which considers the random effects of input variables in Table 1.

$$R_d - A_i \leq 0, \tag{10}$$

Bridges with $a > a_d$ are closed and without traffic. The probability of failure is studied by analyzing the occurrence of failure in the remaining seven bridges, for which a detectable

crack was not previously observed. If we write $P(A|B)$, we are seeking the probability that phenomenon A will occur (failure $a \geq a_{cr}$) in the set (bridges with $a < a_d$ at time t_1), see Fig. 5. When failure occurs in two bridges, then $P(A|B) = 2/7 = 0.286$.

$$P_{f1} = P(A|B), \tag{11}$$

The conditional probability is calculated using (11) with the time step of one year.

III. THE COMPUTATION OF FAILURE PROBABILITY

The required reliability was described by the target reliability index $\beta=2$, which corresponded to the designed probability of failure $P_d = 0.02277$. If P_f reaches magnitude P_d , then all observations (all bridges) from the random selection (200 thousand random realizations) are inspected. In the example described here, the first inspection ($P_f = P_d$) occurred at time $t_1 = 39$ year. Only those observations (186955 random realizations) for which a crack was not detected during previous inspection are included in the calculation of the second path of the probability of failure (first conditional failure probability P_{f1}). If P_{f1} reaches magnitude P_d , a second inspection is performed and random realizations of bridges with crack $a > a_d$ are excluded from the 186955 random realizations. Only those observations (165746 random realizations) for which a crack was not detected during previous inspection are included in the calculation of the third path of the probability of failure (second conditional probability of failure P_{f2}). The calculation procedure for P_{f1} (11) is repeated in this manner after further inspections for the other paths P_{f3}, P_{f4} , see Fig. 6. Inspections are performed when P_d is reached at 39, 54, 71, 90, 113 years of the bridge life.

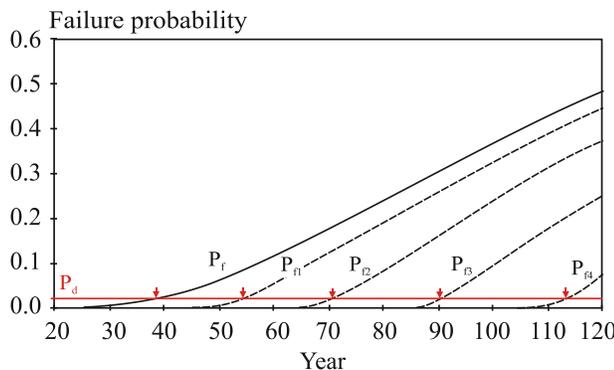


Fig. 6 Dependence of the failure probability on years of operation

Each subsequent path is calculated with fewer random realizations. The original set of 200 thousand random realizations is gradually reduced to 186955, 165746, 138525, 111372 and 85047 random realizations (set B); see blue points in Fig. 9 to Fig. 25. The elimination procedure is described in detail in the following paragraphs.

The first left path P_f represents unconditional probability of failure. In reliability condition (9) calculated on the basis of (8) R_{cr} is strongly dependent on a_0 , see Fig. 7.

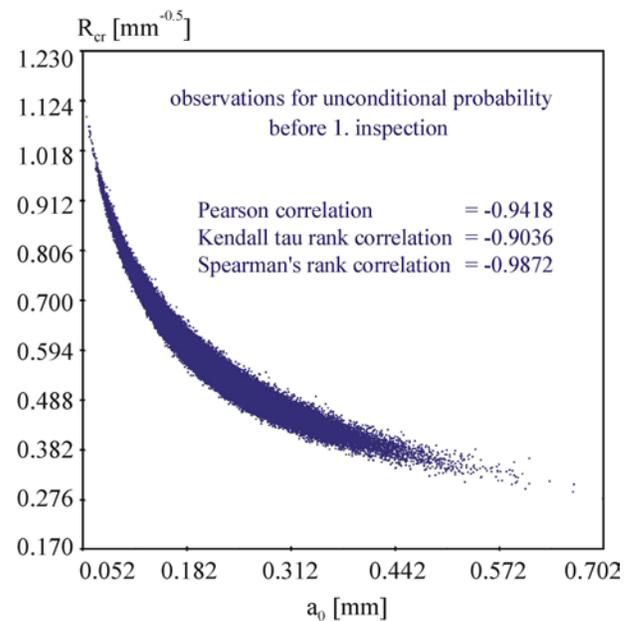


Fig. 7 Dependence between initial crack length a_0 and resistance R_{cr}

The crucial process in calculating path P_{f1} is the exclusion of the random observations, which fulfil condition (10), which is based on the calculation of R_d . Random observations R_d are shown in Fig. 8. Similarly, as shown in Fig. 7, a strong non-linear dependence of R_d on a_0 is evident, see Fig. 8. The advantage of sampling based methods (Monte Carlo, LHS, and Importance Sampling) in comparison to other probability methods [45, 46] is the simple, transparent and natural inclusion of correlation between samples.

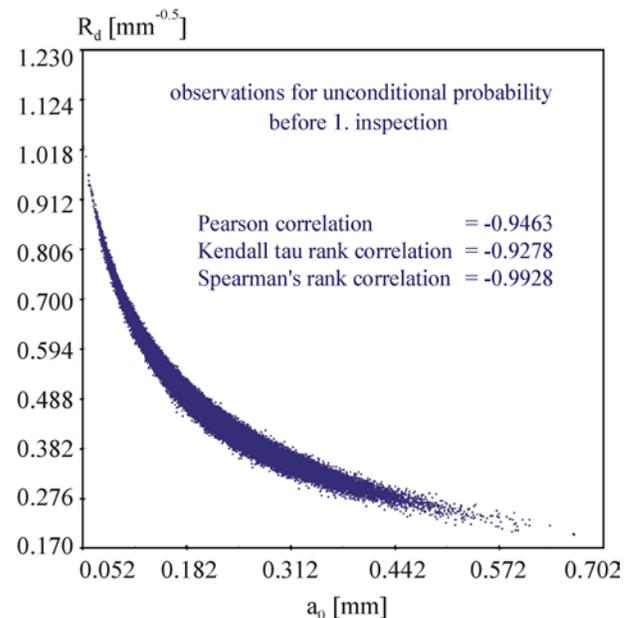


Fig. 8 Dependence between initial crack length a_0 and R_d

The statistical dependencies shown in Fig. 7 and Fig. 8 are strongly non-linear; therefore Spearman's rank correlation coefficient is a more suitable indicator of correlation

dependence than, for e.g. Pearson correlation. Pearson's correlation assesses linear relationships, Spearman's correlation assesses monotonic relationships (whether linear or not). Both Spearman's and Kendall's tau can be formulated as special cases of a more general correlation coefficient.

It can be noted that if Spearman's rank correlation $\text{corr}(a_0, R_d) = -0.99$ and $\text{corr}(a_0, R_{cr}) = -0.99$ then $\text{corr}(R_d, R_{cr}) = 0.99$. If we artificially decrease the correlation between R_d and R_{cr} there is a significant reduction and shortening of the observation interval in the probabilistic analysis.

Fig. 9 to Fig. 12 show random sample distributions of observations without detectable cracks during the first inspection (blue dots) and observations with detectable cracks during the first inspection (red points). Probabilistic analysis and calculation of P_{fi} is performed with the first (left) part of the spectrum (blue points), while the rest of the selection (red points) is excluded.

A shift of the blue and red spectrum is visible from Fig. 9 to Fig. 12. The mean values of excluded (red) observations in Fig. 9, Fig. 10 are higher than the included (blue) observations used for the calculation of P_{fi} . Conversely, the mean values of the red observations in Fig. 11, Fig. 12 are lower than the blue observations. The subsequent calculation of P_{fi} is therefore based on different statistical characteristics of input random variables than P_f . The greatest effect is observed in $\Delta\sigma_E$, where the excluded observations have significantly higher mean values than the included blue observations, whose mean values decrease, see comparison of Fig. 10 and Fig. 4.

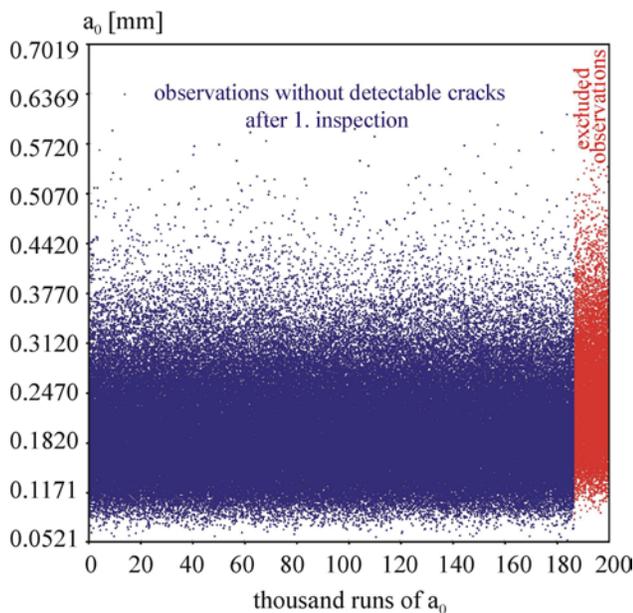


Fig. 9 Selection of LHS samples of a_0 after first inspection

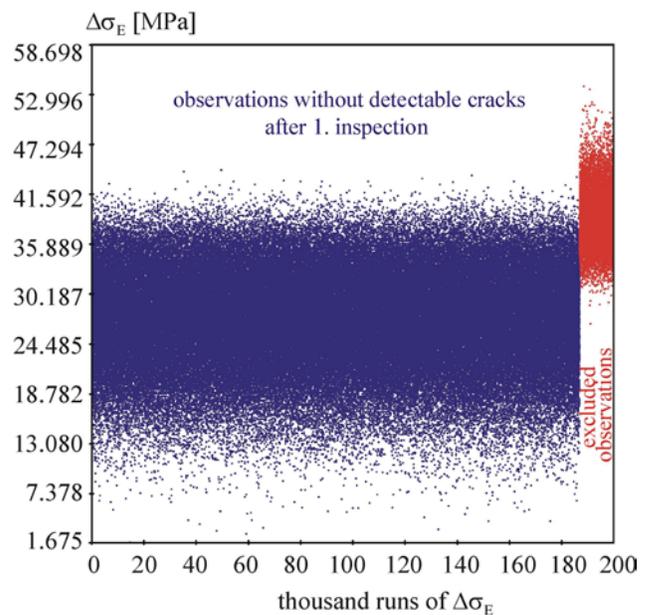


Fig. 10 Selection of LHS samples of $\Delta\sigma_E$ after first inspection

The random variability of $\Delta\sigma_E$ strongly influences the results of the probabilistic analysis in Fig. 6. Introduction of $\Delta\sigma_E$ as a non-random (constant) variable leads to constant inspection intervals. The random variability of $\Delta\sigma_E$ practically takes into account the uncertainty in the weight of vehicle axles. The calculation of the conditional probability P_{fi} is performed with the exclusion of random observations with relatively high values of random realizations $\Delta\sigma_E$. Detectable cracks were observed mainly in heavily loaded bridges, which are excluded from operation or repaired after the first inspection.

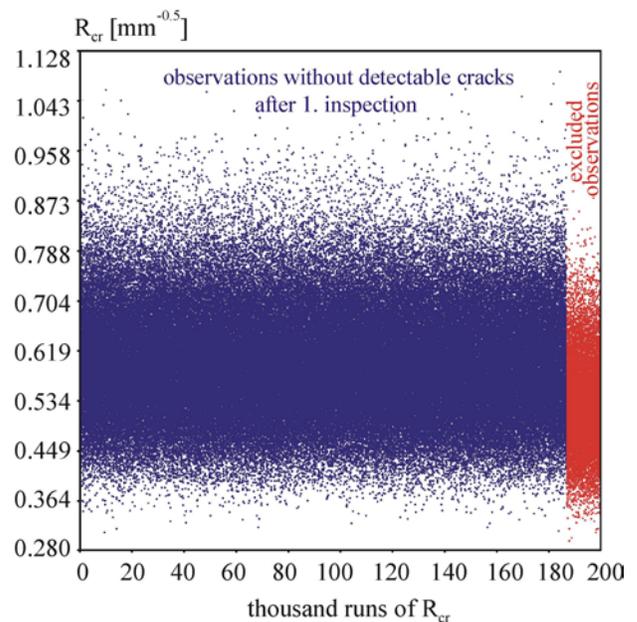


Fig. 11 Selection of LHS samples of R_{cr} after first inspection

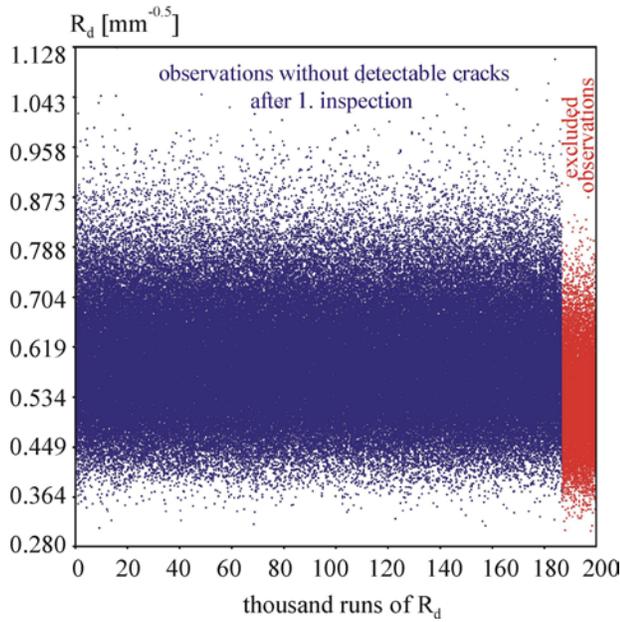


Fig. 12 Selection of LHS samples of R_d after first inspection

Similar conclusions are apparent from Fig. 13 to Fig. 16 as were made on the basis of the analysis in Fig. 9 to Fig. 12, but the set of excluded observations is larger. Because of the exclusion of a higher number of points, the calculation of P_{f2} is less precise than the calculation of P_{f1} and the calculation of P_{f1} is less precise than the calculation of P_f . However, this is insignificant from a practical point of view, which was verified using four million simulation runs of LHS, where the plotted results of the probability of failure are identical with the results in Fig. 6.

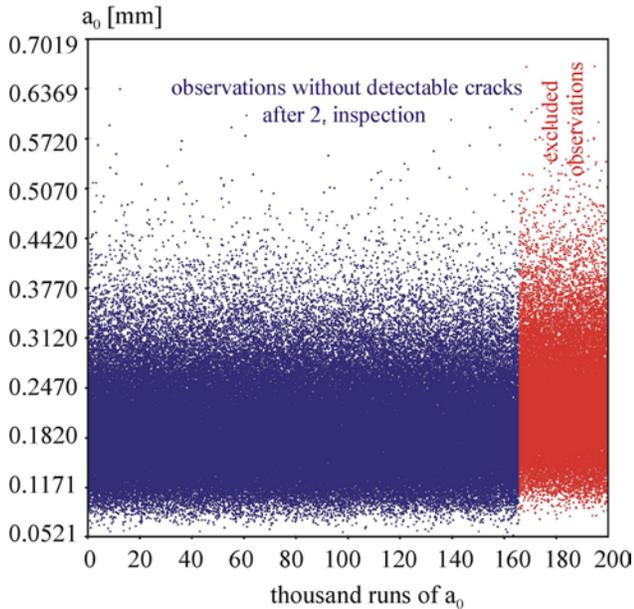


Fig. 13 Selection of LHS samples of a_0 after second inspection

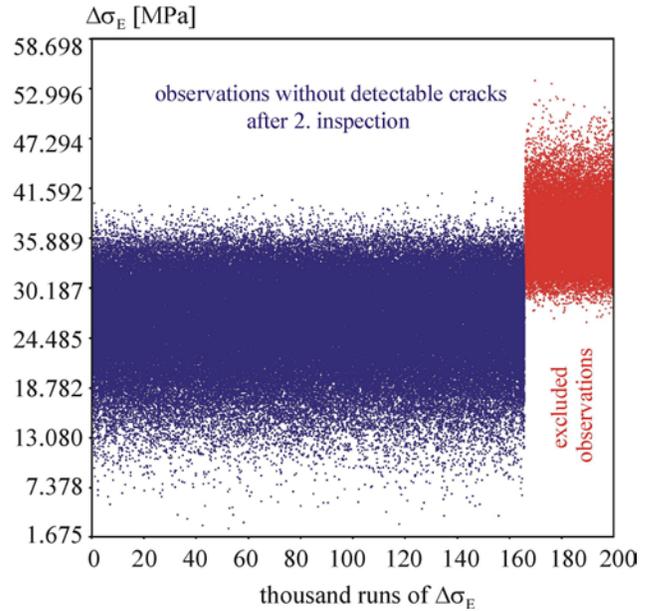


Fig. 14 Selection of LHS samples of $\Delta\sigma_E$ after second inspection

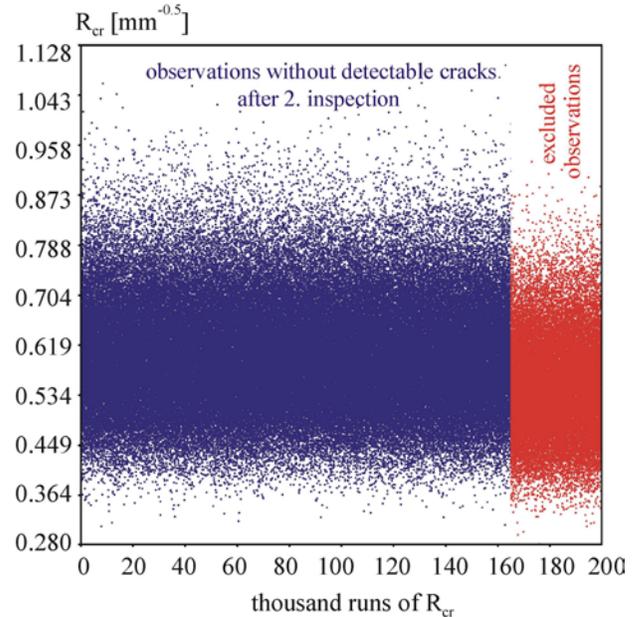


Fig. 15 Selection of LHS samples of R_{cr} after second inspection

It is evident from Fig. 17 to Fig. 20 that the red region of the spectrum is expanding and that the differences between the blue and red spectrum widen. Comparison of Fig. 17 with Fig. 13, Fig. 9 and Fig. 3 shows a decrease in the mean value and a slight decrease in the dispersion of the red spectrum. The mean value of the red spectrum in Fig. 18 is lower than in Fig. 14 and the mean value of the red spectrum in Fig. 14 is lower than in Fig. 10. On the contrary, the mean values of the observation of variables R_{cr} and R_d shown in red slightly increase, see R_{cr} in Fig. 19 vs Fig. 15 and/or Fig. 11; and R_d in Fig. 20 vs Fig. 16 and/or Fig. 12. This trend of increasing the differences between the excluded and included observations continues even after the third inspection; see Fig. 21 to Fig. 24.

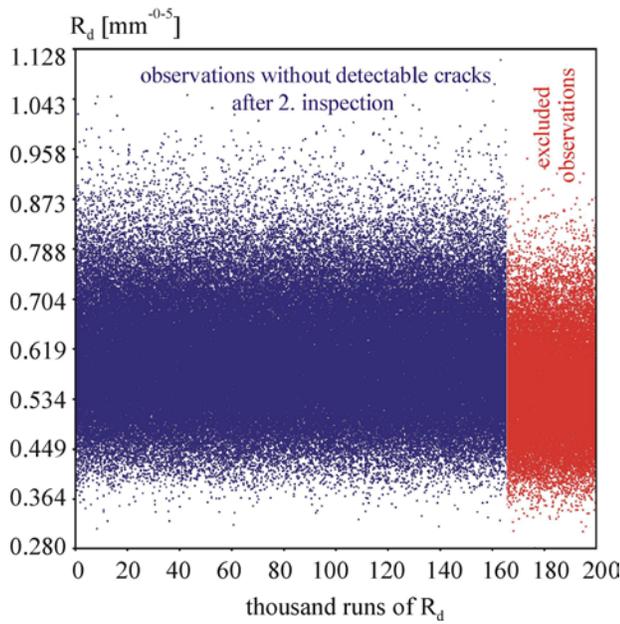


Fig. 16 Selection of LHS samples of R_d after second inspection

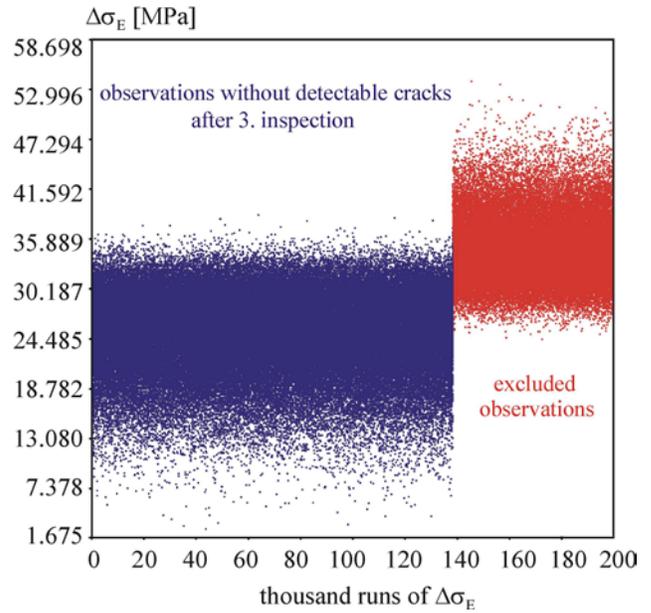


Fig. 18 Selection of LHS samples of $\Delta\sigma_E$ after third inspection

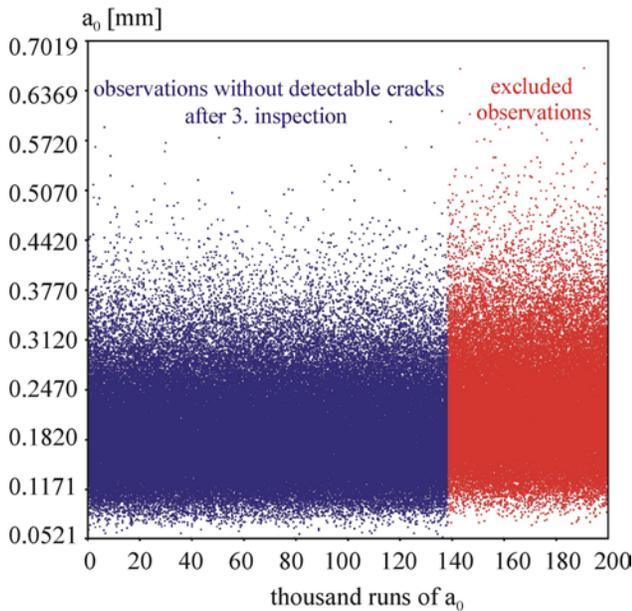


Fig. 17 Selection of LHS samples of a_0 after third inspection

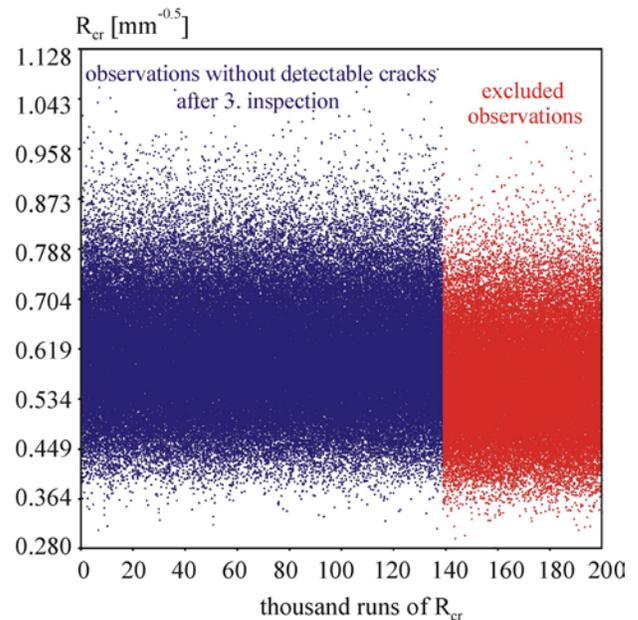


Fig. 19 Selection of LHS samples of R_{cr} after third inspection

Analysis of the differences between the excluded and included sum of observations $N_1+N_2+\dots+N_y$ showed that both sets of data are statistically comparable, i.e. they have the same statistical moments, see Fig. 25. This means that the sum $N_1+N_2+\dots+N_y$ can be substituted with one random variable N , whose mean value and standard deviation would increase with increasing number of years under the rules for the sum of statistically independent random variables with Gauss pdf. For example the mean value for twenty years of operation of the bridge would be calculated as $20 \cdot 10^6$ and standard deviation as $\sqrt{20} \cdot 10^5 = 4.472 \cdot 10^5$. The results of the probabilistic analysis shown in Fig. 6 evaluated with one random variable N or with 120 random variables N_i are identical.

IV. CONCLUSION

The assessment of the fatigue life based on the unconditional and conditional probabilities of failure is presented in the article. The described methodology makes it possible to determine the probability of failure of a bridge for which a detectable crack was not observed during inspection. Otherwise, the bridge must be closed and cracks repaired. In the analysed example, it has been shown that inspections are performed at 39, 54, 71, 90, 113 years of bridge operation. Extension of the inspection intervals occurs due to non-zero standard deviation of the stress peaks range $\Delta\sigma_E$.

Introduction of $\Delta\sigma_E$ as a non-random variable leads to

constant inspection intervals. From a technical point of view, in the case of the present example, a constant inspection interval of 15 years with the first inspection after 40 years of operation of the bridge can be conservatively recommended in the case studied.

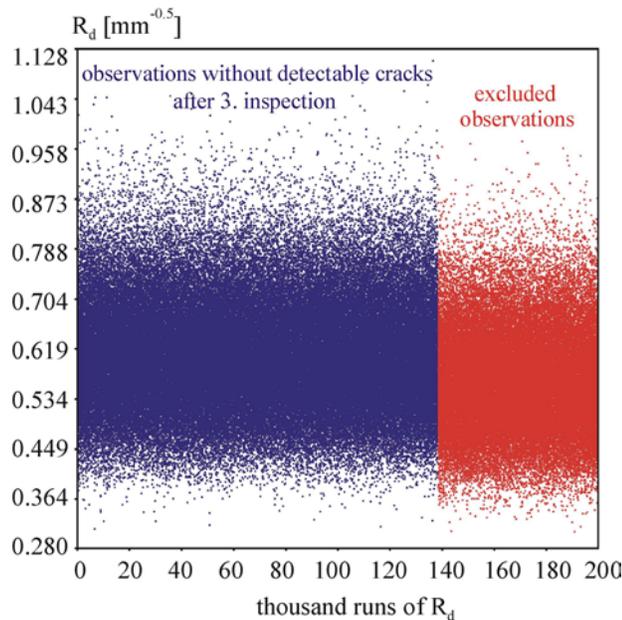


Fig. 20 Selection of LHS samples of R_d after third inspection

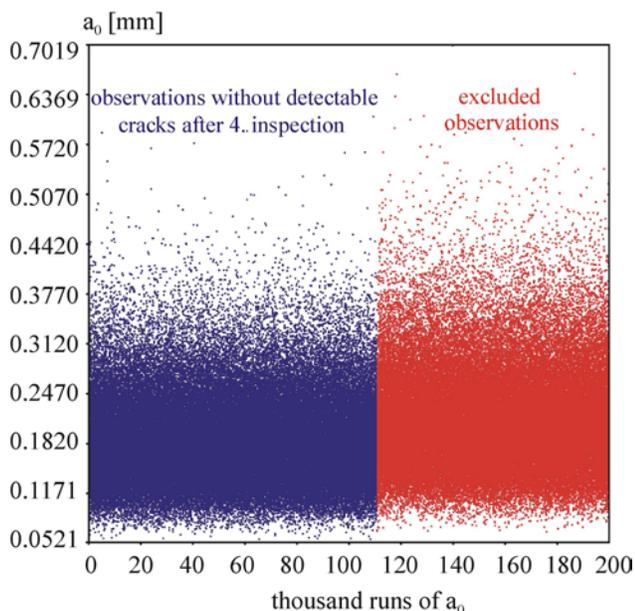


Fig. 21 Selection of LHS samples of a_0 after fourth inspection

In the present example the number of stress peaks per year was considered as one random variable N_i for each year of operation, therefore, 120 statistically independent random variables N_i were considered for 120 years of operation. This is demanding for computer RAM and prolongs computation.

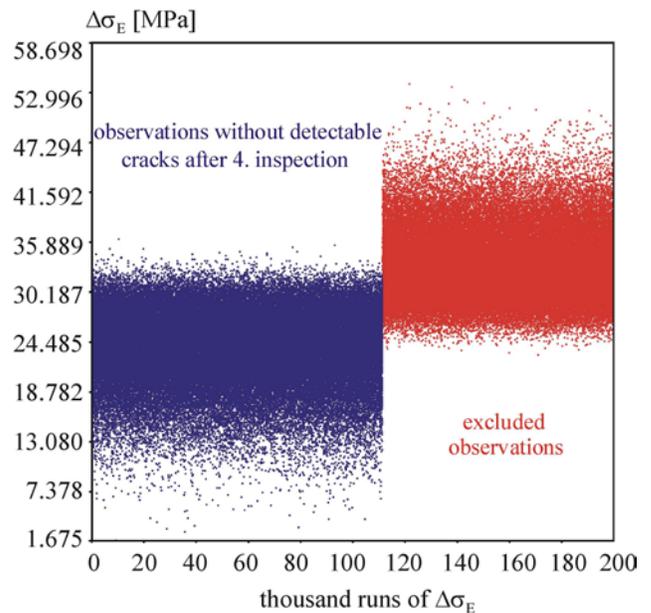


Fig. 22 Selection of LHS samples of $\Delta\sigma_E$ after fourth inspection

The statistical analysis of the excluded and included observation sums $N_1+N_2+\dots+N_y$, performed after each inspection has shown that the sum $N_1+N_2+\dots+N_y$ can be effectively substituted with one random variable with zero mean value and standard deviation of 10^5 , where each random LHS realization is multiplied by the square root of the number of years of operation and added to the number of years of operation multiplied by 10^6 . This transformation permits highly efficient consideration of the number of stress peaks per year using one random variable; it is especially useful for calculating global sensitivity indices subordinated to contrast functions [47] or structural reliability sensitivity indices [48].

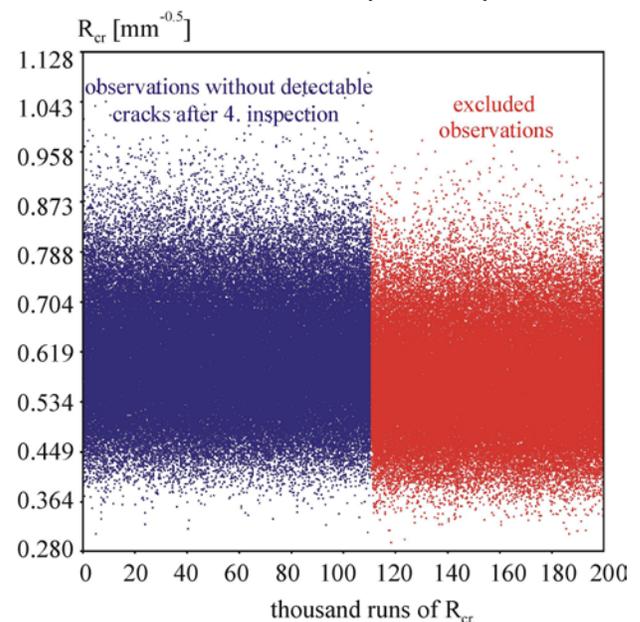


Fig. 23 Selection of LHS samples of R_{cr} after fourth inspection

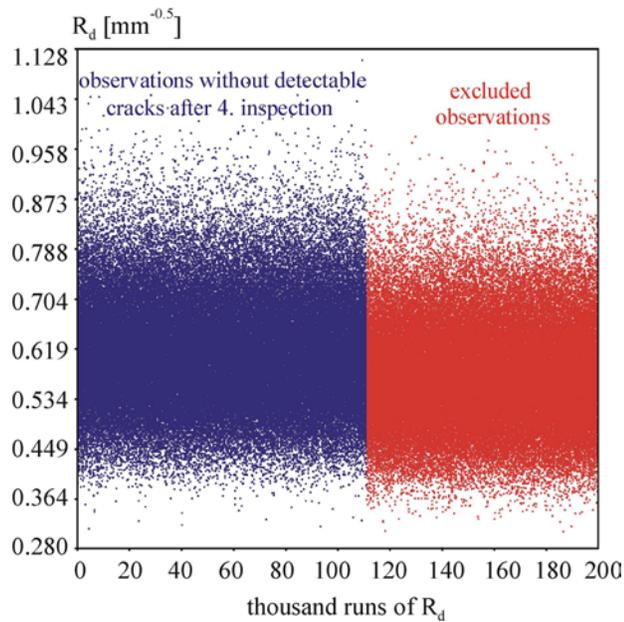


Fig. 24 Selection of LHS samples of R_d after forth inspection

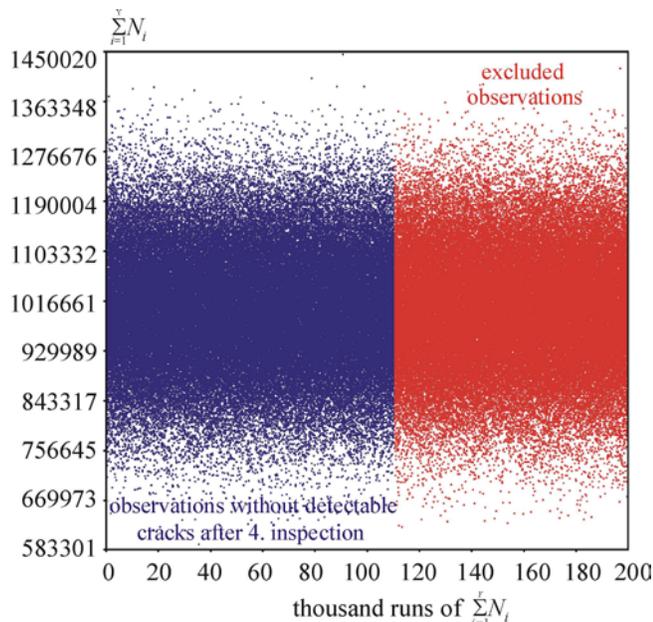


Fig. 25 Selection of LHS samples of sum of N_i after forth inspection

A nearly one hundred percent correlation between the initial crack length a_0 and fatigue resistance R was shown. The random dependence between a_0 and R is nonlinear; therefore three types of correlation dependencies were evaluated: Pearson correlation, Spearman's rank correlation coefficient and Kendall's tau. Spearman's rank correlation coefficient is particularly suitable for measuring the correlation between a_0 and R . Spearman correlation value of -0.99 means that variables a_0 and R are an almost perfect monotone function of each other. This contrasts with Pearson correlation, which has not provide a perfect value because a_0 and R are not connected by a linear function. The fatigue resistance increases very rapidly with decreasing initial crack size, see Fig. 7 and Fig. 8.

It is evident that considering resistances R_{cr} and R_d with separate histograms with no relation to a_0 can lead to significant errors in the probabilistic analysis.

The research results obtained in this article can be used in statistical and probability analyses of fatigue phenomena with conditional probabilities so that inspection intervals are planned as optimally as possible with the lowest cost and maximum lifetime of the bridge.

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