On Simple Algebraic Control Design and Possible Controller Tuning for Linear Systems with Delays

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***Abstract*—**This paper is aimed at possible controller tuning of infinite-dimensional controllers of a predictor (compensator) type obtained from an algebraic-based controller structure design method for linear systems with delays. The design procedure is simple enough so that it can attract practitioners. The controllers are of a generalized proportional-integral(-derivative) (PI(D)) type after a trivial limit approximation, and those obtained for stable controlled processes can be compared to the well-known Smith predictor scheme. Some well-established tuning rules for three study cases are then used and compared; namely, the Chien-Hrones-Reswick and the equalization methods are applied for first and second order plants with input and state delays, and the quasi-continuous shifting procedure with the spectral abscissa minimization versus the triple-dominant-root setting are used to the unstable first-order case. These tuning rules are directly applied to PI(D) laws in the simple feedback structure as well, which is compared to the results for the compensation controllers. The robustness of designed controllers is simply benchmarked via some selected perturbations in the static gain and both delays. Simulation outputs and performance measures are given to the reader to display and quantify the obtained results for clearer comparison.

***Keywords*—**Systems with time-delays, PID control, controller tuning, dead time compensator, dominant pole, spectral abscissa, simulation.

I. Introduction

SYSTEMS with input and/or state time delays (Time Delay Systems, TDSs) have paid considerable and well-deserved attention by scientists and engineers during recent decades [1], [2]. The reason is mainly twofold. First, delays are integral part of a multitude of real-world systems and processes [3]. Second, they can effectively approximate the inertia of a higher order [4]-[6].

Proportional-integral-derivative (PID) controllers have been sufficiently applied in practice for more than seven decades and they have remained to be widely used in industry due to their simplicity, satisfactory control effect, robustness and reliability [7], [8]. However, it is well-known that the use of conventional control laws for TDSs with significant delay values (with respect to time constants) can rapidly deteriorate the feedback control performance and may lead to a poor control response [9], [10]. This is mainly caused by the fact that the feedback control system is no longer finite-dimensional. It is generally inappropriate to use the conventional control methods in which the delays are not considered in the design.

In order to tackle these issues and enhance the overall performance, it is hence necessary to use some advanced structural or tuning techniques, or some approximation techniques. To name just a few, the extended Hermite-Biehler theorem, which expresses the relation between real and imaginary parts of the stable characteristic quasipolynomial and their real root that must satisfy a certain interlacing property, was used to determinate stability regions in the space of PI and PID gains in [11], [12], and [13], respectively, or in a combination with the Padé approximation [14]. Rational approximations were applied within PI/PID controllers' algebraic design by using rings of polynomials and stable-proper rational functions in [15] and [16], respectively. Castaños et al. [17] performed the known D-subdivision method to declare the exact spectral abscissae in the space of PI parameters for first-order linear passive systems with the closed-loop feedback of neutral type. Optimal and optimization techniques suggest themselves: the minimization of the Integral Absolute Error (IAE) compared to dominant pole placement was performed in [18]; Srivastava et al. [19] combined the concept of linear quadratic regulator based PI/PID tuning method together with the dominant pole placement approach to derive the PID parameters analytically for second order plus time delay systems; several heuristic algorithms for tuning of PID controllers for First Order Plus Time Delay (FOPTD) systems were compared in [20].

Another possibility is compensate delays to further enhance control performances. The well-known Smith predictor [21], [22], see Fig. 1, is the most popular deadtime compensation scheme. Its compensation nature can be seen from

$$C_S(s)=\frac{C(s)}{1+C(s)\tilde{G}_0(s)\left(\mathrm{e}^{-\tilde{\tau}s}-1\right)}, \quad (1)$$

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where $\tilde{G}_0(s)\mathrm{e}^{-\tilde{\tau}s}$ represents the transfer function of the controlled system's model, $C_S(s)$ stands for the feedback controller in the predictor structure and $C(s)$ is the equivalent controller in the classical simple feedback loop (see Fig. 2). Notice that *r*, *e*, *u*, and *y* stand for the reference, control error, manipulate input and controlled output signals, respectively, in the figures. However, the Smith predictor is susceptible to stability problems in the face of model imperfections.

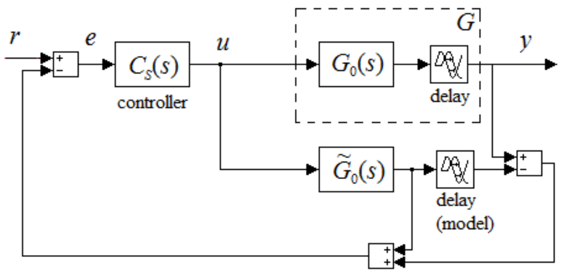


Fig. 1 Smith predictor (delay compensator) scheme

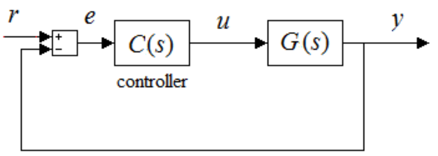


Fig. 2 Simple negative feedback loop scheme

The finite spectrum assignment (FSA) controller [23], unlike the PID or the Smith predictor, ideally allows to assign of the closed-loop poles arbitrarily. The basic idea of the FSA is that the state variables are predicted over the delay period by using a control law that contains a distributed delay term. For instance, for systems with input delay $\tilde{\tau}$, the predicted state $\hat{\mathbf{x}} \in \mathbb{R}^{n\times 1}$ reads

$$\hat{\mathbf{x}}(t+\tilde{\tau}) = \mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\mathbf{x}(t) + \int_{-\tilde{\tau}}^{0} \mathrm{e}^{-\tilde{\mathbf{A}}\theta}\tilde{\mathbf{B}}u(t+\theta)\,\mathrm{d}\theta\,, \tag{2}$$

where $\tilde{\mathbf{A}}$, $\tilde{\mathbf{B}}$ stand for modeled (estimated) state matrices, which is then used to set the feedback controller e.g. as $u(t) = -\mathbf{K}^{\mathrm{T}}\hat{\mathbf{x}}(t+\tilde{\tau})$ where $\mathbf{K} \in \mathbb{R}^{n\times 1}$ is the controller gain matrix, and the superscript "T" means the matrix transpose. The corresponding controller equation in the Laplace transform (with zero initial conditions) reads

$$\left(1 - \mathbf{K}^{\mathrm{T}}\left(s\mathbf{I} - \tilde{\mathbf{A}}\right)^{-1}\left(\mathrm{e}^{-\left(s\mathbf{I}-\tilde{\mathbf{A}}\right)\tilde{\tau}} - \mathbf{I}\right)\tilde{\mathbf{B}}\right)U(s) = -\mathbf{K}^{\mathrm{T}}\mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\mathbf{X}(s)\,, \tag{3}$$

which indicates the compensation behavior and a link to (1) via the difference term on the left-hand side of (3).

This contribution intends to provide the reader with a simple algebraic-based controller structure design method that yields a compensation-type infinite-dimensional controller for systems with input and/or state delays. The work is motivated by the intention to acquire mainly engineers and practitioners with an uncomplicated methodology of control design for delayed systems and to suggest some possible controller tuning ideas. Once the general control law is found, we perform several well-established tuning rules and methods to set adjustable parameters based on the dynamic properties of the controlled system and the controller. Namely, three study cases are presented for controlled processes with input and state delays: the first-order (of inertia) stable and unstable case, and the second-order stable case. Since the designed controllers for the first case have (after a trivial approximation) the PI structure, the habitual Chien-Hrones-Reswick (CHR) method [24] and the Equalization Method (EM) [25] (also called as the balanced one [26]) are utilized for the simplicity. It is i.a. shown by example that the derived controllers are very close to both the tuning rules, in some sense. In the second-order case, both the methods are applied again, and it is found here that the use of the corresponding purely PID controller for the CHR method leads to an unstable closed-loop feedback. Since the designed infinite-dimensional controller for the unstable case is no longer of a PI(D) type, we decided to tune it by the application of the Quasi-Continuous Shifting Algorithm (QCSA) [27] to feedback poles loci of an infinite spectrum. Here, a comparison of the triple dominant root setting and the spectral abscissa minimization (which is very close to the double dominant root setting) is made. Finally, the robustness of controller design and tuning is benchmarked via some perturbations in the static gain and delays of the controlled plant. Several performance and control-quality measures quantify these results.

Notation: $\mathbb{C}$, $\mathbb{R}$, denote the set of complex numbers and real numbers, , respectively. $\mathbb{R}^n$ expresses the *n*-dimensional Euclidean space, $\mathbb{C}^{n\times m}$ is the set of all complex-valued matrices of the dimension $n\times m$. The set of real polynomials is denoted as $\mathbb{R}[s]$, and the set of quasipolynomials as $r_Q[s]$, i.e. $q(s) \in r_Q[s]$ if $q(s) = s^n + \sum_{i=0}^{n}\sum_{j=0}^{k_i} q_{ij}s^i\mathrm{e}^{-\tau_{ij}s}$ where $q_{ij} \in \mathbb{R}$, $\tau_{i0} = 0$, else $\tau_{ij} \geq 0$. $\mathbb{C}_0^- := \{s \in \mathbb{C} \mid \mathrm{Re}(s) < 0\}$ where $\mathrm{Re}(s)$ means the real part of $s \in \mathbb{C}$. Let $F(s): s \in \mathbb{C} \mapsto F \in \mathbb{C}$, then $H_\infty := \left\{F(s): \|F(s)\|_\infty < \infty\right\}$ where $\|F(s)\|_\infty = \sup_{s\in\mathbb{C}^+}|F(s)|$.

II. Preliminaries

A. Controller Structure Design

Consider the simple feedback loop (as in Fig. 2) with the linear delayed controlled plant governed by the transfer function $G(s) = B(s)/A(s)$ where $A(s)$, $B(s)$ are coprime elements of the ring R_{QM} defined as follows [28], [29]:

Definition 1. (R_{QM} ring). $T(s)=t_n(s)/t_d(s)\in R_{QM}$ where $t_n(s)=t_{n0}(s)\mathrm{e}^{-\tau s}$, $t_{n0}(s),t_d(s)\in r_Q[s]$, $\tau\geq 0$; $T(s)\in H_\infty$, and it is formally stable (i.e., all infinite chains of vertical poles are located in $\mathbb{C}_0^-$).

Proposition 1. (R_{QM} divisibility). Any $T_1(s)=t_{1n}(s)/t_{1d}(s)\in R_{QM}$ divides $T_2(s)=t_{2n}(s)/t_{2d}(s)\in R_{QM}$, if and only if all finite zeros $z_i\in\mathbb{C}_0^+$ of $T_1(s)$ are those of $T_2(s)$, the relative order of $T_1(s)$ is less or equal to the relative order of $T_2(s)$, and all formally unstable factors of the numerator of $T_1(s)$ are those of $T_2(s)$.

Let the reference signal and the load disturbance be $r(t)$, $d(t)$, respectively. The control system is stable if and only if

$$A(s)P(s)+B(s)Q(s)=1, \tag{4}$$

where $Q(s),P(s)\in R_{QM}$ are coprime numerator and denominator, respectively, of the controller $C(s)$. Once a particular solution $\{Q_0(s),P_0(s)\}$ of (4) is found, $r(t)$ is asymptotically tracked and $d(t)$ is attenuated if

$$\begin{aligned}&\frac{A(s)P(s)}{F_R(s)}\in R_{QM}\\&\frac{B(s)P(s)}{F_D(s)}\in R_{QM}\end{aligned}, \tag{5}$$

where $F_R(s),F_D(s)\in R_{QM}$ are factorized denominators of $r(s),d(s)\in r_Q[s]$, respectively. Conditions (5) are ensured via the parameterization

$$Q(s)=Q_0(s)\pm T(s)A(s), P(s)=P_0(s)\mp T(s)B(s) \tag{6}$$

where $T(s)\in R_{QM}$ is arbitrary, see [29] for further details and references.

B. PID Tuning Rules for CHR and EM

Let the plant model be of the FOPTD form

$$G(s)=\frac{K_P\,\mathrm{e}^{-\tilde{\tau}s}}{Ts+1}. \tag{7}$$

Consider the known PID controller as

$$C(s)=K_C\left(1+\frac{1}{T_I s}+\frac{T_D s}{T_F s+1}\right), \tag{8}$$

where T_I, T_D, T_F mean, respectively, the integrative, derivative, and filter time constant with $T_F\gg T_D$, and K_C represents the controller gain. Obviously, if $T_D=0$, the PI controller is obtained.

The PI and PID tuning rules according to the well-known CHR method for FOPTD model read, respectively,

$$K_C=\frac{7}{20}\frac{T}{K_P\tilde{\tau}},T_I=\frac{6}{5}T, \tag{9}$$

$$K_C=\frac{3}{5}\frac{T}{K_P\tilde{\tau}},T_I=T,T_D=0.5\tilde{\tau}. \tag{10}$$

The EM (also called as the balanced tuning) aims to minimize the total variation of the control action, $u(t)$, in order to provide the control response, $y(t)$, as close to the plant response as possible by means of constrains of weighted moments of the controller output. The PI tuning rule for the controlled plant model (7) is the following

$$K_C=\frac{1+(1-\theta)^2}{2K_P},\ T_I=T_{ar}\frac{1+(1-\theta)^2}{2}, \tag{11}$$

in which $\theta=\tilde{\tau}/T_{ar}$, where T_{ar} expresses the average residence time; it holds that $T_{ar}=T+\tilde{\tau}$ for (7) [25].

The PID rule for a general linear system with input delay τ is more complex:

$$\begin{aligned}&K_C=\frac{1}{K_P}\left(\frac{1+\sqrt{1+2\theta^2}-2\theta}{1+\sqrt{1+2\theta^2}}\right),\\&T_I=0.5T_{ar}\left(1+\sqrt{1+2\theta^2}-2\theta\right),\\&T_D=\frac{(T_I-T_{aa}-T_{ca})T_{ar}+(T_I+T_{cr})T_{cr}}{T_I},\end{aligned} \tag{12}$$

where

$$\begin{aligned}&T_{ar}=\left[-X(s)\right]_{s=0},X(s)=\frac{G(s)-K_P}{K_P s},\\&T_{aa}=\frac{\left[X'(s)\right]_{s=0}}{T_{ar}},X'(s)=\mathrm{d}X(s)/\mathrm{d}s,\\&T_{cr}=\tau\left(1-K_PK_C\left(1+\frac{\tau}{2T_I}\right)\right),\\&T_{ca}=\frac{0.5\tau^2}{T_{ar}}\left(1-K_PK_C\left(1+\frac{2\tau}{3T_I}\right)\right).\end{aligned} \tag{13}$$

Note that the EM for a FOPTD model tries to keep the Integral Time Absolute Error (ITAE) and the Integral Time Absolute Derivative (ITAD) almost equal, i.e.

$$\int_0^\infty t\left|e(t)\right|\mathrm{d}t \approx T_I\int_0^\infty t\left|\dot{e}(t)\right|\mathrm{d}t\,. \qquad (14)$$

C. PID Tuning Rules for CHR and EM

Let the characteristic quasipolynomial $\Delta(s,\mathbf{p})$ include the number n of adjustable parameters $\mathbf{p}=[p_1, p_2,...p_n]^T \in \mathbb{R}^n$. The goal of the QCSA is to iteratively shift a selected subset s_i, of the (dominant) spectrum to updated desired loci σ_i via the sensitivity matrix $\mathbf{S}\in \mathbb{C}^{n_{sp}\times n}$ as

$$\Delta\mathbf{p}=\mathbf{S}^{+}\Delta\boldsymbol{\sigma}\,, \qquad (15)$$

where the ith element of $\Delta\boldsymbol{\sigma}$ is $\sigma_i = s_i - \delta_i$, for some sufficiently small $\delta_i > 0$, and the (i,j)th entry of $\mathbf{S}$ means $\partial s_i / \delta p_j$. Usually, one wants to reach the minimum spectral abscissa or to get the apriori described poles loci. Note that the double pair of dominant poles is a typical result of the IAE optimization of delayed PID control loops [18], and the triple real pole dominant setting results in nearly optimal solution in the sense of the optimal nominal tuning for precisely known systems and the optimal robust PI tuning for systems with uncertainties [30].

III. Study Cases

A. First Order Stable Plant

Let the controlled process be governed by

$$G(s)=\frac{b\,\mathrm{e}^{-\tau s}}{s+a\,\mathrm{e}^{-\vartheta s}}=\frac{\dfrac{b\,\mathrm{e}^{-\tau s}}{m(s)}}{\dfrac{s+a\,\mathrm{e}^{-\vartheta s}}{m(s)}}=\frac{B(s)}{A(s)}\quad , \qquad (16)$$

$$A(s),B(s)\in R_{QM};\ \tau\geq 0, 0<a\vartheta<0.5\pi,$$

where the condition for ϑ ensures the system stability, and $r(t)$, $d(t)$ be step-wise functions, i.e. $F_R(s)=F_D(s)=s/m_f(s)$. Then, $m(s),m_f(s)\in r_Q[s]$ can be an arbitrary stable (quasi)polynomials of degree one; for the simplicity, say $m(s)=m_f(s)=s+\lambda, \lambda>0$. A stabilizing particular solution of (4) is e.g.,

$$Q_0=1, P_0(s)=\frac{s+\lambda-b\,\mathrm{e}^{-\tau s}}{s+a\,\mathrm{e}^{-\vartheta s}}\,, \qquad (17)$$

If $T(s)$ - according to (6) - is chosen as

$$T(s)=T_0(s)\frac{s+\lambda}{s+a\,\mathrm{e}^{-\vartheta s}}, T_0(s)=\left(\frac{\lambda}{b}-1\right), \qquad (18)$$

one obtains the controller

$$C(s)=\frac{\lambda}{b}\frac{s+a\,\mathrm{e}^{-\vartheta s}}{s+\lambda\left(1-\mathrm{e}^{-\tau s}\right)} \qquad (19)$$

satisfying conditions (5) in a simple form.

It can be easily verified that

$$G_{RY}(s)=\frac{Y(s)}{R(s)}=\frac{\lambda\exp(-\tau s)}{s+\lambda}$$

where $R(s)=r(s)/m(s)$. Controller (19) is of a compensation (predictor) type, see (1), (3) for the comparison. Note that the use of (1) in the nominal case yields $C_S(s)=\left[C(s)\right]_{\tau=0}$, which is a finite-dimensional yet delayed PI structure.

To apply the EM and CHR tuning principles, let us use the following trivial approximation of $C(s)$

$$\vartheta=\tau\to 0\,, \qquad (20)$$

which gives the approximating PI controller $\bar{C}(s)$ of structure (8) with

$$K_C=\frac{\lambda}{b}, T_I=\frac{1}{a}. \qquad (21)$$

Moreover, the tuning rules (9), (11) require the knowledge of T in model (7). In fact, (16) is infinite-dimensional allowing complex poles. A possibility is to take the time constant as

$$T=\frac{1}{|s_0|} \qquad (22)$$

where s_0 means the dominant (rightmost) pole.

Proposition 2. The explicit PI tuning rules for (19) under the approximation (20) according to the CHR method and EM for K_C are, respectively,

$$\lambda=\frac{7aT}{20\tau},\ \lambda=\frac{a\left(1+(1-\theta)^2\right)}{2}. \qquad (23)$$

Proof. Assume $\tilde{\tau}=\tau$, and it holds that $K_P=b/a$. Then, the comparison of (20) with (9) and (11) for K_C gives (23) directly, while T_I is fixed as in (21). ■

Proposition 3. Consider the mutual relations between K_C, T_I the CHR method and EM, and let T_I is given by (21). Then, the explicit PI tuning rules for (19) under the approximation (20) according to the CHR method and EM are

$$\lambda = \frac{7}{24\tau}, \lambda = \frac{1}{T+\tau}, \tag{24}$$

respectively.

Proof. Let $\tilde{\tau} = \tau$ again. Assume conditions (9) first. If T_I from (9) and (21) are matched, one gets $T = 5/6a^{-1}$, which is then substituted to K_C in (9) for $K_P = b/a$, giving rise to

$$K_C = \frac{7}{24}\frac{1}{b\tau}. \tag{25}$$

Eventually, the matching of (21) and (25) for K_C yields (24). The same procedure is applied to (11) and (21) to get the right-hand formula in (24). ■

Remark 1. Unlike $G_{RY}(s)$, the control system itself is infinite-dimensional since the characteristic quasipolynomial includes the factor $s + a\,\mathrm{e}^{-\vartheta s}$ due to (16).

Remark 2. Clearly, there is a single tunable parameter λ for two controller parameters to be determined. However, any attempt to set $T(s)$ does not solve the problem. For instance, if $T_0(s) = (t_1 s + t_0)/(s+\lambda)$ for some suitable real-valued t_0, t_1, it results in the controller of a PID type, not a PI one.

Remark 3. Let us make a note on the closeness of the fixed value of T_I determined by (21) to those required by the CHR method and EM given by (9) and (11). Under the approximating assumption (20), it holds that $T = a^{-1}$; hence, conditions (9) and (11) read

$$T_I = \frac{6}{5}\frac{1}{a}, T_I = \frac{1}{a}, \tag{26}$$

respectively. Contrariwise, whenever $\vartheta = \tau \to \infty$, all the values of T_I approach infinity. Thus, it is evident that all the limit values are very close to each other and the proposed design method almost meets the requirements of the EM and the CHR method.

B. *Second Order Stable Plant*

Consider the same external inputs as in subsection II-A. Let the stable plant model be factorized as

$$G(s) = \frac{b\,\mathrm{e}^{-\tau s}}{\left(s + a_1\,\mathrm{e}^{-\vartheta s}\right)(s+a_2)} = \frac{\dfrac{b\,\mathrm{e}^{-\tau s}}{m(s)}}{\dfrac{\left(s + a_1\,\mathrm{e}^{-\vartheta s}\right)(s+a_2)}{m(s)}}, \tag{27}$$

$$\tau \geq 0, 0 < a_1\vartheta < 0.5\pi, a_2 > 0,$$

where $m(s)$ is a second-order stable (quasi)polynomial.

Let us select two options as follows: $m_1(s) = (s+\lambda)^2$ and $m_2(s) = (s+\lambda_1)(s+\lambda_2)$. Then, a solution of (4) reads

$$Q_0 = 1, P_0(s) = \frac{m_{.}(s) - b\,\mathrm{e}^{-\tau s}}{\left(s + a_1\,\mathrm{e}^{-\vartheta s}\right)(s+a_2)} \tag{28}$$

that can be parameterized by (6) to satisfy (5) via the setting

$$T(s) = \left(\frac{\tilde{\lambda}}{b} - 1\right) A^{-1}(s), \tag{29}$$

where either $\tilde{\lambda} = \lambda^2$ or $\tilde{\lambda} = \lambda_1 \lambda_2$, respectively, which yields

$$C(s) = \frac{\tilde{\lambda}}{b}\frac{\left(s + a_1\,\mathrm{e}^{-\vartheta s}\right)(s+a_2)}{s^2 + \breve{\lambda}s + \tilde{\lambda}\left(1 - \mathrm{e}^{-\tau s}\right)}, \tag{30}$$

where $\breve{\lambda} = 2\lambda$ or $\breve{\lambda} = \lambda_1 + \lambda_2$.

Then $G_{RY}(s) = \tilde{\lambda}\,\mathrm{e}^{-\tau s}/m(s)$ and $C_S(s) = \left[C(s)\right]_{\tau=0}$ again, for the exact plant model. By applying (20) to (30), we have

$$\bar{C}(s) = \frac{\tilde{\lambda}}{b}\frac{(s+a_1)(s+a_2)}{s\left(s+\breve{\lambda}\right)}. \tag{31}$$

Proposition 4. The controller gain, integrative, derivative and filter time constants for (31) are, respectively,

$$\begin{aligned} K_C &= \frac{\tilde{\lambda}\left(\breve{\lambda}a_2 + a_1\left(\breve{\lambda} - a_2\right)\right)}{\breve{\lambda}b}, T_I = \frac{a_1 + a_2}{a_1 a_2} - \frac{1}{\breve{\lambda}}, \\ T_D &= \frac{\left(\breve{\lambda} - a_1\right)\left(\breve{\lambda} - a_2\right)}{\breve{\lambda}\left(\breve{\lambda}a_2 + a_1\left(\breve{\lambda} - a_2\right)\right)}, T_F = \frac{1}{\breve{\lambda}}. \end{aligned} \tag{32}$$

Proof. Formula (8) can also be expressed as

$$\begin{aligned} C(s) &= K_C \frac{T_I s(T_F s + 1) + (T_F s + 1) + T_D T_I s^2}{T_I s(T_F s + 1)} \\ &= \frac{K_C(T_F + T_D)}{T_F} \frac{s^2 + \dfrac{T_I + T_F}{T_I(T_F + T_D)}s + \dfrac{1}{T_I(T_F + T_D)}}{s\left(s + \dfrac{1}{T_F}\right)} \end{aligned}$$

that implies – by its matching with (31) – the following set of algebraic equations

$$\frac{\tilde{\lambda}}{b}=\frac{K_C\left(T_F+T_D\right)}{T_F}, a_1+a_2=\frac{T_I+T_F}{T_I\left(T_F+T_D\right)},$$
$$a_1 a_2=\frac{1}{T_I\left(T_F+T_D\right)}, \breve{\lambda}=\frac{1}{T_F}.$$

After some algebra, the solution of this set gives (32). ■

Tuning rules (10) and (12) are then applied to (32) to get λ or the pair λ_1, λ_2; however, an explicit (analytic) form cannot be obtained. Obviously, there are three conditional equations for one or two unknowns, and the filter constant is fixed from (32).

Remark 4. The FOPTD model (7) is necessary for the CHR tuning method again. Here, one can adopt the idea introduced e.g. [31], according to which T is given by (22) and

$$\tilde{\tau}=\tau+\frac{1}{\left|s_1\right|}, \tag{33}$$

where s_1 means the second most dominant pole of (27). Note, moreover, that $T_{ar}=a_1^{-1}+a_2^{-1}+\tau-\vartheta$ for (27), see (13).

C. First Order Unstable Plant

Assume the model (16) yet with $a\vartheta \notin \left(0, 0.5\pi\right)$. Whenever $m(s)$ is taken as in subsection III-A, the solution of (4) is excessively complicated due to R_{QM} conditions (see Definition 1).

To avoid this, let

$$m(s)=s+a\,\mathrm{e}^{-\vartheta s}+\lambda b\,\mathrm{e}^{-\tau s}, \tag{34}$$

where λ is the tunable parameter. Note that necessary and sufficient stability conditions for (34) were explicitly derived e.g. in [32]. Apparently, the pair $Q_0=1, P_0=\lambda$ is a stabilizing solution of (4) that can be parameterized by the option

$$T(s)=\frac{\lambda_0}{b}\frac{m(s)}{s+\lambda_0}, \lambda_0>0, \tag{35}$$

which gives rise to the controller transfer function

$$C(s)=\frac{1}{b}\frac{\left(\lambda b+\lambda_0\right)s+b\lambda\lambda_0+a\lambda_0\,\mathrm{e}^{-\vartheta s}}{s+\lambda_0\left(1-\mathrm{e}^{-\tau s}\right)}. \tag{36}$$

The characteristic quasipolynomial then reads

$$\Delta\left(s,\lambda,\lambda_0\right)=m(s)\left(s+\lambda_0\right). \tag{37}$$

Note that $C_S(s), G_{RY}(s)$ are no longer finite-dimensional. The use of the QCSA is very simple here since $m(s)$ has only one unknown parameter to be determined, and the remaining factor in $\Delta(s)$ is a polynomial. In order to avoid the influence of the pole $-\lambda_0$, it must hold that $-\lambda_0 \ll \operatorname{Re}\left(s_{0,m}\right)<0$ where $s_{0,m}$ means the rightmost root of quasipolynomial $m(s)$.

IV. Numerical Examples

The presented numerical (simulation) examples are aimed to verify the performance of the designed compensating controllers (19), (30), (36) and their tuning rules in the nominal case and for some selected perturbations. A benchmark with the direct use of CHR and EM tuning laws (9)-(13) for simple PI(D) feedback controllers is also given to the reader.

The following performance measures are utilized: If $r(t)=r=\text{const.}$ and $d(t)=0$, let us define

$$\Delta e_r := \max_{e(t)<0}\left|e(t)/r\right|,$$
$$\mathrm{IAE}_r := \int_0^\infty \left|e(t)\right|\mathrm{d}t,$$
$$T_{95,r} := \max\left\{t_0 : \left|e\left(t_0\right)\right|=0.05r\right\}, \text{while} \left|e(t)\right|<0.05r, \forall t>t_0,$$
$$\mathrm{IAID}_r := \int_0^\infty \left|\dot{u}(t)\right|\mathrm{d}t.$$

Once $d(t)\neq 0$ enters to the control system, values $\Delta e_d, T_{95,d}, \mathrm{IAE}_d, \mathrm{IAID}_d$ are defined analogously as the control reaction measures to the disturbance.

A. Example 1

Consider system (16) with the nominal parameters $a=0.2$, $b=0.6$, $\tau=4$, $\vartheta=0.8$. The system dominant pole equals $s_0=-0.2429$, i.e. $T=4.117$. The CHR and EM tuning rules (9) and (11) yield requirements $K_C=0.2095, T_I=5.1026$ and $K_C=0.1201$, $T_I=4.9409$, respectively. Clearly, the actual fixed $T_I=5$ according to (21) is very close to the required values; thus, controller (19) implicitly satisfies both the tuning rules, in some sense (see Remark 3). Regarding controller parameter λ, formulas (23) give $\lambda=7.205\cdot 10^{-2}$ and $\lambda=0.1257$, respectively, whereas (24) yields $\lambda=7.292\cdot 10^{-2}$ and $\lambda=0.1232$, respectively. The EM does not prove the equality (14) since ITAE = 107.47, ITAD = 66.27 for $\lambda=0.1257$. Note that the use of the pure PI controller with the required constants according to the EM gives ITAE = 44.11, ITAD = 52.12. By simulations, we found that the condition (14) is satisfied for $\lambda=0.2151$ (ITAE = ITAD = 49.11) - it i.a. means that the controlled output is close to the step response of the plant, see Fig. 3 for the comparison.

Simulation results (for $r=\eta(t), d=\eta(t-75)$ where $\eta(t)$ is the Heaviside function) are compared in Table I. The beast and the worst cases are highlighted in green and red, respectively. Since the controller is parametrized only by using λ, only some its values are chosen. The table also contains results when using pure PI controllers set by the EM and the CHR rules. Notice that the higher λ implies the faster response with better disturbance attenuation yet with more aggressive control action. Similarly, it can be observed that the PI controller set by the EM gives better responses compared to that tuned via the CHR method.

Selected control responses $u(t), y(t)$ are displayed in Fig. 4a and Fig. 4b, respectively.

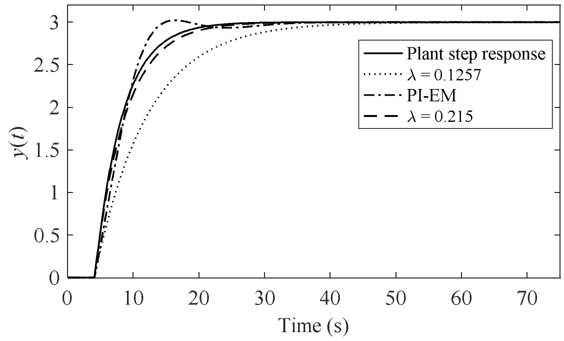


Fig. 3 Step response of (16) vs. control responses

Table I. Performance measures – the nominal case – Example 1

λ	Δe_r	$T_{95,r}$	IAE_r	IAID_r	Δe_d	$T_{95,d}$	IAE_d	IAID_d
$7.2\cdot10^{-2}$	0	45.6	17.8553	0.3457	0.2315	37.9	5.4025	0.3327
0.1257	0	27.8	12.0048	0.3734	0.2168	27.5	3.5866	0.3333
0.2151	0	17.9	8.7017	0.4466	0.2070	21.8	2.5953	0.3333
PI-CHR	0	33.8	13.7519	0.3332	0.2244	30.2	4.1173	0.3332
PI-EM	0.0071	12.6	8.2014	0.461	0.2126	19.7	2.4355	0.3392

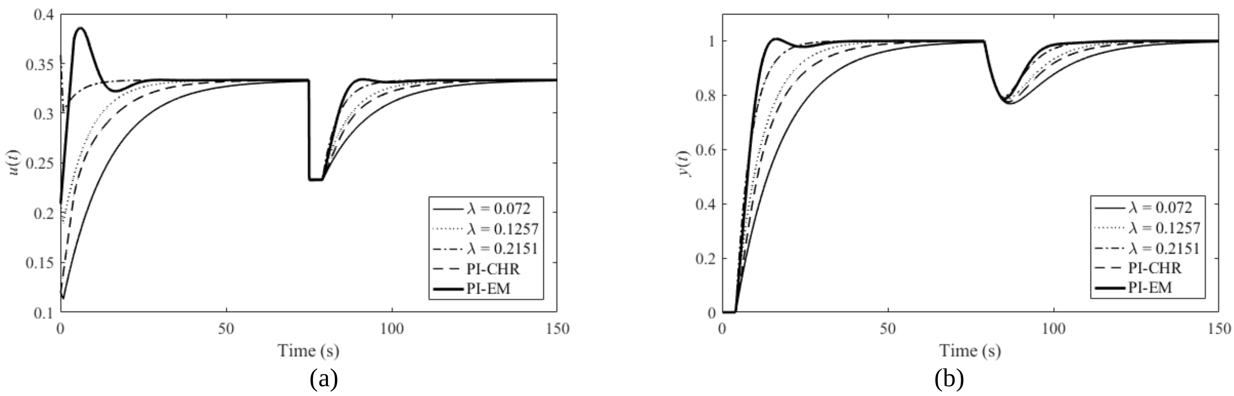


Fig. 4 Selected nominal control responses $u(t)$ (a), $y(t)$ (b) – Example 1

Now, consider the following perturbation in the controlled plant

$$a = 0.2,\ b = 0.66,\ \tau = 4.8,\ \vartheta = 0.72, \tag{38}$$

i.e., the 20% error in the input-output delay and the 10% error is the static gain and the internal delay, while its model is assumed as in the nominal case. Corresponding performance measures for the calculated controllers are benchmarked against each other in Table II, and control responses are displayed in Fig. 5. For perturbation (38), a very good choice is represented by the found balanced value $\lambda = 0.2151$ (in the sense of EM). A lower value of λ gives a slower control response, while the direct PI design via the CHR method yields an aggressive yet not very fast response.

Table II. Performance measures – perturbation (38) – Example 1

λ	Δe_r	$T_{95,r}$	IAE_r	IAID_r	Δe_d	$T_{95,d}$	IAE_d	IAID_d
$7.2\cdot10^{-2}$	0	38	16.2956	0.3163	0.2581	36.1	5.3693	0.3029
0.1257	$1.33\cdot10^{-4}$	21.1	10.9222	0.3438	0.2472	26.4	3.5867	0.3031
0.2151	0.0346	12.8	8.4739	0.5591	0.2393	21	2.6079	0.3101
PI-CHR	0.1567	21.1	9.674	0.5893	0.2442	19.7	2.6321	0.3452
PI-EM	0	26.2	12.5153	0.303	0.2538	28.6	4.1137	0.303

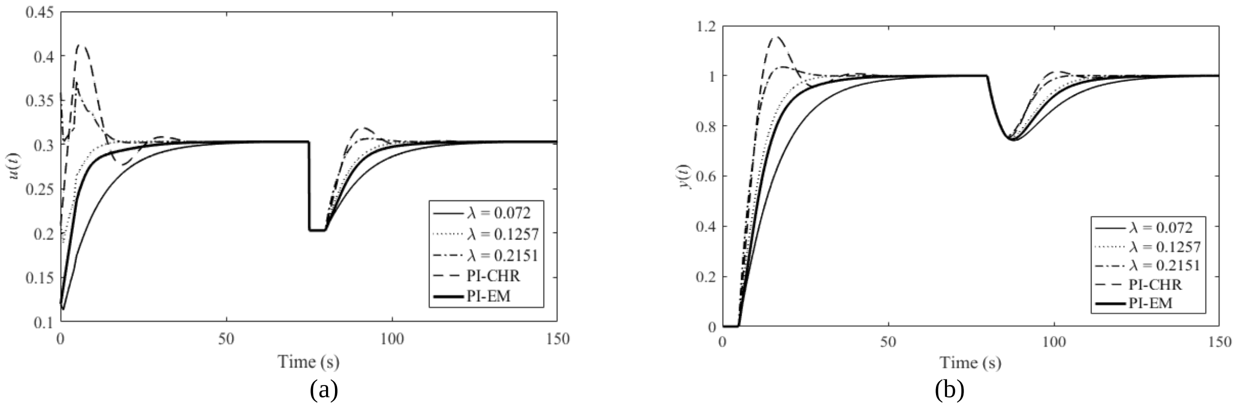


Fig. 5 Selected perturbed control responses $u(t)$ (a), $y(t)$ (b) according to (38) – Example 1

Let us verify the robustness for the opposite perturbation in the plant parameters, i.e.,

$$a = 0.2\,,\ b = 0.54\,,\ \tau = 3.2\,,\ \vartheta = 0.88\,. \qquad (39)$$

The corresponding data can be found in Table III and Fig. 6. Now, surprisingly, the direct PI design via the CHR method seems to be the best choice. However, a compromising solution is $\lambda = 0.1257$ obtained from the EM as in (23).

Table III. Performance measures – perturbation (39) – Example 1

λ	Δe_r	$T_{95,r}$	IAE_r	IAID_r	Δe_d	$T_{95,d}$	IAE_d	IAID_d
$7.2 \cdot 10^{-2}$	0	53.8	19.6794	0.3804	0.2096	39.9	5.5055	0.3689
0.1257	0	34.4	13.322	0.4242	0.1887	28.6	3.5916	0.3703
0.2151	0	23.9	9.6599	0.5713	0.1766	22.6	2.5947	0.3704
PI-CHR	0	22.2	9.0702	0.3901	0.1826	20.6	2.4357	0.3704
PI-EM	0	40.7	15.2378	0.3696	0.1977	31.7	4.1387	0.37

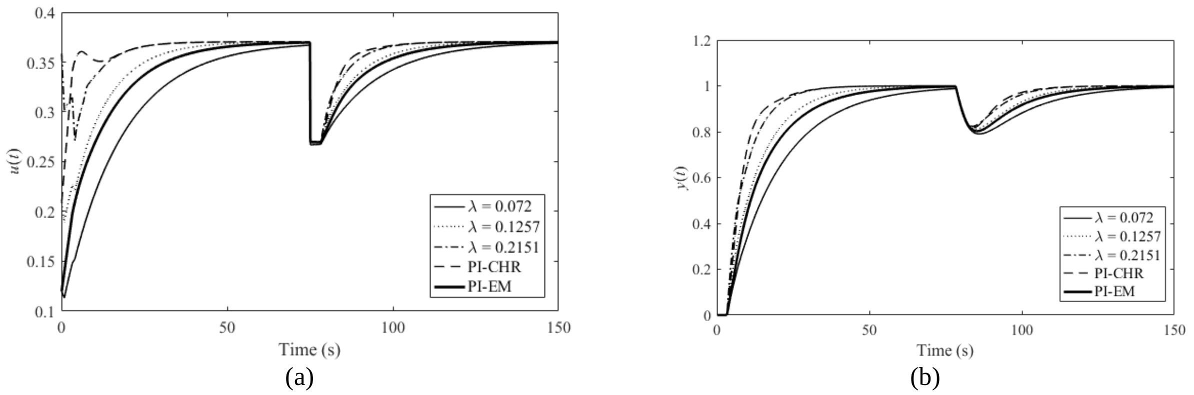


Fig. 6 Selected perturbed control responses $u(t)$ (a), $y(t)$ (b) according to (39) – Example 1

B. *Example 2*

Let (27) be with the nominal parameters $a_1 = 0.2$, $a_2 = 0.5$, $b = 0.6$, $\tau = 4$, $\vartheta = 0.8$. System dominant poles then equal $s_0 = -0.2429$, $s_1 = -0.5$, i.e. $T = 4.117$ and $\tilde{\tau} = 6$ (for the CHR method) in the accordance to (22) and (33), respectively.

Conditions (10) in the sense of the CHR method require the following data:

$$K_C = 0.0686, T_I = 4.117, T_D = 3\,. \qquad (40)$$

By comparison of (40) and (32) with $m_1(s)$, the following values in the sense of the CHR method are, respectively, obtained: $\lambda = 0.1891$, $\lambda = 0.1734$, , $\lambda = 0.0851$. Note that rules for K_C and T_I are close to each other since when K_C is required as in (40), one can simultaneously compute from (32) that $T_I = 4.3553$.

From (12) for the EM, we get the following values:

$$K_C = 0.1057, T_I = 6.9318, T_D = 1.849\,. \qquad (41)$$

Again, by the matching (41) and (32), we have: $\lambda = 0.2526$, $\lambda = 7.3324$, $\lambda = 0.0882$, respectively.

If $m_2(s)$ is considered, the matching of pairs $\{K_C, T_I\}, \{K_C, T_D\}$ for the CHR method yield, respectively: $\lambda_{1,2} = 0.1734 \pm 0.0679\mathrm{j}$, $\lambda_{1,2} = 0.0851 \pm 0.2344\mathrm{j}$. For the EM, one gets the following values: $\{\lambda_1 = 0.0921, \lambda_2 = 14.5727\}$, and $\lambda_{1,2} = 0.0882 \pm 0.2762\mathrm{j}$. The pair $\{T_I, T_D\}$ cannot be compared due to the linear dependence of equations.

Similarly to Example 1, neither compensation controllers nor PID controller tuned by the EM satisfy (14). It was found by simulations that this condition holds for $\lambda = \lambda_1 = \lambda_2 = 0.3265$ (ITAE = ITAD = 61.66). Corresponding control responses and controlled system step response are displayed in Fig. 7. The option $\lambda = 0.3265$ yields almost identical response to the plant step response.

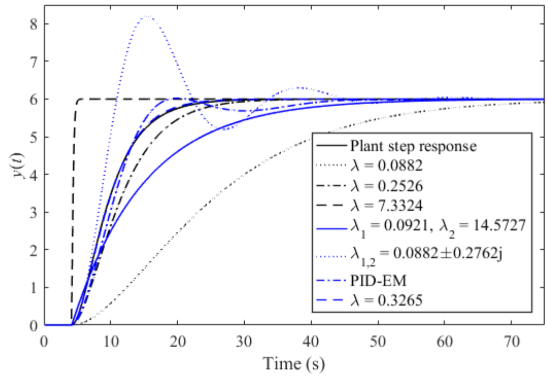
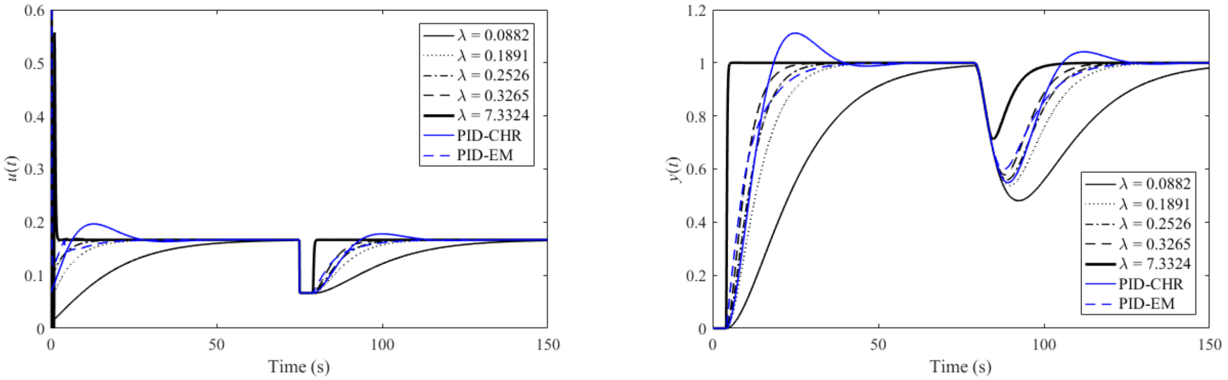


Fig. 7 Step response of (27) vs. control responses

In Table IV, selected corresponding nominal performance measures are given, along with the result for the PID controller tuned by the EM and the CHR method. The selection can be done since $m_1(s)$ has only a single nonzero real parameter. The control responses are further displayed for $m_1(s)$ in Fig. 8, and those for $m_2(s)$ in Fig. 9.

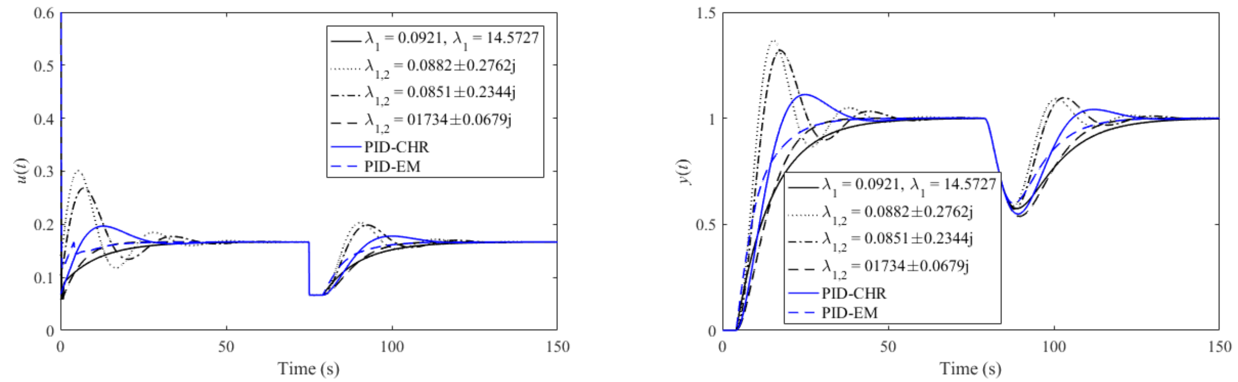
Table IV. Performance measures – the nominal case – Example 2

$\lambda, \lambda_{1,2}$	Δe_r	$T_{95,r}$	IAE$_r$	IAID$_r$	Δe_d	$T_{95,d}$	IAE$_d$	IAID$_d$
0.0882	0	57.8	26.5506	0.1657	0.5193	61.7	15.9355	0.1653
0.1891	0	29.3	14.6263	0.1667	0.4624	37.9	8.7452	0.1667
0.2526	0	22.8	11.9677	0.1712	0.4408	32.7	7.1506	0.1667
0.3265	0	18.5	10.1756	0.2156	0.4218	29.4	6.0753	0.1667
7.3324	$1.11 \cdot 10^{-4}$	4.7	4.3237	201.0132	0.2871	19.4	2.5637	0.1666
0.0921, 14.5727	0	36.6	14.9659	4.5211	0.4255	42.5	8.9478	0.1665
0.0882 ± 0.2762j	0.3668	31.5	12.1113	0.5898	0.4137	29.4	5.1084	0.2824
0.0851 ± 0.2344j	0.3196	35.8	12.6426	0.465	0.4277	32.6	5.7852	0.2603
0.1734 ± 0.0679j	$3.28 \cdot 10^{-4}$	26.9	14.0597	0.167	0.4622	36.2	8.4017	0.1667
PID-CHR	0.1118	33	12.7818	0.234	0.4514	27.9	7.005	0.1917
PID-EM	0	25.2	10.9817	5.9234	0.4012	33.5	6.5586	0.1667



(a) (b)

Fig. 8 Selected nominal control responses $u(t)$ (a), $y(t)$ (b) – Example 2, $m_1(s)$



(a) (b)

Fig. 9 Selected nominal control responses $u(t)$ (a), $y(t)$ (b) – Example 2, $m_2(s)$

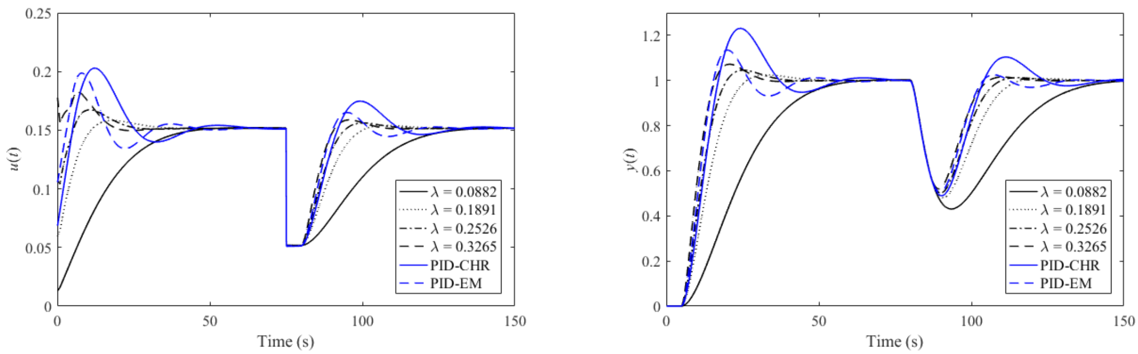
The higher value of λ is, the more aggressive the control response is obtained. The use of $m_2(s)$ brings about a slow or a periodic response that can improve the disturbance attenuation yet suffers from a reference-response overshoot. If $\lambda = 0.3265$, the control action is almost constant. Pure PID controllers give comparable results to the use of $m_2(s)$; the controller designed by the EM is more aggressive than that tuned by the CHR method with a very high initial control action.

Let us now perturb the controlled plant such that the model differs from the real plant. The perturbation is analogous to (38) where $a_1 = a$. The corresponding results are given in Table V and Figs. 10 and 11 for $m_1(s)$ and $m_2(s)$, respectively.

Apparently, a value lying between the EM result $\lambda = 0.2526$ and the balanced tuning $\lambda = 0.3265$ represents a very good tuning choice in the case of perturbation (38). For the option $m_2(s)$ and for the use of standard PID controllers, the same statements as in the nominal case hold true. Note that the value $\lambda = 7.3324$ causes an unstable feedback.

Table V. Performance measures – perturbation (38) – Example 2

$\lambda, \lambda_{1,2}$	Δe_r	$T_{95,r}$	IAE_r	IAID_r	Δe_d	$T_{95,d}$	IAE_d	IAID_d
0.0882	0.0034	47.6	24.4174	0.1523	0.5693	56.2	16.0164	0.1521
0.1891	0.0248	23	13.9914	0.1654	0.5191	34.7	8.9936	0.1566
0.2526	0.0459	17.9	11.9065	0.189	0.4985	30.2	7.4273	0.1611
0.3265	0.0723	17.8	10.6117	0.2606	0.4805	27.1	6.3525	0.1669
7.3324	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
0.0921, 14.5727	0	29.5	13.6185	4.5328	0.4802	39.1	8.9532	0.1515
0.0882 ± 0.2762j	0.5981	69.1	18.8183	0.8728	0.4603	55	8.21	0.4006
0.0851 ± 0.2344j	0.5275	64.6	18.3313	0.6709	0.5044	59.9	9.1611	0.3661
0.1734 ± 0.0679j	0.0415	21.7	13.9055	0.1738	0.519	33.5	8.8265	0.1601
PID-CHR	0.2312	46.5	14.9993	0.2832	0.5106	43.7	8.6158	0.2105
PID-EM	0.1349	37.6	12.1762	0.291	0.494	26.5	6.8744	0.1959



(a) (b)

Fig. 10 Selected perturbed control responses $u(t)$ (a), $y(t)$ (b) according to (38) – Example 2, $m_1(s)$

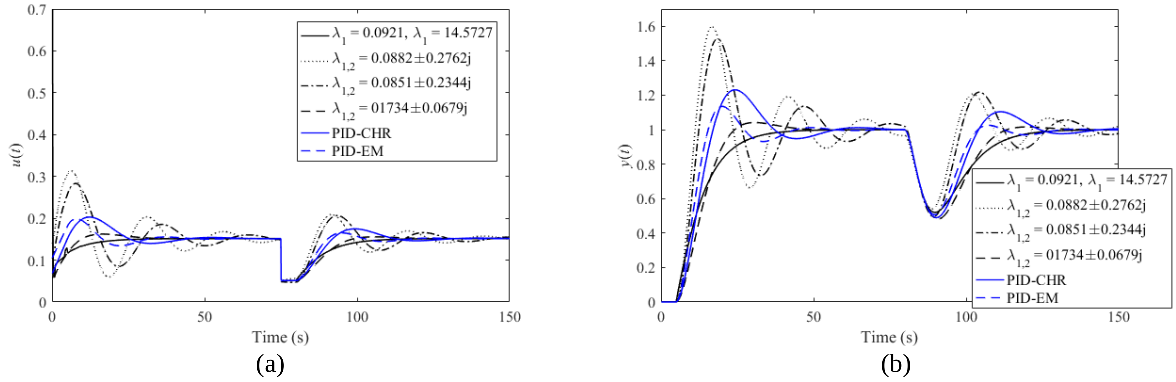


Fig. 11 Selected perturbed control responses *u*(*t*) (a), *y*(*t*) (b) according to (38) – Example 2, $m_2(s)$

Table VI along with Figs. 12 and 13 provide the reader with results for perturbation (39).

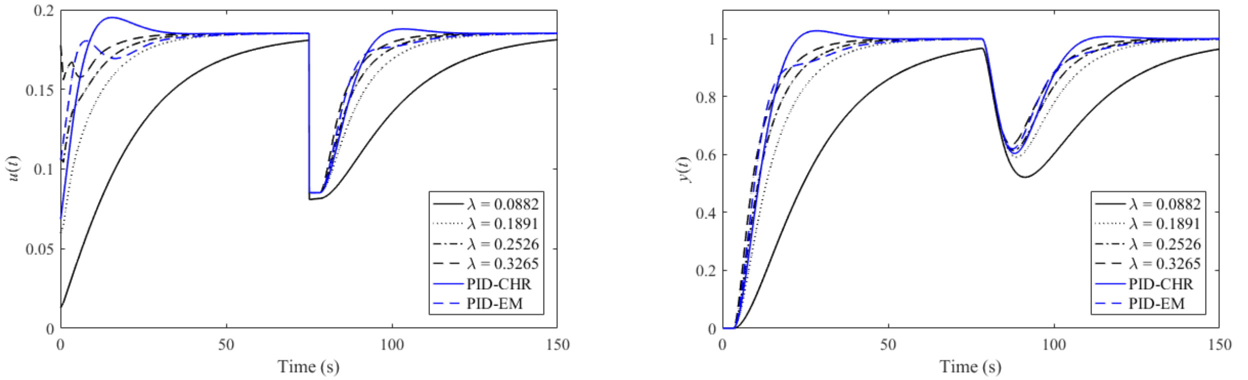
For this perturbation, similar statements to perturbation (38) can be formulated. Moreover, the periodic solutions $\lambda_{1,2} = 0.0882 \pm 0.2762\mathrm{j}$, $\lambda_{1,2} = 0.0851 \pm 0.2344\mathrm{j}$ would be good options as well, especially when rejection the disturbance; however, there is a significant overshoot (i.e., control error) here. It is also interesting to see the difference between these two close settings. Namely, the former option provides a relatively fast reference tracking; whereas, the latter one reduces the error due to input disturbance more effectively. If one compares data in Table VI, it can be observed that a complex conjugate pair $\lambda_{1,2}$ with a small non-zero imaginary part can enhance performance measures (see $\lambda = 0.1891$ versus $\lambda_{1,2} = 0.1734 \pm 0.0679\mathrm{j}$). Last but not least, it is worth noting that the PID controller designed by the CHR method yields acceptable performance for this perturbation.

C. Example 3

Assume (16) with $a = -0.2$, $b = 0.6$, $\tau = 4$, $\vartheta = 0.8$. It can be proved that $m(s)$ as in (29) is stable for $1/3 < \lambda < 0.564$, see [32] for more details, and it has a minimum abscissa $\mathrm{Re}(s_{0,m}) = -0.0794381$ at $\lambda = 0.35486789$. This quasipolynomial root is simultaneously real and double; therefore, the triple real feedback pole is reached when $\lambda_0 = 0.35486789$. Let us set $\lambda_0 = 1$ in (35) to get a double real dominant pole, due to a less dominancy of the real pole $s_1 = -1$. Results for both the settings compared to another trial (periodic) setting, in the nominal case, are given to the reader in Table VII and Fig. 14. Perturbations (38) and (39) are benchmarked in Tables VIII and IX, and in Figs. 15 and 16, respectively. Note that it is set that $d = \eta(t-125)$ (the nominal case), and $d = \eta(t-200)$ (for both the perturbations). The value for ITAE is computed by using (14) for the disturbance-free case.

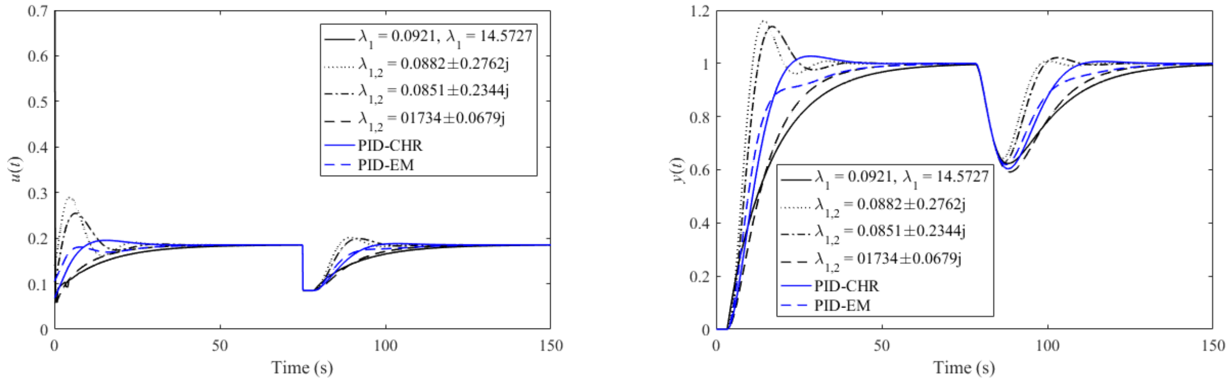
Table VI. Performance measures – perturbation (39) – Example 2

$\lambda, \lambda_{1,2}$	Δe_r	$T_{95,r}$	IAE_r	IAID_r	Δe_d	$T_{95,d}$	IAE_d	IAID_d
0.0882	0	70.5	28.9349	0.1808	0.4783	68.7	16.0691	0.1812
0.1891	0	37.9	16.2297	0.1851	0.4088	41.7	8.7387	0.185
0.2526	0	30.8	13.289	0.1897	0.3862	36.1	7.1482	0.1852
0.3265	0	26.1	11.3	0.2478	0.3664	32.4	6.0745	0.1852
7.3324	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
0.0921, 14.5727	0	44.3	16.5619	4.5639	0.3765	46	8.9464	0.1847
0.0882 ± 0.2762j	0.1609	18.8	8.7793	0.4538	0.3563	20.6	6.5533	0.2273
0.0851 ± 0.2344j	0.1389	22.3	9.6231	0.3513	0.37	21.8	4.3283	0.2186
0.1734 ± 0.0679j	0	35.4	15.5981	0.1851	0.4082	39.9	8.3951	0.1851
PID-CHR	0.0272	19.9	11.833	0.2056	0.3963	29.2	6.1782	0.1908
PID-EM	0	33.6	12.1768	0.3696	0.3805	34.9	6.5533	0.37



(a) (b)

Fig. 12 Selected perturbed control responses $u(t)$ (a), $y(t)$ (b) according to (39) – Example 2, $m_1(s)$

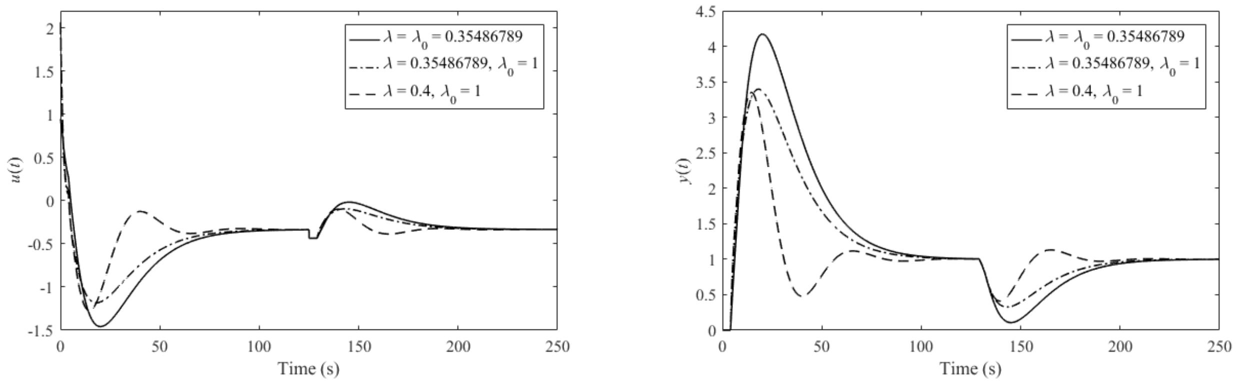


(a) (b)

Fig. 13 Selected perturbed control responses $u(t)$ (a), $y(t)$ (b) according to (39) – Example 2, $m_2(s)$

Table VII. Performance measures – the nominal case – Example 3

λ, λ_0	Δe_r	$T_{95,r}$	IAE_r	IAID_r	Δe_d	$T_{95,d}$	IAE_d	IAID_d	ITAE
0.3549, 0.3549	3.1707	97.4	115.48	4.4721	0.8928	78.3	31.584	1.1706	3574.7
0.3549, 1	2.3946	91.1	86.48	6.0743	0.6735	71.9	23.169	1.0142	2517.5
0.4, 1	2.3505	75.6	50.85	6.88	0.5888	50.7	11.717	1.1404	1101.2

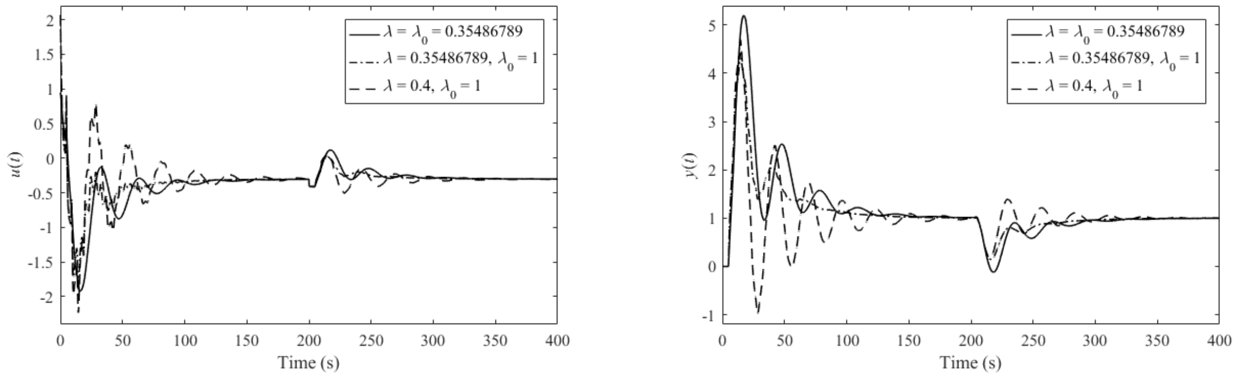


(a) (b)

Fig. 14 Selected nominal control responses $u(t)$ (a), $y(t)$ (b) – Example 3

Table VIII. Performance measures – perturbation (38) – Example 3

λ, λ_0	Δe_r	$T_{95,r}$	IAE_r	IAID_r	Δe_d	$T_{95,d}$	IAE_d	IAID_d	ITAE
0.3549, 0.3549	4.1958	149.2	107.09	7.8417	1.1210	115.8	31.2291	1.7891	4013.3
0.3549, 1	3.4249	126.3	80.83	14.8782	0.8556	91.1	23.0724	1.3512	2706.4
0.4, 1	3.7061	167.5	99.05	19.3616	0.8219	113	21.2443	2.6948	4047

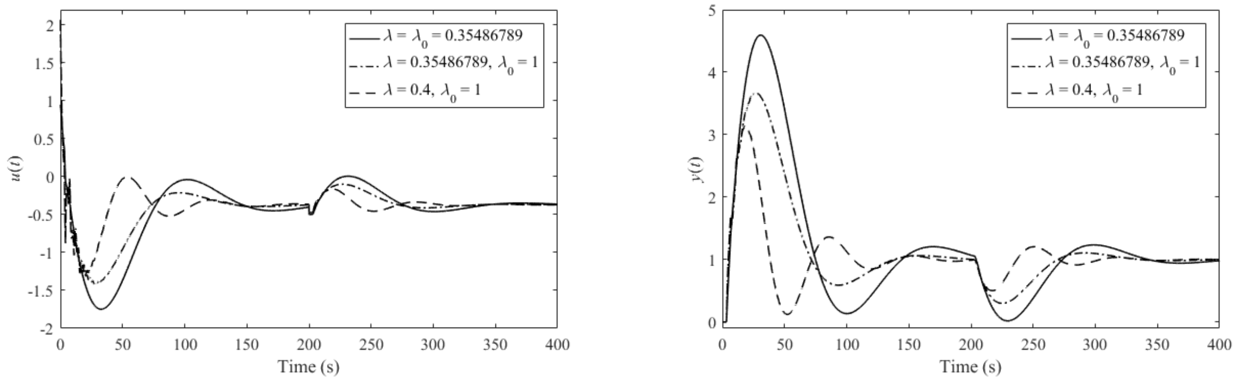


(a) (b)

Fig. 15 Selected perturbed control responses $u(t)$ (a), $y(t)$ (b) according to (38) – Example 3

Table IX. Performance measures – perturbation (38) – Example 3

λ, λ_0	Δe_r	$T_{95,r}$	IAE_r	IAID_r	Δe_d	$T_{95,d}$	IAE_d	IAID_d	ITAE
0.3549, 0.3549	3.5936	213.2	201.159	5.8094	0.9802	179.4	53.447	1.6015	10605
0.3549, 1	2.6654	173.7	125.764	10.0408	0.7011	116.6	31.9775	1.2235	5346.8
0.4, 1	2.1526	160	79.228	10.7607	0.496	94.9	17.6711	1.286	3231.9



(a) (b)

Fig. 16 Selected perturbed control responses $u(t)$ (a), $y(t)$ (b) according to (39) – Example 3

In the nominal case, advantages of the triple dominant root setting (minimizing the spectral abscissa, in addition) have not been proofed. A complex conjugate pair of dominant poles, contrariwise, gives the best results; except for the smoothness of the control action when reference tracking.

Under perturbation (38), the double real pole with the minimum abscissa is definitely the best (sufficiently robust) solution. The periodic solution is effective to reject the disturbance, yet it is rather aggressive.

In the sense of selected performance measures, perturbation (39) yields results similar to the nominal case.

V. Conclusion

A comparative study on algebraic-based controller PID-like design for time-delay systems followed by the application of some tuning techniques has been presented. The obtained controllers have a predictor-compensation structure. Three prototypal controlled plants with input-output and internal delays have been considered. Explicit controller tuning rules have been derived for a stable first-order plant using a simple approximation and two well-established tuning principles; namely, via the Chien-Hrones-Reswick and the balanced (equalization) ones. The same principles have been applied to a second order stable plant; however, tuning formulas could not be obtained by analytic means. A quasi-continuous shifting procedure has been introduced to tune poles loci for the feedback loop with an unstable first-order delayed system. For all these three cases, extensive numerical examples have been presented. Tuning rules for compensation controllers have been applied to the nominal case and two selected perturbations, and compared to the direct use of standard PI or PID controllers. These examples have shown that algebraically derived controllers are more robust against perturbations compared to standard PI(D) rules. Amazingly, more degrees of freedom might not bring better control responses. By using several performance measures, we judge that an experimentally found balanced tuning (satisfying the equality of some integral criteria) represents the best option for stable control plants. For the unstable case, the triple real dominant roots setting has not proved to be a suitable one; however, a double real pole with the minimized spectral abscissa has given a robust performance in some sense.

REFERENCES

[1] J.-P. Richard, “Time-delay systems: An overview of some recent advances and open problems,” *Automatica*, vol. 39, no. 10, pp. 1667-1694, 2003.

[2] R. Sipahi, T. Vyhlídal, S.-I. Niculescu, and P. Pepe (eds.), *Time Delay Systems: Methods, Applications and New Trends*. New York: Springer, 2012.

[3] E. Fridman, *Introduction to Time-Delay Systems: Analysis and Control*. Basel: Birkhäuser, 2014.

[4] S. Skogestad, “Simple analytic rules for model reduction and PID controller tuning,” *Model. Identif. Control*, vol. 25, no. 2, pp. 85-120, 2004.

[5] R. Prokop, J. Korbel, and O. Líška,, “A novel principle for relay-based autotuning,” *Int. J. Math. Models Meth. Appl. Sci.*, vol. 7, no. 5, pp. 1180-1188, 2011.

[6] M. Kubalčík and V. Bobál, “Predictive control of higher order systems approximated by lower order time-delay models,” *WSEAS Trans. Syst.*, vol. 11, no. 10, pp. 606-617, 2012.

[7] K. J. Aström and T. Hägglund. *Advanced PID Control*. Research Triangle Park, NC: ISA, 2006.

[8] R. Vilanova and A. Visioli, *PID Control in the Third Millennium*. London: Springer, 2012.

[9] R. C. Panda, C. C. Yu, and H. P. Huang, “PID tuning rules for SOPDT systems: review and some new results,” *ISA Trans.*, vol. 43, no. 2, pp. 283–295, 2004.

[10] Y. Xia, M. Fu, and P. Shi, *Analysis and Synthesis of Dynamical Systems with Time-Delays*. Berlin, Heidelberg: Springer-Verlag, 2009.

[11] G. J. Silva, A. Datta, and S. P. Bhattacharyya, “PI stabilization of first-order systems with time delay,” *Automatica*, vol. 37, no. 12, pp. 2025-2031, 2001.

[12] S. Elmadssia, K. Saadaoui, and M. Benrejeb, “PI controller design for time delay systems using an extension of the Hermite-Biehler theorem,” *J. Indust. Eng.*, vol. 2013, art. ID 813037, 2013.

[13] R. Farkh, K. Laabidi, and M. Ksouri, “Stabilizing sets of PI/PID controllers for unstable second order delay system,” *Int. J. Autom. Comput.*, vol. 11, no. 2, pp. 210-222, 2014.

[14] A. Roy and K. Iqbal, “PID controller design for first-order-plus-deadtime model via Hermite-Biehler theorem,” in *Proc. American Control Conference*, Denver, Colorado, USA, 2003, pp. 5286–5291.

[15] P. Dostál, F. Gazdoš, and V. Bobál, “Design of controllers for processes with time delay by polynomial method,” in *European Control Conference (ECC) 2007*, Kos, Greece, 2007, pp. 4540-4545.

[16] R. Prokop and J.-P. Corriou, “Design and analysis of simple robust controllers,” *Int. J. Control*, vol. 66, no. 6, pp. 905-921, 1997.

[17] F. Castaños, E. Estrada, S. Mondié, and A. Ramírez, “Passivity-based PI control of first-order systems with I/O communication delays: a frequency domain analysis,” *Int. J. Control*, in press.

[18] P. Zítek, J. Fišer, and T. Vyhlídal, “IAE optimization of PID control loop with delay in pole assignment space,” *IFAC-PapersOnLine*, vol. 49, no. 10, pp. 177–181, 2016.

[19] S. Srivastava, A. Misra, S. K. Thakur, and V. S. Pandit, “An optimal PID controller via LQR for standard second order plus time delay systems,” *ISA Trans.*, vol. 60, no. 1, pp. 244-253, 2016.

[20] K. Sundaravadivu, S. Sivakumar, and N. Haripriasad, “2DOF PID controller design for a class of FOPTD models – An analysis with heuristic algorithms,” in *Procedia Computer Science*, vol. 48, 2015, pp. 90-95.

[21] O. J. M. Smith, “Closer control of loops with dead time,” *Chem. Eng. Prog.*, vol. 53, no. 5, pp. 217-219, 1957.

[22] M. R. Matausek and A. D. Micic, “A modified Smith predictor for controlling a process with integrator and long dead-time,” *IEEE Trans. Autom. Control*, vol. 41, no. 8, pp. 1199-1203, 1996.

[23] A. Z. Manitius and A. W. Olbrot, “Finite spectrum assignment problem for systems with delays,” *IEEE Trans. Autom. Control*, vol. 24, no. 4, pp. 541–553, 1979.

[24] K. L. Chien, J. A. Hrones, and J. B. Reswick, “On the automatic control of generalized passive systems,” *Trans. Am. Soc. Mech. Eng.*, vol. 74, no. 2, pp. 175-185, 1952.

[25] R. Gorez, and P. Klán, “Nonmodel-based explicit design relations for PID controllers,” *IFAC Proc. Vol.*, vol. 33, no. 4, pp. 133-140, 2000.

[26] P. Klán and R. Gorez,, “Simple analytic rules for balanced tuning of PI controllers,” *IFAC Proc. Vol.*, vol. 36, no. 18, pp. 47-52, 2003.

[27] W. Michiels, K. Engelborghs, P. Vansevevant, and D. Roose, “Continuous pole placement for delay equations,” *Automatica*, vol. 38, no. 6, pp. 747-761, 2002.

[28] L. Pekař, “A ring for description and control of time-delay systems,” *WSEAS Trans. Syst.*, vol. 11, no. 10, pp. 571-585, 2012.

[29] L. Pekař and R. Prokop, “The revision and extension of the RMS ring for time delay systems,” *B. Pol. Acad. Sci - Tech. Sci.*, vol. 65, no. 3, pp. 341-350, 2017.

[30] M. Huba, D. Vrancíć, and T. Huba, “Evaluating performance limits in FOTD plant control,” *IFAC-PapersOnLine*, vol. 49, no. 6, pp. 218-225, 2016.

[31] M. Vítečková, A. Víteček, and L. Smutný, “Simple PI and PID controllers tuning for monotone self-regulating plants,” *IFAC Proc. Vol.*, vol. 33, no. 4, pp. 259-264, 2000.

[32] L. Pekař, R. Prokop, and R. Matušů, “A stability test for control systems with delays based on the Nyquist criterion,” *Int. J. Math. Models Meth. Appl. Sci.*, vol. 7, no. 5, pp. 1213-1224, 2011.