An Improved Motion Controller of a Mobile Robot Based on a Hyperchaotic System

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***Abstract*—**In this paper, an improved technique of a mobile robot's motion control, which is based on a hyperchaotic system, is studied. The proposed motion control strategy of the mobile robot is based on a chaotic path planning generator in order to cover a terrain faster, in regard to other previous works, and also with unpredictable way. The simulation results for the proposed motion control strategy prove that greater terrain coverage can be achieved.

***Keywords*—**Coverage rate, hyperchaotic system, mobile robot, motion controller.

I. Introduction

IN the last decades the research field of autonomous mobile robots has become a topic of great interest because of its ever-increasing applications in various activities. Floor-cleaning devices, robots for space missions, industrial transportation, and fire fighting devices have been developed accenting autonomous mobile robots as very useful tools in industrial and civil life [1], [2]. Also, many military activities, which put human integrity in risk, such as the surveillance of terrains, the terrain exploration for explosives or dangerous materials and the patrolling for intrusion in military facilities, have driven to the development of intelligent robotic systems [3]-[5].

In all the aforementioned missions robotic systems must have some very important features such as the perception and identification of the target, the positioning of the robot on the terrain and the updating of the terrain's map. However, the most useful feature, determining the success of these military missions, is the path planning. For this reason many research teams try to find out the way to generate a trajectory, which will guarantee that the robot will cover the entire terrain.

Furthermore, in some cases, such as the case of patrolling for intrusion, the path of the robot must be as much difficult to be predicted by the intruder as possible. So, the mission of patrolling a terrain with a mobile robot is an issue that has to do with finding a plan which must satisfy three major targets: the unpredictability of the trajectory, the scan of the entire terrain and the fast scanning of the robot's workplace. These are the basic requirements for selecting the most suitable autonomous mobile robots for the specific kind of missions.

These characteristics were the beginning of using nonlinear dynamical systems in the development of autonomous mobile robots, especially in the last two decades [6]-[8]. As it is known, nonlinear systems have a very rich dynamic behavior, showing a variety of chaotic phenomena. This chaotic behavior is the reason for which nonlinear systems have been used in many other engineering fields, such as communications, cryptography, random bits generators and neuronal networks [9]-[12].

The aim of using nonlinear systems in autonomous robots is achieved by designing controllers, which ensure chaotic motion. Signals, which are produced by chaotic systems or circuits, are used to guide autonomous robots for exploration of a terrain for vigilance, search or de-mining tasks. The main feature of chaotic systems, which is the unpredictability, is a necessary condition in the previous mentioned tasks. In literature well-known chaotic systems, such as Arnold dynamical system, Standard or Taylor-Chirikov map, Lorenz system, Chua circuit, Logistic map, and other, have been used [3], [4], [13]-[18].

In this work, an improved motion control strategy of a mobile robot is studied, in order to generate the most unpredictable trajectory as well as the fast covering of the whole terrain. This is implemented by using a hyperchaotic dynamical system. By sampling signals that are produced from the system in various time periods better results can be produced in regard to other published relative works.

The rest of the paper is organized as follows. In the next section the basic features of the chosen hyperchaotic system are presented. The mobile robot model, which is adopted, is described in Section 3. The simulation results of the proposed motion control scheme and its analysis are presented in Section 4. Finally, Section 5 includes the conclusions of this work.

II. The System

In 2014, Li and Sprott proposed the 4-D simplified Lorenz system (1) with coexisting hidden attractors [19].

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$$\begin{cases} \dot{x}_1 = x_2 - x_1 \\ \dot{x}_2 = -ax_1x_3 + x_4 \\ \dot{x}_3 = x_1x_2 - 1 \\ \dot{x}_4 = -bx_2 \end{cases} \quad (1)$$

Also, according to Ref. [19], system (1) has a maximum hyperchaotic behavior for a = 2.6 and b = 0.44, where the Lyapunov exponents are (LE_1, LE_2, LE_3, LE_4) = (0.0704, 0.0128, 0, −1.0832) and the Kaplan–Yorke dimension is D_{KY} = 3.0768.

In this work a modification of system (1) has been used. The new system (2) has been produced by replacing the nonlinear term x_1x_2 in the third equation of system (1) with $x_1\sinh(x_2)$.

$$\begin{cases} \dot{x}_1 = x_2 - x_1 \\ \dot{x}_2 = -ax_1x_3 + x_4 \\ \dot{x}_3 = x_1\sinh(x_2) - 1 \\ \dot{x}_4 = -bx_2 \end{cases} \quad (2)$$

It can be found that system (2) has no real solutions and thus no equilibrium points, when a and b are nonzero. Therefore, any attractors of the system are hidden. This means that the system has a basin of attraction that does not intersect with small neighborhoods of any equilibrium point. This feature makes this kind of systems very useful in applications like chaotic cryptography or chaotic path planning because it increases the system's unpredictability.

The bifurcation diagram of the variable x_1, for a = 2.6, when the trajectories cut the plane x_2 = 0 with dx_2/dt < 0, as well as the spectrum of system's Lyapunov, by changing the value of the parameter b, in order to investigate the dynamics of system (2), while keeping the initial conditions as $(x_1, x_2, x_3, x_4)_0$ = (0.1, 0.2, 0.5, 0.2), are depicted in Fig. (1). So, the proposed system (2) is integrated numerically using the classical fourth-order Runge–Kutta integration algorithm. For each set of parameters used in this work, the calculations are performed using variables and parameters in extended precision mode. Also, the Lyapunov exponents are calculated by using the Wolf's algorithm [20].

As it can be seen from the bifurcation diagram (Fig. 1(a)) the system has a rich dynamical behavior. There are some small windows of limit cycles and big regions of chaos when varying the parameter b. Also, other interesting phenomena related with chaos, such as quasiperiodic route to chaos as well as crisis phenomena are observed.

The spectrum of the three largest Lyanpunov exponents (Fig. 1(b)) confirms the system's dynamical behavior as it was discovered from the bifurcation diagram. Furthermore, system (2) exhibits the maximum hyperchaotic attractor, for b = 0.144 (Fig. 2), where the Lyapunov exponents are (LE_1, LE_2, LE_3, LE_4) = (0.3401, 0.0220, 0, −1.8040) and the Kaplan–Yorke dimension is D_{KY} = 3.2003, which is greater than system's (1).

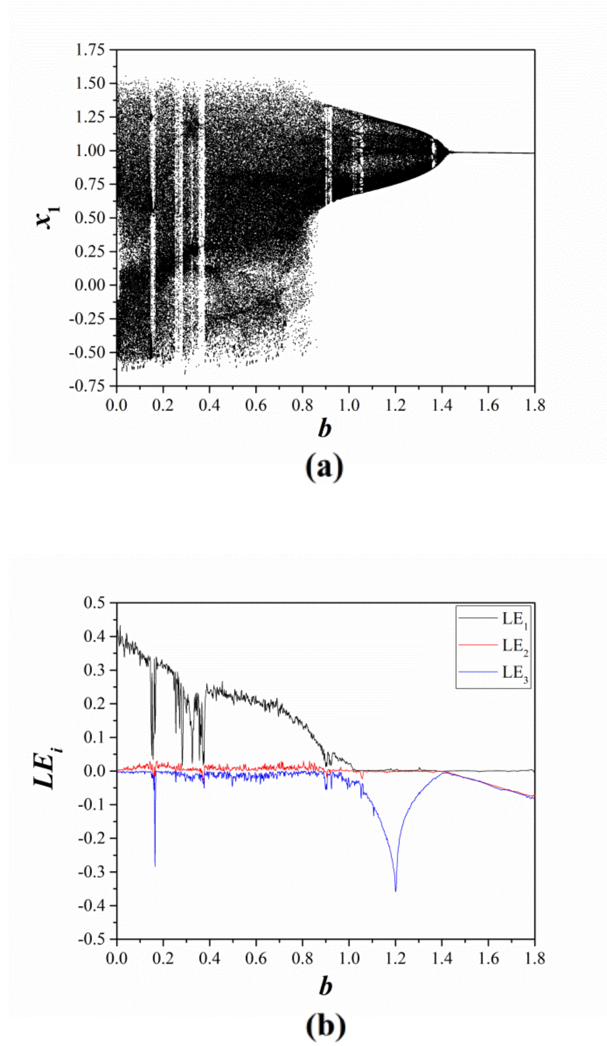

Fig. 1 (a) Bifurcation diagram and (b) the spectrum of the three largest Lyapunov exponents, of the system (2), for a = 2.6, when varying the value of the bifurcation parameter b from 0 to 1.8

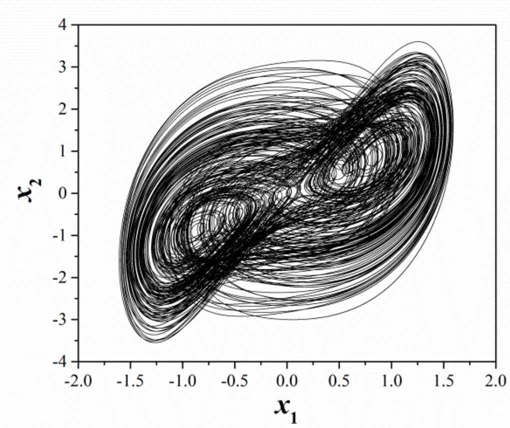

Fig. 2 System's (2) hyperchaotic attractor for a = 2.6 and b = 0.144 in x_1-x_2 plane

III. The Mobile Robot Model

Many works on kinematic control of chaotic robots are based on a typical differential motion with two degrees of freedom, composed by two active, parallel and independent wheels and a third passive wheel [21]. The active wheels are independently controlled on velocity and rotation sense.

In this work the aforementioned mechanism has been adopted for the kinematic control of the robot. So, the proposed mobile robot's motion is described by the linear velocity $v(t)$ [m/s], the angle $\theta(t)$ [rad] describing the orientation of the robot, and the angular velocity $w(t)$ [rad/s]. The linear velocity provides a linear motion of the medium point of the wheels axis, while the direction velocity provides a rotational motion of the robot's over the same point. In Fig. 3 the description of the robot motion on a plane is shown. The robot's motion control is described by the following system equations:

$$\begin{pmatrix} \dot{X}(t) \\ \dot{Y}(t) \\ \dot{\theta}(t) \end{pmatrix} = \begin{pmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v(t) \\ w(t) \end{pmatrix} \tag{3}$$

where, $\{X(t), Y(t)\}$ is the robot's position on the plane. Also, $v(t) = 1/2(v_r(t) + v_l(t))$ is the linear velocity of the robot, $v_r(t)$ is the linear velocity of the right wheel, $v_l(t)$ is the linear velocity of the left wheel, $w(t) = (v_r(t) - v_l(t))/L$ is the angular velocity and L is the distance between the two wheels. Furthermore, it must be mentioned that in the case in which the robot reaches the borders of the terrain, the robot stops and waits the next direction order to move.

The mobile robot navigation equations were solved numerically by using the fourth order Runge-Kutta algorithm. In the equations of linear velocity and angular velocity of the robot, linear velocity of right wheel ($v_r(t)$) and linear velocity of left wheel ($v_l(t)$) are replaced by chaotic signals $x_3(t)$ and $x_4(t)$, respectively. Then the mobile robot's equations become:

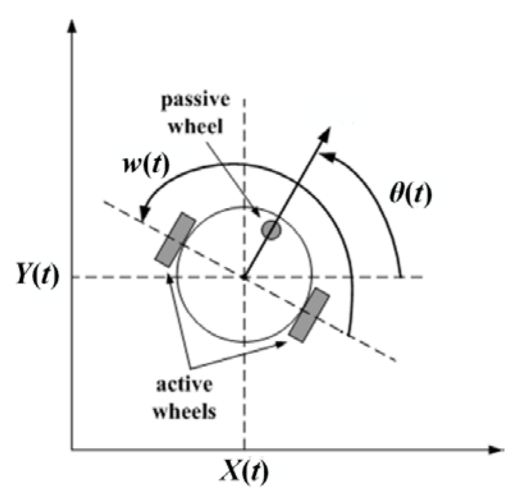

Fig. 3 The description of the robot motion on a plane

$$v(t) = \frac{1}{2}\left(x_3(t) + x_4(t)\right) \tag{4}$$

$$w(t) = \frac{x_3(t) - x_4(t)}{L} \tag{5}$$

By combining systems (2) and (3), the following dynamics is obtained.

$$\begin{cases} \dot{x}_1 = x_2 - x_1 \\ \dot{x}_2 = -a x_1 x_3 + x_4 \\ \dot{x}_3 = x_1 \sinh(x_2) - 1 \\ \dot{x}_4 = -b x_2 \\ \dot{X} = v(t)\cos\theta(t) \\ \dot{Y} = v(t)\sin\theta(t) \\ \dot{\theta} = w(t) \end{cases} \tag{6}$$

The system (6) describes the mobile robot navigation based on the new hyperchaotic system (2).

IV. Numerical Simulations

To test the proposed control strategy of a mobile robot the results of the numerical simulations are presented in details in this paragraph. For this reason the terrain coverage, using the known coverage rate (C), which represents the effectiveness, as the amount of the total surface covered by the robot running the algorithm, is used.

The coverage rate (C) is given by the following equation:

$$C = \frac{1}{M} \cdot \sum_{i=1}^{M} I(i) \tag{7}$$

where, $I(i)$ is the coverage situation for each cell in which the terrain has been divided [22]. This is defined by the following equation:

$$I(i) = \begin{cases} 1, & \text{when the cell } i \text{ is covered} \\ 0, & \text{when the cell } i \text{ is not covered} \end{cases} \tag{8}$$

where, $i = 1, 2, ..., M$. The robot's workplace is supposed to be a square terrain with dimensions $10\text{m} \times 10\text{m} = 100\text{m}^2$. The dimension of each cell is $0.25\text{m} \times 0.25\text{m} = 0.0625\text{m}^2$. Furthermore, a second interesting evaluation criterion is the coverage time of the terrain, which is the time or the minimum number of robot's motion commands, in order the entire terrain to be covered.

The chaotic mobile robot trajectories shown in Fig. 4 can be obtained by solving the system dynamics (6) by using the fourth order Runge-Kutta algorithm, for $N = 10{,}000$ iterations with step $\Delta t = 0.01$ and by taking the parameter values and initial conditions as: $a = 2.6$ and $b = 0.144$, $(x_1, x_2, x_3, x_4)_0 = (0.1, 0.2, 0.5, 0.2)$, $(X, Y, \theta)_0 = (5, 5, 0)$ and $L = 0.08$m.

A more clear view in this figure revealed the basic problem, which this approach introduces to the robot motion. The robot makes many spiral movements, consuming a lot of energy and delaying the achievement of its mission, which is to cover as much surface as possible in a less time. This problem has also been presented in a number of related works [4], [14], [18].

In this work a relative simple solution to this problem is proposed. In order to avoid the autocorrelation of the chaotic signals (x_3 and x_4), which is in fact responsible for this kind of motion, the sampling of the signals in various time periods has been done. So, in Fig. 5 the color scale maps of the terrain's $40 \times 40 = 1600$ cells versus the number of visiting times for M = 10,000 motion commands and for various normalized sampling times (every 1, 100, 200, 300, 400, 500 points) are presented.

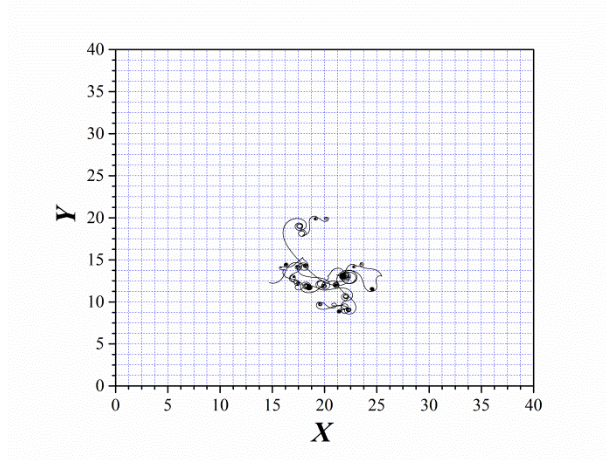

Fig. 4 The mobile robot's motion path for M = 10,000 motion commands

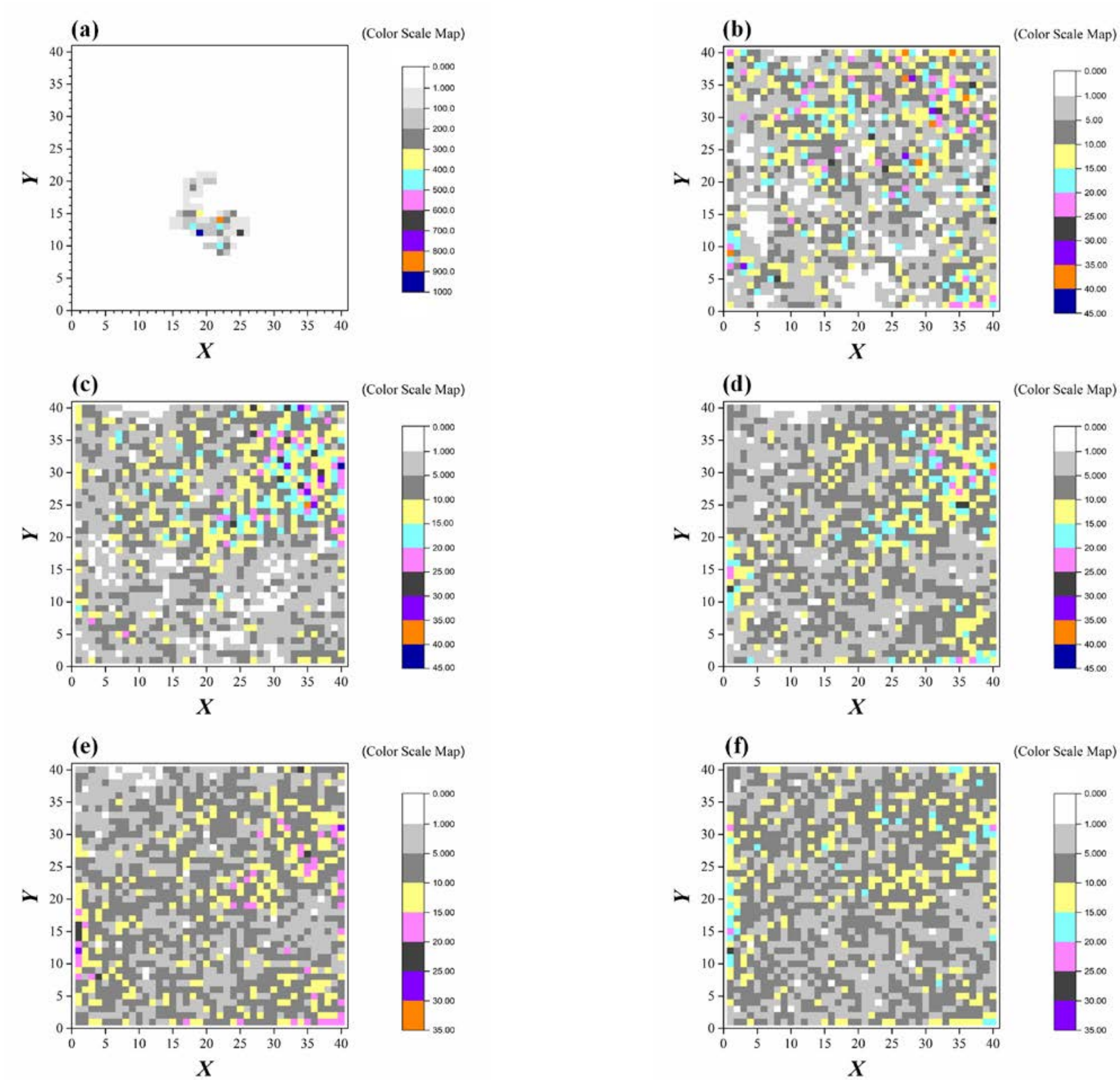

Fig. 5 Color scale maps of the terrain's cells versus the time of visiting for M = 10,000 motion commands and for various sampling times (a) 1, (b) 100, (c) 200, (d) 300, (e) 400, and (f) 500 points. White color corresponds to the uncovered cells.

The number of the uncovered cells as well as the coverage rate after M = 10,000 motion commands, for the chosen sampling times, is presented in Table 1. Also, the coverage rate versus the number of motion commands, for the robot with the proposed chaotic motion controller, is presented in Fig. 6.

Table 1. The number of uncovered cells and the coverage rate in the chosen sampling times.

Sampling Time (no. of points)	**Uncovered Cells**	**Coverage Rate (%)**
1	1,533	4.19
100	194	87.88
200	132	91.75
300	57	96.44
400	35	97.81
500	16	99.00

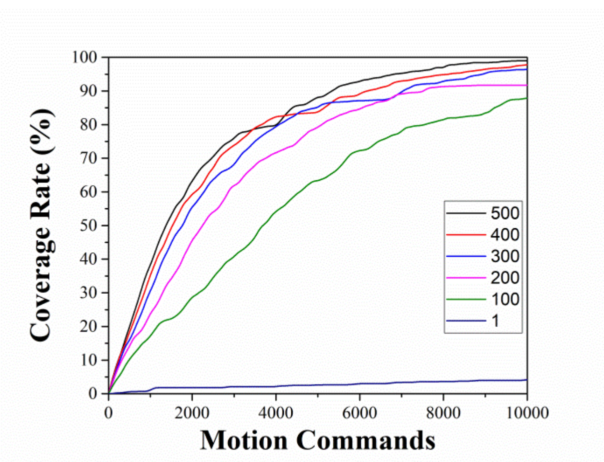

Fig. 6 Coverage rate versus the number of motion commands, for the various sampling number of points

V. Conclusion

In this work an improved chaotic motion controller for a mobile robot was presented. For the aim of this approach a 4D hyperchaotic dynamical system was chosen. This approach is followed in order to generate the most unpredictable trajectory, as well as the trajectory with the higher coverage rate of a specific terrain. The results of the comparative study for various sampling times prove that for the same motion commands the coverage rate increases with the increase of the sampling time. As a consequence when the sampling time is large (the sampling was done every 500 points) almost the whole terrain was covered. As a future work the experimental study of the specific approach has been planned.

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