A Novel Adaptative Mesh−Free Method using Prandtl's Equation in the Boundary Layer

Asmae Mnebhi-Loudyi,El Mostapha Boudi,and Driss Ouazar

Abstract—In this paper,we propose a new adaptative Meshfree Method applied in the boundary layer. Traditional methods such as MEF (Finite Elements Method), VFM (Volume Finite Method), and FD(Finite Difference Method) are based on the extraction of too many parameters, that have an influence in the resulting accuracy and also in time processing. Only one parameter is used in the proposed method to compute Meshless nodes which is related to a local RBF method. Generation of nodes is done using a so− called "Prandtl equations". Furthermore, by means of Radial Basis Function in Finite Difference method(RBF-FD) the generated nodes will be modeled into independent models: flat plate, and circular disk. This last, will provide nodes and fluid flow velocity. Experimental results demonstrate that the proposed Meshless method surpass traditional methods.

Keywords—Mesh-Free, Meshless, Traditional methods, Prandtl equation, local RBF, and RBF-FD.

I. Introduction

MESH-FREE or Meshless method is a new special category of numerical simulation methods for physics problems. The principal idea in this method is to treat the problems without discretization of the domain, and use only one parameter, which is the scattered-nodes. The main characteristic and the object of this methods is to facilitate the resolution of the problems in a large deformation or with a complex geometry. Moreover, it avoid cumbersome calculations and remeshing, than the traditional method such as the Finite Difference method(FD), Volume Finite method(VF), and the Finite Element methods(MEF).
The study showed the Meshless method to be very reliable because they offer better approximations of physical problems. The Meshless have used for the first time in order to solve polynomial interpolation problems [2] and solve partial differential equations [13], [12].

However, in our work, we will only present the methods using the Basis Radial Functions in Finite Difference. The Radial Basis Function (RBF) is the strongest formulation that is known by the Kansa [5] method and it is very popular in the Meshless community. It has different types of Radial Basis Function (RBF). In this work, we are interested in the Multi-Quadric (MQ) functions. It is one type of Radial Basis Functions (RBF).

The advantages of (RBF) are the simplicity of their algorithm, implementation, precision, and efficiency. Although, they have limitations in large-scale applications because of the dense and bad conditioned coefficient method when the number of nodes rises [2].

It is for this reason, to overcome the disadvantages of the overall, the researchers have suggested to use local RBFs. The local RBFs methods have been invented by different authors and they have different types of local RBFs. For example, the local Radial Basis Function Differential Quadrature method (RBF-DQ), which was proposed by Shu [3], [4], and the Radial Basis Function of Finite Differences (RBF-FD), which was introduced by Tolstykh in 2000 [1] and Wright in 2006 [3], who found it very efficient to solve the fluid flow problems, such as convection−diffusion equations [8], [7], the incompressible Navier−Stokes equations [4], [6], elliptic equations [14], Hamilton−Jacobi equations [9], and shallow water equations [10].

In this paper is organized as follows: In the first part of this work, we present the Meshless method such as (RBF-FD) method. Next, in the second, we propose the problem which will be treated later in the fourth and fifth part. In the fourth part, they defined the Navier-Stokes where it considered the laminar boundary layer, Newtonian fluids, incompressible in (2D) flow over a flat plate and circular disk at zero degree incidence, with these approximations we obtain the Prandtl's equation. After that, the numerical result obtained will be treated by Matlab for Meshless and will be compared by Ansys (Fluent) [16]. Finally, we finish my work by a conclusion.

II. The formulation of the RBF Method in Finite Difference Mode (RBF-FD)

A. Definition

The RBF-FD method is to approximate the derivative of the function at a point that is based on the linear combination.

B. The formulation of the Meshless method (RBF - FD) :

In the first order (1D), the classical finite difference method for each node x_i the function u which corresponds to this node

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can be written through the following equation:

$$\frac{d^k u(x_i)}{dx^k} \cong \sum_{j=1}^{N} W_{i,j}^k u(x_j)\ . \tag{1}$$

where: $W_{i,j}^k$ are the weights coefficients and are computed by the polynomial interpolation or Taylor series. N are numbers of nodes which are found in the domain and $u(x)$ is a function of node x_i.

The standard RBF interpolation for a set of distinct points x_j; $j = 1, 2...N$ is given by [2]:

$$u(x) = \sum_{j=1}^{N} W_j \phi(\|x - x_j\|) + \beta \tag{2}$$

where: $\phi(\|x - x_j\|)$ is the radial basis function, $\|.\|$ is the euclidean norm between $x = (x_i, y_i)$ and $x_j = (x_j, y_j)$; the expension coefficients W_j and β are determined from the interpolation conditions and the constraints.

The equation (2) was written in the Lagrange by following form:

$$u(x) = \sum_{j=1}^{N} \chi(\|x - x_j\|) u(x_j) \tag{3}$$

where the conditions satisfies :

$$\chi(\|x - x_j\|) = \begin{cases} 1, & if \quad k = j \\ 0, & if \quad k \neq j \end{cases} \qquad k = 1, 2, ..., N$$

The differential operator L at node x_1 is given by the following form:

$$Lu(x_1) = \sum_{j=1}^{N} L\chi(\|x_1 - x_j\|) u(x_j) \tag{4}$$

The RBF−FD weights are obtained by using the equations(1) and (6):

$$W_{1,j}^{(L)} = L\chi(\|x_1 - x_j\|) \tag{5}$$

In the case(2D), the weights are obtained by solving the following linear system [2], [11] and [10]:

$$\begin{bmatrix} \phi & e \\ e^T & 0 \end{bmatrix} \begin{bmatrix} W \\ \beta \end{bmatrix} = \begin{bmatrix} L\phi_1 \\ 0 \end{bmatrix} \tag{6}$$

where: $\phi_{(i,j)} = \phi(||x_1 - x_j||)$, $i, j = 1, 2, .., N$, $e_i = 1, 2, .., N$, L corresponding of differential operator, and $L\phi_1$ represents the column vector, $L\phi_1 = [L\phi||x - x_1||||x - x_2||...||x - x_N||]^T$ at node x_1 and β is a scalar parameter whose imposes the condition:

$$\sum_{i=1}^{N} W_{1,j}^L = 0 \tag{7}$$

The linear system can be written by the following equation:

$$[A][u] = [F] \tag{8}$$

The objective is to calculate the matrix, u is the vector of the unknown function at all the interior nodes, the matrix $[A]$ (must be invertible) which depends on the type of function(RBF), where the $[A] = \begin{bmatrix} \phi & e \\ e^T & 0 \end{bmatrix}$, $[u] = \begin{bmatrix} W \\ \beta \end{bmatrix}$ and$[F] = \begin{bmatrix} L\phi_1 \\ 0 \end{bmatrix}$.

III. Problem of position

Consider the laminar flow of a fluid, the boundary layer over a flat plate at zero degree incidence with a speed at U_∞ and the boundary layer over the circular disk. The Prandtl′s equation reduced by the Navier−Stokes in the Cartesian coordinate and in the cylindrical coordinates. Then, try to solve these equations by the Meshless methods(RBF-FD).

IV. Prandtl′s equation

A. The boundary Layer

Consider the flow of a fluid with a speed at U_∞, and a temperature T_∞ over a flat plate at a temperature T_p. In the vicinity of the wall where the effects of viscosity have important and cannot be neglected in all regions. The values of speed and temperature are different from those of the potential flow and vary depending on the distance to the wall y. The zone gradients of speed and temperature is called the boundary layer. It results from an exchange of momentum and heat between the fluid and the wall. The thickness is usually small compared to the entire flow. They have two types of boundary layer: the dynamic boundary layer and the boundary layer thermal. In this article, they worked on the dynamic boundary layer and they nommed Prandtl equation′. It never changed with the temperature; and as shown in Fig.1.

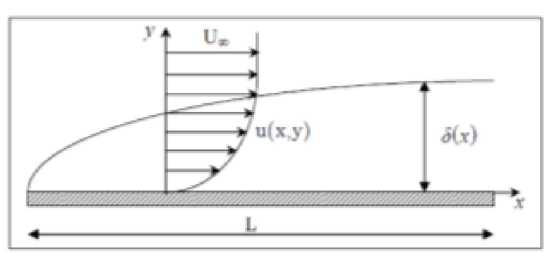

Fig. 1. Laminar Boundary Layer For Flow over a flat plate

B. Prandtl′s equation

1) Prandtl′s equation flow over a flat plate: The fundamental concept of the boundary layer was suggested by L. Prandtl (1904) [15], it was defined the boundary layer of fluid developing in flows with very high Reynolds numbers Re.
Navier−Stokes equations in Cartesian Coordinates (2D):

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{9}$$

Momentum in direction(Ox):

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})\ . \tag{10}$$

Momentum in direction(Oy):

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})\ . \tag{11}$$

where (x,y) are the Cartesian coordinates with the associated fluid velocities u, v and ρ is the fluid density, p is the fluid pressure and ν is the kinematic viscosity.
This important assumption reduces the Navier-Stokes equations yet again. Prandtl used the concept of dimensional analysis from which he found the similarity parameters.
The Prandtl's hypotheses are:
•In the boundary layer region the geometric and kinematic distortions are of the similar order.
•For the longitudinal momentum, pressure and viscosity forces are of the same order in the boundary layer region when plotted against nondimensional coordinates.

The Prandtl equation for the 2D of stationary and incompressible flow are:

$$\begin{cases}\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\\ u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}\\ \frac{\partial p}{\partial y}=0\end{cases} . \tag{12}$$

In the application of the Prandtl model and since the longitudinal of pressure is constant at the edge of the boundary layer. The speed for the boundary layer is governed by the equations :

$$\begin{cases}\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\\ u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}\end{cases} . \tag{13}$$

with the boundary conditions are:

$$\begin{cases}u=U_\infty\ \ ,\quad for\quad x=0\\ u=v=0\ \ ,\quad for\quad y=0,\ \ x\succ 0\\ u=U_\infty\ \ ,for\quad y\to\infty,\ \ all\ x\end{cases} . \tag{14}$$

2) Prandtl′s equation laminar flow over a Circular disk: : The circular plate of radius to the axial symmetry. The flow due to a continuous circular disk which associated system of coordinates.
The Navier-Stokes in Cylindrical Coordinates:
Continuity equation:

$$\frac{1}{r}\frac{(r\partial u)}{\partial r}+\frac{1}{r}\frac{(\partial v)}{\partial\theta}+\frac{\partial w}{\partial z}=0 \tag{15}$$

Momentum in direction(Or):

$$u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial\theta}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{1}{r^2}\frac{(\partial^2 u)}{\partial\theta^2}+\frac{\partial^2 u}{\partial z^2}-\frac{2}{r^2}\frac{\partial v}{\partial\theta})\ . \tag{16}$$

Momentum in direction(Oθ):

$$u\frac{\partial v}{\partial r}+\frac{uv}{r}+\frac{v}{r}\frac{\partial v}{\partial\theta}+w\frac{\partial v}{\partial z}=-\frac{1}{r}\frac{1}{\rho}\frac{\partial p}{\partial\theta}+\nu(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{1}{r^2}\frac{(\partial^2 v)}{\partial\theta^2}+\frac{\partial^2 v}{\partial z^2}+\frac{2}{r^2}\frac{\partial u}{\partial\theta})\ . \tag{17}$$

Momentum in direction(Oz):

$$u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial w}{\partial\theta}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{(\partial^2 w)}{\partial\theta^2}+\frac{\partial^2 w}{\partial z^2})\ . \tag{18}$$

where $(r,,\theta,z)$ are the Cylindrical Coordinates with the associated fluid velocities u, v,w and ρ is the fluid density, p is the fluid pressure and ν is the kinematic viscosity.

Boundary Layer in Cylindrical coordinates:

Consider the flow of a fluid with a speed at U_0. In the vicinity of the wall where the effects of viscosity have important and cannot be neglected in all regions. The potential flow and vary depending on the distance to the wall z. The zone gradients of speed and temperature is called the boundary layer. It results from an exchange of momentum and heat between the fluid and the wall. The thickness is usually small compared to the entire flow. In this article, they worked on the dynamic boundary layer and they nommed Prandtl equation′ in the circular plate and as shown in Fig.2.

The circular plate of radius to the axial symmetry. The flow due to a continuous circular plate which associated system of coordinates.

For the 2D of stationary and incompressible flow are:

$$\begin{cases}\frac{\partial(ru_r)}{\partial z}+\frac{\partial(ru_\theta)}{\partial z}=0\\ u_r\frac{\partial u_r}{\partial z}+\nu\frac{\partial u_r}{\partial r}=\nu\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u_r}{\partial r})\end{cases} . \tag{19}$$

Let R be the radius of the disk and u_0 be the velocity in the flow. The boundary-layer equations in the case of a circular plate and where the axymetric $\frac{\partial}{\partial\theta}=0$. The longitudinal pressure is constant at the edge of the boundary layer. The

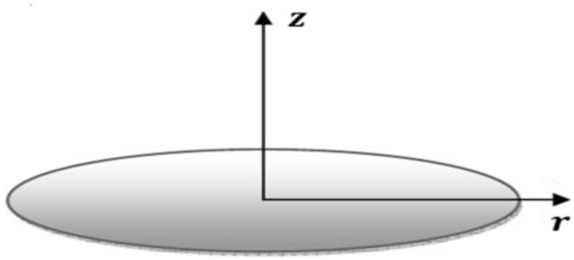

Fig. 2. representative scheme1:circular plate

speed for the boundary layer is governed by the system takes the form:

$$\begin{cases} \frac{\partial(ru_r)}{\partial r} + \frac{\partial(ru_z)}{\partial z} = 0 \\ u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_r}{\partial z} = \nu \frac{\partial^2 u_r}{\partial z^2} \end{cases} . \quad (20)$$

The circular disk geometry is shown in Fig.3.

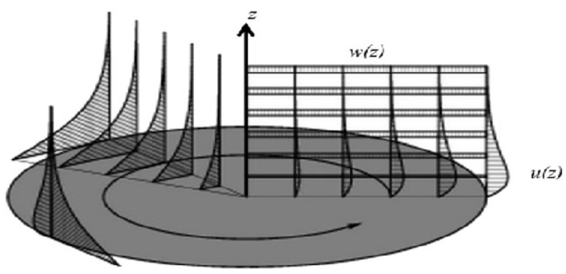

Fig. 3. Flow structure over a Circular Disk

where $x = Rcos\theta$, $z = Rsin\theta$ and $R = \sqrt{x^2 + z^2}$. Since we are dealing with an axisymmetric flow the problem can be further simplified by focusing only on the radius of the shape.

V. Adaptative Meshless method (RFD-FD) based on Prandtl's equation

A. Adaptative Meshless method (RFD-FD) based on Prandtl′s equation flow over a flat plate:

For to solve the equation (9) by Meshless method (RBF-FD) on Prandtl's equation is given by:

$$\sum_{j=1}^{N} W_{i,j}^{(x)} u_j + \sum_{j=1}^{N} W_{i,j}^{(y)} v_j = 0 . \quad (21)$$

where: N is the total number of nodes and boundaries in the support region for the node x_i and $W_{i,j}^{(x)}$, $W_{i,j}^{(y)}$, are the weights coefficients of the RBF-FD; u and v are the components of the velocity field.

which can be written in matrix form:

$$\begin{bmatrix} W_{1,1}^{(x)} & \cdots & W_{1,N}^{(x)} \\ \vdots & \ddots & \vdots \\ W_{N,1}^{(x)} & \cdots & W_{N,1}^{(x)} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} + \begin{bmatrix} W_{1,1}^{(y)} & \cdots & W_{1,N}^{(y)} \\ \vdots & \ddots & \vdots \\ W_{N,1}^{(y)} & \cdots & W_{N,1}^{(y)} \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} . \quad (22)$$

The equation (13) was solving by the Meshless method by:

$$u_i \sum_{j=1}^{N} W_{i,j}^{(x)} u_j + v_i \sum_{j=1}^{N} W_{i,j}^{(y)} v_j = \nu \sum_{j=1}^{N} W_{i,j}^{(yy)} u_j . \quad (23)$$

Storing the vector u and v, are given by:

$$-u_i \sum_{j=1}^{N} W_{i,j}^{(x)} u(x_j) + \nu \sum_{j=1}^{N} W_{i,j}^{(yy)} u(x_j) = v_i \sum_{j=1}^{N} W_{i,j}^{(y)} v(x_j) . \quad (24)$$

By writing the preceding equation in matrix as the following forms:

$$-\begin{bmatrix} u_1 W_{1,1}^{(x)} & \cdots & u_1 W_{1,N}^{(x)} \\ \vdots & \ddots & \vdots \\ u_N W_{N,1}^{(x)} & \cdots & u_N W_{N,1}^{(x)} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} + \nu \begin{bmatrix} W_{1,1}^{(yy)} & \cdots & W_{1,N}^{(yy)} \\ \vdots & \ddots & \vdots \\ W_{N,1}^{(yy)} & \cdots & W_{N,1}^{(yy)} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} = \begin{bmatrix} v_1 W_{1,1}^{(y)} & \cdots & v_1 W_{1,N}^{(y)} \\ \vdots & \ddots & \vdots \\ v_N W_{N,1}^{(y)} & \cdots & v_N W_{N,1}^{(y)} \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_N \end{bmatrix} . \quad (25)$$

The previous equation is given by :

$$\begin{bmatrix} -u_1 W_{1,1}^{(x)} + \nu W_{1,1}^{(yy)} & \cdots & -u_1 W_{1,N}^{(x)} + \nu W_{1,N}^{(yy)} \\ \vdots & \ddots & \vdots \\ -u_N W_{N,1}^{(x)} + \nu W_{N,1}^{(yy)} & \cdots & -u_N W_{N,1}^{(x)} + \nu W_{N,1}^{(yy)} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} \begin{bmatrix} v_1 W_{1,1}^{(y)} & \cdots & v_1 W_{1,N}^{(y)} \\ \vdots & \ddots & \vdots \\ v_N W_{N,1}^{(y)} & \cdots & v_N W_{N,1}^{(y)} \end{bmatrix}^{-1} = \begin{bmatrix} v_1 \\ \vdots \\ v_N \end{bmatrix} . \quad (26)$$

where: N is the total number of nodes and boundaries in the support region for the node x_i and $W_{i,j}^{(x)}$, $W_{i,j}^{(y)}$, $W_{i,j}^{(xx)}$and$W_{i,j}^{(yy)}$ are the weights coefficients of the RBF-FD; u and v are the components of the velocity field.

B. Adaptative Meshless method (RFD-FD) based on Prandtl′s equation flow over a Circular Disk:

For to solve the equation (20) by Meshless method (RBF-FD) on Prandtl's equation is given by:

$$\frac{u_i}{r} + \sum_{j=1}^{N} W_{i,j}^{(r)} u_j + \sum_{j=1}^{N} W_{i,j}^{(z)} w_j = 0 . \quad (27)$$

where: N is the total number of nodes and boundaries in the support region for the node x_i and $W_{i,j}^{(x)}$, $W_{i,j}^{(y)}$, are the weights coefficients of the RBF-FD; u and w are the components of the velocity field.

which can be written in matrix form:

$$\frac{1}{r}\begin{bmatrix}u_1\\ \vdots\\ u_N\end{bmatrix}+\begin{bmatrix}W^{(r)}_{1,1} & \cdots & W^{(r)}_{1,N}\\ \vdots & \ddots & \vdots\\ W^{(r)}_{N,1} & \cdots & W^{(r)}_{N,1}\end{bmatrix}\begin{bmatrix}u_1\\ \vdots\\ u_N\end{bmatrix}+ \begin{bmatrix}W^{(z)}_{1,1} & \cdots & W^{(z)}_{1,N}\\ \vdots & \ddots & \vdots\\ W^{(z)}_{N,1} & \cdots & W^{(z)}_{N,1}\end{bmatrix}\begin{bmatrix}w_1\\ \vdots\\ w_N\end{bmatrix}=\begin{bmatrix}0\\ \vdots\\ 0\end{bmatrix}. \quad (28)$$

The second equation of (20) was solving by the Meshless method by:

$$u_i\sum_{j=1}^{N}W^{(r)}_{i,j}u_j+w_i\sum_{j=1}^{N}W^{(r)}_{i,j}w_j=\nu\sum_{j=1}^{N}W^{(zz)}_{i,j}u_j . \quad (29)$$

Storing the vector u and w, are given by:

$$-u_i\sum_{j=1}^{N}W^{(x)}_{i,j}u(x_j)+\nu\sum_{j=1}^{N}W^{(yy)}_{i,j}u(x_j)=w_i\sum_{j=1}^{N}W^{(y)}_{i,j}u(x_j) . \quad (30)$$

By writing the preceding equation in matrix as the following forms:

$$-\begin{bmatrix}u_1W^{(r)}_{1,1} & \cdots & u_1W^{(r)}_{1,N}\\ \vdots & \ddots & \vdots\\ u_NW^{(r)}_{N,1} & \cdots & u_NW^{(r)}_{N,1}\end{bmatrix}\begin{bmatrix}u_1\\ \vdots\\ u_N\end{bmatrix}+\nu \begin{bmatrix}W^{(yy)}_{1,1} & \cdots & W^{(yy)}_{1,N}\\ \vdots & \ddots & \vdots\\ W^{(yy)}_{N,1} & \cdots & W^{(yy)}_{N,1}\end{bmatrix}\begin{bmatrix}u_1\\ \vdots\\ u_N\end{bmatrix} = \begin{bmatrix}w_1\\ \vdots\\ w_N\end{bmatrix}\begin{bmatrix}W^{(z)}_{1,1} & \cdots & W^{(z)}_{1,N}\\ \vdots & \ddots & \vdots\\ W^{(z)}_{N,1} & \cdots & W^{(z)}_{N,1}\end{bmatrix}\begin{bmatrix}u_1\\ \vdots\\ u_N\end{bmatrix}. \quad (31)$$

The previous equation is given by :

$$\begin{bmatrix}-u_1W^{(r)}_{1,1}+\nu W^{(zz)}_{1,1} & \cdots & -u_1W^{(x)}_{1,N}+\nu W^{(zz)}_{1,N}\\ \vdots & \ddots & \vdots\\ -u_NW^{(r)}_{N,1}+\nu W^{(zz)}_{N,1} & \cdots & -u_NW^{(x)}_{N,1}+\nu W^{(zz)}_{N,1}\end{bmatrix} \begin{bmatrix}u_1\\ \vdots\\ u_N\end{bmatrix}\begin{bmatrix}u_1W^{(z)}_{1,1} & \cdots & u_1W^{(z)}_{1,N}\\ \vdots & \ddots & \vdots\\ u_NW^{(z)}_{N,1} & \cdots & u_NW^{(z)}_{N,1}\end{bmatrix}^{-1}=\begin{bmatrix}w_1\\ \vdots\\ w_N\end{bmatrix}. \quad (32)$$

where: N is the total number of nodes and boundaries in the support region for the node x_i and $W^{(r)}_{i,j}$, $W^{(z)}_{i,j}$, $W^{(rr)}_{i,j}$and$W^{(zz)}_{i,j}$ are the weights coefficients of the RBF-FD; u and v are the components of the velocity field.

VI. Numerical Results

A. *Example1*

1) Data: Consider the plate length $L = 40cm$, width $l = 10cm$. The fluid chosen is air at a temperature of $T = 15C$ with $\rho = 1.225kg.m^{-3}$, $U_{inf} = U(initial) = U = 1m/s$ and $\mu = 1.7894*10^{-5}Pa.s$. The speed of airflow around the plate is chosen so that the Reynolds number inferior of 10^6(laminar flow).

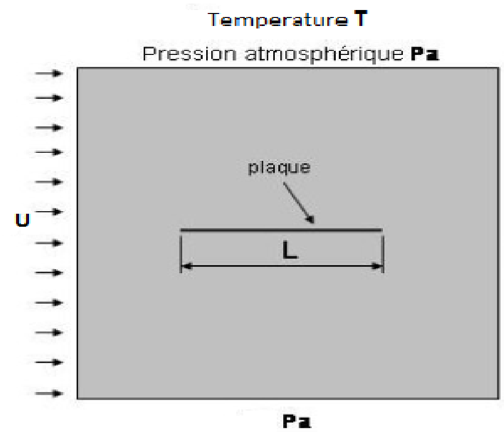

Fig. 4. representative scheme

2) Mesh.: The Meshless method is given by the Fig.3 with the following data :

Number of node (ox): $nx = 15$, Number of node (oy): $ny = 15$, Number of node: $N = 225$, Density: $\rho = 1.225\ kg.m^{-3}$, initial Vitesse: $U = 1\ m/s$, dynamic viscosity of the fluid: $\mu = 1.7894 * 10^{-5}\ kg/ms$, Reynolds' Number : $Re = 2.7383 * 10^4$, the plate length $L = 0.4\ m$, width $l = 0.1\ m$, step of mesh according to the x direction: $dx = L/(nx - 1)$, step of mesh according to the y direction $dy = l/(ny - 1)$.

3) Comparing of Results.: Comparing the result between the Meshless method and the Finite Element Mehod(MEF).

Finite element methods(MEF) results:

In the first Fig.5 the line has inclined in the middle and in the edge have small elements. In this case, the Node=400 and Elements:360. In the second Fig.5, by remeshing , the elements have equal, the Node=256 Element=225.
In this work, the Finite Element Method (MEF) have programming in ANSYS(Fluent). For mesh, we need remeshing as shown in Fig.5.

To solve the momentum equation, the Finite Element Method by Ansys [16] is obtained by the Fig.6.

Meshless results:

The resolution of the meshless method by the continuity equation is shown in the Fig.7 and Fig.8, and the node is a

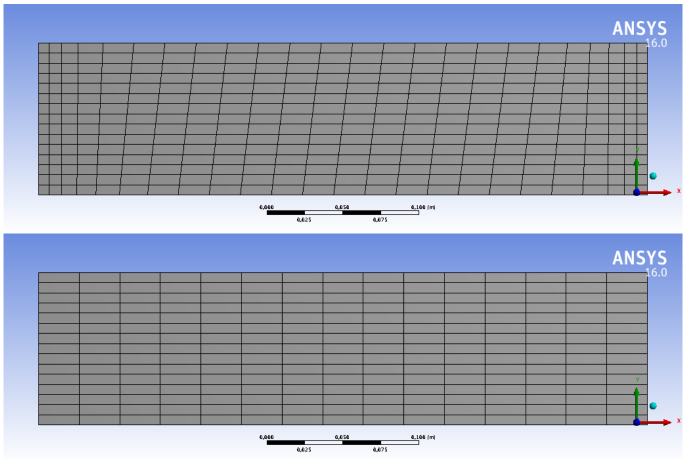

Fig. 5. **Mesh of the plate in MEF**

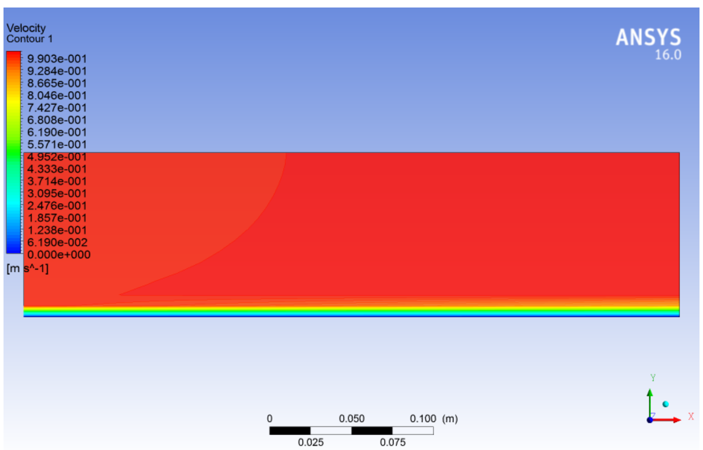

Fig. 6. Ecoulement velocity ANSYS

divised uniform.

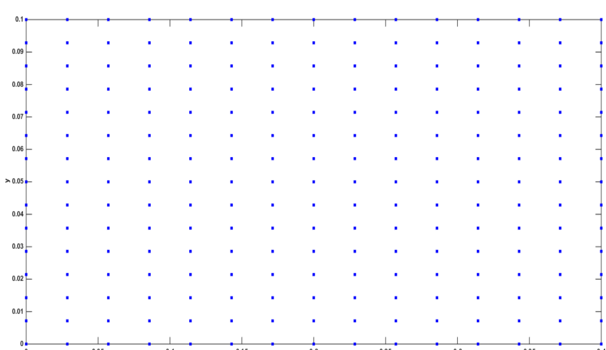

Fig. 7. mesh of the plate Node number $N = 225$

By the Meshless method, the momentum equation is programmed in Matlab, the result obtained is shown in the Fig.9.

Discussion:

The Meshless method proves to be frugal in terms of time and node, as well as it avoids remeshing compared by a finite element method which shows errors. The Meshless method provides precision comparing to the two methods.

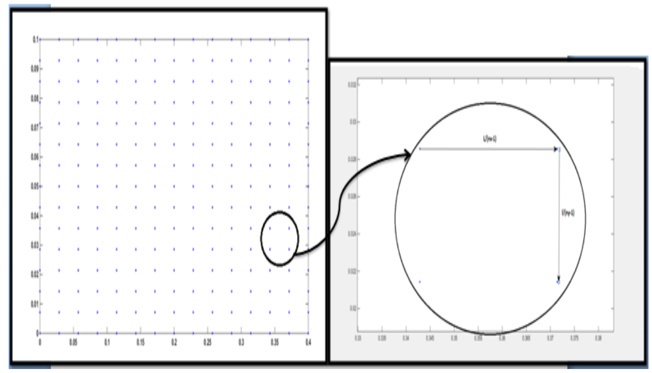

Fig. 8. mesh of the plate Node number $N = 225$

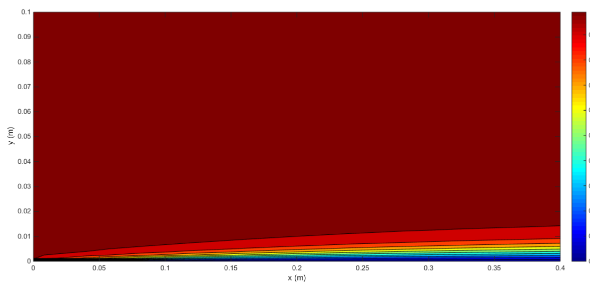

Fig. 9. Ecoulement velocity by Meshless.

B. *Example2*

1) Data.: Consider fluid flowing through a circular disk of constant radius as illustrated below. The radius $D = 2m$. Consider the inlet velocity to be constant over the cross-section and equal to $u0 = 1m/s$. Take density $\rho = 1.225\ kg.m^{-3}$, dynamic viscosity of the fluid: $\mu = 1.7894 * 10^{-5}\ kg/ms$.Show Fig.10.

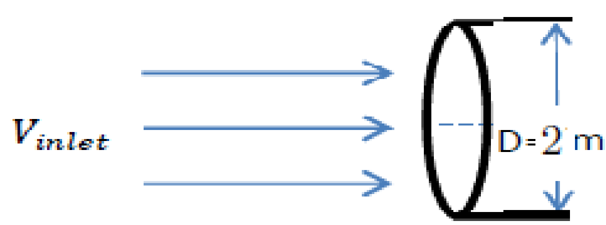

Fig. 10. representative scheme: numerical circular disk

2) Comparing of Results.: Comparing the result between the Meshless method and the Finite Element Method(MEF).

Finite element methods(MEF) results:

In this case, the Node=3392 and Elements:2842.
In this work, the Finite Element Method (MEF) have programmed in ANSYS(Fluent). For mesh, we need remeshing as shown in Fig.11.

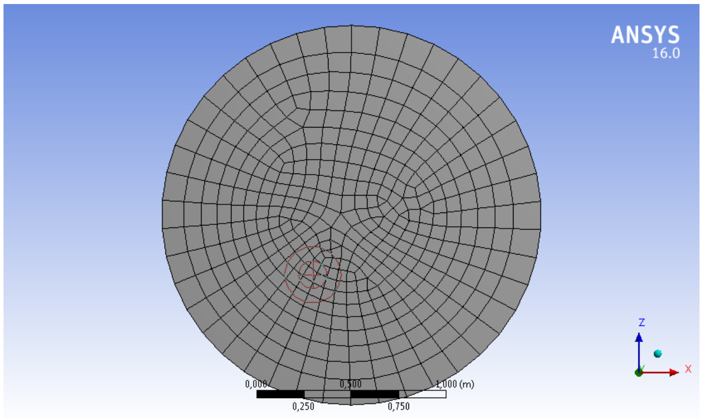

Fig. 11. **Mesh of the circular disk in MEF**

To solve the mumentum equation, the Finite Element Method by Ansys [16] are obtained by the Fig.12.

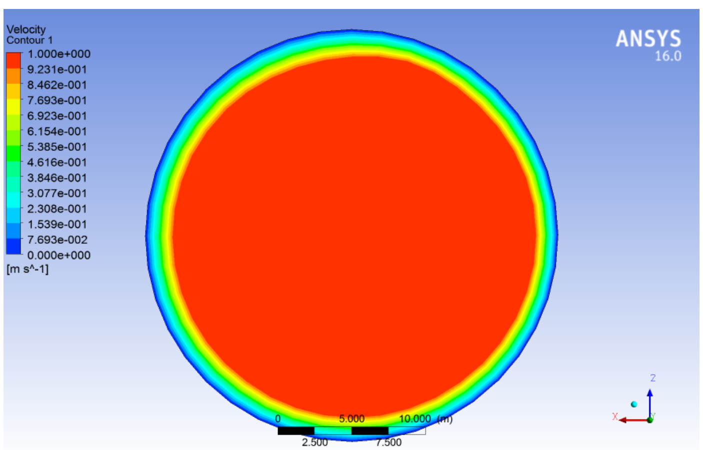

Fig. 12. Ecoulement velocity ANSYS of circular disk

Meshless results:

The resolution of the meshless method by the continuity equation is shown in the Fig.13, and the node $N = 88$ is a divised uniform.

By the Meshless method, the mumentum equation is programmed in Matlab, the result obtained is shown in the Fig.14 .

Discussion:
The Meshless method proves to be frugal in terms of time and node, as well as it reduced number of nodes shown especially in the circular disk compared by a finite element method which shows more errors in figure 11.

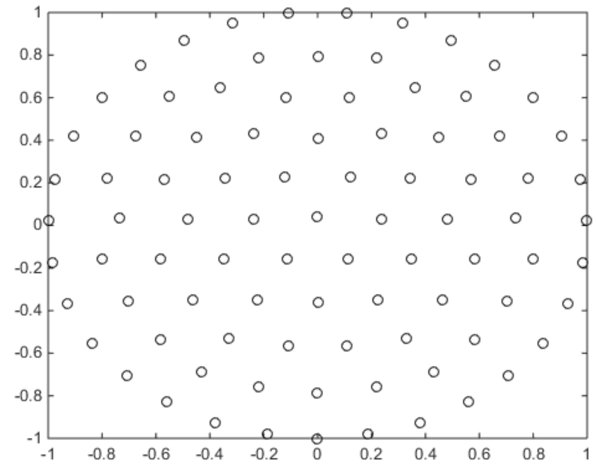

Fig. 13. mesh of circular disk node number $N = 88$

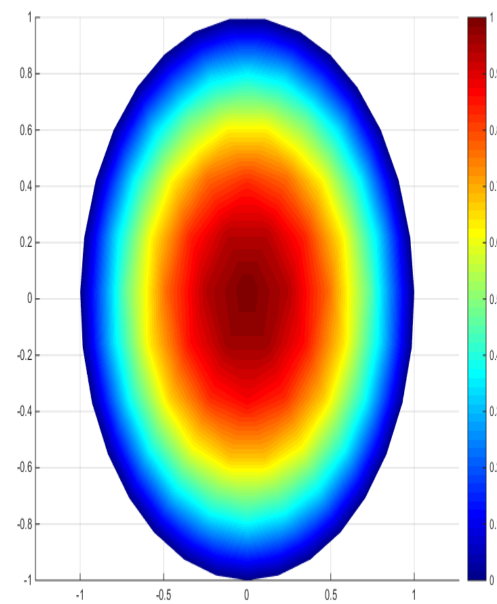

Fig. 14. Ecoulement velocity circular disk by Meshless

The Meshless method is the method which specifies more precision comparing of the two methods.

VII. Conclusion

In this paper, we presented a new Mesh-Free method based on Prandtl's equation and a local RBF method. The main objectif is to extract nodes with less parameters. In our case, we use only one parameter. In one hand, Prandtl's equation was responsible to extract the raw nodes. In the other hand, RBF−FD modeled these equations in order to compute scattered nodes and fluid flow velocity, which ensure a low complexity and good precision. The obtained average velocity was $v = 0.9\ m/s \simeq 1\ m/s$, which considered a good performance. Simulation results showed that the proposed

Meshless Method exceed others compared the conventional methods as well as MEF.

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