On the structural assessment of masonry vaults and domes

Alessandro Baratta

Abstract—Basic properties of masonry do not allow to rely on tensile strength, and flexural strength cannot be trusted on. Nevertheless in 2D walls and in double curvature vaults, a particular organization of the vault apparatus can in some instances, through the action of compression and friction, give place to a equilibrium pattern including tension, which explains the unexpected good performance of some walls and cupolas.

Keywords—Domes, Masonry texture, Membrane equilibrium, Structural assessment

I. INTRODUCTION

MASONRY is the main material mankind has exploited to provide itself a shelter. Homes, temples, offices, markets and so on are built by some kind of masonry since the beginning of civilization. Walls are the main way loads are transferred to foundations and to underlying soil, but horizontal floor structures require some more skill, since masonry, due to its very poor, unreliable, inhomogeneous and time-degrading tensile strength, is not able to resist bending moments.

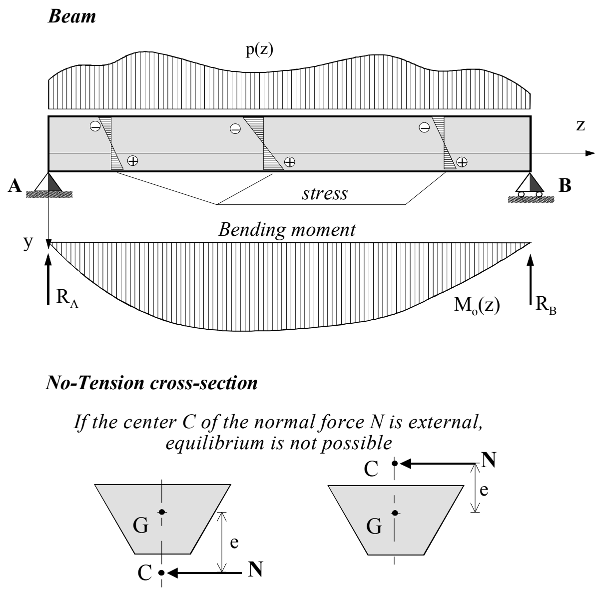

Fig. 1: Beam requires compression/tension stresses to work

This is the reason why masonry buildings are often complemented by wood or, more recently, steel systems to cover spaces, providing beam elements resisting by pure flexure.

In a simply supported beam, equilibrium is sustained by bending moments, which in turn require that compression stresses are coupled with tension (Fig. 1a); so if some beam has ever been attempted it is soon realized that failure is inexorable.

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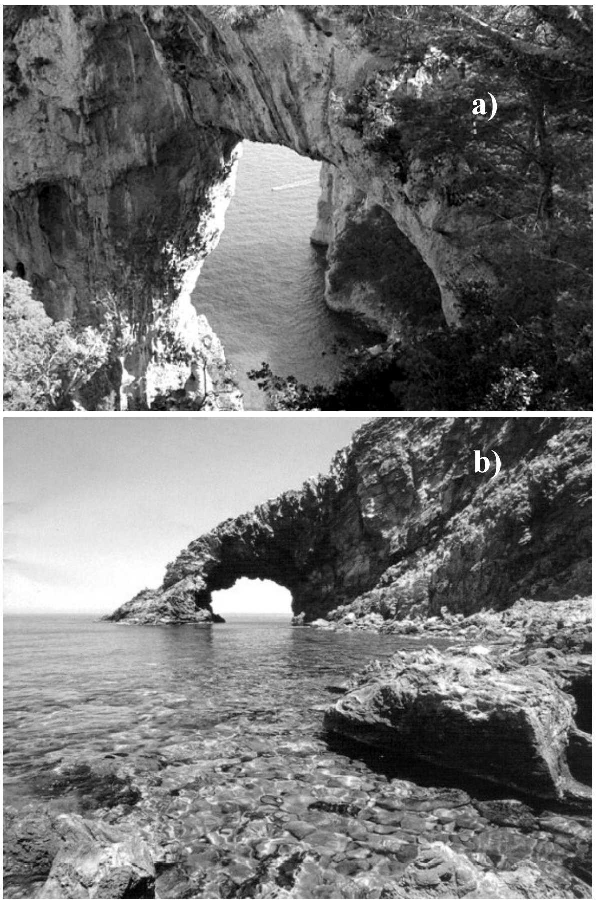

Fig. 2: Natural Arches: a) Capri (Italy); b) The "Elephant Arch" in Pantelleria (Italy)

On the other side, early ante-literam architects learned from nature that it is possible to overpass empty spaces by stones: natural arches are encountered everywhere in the world (Fig. 2). So, the first character that is acquired at glance is that the masonry should be “curve”. If one considers a curved beam (the *pseudo-arch* in Fig. 3) one finds that the only difference is the insurgence of a compressive normal force on the cross-section, which mitigates, but does not cancel, the need for tension (Fig. 3).

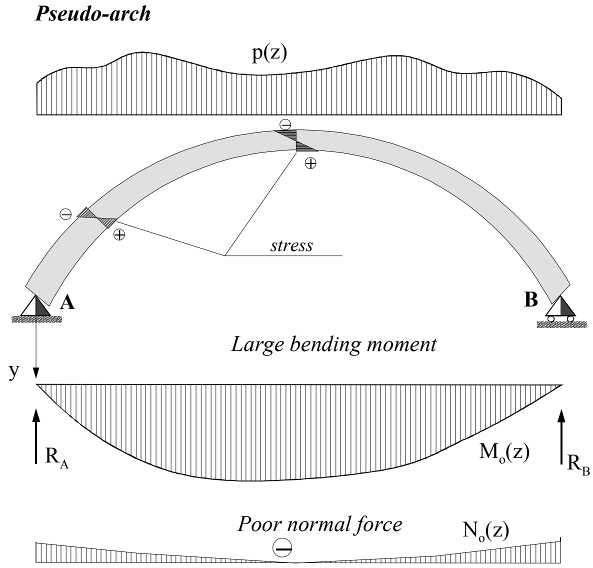

Fig. 3: Curvature of the beam does not help by itself to cancel tension

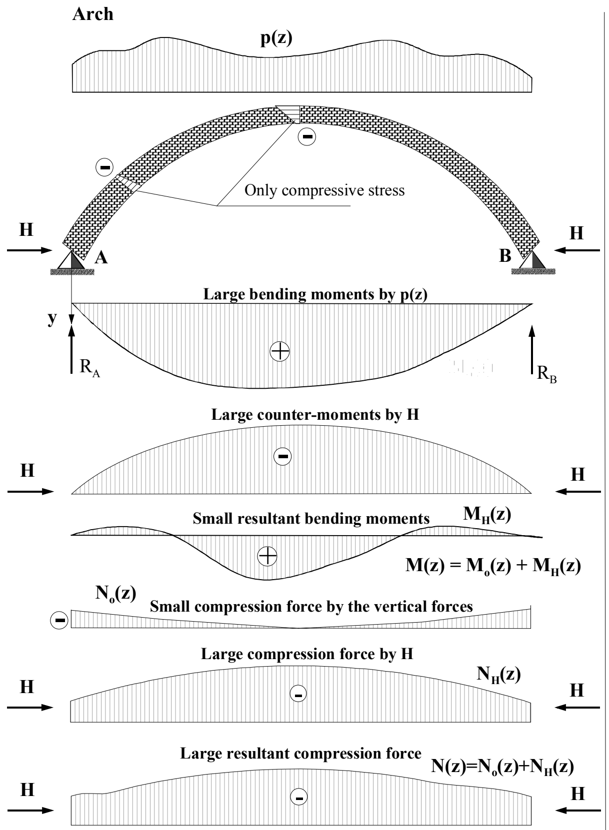

Fig. 4: True arch: the action of the thrust force H increases the normal force and mitigates the bending moment. A small eccentricity $e = M/N$ results

The bending moments remain the same, the normal force is small, the eccentricity is large and the center of the force is generally out of the cross section: equilibrium cannot subsist unless tension is resisted where it is necessary in the structure.

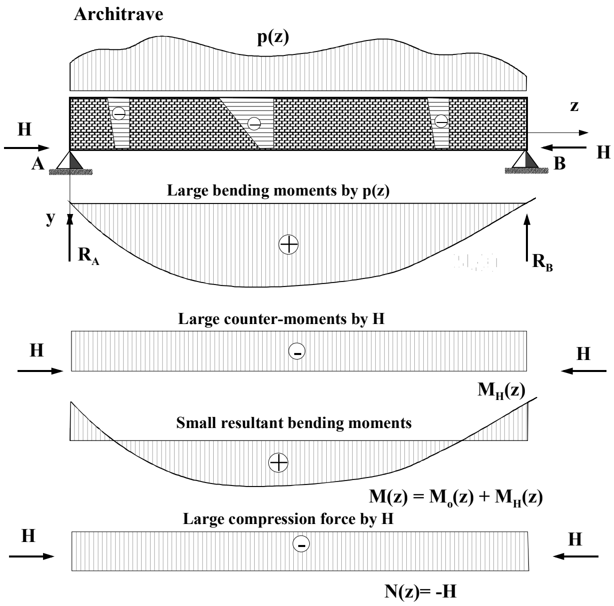

Fig. 5: Similar to the arch behaviourt, a beam with horizontally contrasted supports results in an *architrave.* The eccentricity is small and the center C of the normal force is in the interior of the cross-sections.

If the system is horizontally fixed at both ends, a new entity is born, the *horizontal thrust H*, which drastically changes the static regime and a true *arch* is realized. The thrust force is the key for the arch statics. It acts, in fact, producing larger compressive normal forces and strong counter-moments (Fig. 4), thus mitigating the flexure and enhancing compression. The effect is quite independent on apparent curvature: the important fact is that a horizontal force exists able to produce counter-moments with respect to active load. The architrave is nothing else than an arch with the appearance of a beam (Fig. 5). The typical condition is a composite compression-flexure stress, where the compression is large and the flexure is small, so that the eccentricity with respect to the central line is strongly reduced and the center of force enters in the interior of the cross section: equilibrium is now possible by purely compressive stresses (Figs. 4, 5).

So, Man learned that it was possible to cover spaces with stones. Anyway, he found and inhabited also large caves, so the attempt to reproduce nature (a strong impulse in Architecture, as testified also in recent times by the Gaudì's opera, see e.g. [1]) may be has pushed to realize double-curvature roofs. This activity gradually resulted in a success, with larger and larger spans being covered, thus leading to the early architecture and to its developments up to our times.

The historical development of Structural Mechanics is exhaustively reconstructed in the book by E. Benvenuto [2]. A very interesting and complete historical survey on the conception, realization and progress in the masonry vaults technology can be found in [3] and in [4,5]. Here an observation by Thomas Young is reported, namely: "*The construction of the dome is less difficult than that of an arch since the tendency of each arch to fall is counteracted not only by the pressure of the parts above and below but also by the resistance of those which are situated on each side*…..".

Further extensive studies have been developed by the author with the Naples research group on the No-Tension treatment of bodies made of masonry or non-cohesive materials [6-17].

That double curvature surfaces are easier to be built than simple arches or barrel vaults is a fact that merits further specification (see e.g. [6,7].

II. The Masonry as a Material

Masonry is not properly a "material" in the strict sense of the word. It consists in the (generally man-made) assemblage of a basic component (the stones) simply laid on each other or, more often, jointed by mortar. Stones and mortar may have very variable mechanical properties, and the way in which the stones are organized in the masonry volume (the masonry "texture") may be very different, and is subject to the skill and the creativity of the designer and/or of the builder.

So, "masonry" has not a uniquely defined object, and it is very difficult to set up a mechanical model able to closely reproduce the properties of masonry, fitting all the possible variety of masonry assortment and texture.

Anyway, in all structural analyses the engineer is forced to balance the trend to reproduce the material (and consequently the structural behaviour) as closely as possible, with the practical manageability of the analytical tools. Linear theory of structures applied to steel, reinforced concrete and even to masonry, is a successful example of such effort. In all cases the basic theory should include the major features of the behaviour, possibly neglecting many details that poorly influence structural safety assessment, and/or are uncontrollable. The small tensile strength in concrete, for instance, not only yields a poor contribution to the structure performance, but since it is a highly uncertain parameter in the concrete mass of a building, it increases uncertainty of the analysis' results: so it is preferred to adapt linear theory by neglecting tensile strength rather than to exploit cumbersome procedures yielding results depending on uncontrollable parameters.

The first step is then to identify the major properties, that are more or less common to all masonry types. The basic knowledge can be achieved through simple experiments. Uni-axial compression/tension tests can be performed on some Representative Volume Element (RVE) of a typical masonry (Fig. 6).

After some experiments, it is possible to conclude that (Fig. 6a): i) the masonry has different elastic moduli in tension (E_t) and compression (E_c); ii) the masonry has different limit stresses in tension (σ_t) and compression (σ_c); iii) the limit stress in tension is much smaller than the limit strength in compression ($\sigma_t << \sigma_c$); iv) the behaviour at failure in compression has some degree of ductility; v) the behaviour at failure in tension is definitely brittle, so tensile strength cannot be recovered absolutely.

Moreover, surprisingly (Fig. 6b), the limit strength in compression of masonry is larger than the strength of the weak element (the mortar) and is bounded from above by the limit strength of the strong component (the stones); this is due to some complex phenomenon of stress interaction and transverse deformation of mortar with respect to stones. It is also easy to understand that if the axis of the stress is rotated by an angle, say 90°, the results of the experiment may significantly change, in particular as regards the tensile strength. Some similar conclusions can be drawn from biaxial tests (see e.g. [18,19]). Experimental limit strength domains are of the type in Fig. 7, showing a high capacity in compression and a very poor limit in tension without ductility.

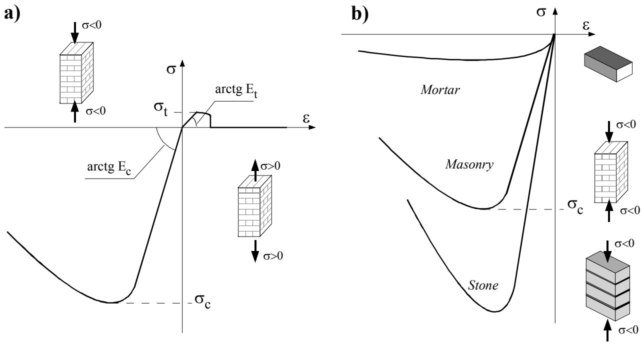

Fig. 6: a) A typical test of compression/tension on a masonry specimen; b) The limit strength in compression is in between the strength of mortar (small) and the strength of the bricks (large).

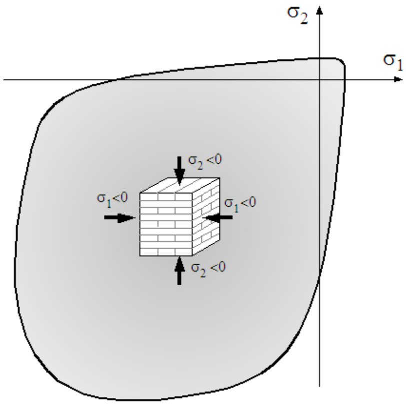

Fig. 7: Synthesis of biaxial tests on masonry prisms. Limit domain (see e.g. Hegemier [18] and Page [19].

Summing up, masonry is a non-linear material, strongly hetero-resistant, anisotropic with respect to tensile strength, with compliance coefficients depending on the orientation of the stress axes and different in compression and tension, and with brittle failure at the tension threshold. If one needs to confer masonry some reliable tensile strength, contemporary technology allows effective reinforcement by applying composite materials (see e. g. [20,21]).

III. Effect of Masonry Texture

The influence of the texture on the masonry performance can be illustrated by the following example.

Assume that a panel is built by regular bricks with interposed poor mortar joints, lacking any adhesive force. Consider that bricks are set according to the following two patterns (Fig. 8a,b). If there is no vertical compression both panels are free to expand laterally without encountering any resistance (Fig. 8c). If a vertical compression is applied, the panel in Fig. 8a still can freely separate; by contrast an horizontal tensile pseudo-strength becomes active in the panel in Fig. 8b, because of friction and interlocking of bricks with each other.

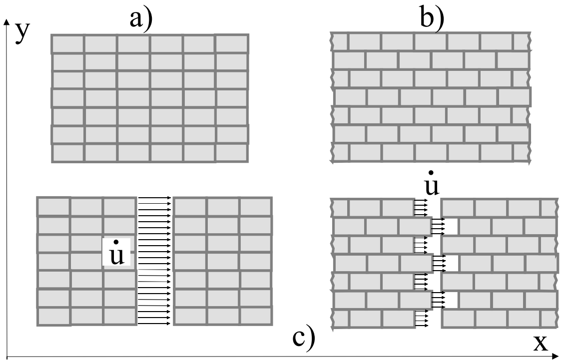

Fig. 8: Masonry element: a) Aligned bricks; b) Staggered bricks; c) Free lateral expansion for both panels

The failure mechanism in Fig. 9 can be studied for the bi-dimensional masonry plane element in Fig. 8b having a friction coefficient *f*, a joints stagger *s* (Fig. 9) and a row-density ω defined as the ratio of the number of block rows in the panel height *H* to the height *H*. In Fig. 9, $\omega = 7/H$.

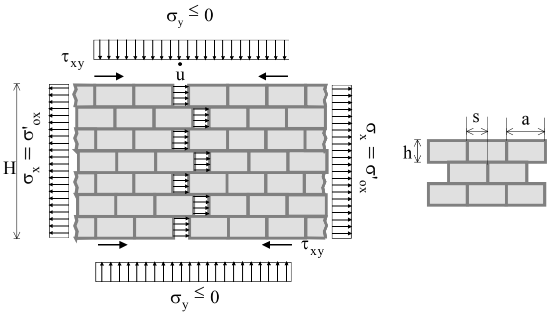

Fig. 9: Masonry element: Failure mechanism under compression and limit tensile forces; stagger parameter.

The wall is subjected to vertical compression stresses σ_y orthogonal to the joints direction and horizontal tractions σ_x parallel to the joints. It is possible to prove [22] that the horizontal tensile strength σ'_{ox} is given by (Fig. 9)

$$\sigma'_{ox} = -f\sigma_y s\omega \tag{1}$$

The ratio between the compressive stress on the joints and the transverse tensile strength is

$$\left|\frac{\sigma'_{ox}}{\sigma_y}\right| = fs\omega \tag{2}$$

If the length of the stone is *a*, *s* is of the order *a*/2. Usually $a > 2h$ (very often $a > 4h$), with *h* the thickness of the brick, and so $s > h$. On the other side, $\omega \approx 1/h$, so that $s\omega > 1$ (very often $s\omega > 2$). With the help of mortar and/or of roughness of the interface between stones, *f* may possibly be rather large ($f = 0.5 \div 0.8$), and the ratio in (2) is frequently larger than 1, i.e. the tensile strength in the direction parallel to joints is larger than the acting compressive stress.

It can be also proved that a pretty ductility is associated to the tensile strength σ'_{ox} . With reference to the diagrams in Fig. 3, applying a safety coefficient γ to the limit resistance σ'_o , the loss in strength is balanced by a gain in ductility (Fig. 18). In other words if σ'_a is the admissible stress and δ_a is the maximum ductility, one can write

$$\sigma'_a = \sigma'_o/\gamma \;\; ; \;\; \delta_a = \frac{\varepsilon'_{oa}}{\varepsilon'_a} = 1 + \frac{\varepsilon'_r}{\varepsilon'_o}\left(\gamma - 1\right) \qquad (3)$$

A fundamental observation is that (1) not only expresses the tensile resistance of the masonry element, but also puts to evidence that the tension can be contrasted in function of the static needs by means of a skilled orientation of the texture of the masonry blocks and of the mortar joints. After recognizing that by the combined effect of compression and friction the lines of the mortar joints are probably the lines where original designers and builders intended to provide tensile strength in the masonry mass, it can be conceived that a technical practice had spread out, very similar to the modern technology of reinforced concrete where the structural designer inserts steel bars in way to balance tension along stretched lines.

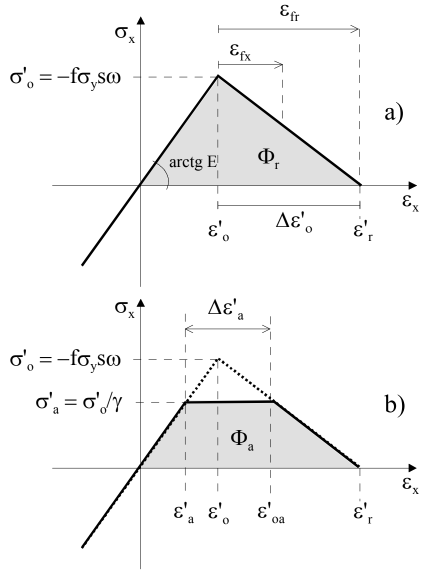

Fig. 10: a) Stress vs. deformation in the tension range,
b) conventional diagram with variable ductility.

Many examples proving that clever architects were aware of this effect when designing vault structures can be illustrated as for instance in so-called *cantilever stairs* (see e.g. [23-25]).

Masonry elements or components behaving like rigid blocks under dynamic action may be analysed by worst scenario approaches [26-28].

IV. Cantilever stairs

In the static analysis of a vaulted staircase, like in Fig. 11, it is possible to recognize three basic typological components: the landings, the angle connections, and the flights of stairs (two or three depending on the structure morphology). The structure is supported by the outside walls system which represents the stairs box.

Looking at the section of a vaulted stair in Fig. 12a, such structural conformation suggests an apparent paradox: despite the fact that masonry is not effective in sustaining tension stresses and bending, it should work as a cantilever, or however it is an incomplete vault which lacks the counter-thrust from the missing part of the arch (Fig. 12b) and so being prone to lose the equilibrium state (Fig. 12c).

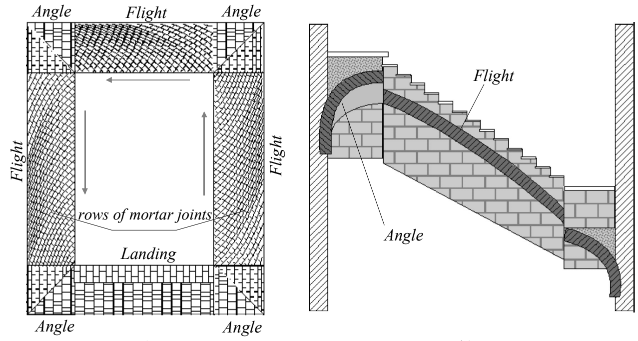

Fig. 11: Vaulted stairs: a) Planimetric view; b) Longitudinal section

It is quite obvious that the solution of the contradiction goes pursued abandoning the search of improbable plane patterns and by investigating three-dimensional equilibrium paths accounting for the space articulation of such structural organisms, searching stress fields in equilibrium and compatible with the resistant abilities of the masonry material as usually interwoven in the case of "cantilever" stairs.

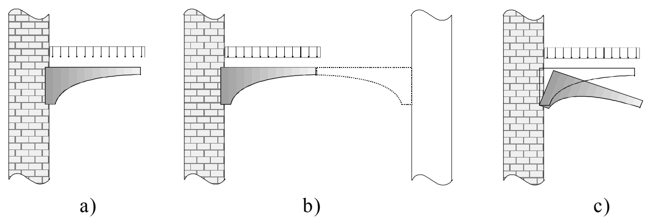

Fig. 12: Transverse sections of vaulted stairs: a) Section and particular of one step; b) "Half barrel vault" model, c) Improbable "cantilever" behaviour.

After identifying the basic internal force distributions through which the stairs can equilibrate their own weight and live loads, and the correlation that was intended by the original builders between statics and masonry tissue, it is also possible to design the reinforcement of the vaults, that shall be designed in way to sustain the possible equilibrium paths. Apart from complex FEM analyses (see e.g. [29]), it is possible to identify simplified equilibrium patterns that are compatible with the load-carrying capacity of the structure [30, 31]. All these approaches, FEM and/or simple 3D-beam, agree in identifying isostatic tension lines that approximately agree with the proceeding of the rows of mortar joints (Fig. 13), that are compressed in the orthogonal direction, thus developing a tension capacity along their lines of action; thus proving that the statics of these stairs are strictly connected with the vault apparatus. It is also possible to use this argument in an inverse fashion, i.e. to infer isostatic lines proceeding from the observation of the masonry texture. Any double curvature cover, in fact, is a highly hyperstatic system, which means that it can select its own pattern in a large set of possible equilibrium paths. So texture and vault apparatus are a tool by which, apart from the shape of the vault (barrel vault, rib vault, groin vault, etc.) the architect can steer the structure to work in some preferred way.

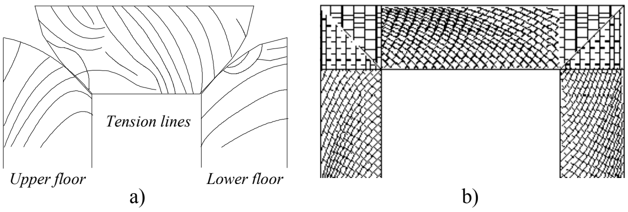

Fig. 13: Comparison of tension isostatic lines with the mortar rows. a) Tension lines calculated by a FEM (linear) procedure; b) Mortar rows in the flights

V. Tension in Spherical Domes

Consider the axial-symmetric hemispherical dome with radius R and thickness t (Fig. 14a), supporting its own weight w, where it is well known that in the classical solution, tension should be active along the parallel lines after some degree of the zenith angle $\varphi = 51.8°$.

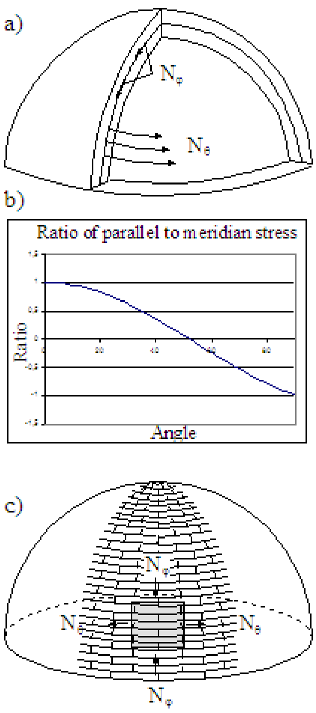

Fig. 14: a) Spherical dome; b)Ratio of parallel to meridian stress resultant. N_φ is everywhere compressive for any φ, and N_θ is a tensile stress for $\varphi > 51.8°$; c) The friction pattern for tensile strength yields a admissible stress if $fs\omega > 1$.

Here the meridian stress N_φ and the hoop stress N_θ are [32]

$$\begin{cases} N_\varphi = -\dfrac{wR}{1+\cos\varphi} \\ N_\theta = wR\left(\dfrac{1}{1+\cos\varphi} - \cos\varphi\right) \end{cases} \tag{4}$$

with $w = \gamma t$ and γ the unit weight of the material constituting the shell.

The ratio is

$$\frac{N_\theta}{N_\varphi} = 1 - \cos\varphi - \cos^2\varphi \tag{5}$$

The ratio is plotted in Fig. 14b, whence one can see that the ratio is always not larger than 1. So, if masonry is organized by staggered regular bricks –as often happens– tension could generally be faced by the friction mechanism as illustrated in Sec. 2 (Fig. 14c).

Anyway, equilibrium can be found by some other membrane surface other than the mean surface of the shell, provided it is included in the thickness between the (spherical) intrados and extrados.

Considering a revolution membrane surface having an elliptic profile with radii a and b, included in the interior of the hemisphere (Fig. 15a) the internal forces equilibrating the weight of the spherical dome can be found as follows

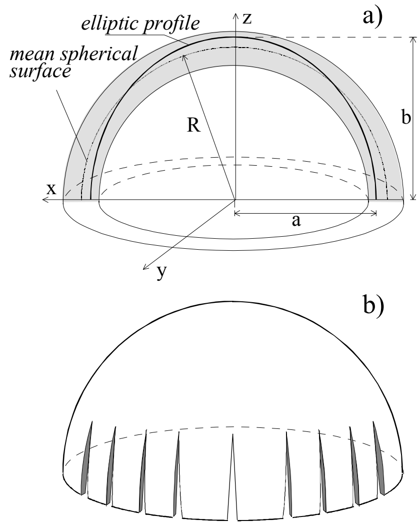

Fig. 15: a) The elliptic membrane surface included in the dome thickness; b) Possible physiological fractures

Consider the spherical cap above the center angle β, whose weight is

$$W = 2\pi w R^2 (1 - \cos\beta) \tag{6}$$

The angle β is related to the zenith angle φ by the relationships

$$sin\beta = \frac{a\,sin\varphi}{\sqrt{a^2\,sin^2\varphi + b^2\,cos^2\varphi}}$$

$$cos\beta = \frac{b\,cos\varphi}{\sqrt{a^2\,sin^2\varphi + b^2\,cos^2\varphi}} \qquad (7)$$

$$d\beta = \frac{ab}{a^2\,sin^2\varphi + b^2\,cos^2\varphi}\,d\varphi$$

The radii of curvature of the ellipsoidal surface are ([30], p.40)

$$r_1(\varphi) = \frac{a^2b^2}{\left(a^2\,sin^2\varphi + b^2\,sin^2\varphi\right)^{3/2}}$$

$$r_2(\varphi) = \frac{a^2}{\left(a^2\,sin^2\varphi + b^2\,sin^2\varphi\right)^{1/2}} \qquad (8)$$

so that

$$sin\beta = \frac{ar_2(\varphi)sin\varphi}{a^2} = \frac{r(\varphi)}{a}$$

$$cos\beta = \frac{b}{a}\frac{r(\varphi)}{a\,tan\varphi} \qquad (9)$$

$$r(\varphi) = r_2(\varphi)sin\varphi$$

and

$$d\beta = \frac{a}{b}\frac{r_1(\varphi)}{r_2(\varphi)}d\varphi \qquad (10)$$

The equilibrium versus the vertical translation can be written

$$2\pi r_2 N_\varphi(\varphi)sin^2\varphi + W = 0 \qquad (11)$$

and

$$N_\varphi(\varphi) = -\frac{wR^2(1-cos\beta)}{r_2(\varphi)sin^2\varphi} \qquad (12)$$

The ellipsoidal membrane shall now sustain the weight w of the spherical shell, that transforms in the weight w^* on the ellipsoid setting

$$w^* r_1(\varphi)d\varphi r(\varphi)d\theta = wRd\beta r_s d\theta \;\; ; \;\; r_s = R\,sin\beta \qquad (13)$$

whence

$$w^* = wR^2\frac{1}{br_2(\varphi)} \qquad (14)$$

The equilibrium along the outward normal to the (ellipsoidal) membrane yields

$$\frac{N_\varphi(\varphi)}{r_1(\varphi)} + \frac{N_\theta(\varphi)}{r_2(\varphi)} = p_n(\varphi) =$$

$$= -w^*\,cos\varphi = -w\frac{R^2}{br_2(\varphi)}cos\varphi \qquad (15)$$

and

$$N_\theta(\varphi) = -\left(w\frac{R^2}{b}cos\varphi + N_\varphi(\varphi)\frac{r_2(\varphi)}{r_1(\varphi)}\right) \qquad (16)$$

Ellipsoidal stress surface can be active in order to mitigate tension hoop stresses, possibly after some fractures have opened (Fig. 15b), that can be considered physiological if masonry has some degree of ductility in the parallel direction, as in the friction strength mechanism illustrated in Sec. 2. In Fig. 16a various membrane stress surfaces are plotted, with different ratios *a*/*b*.

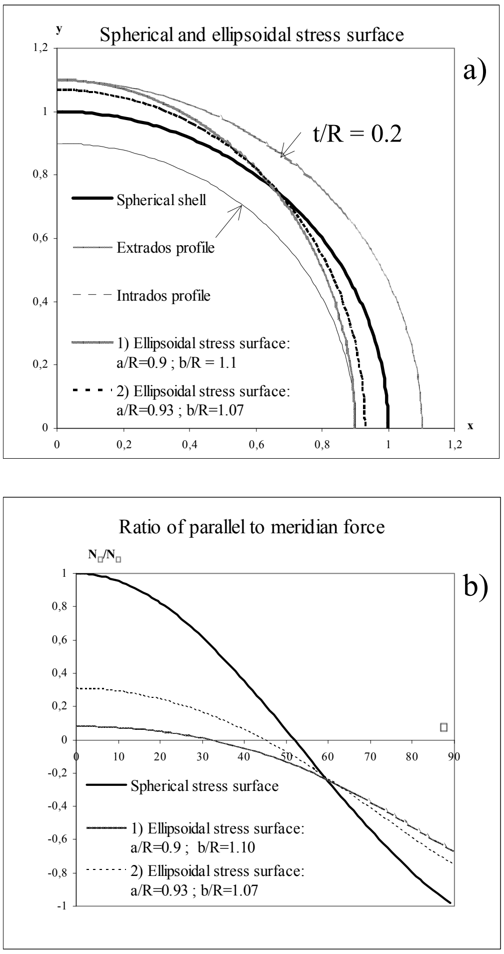

Fig. 16: a) Ellipsoidal membrane surfaces for different ratios of the ellipse radii a and b to the radius R of the spherical dome; b) Ratio of N_θ to N_φ for different shapes of the elliptic profile.

Note that such surfaces make sense provided that they remain included in the thickness of the spherical shell, i.e. if $t \geq 2(R-a)$ and $t \geq 2(b-R)$, with $b \geq a$, since it is assumed that the interface in the meridian direction is no-tension. The plots in Fig. 16b prove that the ratio of the parallel to the meridian normal force can be mitigated, and also be near 0.4 and smaller, with increasing the ratio b/a, a value that is very often in the range of the ratio σ'_{ox}/σ_y in (2), so that one can conclude that tensile hoop stress most times does not cause any problem. Consider that both in the spherical and in the elliptic membranes, the stress surface is a complete semi-ellipsoid, with φ = 90° at y=0, so that the equilibrium solutions do not require any thrust force at the bottom support y = 0.

Anyway, it has been proved in [33] that a membrane surface included in the thickness of the dome can be found without hoop tension, provided that a adequate counter-thrust force can be exerted at the bottom of the dome. In Fig. 17a it is illustrated how the spherical and elliptic membranes only transfer vertical actions on the basement, v_s and v_e respectively, while a no-tension profile requires that the base support can support a horizontal force h_n (Fig. 17b).

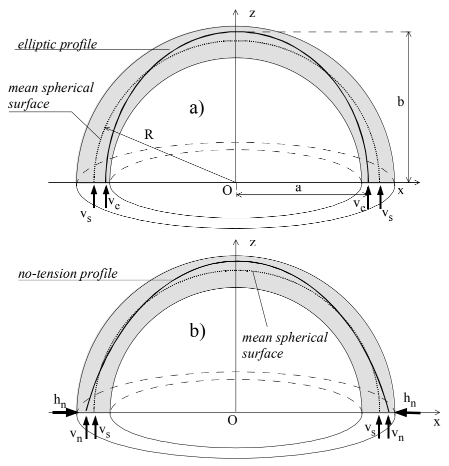

Fig. 17: No-thrust and no-tension stress surfaces: a) The basement of the dome is not subject to thrust action, but lower parallel lines are under tension; b) If a no-tension solution is adopted, the support of the dome is subject to a horizontal thrust force. Tension in the parallel lines is transferred to the basement.

VI. The masonry apparatus. An help to intuition

Reading masonry texture in a vault can help in understanding its equilibrium asset. The first element is indeed its geometry, a cross vault yields a equilibrium pattern different than a barrel vault, and so on. But a double-curvature surface, apart from its particular conception is anyway a highly hyperstatic system, and the equilibrium is never uniquely determinate. So the way the stones are jointed all together is a key to understand what equilibrium path would stresses run through, and/or what path would the builder have preferred to drive the vault to accommodate in.

So, consider for instance the two vaults in Fig. 18a and in Fig. 18b, having the same geometry, but in vault a) the mortar rows are parallel to the base perimeter, while in the vault b) the mortar rows are normal to the perimeter. The postulate is that compression normal to the mortar rows is the preferred equilibrium path for the vault, and that this is the tool for the original builder to steer the vault into a (his own) objective static asset. If the preferred direction for compression is normal to the perimeter, it is expected that compression acts along the arrows drawn in Figs. 18, a) and b), so that the vault gains a tensile capacity in the direction orthogonal to the arrows. It is easy to understand that this produces an effect on the thrust the vault exerts on the base supports. Consider in fact that in both cases the vault is made by four gores. In the case a) compression is directly transferred to the sides of the basement, while lateral dilatation and the diffusion of stresses to the corners is contrasted by internal tensile strength; so two opposite gores tend to directly sustain each other, and the distribution of the horizontal thrust force tends to concentrate towards the middle of the sides (Fig. 18c). By contrast, in case b) compression is active in the direction parallel to the base sides, and the gores tend to support each other along the diagonal lines, while the orthogonal dilatation and diffusion of stress are now contrasted by tensile strength in the direction orthogonal to the sides; so all forces tend to converge in the corners, and the distribution of the horizontal thrust force tends to concentrate to the corners (Fig. 18d).

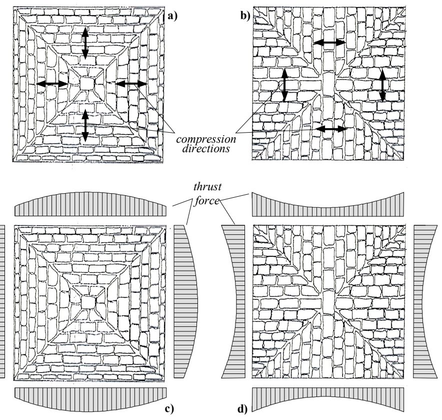

Fig. 18: Influence of the vault apparatus on the static behaviour of vaults. The difference in the apparatus in Figs. a) and b) yields different equilibrium pattern and a different distribution of the thrust force as in Figs. c)- d).

In other words, by acting on the masonry apparatus it is possible that, with the same geometry, a structure may be realized that works like a cloister vault rather than like a groin vault or viceversa. Which means that it may be not wise to analyze the statics of a vault only on the basis of its geometry.

Anyway, a skilled design of apparatus is also a tool to build vaults without formworks [34].

VII. Conclusions

Historical masonry vaults and/or cupolas exhibit a large variety of typological assets. Often masonry is well operated, with strong stones and effectively adhesive mortar; in many cases masonry is in worse working order; in other cases a poor masonry is encountered.

Anyway, double-curvature structures can appeal to many equilibrium patterns to sustain at least their own weight plus some light additional loads. So they are, in general, stable systems, provided that their supports are strong and able to contrast thrust forces. Vaults are in general characterized by their shape, and a lot of types can be listed (see e.g. [35]), that have been conceived to be included in any simple or complex architectural design. But the equilibrium paths are also driven by the way masonry is interwoven. In some cases, a masterly design of the masonry tissue and of the vault apparatus may help in improving the structure's stability, and sometimes even in preventing fractures, as discussed and illustrated in Sec. 3.

It should be realized, by contrast, that fractures are almost always a physiological feature of masonry; since almost always it has not significant tensile strength, it cannot expand by tension and, when necessary to comply with congruence of the overall deformation, dilatation is provided by fractures.

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