An Innovative Modular System for the Building of Timber Cylindrical Roofs

Dora Foti and Domenico De Tommasi

***Abstract*—**This article aims to analyze the mechanical characteristics of an innovative constructive system for arches and vaults made of hollow wooden boxes designed with interlocking joints. This new technology ensures at the same time good mechanical performances and fast installation and servicing. Experimental studies have been conducted on the base material, on the individual blocks and, thereafter, on an arch built in scale 1:1 to evaluate the displacements, the deformations and the maximum load to failure and to examine its compatibility with the provisions of the code on force. Finally a limit analysis has been developed to model the behavior of the structural system under the testing load.

***Keywords*—**Modular system, Wood structures, Vaults and arches.

I. INTRODUCTION

THE wood always turns out to be a building material that creates magnificent constructions allowed not only in the past, but also in recent times. From a static point of view the wood can show excellent results also better than masonry, reinforced concrete and iron, thanks to new technical advances [1], [2].

For various reasons, however, the use of wood in construction, at least in Italy, has always been decreasing, first with the use of steel as building material, and then of reinforced concrete. It totally disappeared for slabs, except in particular cases. These lacks are due also to a vacuum in the normative code of our country, which has been filled only recently. So far, in Europe the code on timber constructions is still evolving. In Europe significant is the final version of Eurocode 5, which is divided into two parts, "Rules and general rules for buildings" and "Structural fire design" [3].

Wood is strong, lightweight, very easy to use and to manufacture into new products. Furthermore today the selection of environmentally responsible building products and systems is a central aspect of sustainable building, and wood possesses positive attributes in this field; not only it is the single major renewable building material, but it also requires less energy to manufacture than any other building material [4].

A key aspect in the design of timber structures deals with the choice of the mechanical and structural models that properly describe the behavior of the materials and the structural systems themselves. According to all code prescriptions of the latest European edition, actions must be assigned to one of the classes of duration of load, which are characterized by the effect of a constant load for a certain period of time in the life of the structure. In this way it is possible to estimate the interaction between the typical temporal variation of the load in the time and the material properties.

Scientific studies have also highlighted a legislative gap regarding the evaluation of the loads capacities of timber-concrete composite joints exclusively entrusted to standard shear tests on "push out" joints [5].

In recent years several researches are trying to fill the gap, present both in literature and in the legislation, on the evaluation of the effective seismic behavior of special structures, like the one analyzed in the present paper, and not only in wood [6].

From a seismic point of view studies aimed at evaluating the seismic response of traditional timber structures are widely spreading both in Italy [7] and abroad. Their behavior highly depends on the conditions of the connections of the structure. For example in Taiwan timber structures are commonly used in highly seismic zones [8]. In fact, the wood is a very particular and efficient material, different from the most commonly building materials, its structure has different and interesting characteristics, almost conflicting, as for example its notable resistance under both compressive and tensile loads that becomes nearly unique if compared with its limited weight density. All this leads to realize structures absolutely suitable to the seismic safety.

It is also important to consider the degradation in the time of the mechanical characteristics of timber structures depending on the type of wood [9]. Generally elements in wood, due to the nature of the material, can be subject to a reinforcement intervention for several reasons: increment of the dead loads, degradation of the mechanical properties of the element or just to reduce the excessive displacements. In particular, in recent years also techniques born to reinforce masonry elements have been tested on the wood such as Fiber Reinforced Polymers (FRP) [10], which has given excellent results in both construction techniques in the case of structural elements subject to bending. Nevertheless, some of these techniques have been used to consolidate existent wood

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beams where it is not possible to realize, for various reasons, a complete replacement of the wood element. In this perspective the introduction of composite materials as reinforcements for wood elements subject to bending loads or shear loads is of great interest.

In this field of restoration and seismic improvement, there are many researches that devise and develop vulnerability studies and consequent actions for all the buildings types, residential and monumental, covered with timber roofs [11], [12]. In fact, it is known that the latters influence the global seismic response. This is also true for new large-span timber structures, such as roof structures of arenas or halls often composed of a primary structure carrying a secondary structure in the form of purlins [13]. The primary structure often consists of single-span members, e.g. pitched cambered glulam beams, trussed beams or three hinged frames. The purlins can be realized as simply supported beams, continuous beams, gerber beams and lap-jointed purlins.

A. *The arch system*

The arch is a structure that transforms the load mainly in normal stress. Generally the stresses due to shear and bending moment are present in minimum part. The static behavior of the arch comes from the shape, the production method, the materials, the actual boundary conditions and the quality of execution.

Generally the most common causes of instability arise from the inadequacy of the cross section, the crushing phenomena of some parts than others, the sliding of the laying surface of some blocks, the subsidence of the laying surface of the arch system for translation or for rotation of the support walls.

The behavior of arches (those in masonry) in presence of horizontal actions is not trivial because it lacks a systematic observation of damage after the earthquake. Some considerations on the response of a single arch are explained by the Italian code DPCM 09.02.11 [14] and in particular in respect of the vertical loads.

In general, the arch-pier system (or barrel vault walls) reaches the crisis for the loss of balance if the loads are mainly in the keystone. In this case two conditions may occur: the first where five hinges are generated, one of which in the keystone, so in the intrados a crack opens and the structure is divided into four segments from the five hinges; these segments can be assumed rigid. In the second case, when there are horizontal seismic actions, the system will damage non-symmetrically, with the formation of four hinges, two in the piers and two in the arch, with a crack at the intrados slightly displaced with respect to the keystone and one on the extrados at the haunch. In both loading conditions the formation of the sliding mechanisms are rare, because the friction between the segments constitutes an effective contrast, being the forces mostly orthogonal to the planes of contact of the segments themselves.

The construction system for wood arches and vaults presented in this article is based on wooden hollow blocks [15]. The shape of the blocks produces a high save of material as well as the possibility of having rooms to accommodate facilities. The system is good to satisfy the necessity to cover spans (some of considerable size) with a structure that does not require the inclusion of intermediate supports, so that it results lightweight and easy to install by mean of its mechanical junctions. The present experimental research aims to evaluate the mechanical behavior of the system and define a simple and reliable design method. A limit analysis has been also performed on a model of the timber vault to get the behavior of the arch under the load.

II. EXPERIMENTAL ANALYSIS

The system for the construction of arches and vaults considered in the present study is based on wooden hollow blocks. The shape of the blocks produces a high save of material as well as the possibility of having rooms to accommodate facilities. The system arises from the necessity to cover spans (some of considerable size) with a structure that does not require the inclusion of intermediate supports so that it results lightweight and easy to install through mechanical junctions.

The box-shaped blocks are made with plywood panels from pine (9 layers for a total thickness of 18 mm), and from birch (13 layers for a total thickness of 18 mm). The panels are then combined to form a closed box structure which has a special feature: the connecting system (Fig. 1) consists of a tooth and a cut and this allows a simple assembly of the vault. In particular, the connecting system is constituted by a male-female joint that fits each other block. Important characteristic of the block is its modularity. Only the key block is designed with two "female" links (Fig.1a) in order to accommodate the standard elements.

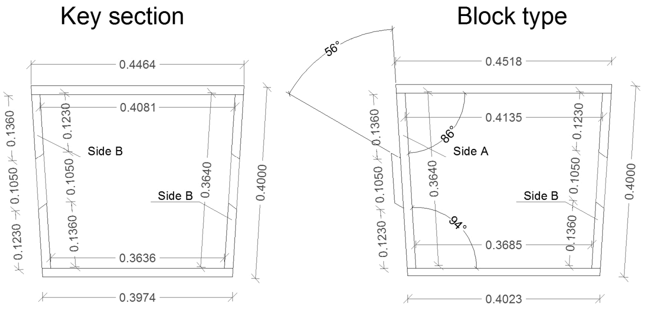


Fig. 1 executive drawings of the blocks.

The test program was divided into three phases:

1- characterization tests on the material "wood";

2- mechanical characterization of the blocks consisting in compression tests on individual elements, couples and triplets;

3- load tests on the vault structure.

A. *Tests on the material*

The characterization tests were performed on the wood elements in birch plywood, manufactured according to UNI EN 1058:1997. Since the thickness of the plywood is 18 mm, three sheets of plywood were used for each specimen. The sheets, after a process of scraping, assumed the thickness of 15 mm each; then they were bonded together to create a specimen having a total thickness of 45 mm (Fig.2).

The results of the compression tests, carried on three specimens, showed an average value of the ultimate compressive stress equal to 27.56 kN/mm^2. In Table 1 the results of the tests on the base material are shown.

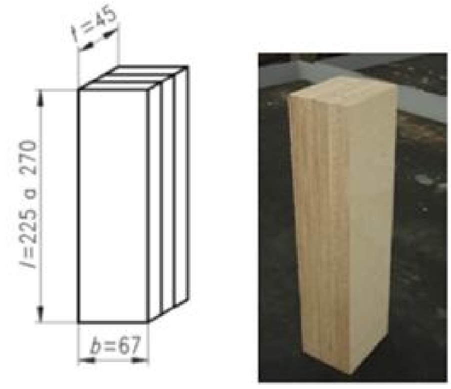


Fig. 2 specimen utilized in the tests on the material

Table 1 compressive strength of birch plywood specimens

Specimens in multilayered birch plywood	
Specimen n°	Failure stress [N/mm^2]
1	28.40
2	27.30
3	27.00

B. *Tests on block specimens*

The tested blocks, as described before, have the particularity of a hollow section in order to give lightness and ease of assembly. The different panels constituting the block have been assembled using the same glue to fast setting the adhesive for cold hardwoods, according to EN 204 - D2 [16].

The compression test load was applied by a hydraulic test machine on cubic specimens with dimensions 400mmx400mmx400 mm. The tests were performed on:

1. three specimens consisting of a single cubic specimen (Fig.3a);
2. three specimens each consisting of a pair of blocks (Fig. 3b);
3. three specimens each consisting of three blocks.

From the tests the following average values of the compressive ultimate forces were obtained: 358.7 kN, 237.3 kN and 210 kN, respectively for the first, second and third test. It was also checked that the block collapsed when the strength of the material was reached.

As regards the dynamic tests on rigid blocks, the behavior has been demonstrated to be highly uncertain [17], [18] and worst scenario approaches should be rather preferred [19]. On the basis of these tests it was also observed that the block exclusively behaved with the concave extrados and intrados surfaces (Fig. 4).

The low dispersion of the values obtained shows that the defects of the blocks (which is possible in a semi-traditional production) moderately affect the value of the tensile strength, and then the failure stresses of the system.

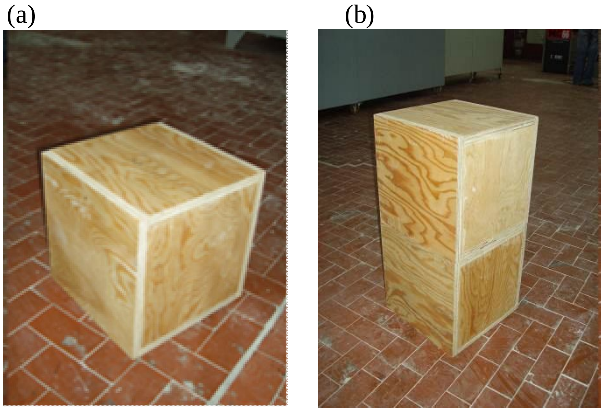


Fig. 3 specimen to be used for testing; a) coupled specimen and b) single specimen

Fig. 4 block specimens after the failure

C. *Tests on the timber vaults*

After the characterization tests on the material and on the blocks, a load test on a real vault has been performed. To this end, a round arch was made in scale 1:1. The arch consisted of 18 wooden blocks and one key block, to a depth of two blocks, with a span of 6 m and a covering of 2 m (Fig. 5).

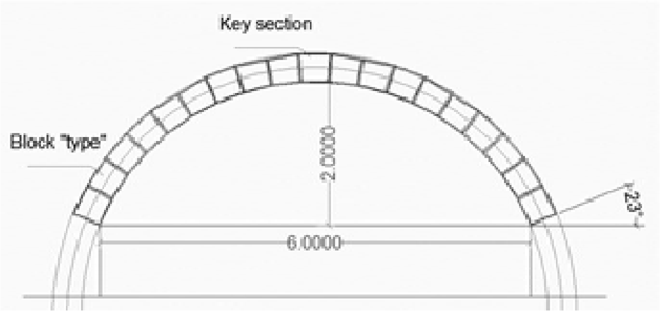


Fig. 5 executive drawing of the arch

The vault described above was built by Intini srl, whereas the experimental tests were carried out by the Department of Civil Engineering and Architecture of the Polytechnic of Bari, Italy.

Fig. 6 shows the placement of the key blocks during the construction phase of the arch.

To protect the vault from atmosphering agents, a protective sheet with a weight of 50 kN was placed. For a proper placement of the slinging, special wooden tracks were made to guarantee the strings to slide but not slip from the assigned position. The loads were applied by means of 1000mmx1200mmx1200mm tanks that were filled with water (Figs. 7-8). The readings of the load were obtained using a graduated scale applied on the tanks.

Fig. 6 positioning of the key block

Fig. 7 tested arch: details with the low tanks of the positioning of the displacement transducers

Fig. 8 detail of a loading tank.

The arch was instrumented with five touch transducer of 75 mm, named Pot 1-2-3-4-5, and by two inductive transducers by 100 mm, named Pot 6-7 (Fig. 9 a, b). The instrumentation was connected to a data acquisition board.

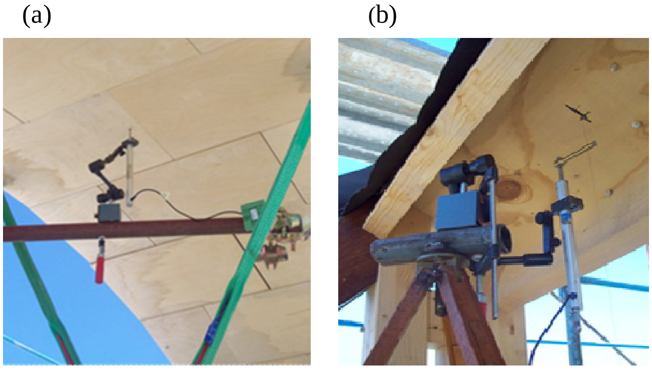


Fig. 9 displacement transducers utilized in the tests on the real structure; (a) touch transducer at the key section; (b) inductive transducer at a shoulder.

Two series of loading tests were performed on the arch, one with a symmetrical load keeping the arch in the elastic field, and one with the load applied in an asymmetrical position and incremented up to the collapse of the arch.

During the first experimental tests the symmetrical loading condition was realized filling the three tanks. Therefore, three symmetrical incremental vertical forces with a maximum value of 30 kN have been applied on the arch; this load condition was safer with respect to the real service load condition for the following reasons:

- the value of the modulus of the external force is larger in the experimental phase respect to the design phase, 30 kN > 20.7 kN, where 20.7 kN is the design load;
- the distributed static load conditions are less demanding than the concentrate ones assumed in the design phase. Also for an equal resultant it is clear that the collapse should not be achieved.

A second experimental test was carried out applying a non-symmetrical loading condition. To this aim only a lateral tank was filled with water and positioned as shown in Fig. 11. In this case the structure was brought to collapse and the ultimate load was equal to 10 kN.

III. Discussion of the Results

Following the Italian Code on force (D.M. 2008) [20], the vault should be able to support a vertical load corresponding to a uniform load equal to 2.65 kN/m^2.

During the test a symmetrical load condition of three incremental steps up to a vertical maximum load of 30 kN has been considered. A first test load was made with the attainment of half of the maximum value, that is equal to 15 kN, and a subsequent unloading. As a consequence of this test in the key section of the arch a residual vertical displacement of 2.53 mm was obtained. It reduced to 2.50 mm after 24 hours.

Then a second test has been performed to reach the maximum load of 30 kN. The arch reached the maximum load of 30 kN without generating phenomena of crisis.

With regard to the deformation data read by the displacement transducers, starting from a residual deformation of 2.50 mm under the maximum load, it was reached a deflection at the key section of 8 mm and a residual deformation of 3.68 mm (Fig. 10). It is possible to notice that when the load reached the value of 12 kN the transducer in the key section highlighted "a jump” of about 2 mm, which gave, when unloading, a corresponding residual value.

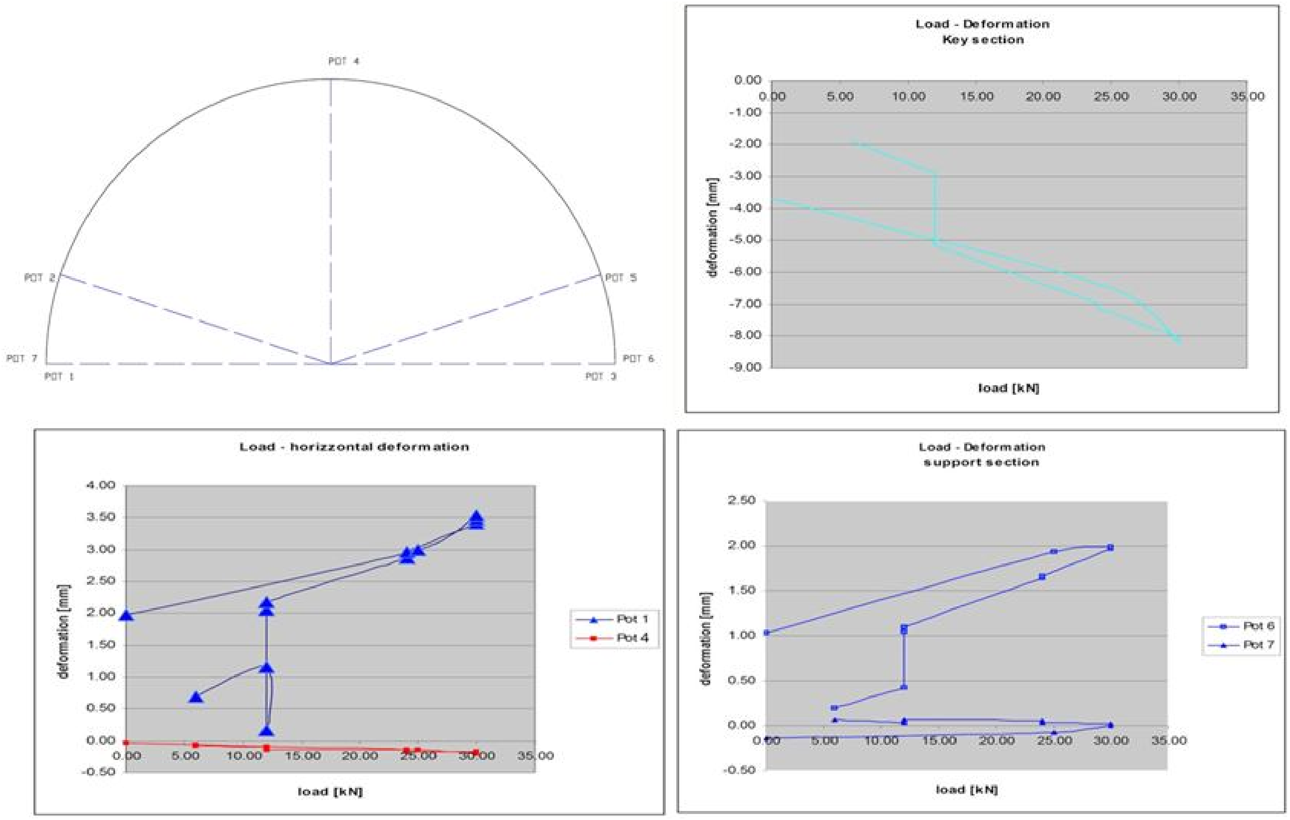


Fig. 10 load-deformation plot at the key section and at the supports during the loading test

Finally, the behavior of the arch for the asymmetric loading condition was analyzed. As it is well known, this condition is particularly heavy for vaults and arches. The tests carried out on the structure under study confirmed this general situation.

In more detail the test with non-symmetric load condition has shown that the system behaved in a way similar to masonry arches. In particular, as shown by Heyman [21], masonry arches collapse for the insurgence of hinges due to the poor tensile strength of masonry. That is to say that an arch reaches the collapse when a mechanism is formed, i.e. a number of hinges both in the extrados and in the intrados.

A number of papers are available in literature providing analysis tools for the analysis of masonry constructions [22], [23], confirming that the limit analysis is a powerful method for describing the collapse of masonry arches and could be extended with no error to the study of wooden arches too.

In the present case, a kinematic mechanism of collapse was reached with the achievement of four hinges in the arch, alternate at the extrados and intrados in the asymmetric loading test condition (see Fig. 11).

Fig.11 shows the mechanism that occurred in the arch at the collapse for the presence of the four hinges.

A. *Limit analysis of the behavior of the arch*

An arch is formed by a continuous sequence of ideal resisting elements subject to stress states of buckling and shear that in general vary from element to element. It can be said that the generic deformation in a single resisting block is null or is a deformation of detachment. When, due to external stress, deformations of detachment are generated, the arch can be considered as an articulated system consisting of multiple rigid elements. Similar considerations can be done for the wooden arch of the present research. The achievement of such a limit state that transforms the arch in a "mechanism" indicates the value of the load-bearing capacity that such a structure can support.

The following analysis is usually applied to masonry arches, but their behavior can be extended to wooden arches and vaults due to the reduced tensile strength of wood, typical of masonry too.

The structural elements in simple or double curvature generally forgiveness their functionality after the formation of hinges. This happens precisely because of the poor tensile strength of the masonry, which can lead to activate the mechanisms of collapse.

The arches reach the crisis when forming a mechanism, i.e. a number of plastic hinges, both in the extrados and in the intrados, sufficient to make the arches labile. In particular, in the case presented in this research it has been utilized a kinematic mechanism of collapse with fixed supports, asymmetrical, due to the presence of not symmetrical load: it is generated by the formation of four hinges in the arch, alternate on the extrados and intrados (fig. 12).

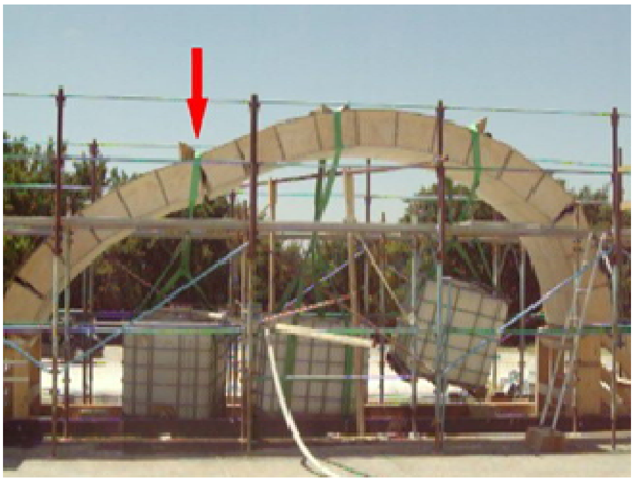


Fig. 11 collapse mechanism and achievement of the hinges in the arch with an asymmetric load condition

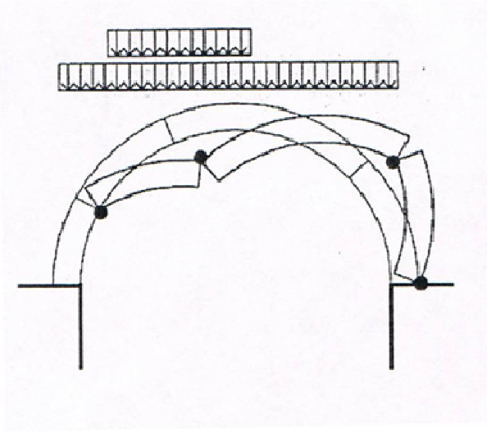


Fig. 12. kinematic mechanism of collapse with fixed supports, asymmetrical

It has thus been carried out simulations of the limit state behavior of the arch utilizing a commercial software [24] which is based on a finite element modeling (FEM). An asymmetric failure mechanism, the only one that has led the structure to the failure, has been simulated by mean of an incremental analysis, in order to validate the experimental tests performed.

In the study of the system the following assumptions have been considered:

1. system with blocks with no tensile strength; the compressive behavior is rigid-plastic and the strength is fixed by the type of material utilized;
2. negligible shear strength;
3. coefficient of friction μ in correspondence of the contact surfaces of the blocks equal to 0.45;
4. filled blocks, (this hypothesis derives from the characteristics of simulation of the software and justified because, in the limit analysis method, it is believed that no local breakdown of the individual block is generated), and

then with a specific weight equivalent equal to 2.73 kN/m;

5. characteristic compressive strength of wood equal to 15 kN/mm^2.

For the modeling, moreover, it has been considered the load due to the weight of the structure evaluated considering the panels with a thickness equal to 18 mm, and of a waterproofing layer with a weight equal to 0.6 kN/m^2.

In the modeling plate/shell - quad4 elements with a biaxial stress state and brick elements have been used, as well as fix joints at the base of the arch.

The vertical load was positioned just as in the experimental test, concentrated at a distance from the support equal to one third of the total length of the bay (Fig. 13). The load condition shown does not include the self-weight and the waterproofing layer.

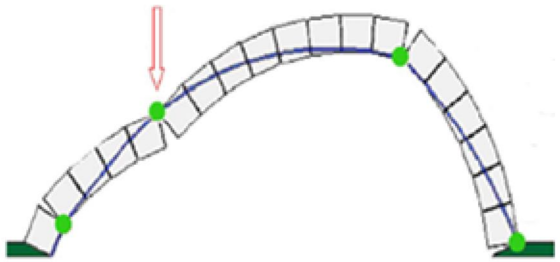
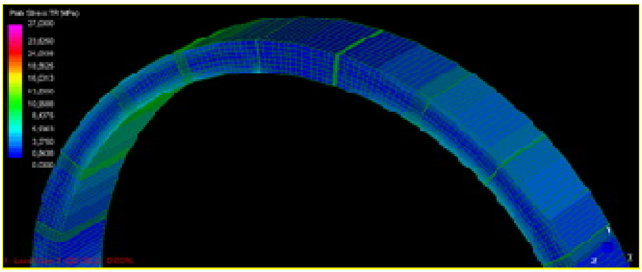


Fig. 13 model of the arch at failure with the generation of kinematic hinges under an asymmetric load

A collapse load equal to 8.60 kN has been reached; it is very close to the failure load experimentally found, equal to 10 kN.

In the post-processing, through the display of strains, displacements and stresses, it is shown the exact behavior of the arch loaded up to the collapse, giving a numerical validation to the experimental tests (Fig. 14). Post-processing has been developed utilizing a commercial software [25] and implementing a bi-dimensional model of the arch.



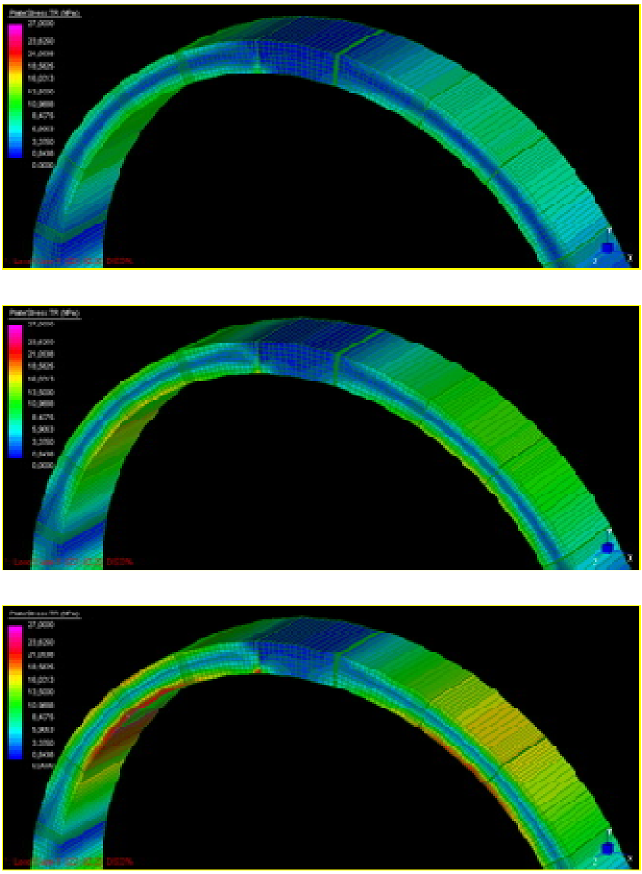


Fig. 14 Sequence of 4 pictures of the evolution of the designed arch subjected to asymmetric loading

IV. CONCLUSIONS

This work represents a first approach to the study of an innovative system to make timber roof structures, which provide the required standard prescriptions especially regarding the eco-compatibility.

The wood is a perfectly eco-sustainable material, from its extraction, through the production and processing, until its use and disposal. The construction of small halls or rooms with wood vaults utilizing this new construction technique will combine the easy capacity of assembly to the material savings, and will take advantage of the low thermal conductivity of the material that ensures excellent thermal insulation.

The system studied offers multiple applications, which range from the easy use for aesthetic purposes, to the structural purposes.

It is easy to understand how the system is aesthetically pleasing; moreover, if properly introduced in certain contexts, it also allows a recovery of the techniques of traditional building materials.

The tests show how the system is able to accommodate heavy loads in total safe, at least for the uniform load condition. It is a more competitive constructive technique with respect to those structures developed in the more expensive laminated wood, especially with regard to the applications such as covering of large areas.

The analyzes conducted allowed to get a large amount of information useful to understand the behavior of such structures. In particular, the investigations carried out considering different levels of detail and modeling tools have made possible to clarify what are the lacks and benefits of different approaches.

It was shown that, even using a simplified plane model of the arch, it is possible to identify the areas where the cracks are formed. A more accurate modeling can be done by a non-linear analysis, if necessary with mechanisms of damage of the material. It would be desirable to increase the analysis considering for example the non-linearity resulting from the connections and the degree of ductility of the these. In this paper, the mechanism of damage analyzed considers a limit surface defined by parameters of tensile and compressive strengths of the wood.

From the analyzing it emerges that the two-dimensional model of the timber arch is a highly valid tool for the localization of the main problems. However it is possible also to notice that the plane model is able to well reproduce the trend to vertical displacements.

On the whole, the knowledge acquired from this work constitutes the necessary basis for performing analyzes aimed to identify possible measures to improve the structure, as well as for further and more detailed structural analyzes both in the non-linear and dynamic fields.

It should be emphasized that the system collapsed not for the material failure, but because of its connections and the joints between the various wooden blocks that must be improved and modified.

Further testing campaigns will aim to study the problem of asymmetric loads, which can destabilize the system. It is sufficient to think about the effects of the earthquake, which is a dominant aspect in recent years, but especially the wind effects, real problem for lightweight structures.

ACKNOWLEDGMENT

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