Minimising the actuating power of vertical transport installations by optimisation of dynamic and kinematics parameters

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Abstract— The specific energy consumption is mainly influenced by kinematics and dynamic measures of vertical transport installations as well as by the compatibility of different composing parts and their subcomponents. The optimisation of kinematics and dynamic parameters characterising a transport cycle is decisive considering the energy consumption. Also considering the operation of the vertical transport installations, as well as the character of the variation of kinematics and dynamic parameters during a race, it has been considered that one of the adequate optimisation methods of these parameters is the calculus of variations. In order to apply this calculus, the definition of the optimisation functional and restrictions is imposed. The acceleration and deceleration periods during each race of a vertical transport installation may be considered as periods of transitional processes where kinematics and dynamic measures variations take place (acceleration, speed and forces) as well as some electric measures (actuating motor's current). One of the basic performance parameters of the operation of the vertical transport installations is the specific energy consumption during a cycle. It therefore means that the optimisation of the transport cycle related to this parameter may be realised using a functional with a function under the integral depending on the electric energy consumption during a race.

Keywords— acceleration, dynamically balanced, optimization, power, statically balanced installation, tachograms, unbalanced installations.

I. Kinematics Parameters Optimisation

TWO constant acceleration phase tachograms are used in the case of reduced transport systems and are characterised by the lack of a constant speed period.

A. Constant acceleration tachograms

Therefore, the process is composed of only two periods of time: the acceleration phase t1 and the deceleration phase t2 (figure 1).

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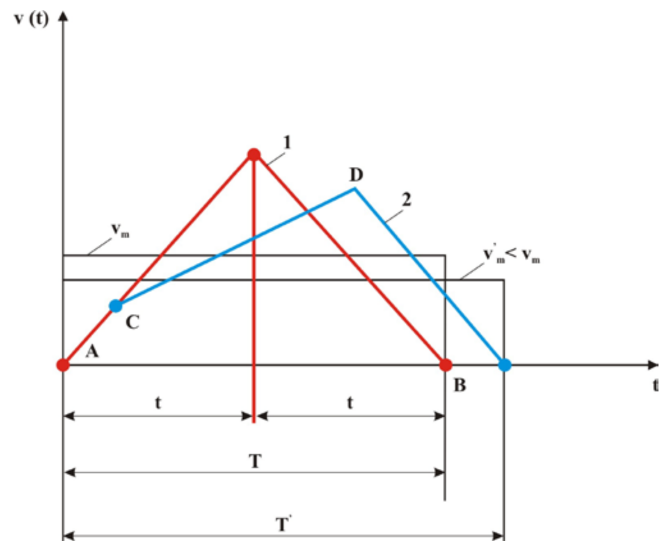


Fig.1 Speed variation trajectories for the two phase tachogram

The discovery of a law of variations is imposed either for i(t) or for a(t), for which the transition of the system from the point of balance A to the point of balance B to be realised in the shortest period of time possible. According to figure 1, for the transition in time of the system from point A to B, trajectory 1 needs to be followed. The speed of movement needs to be maximum:

$$V_m T = H \tag{1}$$

where, H is the distance undergone. If H is constant and $T = min$, then: $V_m = V_{m\max}$.

In order for the average speed to have a maximum value, the acceleration is imposed to be maximum a_{max}. It is also valid for the deceleration period t_2. If on one part of the trajectory (for instance CD), the acceleration is smaller than the maximum admitted one, the average speed decreases therefore increasing the period of the transitional process T' (line 2). In this case, for the optimum process, the acceleration is a staircase function:

$$\left.\begin{aligned} a(t) &= a_m; \quad 0 < t < \frac{T}{2} \\ a(t) &= -a_m; \quad \frac{T}{2} < t < T \end{aligned}\right\} \tag{2}$$

The law variation of speed and space is obtained by integrating the equations of movement variation considering the equations (2):

$$v(t) = \int_0^T a(t)\, dt \tag{3}$$

$$h(t) = \int_0^T v(t)\, dt \tag{4}$$

Therefore:

$$\left.\begin{aligned} v(t) &= a_m t \\ h(t) &= \frac{1}{2} a_m t^2 \end{aligned}\right\} \quad 0 < t < \frac{T}{2} \tag{5}$$

$$\left.\begin{aligned} v(t) &= a_m (T - t) \\ h(t) &= a_m \left(Tt - \frac{T^2}{2} - \frac{T^2}{4} \right) \end{aligned}\right\} \quad \frac{T}{2} < t < T \tag{6}$$

For the determination of the period of time T, the limit condition is used $h(T) = H$. In this way $h(T) = a_m \left(T^2 - \frac{T^2}{2} - \frac{T^2}{4} \right) = a_m \frac{T^2}{4}$ and:

$$T = 2\sqrt{\frac{H}{a_m}} \tag{7}$$

For $t = \frac{T}{2}$, speed v reaches the maximum value:

$$V_{\max} = a_m \frac{T}{2} = a_m \sqrt{\frac{\mathrm{H}}{\mathrm{a_m}}} \tag{8}$$

B. Variable acceleration tachograms

The continuous variation of the acceleration will be replaced with a variation in steps within the same phase of the trajectory for a period of time *T* of the process (figure 2), in a finite number of equal intervals with a duration of:

$$\tau = \frac{T}{n} \tag{9}$$

It is supposed that acceleration a (as a command value) is constant within each sub-interval, with values comprised between $a_1, a_2, \ldots, a_n$. By divide the acceleration, the following may be written:

$$W^* = \sum_{i=1}^{n} a_i^2 \tau \tag{10}$$

depending on n variables.

Out of all the staircase functions a(t) the chosen one is that for which the minimum of the W^* sum is obtained and simultaneously ensuring the compliance with the limit conditions:

$$\left.\begin{aligned} v(0) &= 0; \quad v(T) = v_k = 0 \\ h(0) &= 0; \quad h(T) = h_k = H \end{aligned}\right\} \tag{11}$$

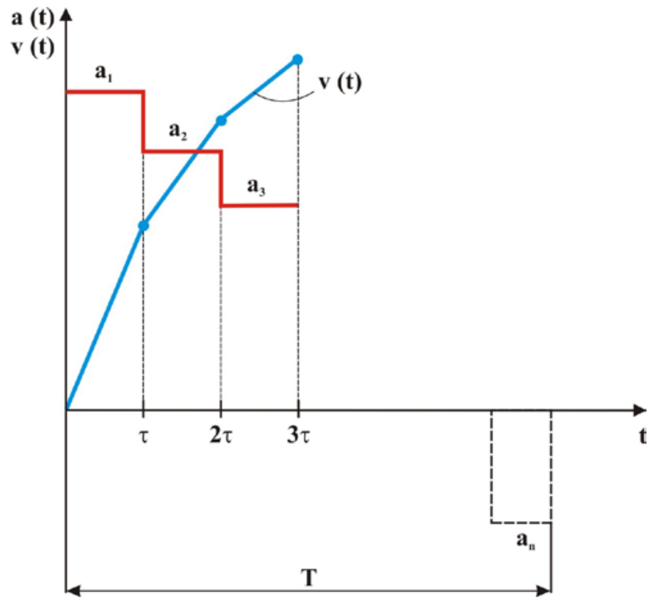


Fig.2 Variation in steps of the acceleration within the same phase

Permanently decreasing the duration of intervals τ, as a result of a limit transition, a continuous dependence a(t) will be obtained which minimises the integral *W*. Therefore, this is the optimum command condition.

The speed v given by relation (3), considering the initial condition $v(0) = 0$, varies according to a dotted line (figure 2), consisting of parts of lines the coordinates of which $t = 0$, $t = \tau$, $t = 2\tau$, ..., $t = T$ are:

$$\left.\begin{aligned} v_0 &= v(0) = 0; \\ v_1 &= v(\tau) = a_1 \tau; \\ v_2 &= v(2\tau) = (a_1 + a_2)\tau; \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ v_n &= v(n\tau) = \sum_{i=1}^{n} a_i \tau = 0 \end{aligned}\right\} \tag{12}$$

The movement h will be composed of sections of parabola. Considering the initial condition $h(0)=0$, based on relation (12) h_i ordinates in *points* $t = 0$, $t = \tau$, $t = 2\tau$, ..., $t = T$ are obtained.

$$\left.\begin{array}{l} h_0 = h(0) = 0; \\ h_1 = h(\tau) = \frac{\tau}{2}v_1 = \frac{\tau^2}{2}a_1; \\ h_2 = h(2\tau) = \frac{\tau}{2}v_2 = \tau^2 a_1; \\ h_3 = h(3\tau) = \frac{\tau}{2}v_3 = \tau^2(2a_1 + a); \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ h_n = h(T) = \frac{\tau}{2}v_n + \tau^2\sum_{k=1}^{i-1}(n-k)a_k = h_k \end{array}\right\} \tag{13}$$

In order to determine the conditioned extreme of sum W^* considering the relations (12) and (13), it is sufficient enough to determine the unconditioned extreme of the auxiliary function *V*:

$$V = W^* + \lambda_1 v_k(a_i) + \lambda_2 h_k(a_i) \tag{14}$$

where λ_1 and λ_2 undetermined Lagrange multipliers, determining the limit conditions.

Therefore,

$$V = \tau\sum_{i=1}^{n}a_i^2 + \lambda_1\tau\sum_{i=1}^{n}a_i + \lambda_2\frac{\tau^2}{2}\sum_{i=1}^{n}a_i + \lambda_2\tau^2\sum_{i=1}^{n-1}(n-i)a_i \tag{15}$$

The conditions needed for the extremes, is expressed by the system $\frac{\partial V}{\partial a_i} = 0;\quad i = 1,\ldots,n$.

Considering the expression (15), the following are obtained:

$$\frac{\partial V}{\partial a_i} = 2\tau a_i + \lambda_1\tau + \frac{\lambda_2}{2}\tau^2 + \lambda_2\tau^2(n-i) = 0 \tag{16}$$

From where:

$$a_i = -\frac{\lambda_1}{2} - \frac{\lambda_2}{4}\tau - \frac{\lambda_2}{2}\tau(n-i) \tag{17}$$

If for T = ct, the duration of the interval τ decreases unlimited, and the number of intervals n tends towards infinity, then a_i passes into *a(t)*, and τ_i in *t*. Considering $n\cdot\tau = T$, it results:

$$a(t) = -\frac{\lambda_1}{2} - \frac{\lambda_2}{2}(T-t) \tag{18}$$

In order to determine the Lagrange multipliers the following limit conditions are applied:

For $t = 0; a(0) = a_a$; and $t = t_a; a(t_a) = 0$, where:

a_a - is the initial value and the largest of the command measure (acceleration); considering an optimum process it varies linearly from $+a_a$ to $-a_a$;

t_a - is the moment the acceleration passes through the neutral.

The following equation system results applying these conditions for expression (18):

$$\left.\begin{array}{l} a_a = -\frac{\lambda_1}{2} - \frac{\lambda_2}{2}T \\ 0 = -\frac{\lambda_1}{2} - \frac{\lambda_2}{2}(T - t_a) \end{array}\right\}$$

Solving the system according the unknown λ_1 and λ_2, it results: $\lambda_1 = 2a_a;\quad \lambda_2 = -\frac{2a_a}{t_a}$.

Considering that $T = 2t_a$ and $a(T) = -a_a$, equation (18) becomes $a(t) = -a_a + \frac{a_a}{t_a}(2t_a - t)$.

Therefore, expression (18) may be written:

$$a(t) = a_a\left(1 - \frac{t}{t_a}\right) \tag{19}$$

It is observed that the optimum law of variation of acceleration both during acceleration as well as during deceleration is limited, imposing a parabola variation of speed during these periods.

Integrating equation (19), speed, space and energy dissipated during transitional starting and breaking periods, laws of variations are obtained:

$$v(t) = \int_0^t a(t)\,dt = \int_0^t\left[a_a\left(1-\frac{t}{t_a}\right)\right]dt = a_a t\left(1-\frac{t}{t_a}\right) \tag{20}$$

$$h(t) = \int_0^t v(t)\,dt = \int_0^t\left[a_a t_a\left(1-\frac{t}{t_a}\right)\right]dt = a_a\frac{t_2}{2}\left(1-\frac{t}{3t_a}\right) \tag{21}$$

$$W(t) = \int_0^t a^2(t)\,dt = \int_0^t\left[a_a\left(1-\frac{t}{t_a}\right)\right]^2 dt = a_a^2 t_a\left(1-\frac{t}{t_a}+\frac{t^2}{3t_a}\right) \tag{22}$$

The t_a and a_a constancies are determined from the limit conditions:

$$v(T) = a_a T\left(1-\frac{T}{t_a}\right) = 0 \tag{23}$$

$$h(T) = a_a\frac{T^2}{2}\left(1-\frac{T}{3t_a}\right) = H \tag{24}$$

From the above equation it results:

$$t_a = \frac{T}{2}; \quad a_a = 6\frac{H}{T^2} \tag{25}$$

The above case presents the starting and breaking transitional processes considering a two phase tachogram. Introducing the speed limit imposed by the operational norms of vertical transport installations, $v(t) \le v_{\max\, adm}$ then the tachogram transforms into a three phase one where $t_a^{'} < t_a$ (figure 3).

It is observed that the duration of the transitional periods t_1 (acceleration) and t_3 (deceleration) depend on the level of the maximum adopted speed, namely on the ordinate intersected by the optimum variation curve of the speed (parabola) with a horizontal line corresponding to the maximum speed.

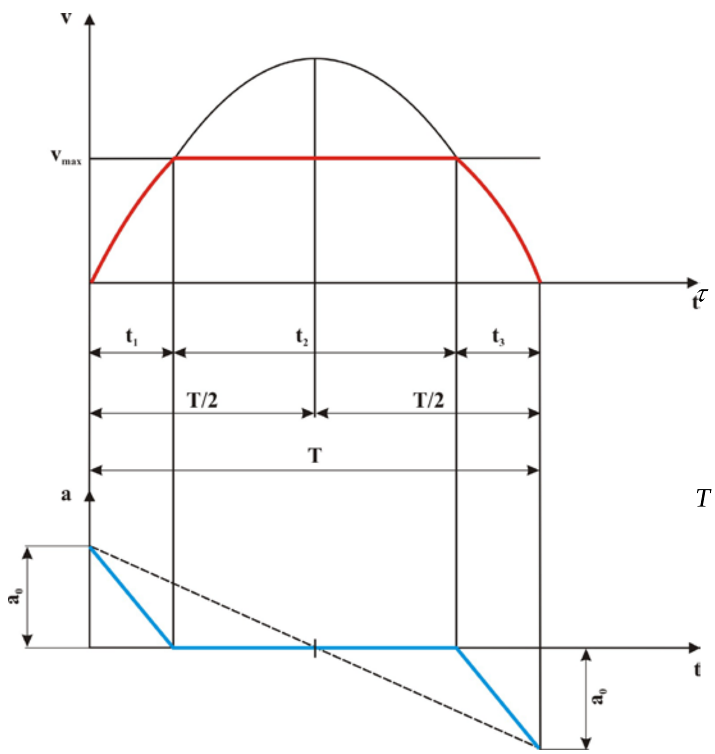


Fig.3 Speed limit tachogram

In the same time, it results that the acceleration needn't be kept at a constant level, imposing a smooth linear variation.

II. Dynamic Parameters Optimisation

A method for the optimisation of the electric operation is constituted by adopting a trapezoidal tachogram and considering a constant static torque according to the criteria of equivalent power. The objective was the development of an optimum trapezoidal tachogram for the minimisation of the equivalent power (figure4). The mathematical model used is based on relative coordinates with the purpose of generalising the results.

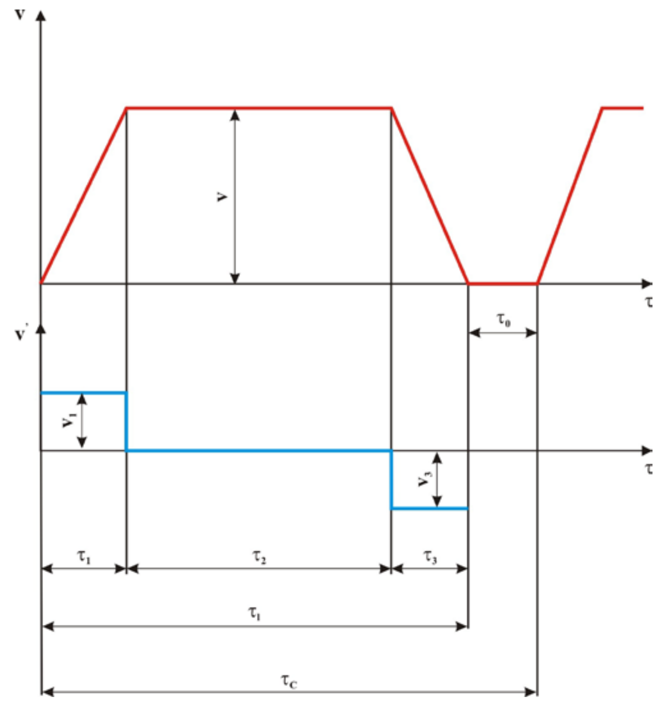


Fig.4 Analysed tachogram

The following have been considered for the time reference:

$$\tau = \frac{t}{T} \tag{26}$$

where:

T represents the mechanical time constancy:

$$T = \frac{I \cdot \omega_N}{M_N} = \frac{m \cdot v_N}{F_N} \tag{27}$$

where:

- I is the inertia moment of moving elements;
- m the weight of the moving elements;
- M_N and F_N the peripheral momentum and force;
- ω_N and v_N the angular and peripheral speed of the operating mechanism.

For the speed, torque and power, their nominal values have been considered as reference values:

$$v = \frac{\omega}{\omega_N} = \frac{v}{v_N}; \quad \mu = \frac{M}{M_N} = \frac{F}{F_N}; \quad \rho = \frac{P}{P_N} \tag{28}$$

The following relations result for the movement and acceleration:

$$t = \frac{\theta}{T\omega_N} = \frac{H}{Tv_N}; \quad v^{'} = \frac{\omega^{'} T}{\omega_N} = \frac{v^{'} T}{v_N} \tag{29}$$

Therefore the movement equation in absolute measures $M = M_s + \left|\frac{d\omega}{dt}\right|$ may be written in relative measures as:

$$\mu = \mu_s + \frac{dv}{d\tau} \tag{30}$$

The expressions of speed and space are:

$$v = \int v' dt; \quad h = \int v dt \tag{31}$$

The power in relative measures is:

$$\rho = \mu\, v = \left(\mu\, s + v'\right) v \tag{32}$$

The total movement of a trapezoidal tachogram, after making the integrals (31) is:

$$x_0 = \frac{1}{2} v\tau_1 + v\tau_2 + \frac{1}{2} v\tau_3 \tag{33}$$

Introducing a dimensional variables:

$$\alpha = \frac{\tau_1}{\tau_2}; \quad \beta = \frac{\tau_3}{\tau_1}; \quad 0 < \alpha; \quad \beta < 1 \tag{34}$$

The periods of the tachogram become:

$$\tau_1 = \alpha\tau_2; \quad \tau_3 = \beta\tau_1; \quad \tau_2 = \left[1 - (\alpha - \beta)\right]\tau_1 \tag{35}$$

And the regime movement and speed will be:

$$x_0 = \left[1 - \frac{1}{2}(\alpha + \beta)\right] v\,\tau_1 \tag{36}$$

$$v = \frac{x_0}{\tau_1} \cdot \frac{1}{1 - \frac{1}{2}(\alpha + \beta)} \tag{37}$$

The corresponding accelerations for the two ends of the tachogram are:

$$\begin{aligned} v_1' &= \frac{\mathrm{v}}{\tau_1} = \frac{1}{\alpha\left[1 - \frac{1}{2}(\alpha + \beta)\right]} \cdot \frac{\mathrm{x}_0}{\tau_1^2} \\ v_3' &= \frac{\mathrm{v}}{\tau_3} = \frac{1}{\beta\left[1 - \frac{1}{2}(\alpha + \beta)\right]} \cdot \frac{\mathrm{x}_0}{\tau_1^2} \end{aligned} \tag{38}$$

Equivalent torque:

$$\mu_{ech}^2 = \frac{1}{\tau_c}\int_0^{\tau c} \mu^2 d\tau = \frac{\varepsilon}{\tau_1}\int_0^{\tau l} (\mu_s + \mathrm{v}')^2 d\tau \tag{39}$$

Where ε is the connection period:

$$\varepsilon = \frac{\tau_l}{\tau_c} \tag{40}$$

For a trapezoidal tachogram, it results the following equivalent torque:

$$\mu_{ech}^2 = \varepsilon\left[\mu_0^2 \frac{\frac{1}{\alpha} + \frac{1}{\beta}}{\left[1 - \frac{1}{2}(\alpha + \beta)\right]^2} \cdot \frac{\mathrm{x}_0^2}{\tau_1^4}\right] \tag{41}$$

Equivalent power depending on the torque and speed:

$$p_N = v\sqrt{\frac{1}{\tau_c}\int_0^{\tau c} \mu^2 d\tau} \quad p_N^2 = \varepsilon \mathrm{x}_0^4 \left[\frac{\mu_0^2 \cdot \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)}{\tau_1^4 \cdot \tau_0^2 \cdot \left[1 - \frac{1}{2}(\alpha + \beta)\right]^4}\right] \tag{42}$$

The extreme of the equivalent power is obtained for $\alpha = \beta$:

$$p_N^2 = \varepsilon\left[\mu_0^2 \frac{1}{(1-\alpha)^2} + \frac{2}{\alpha(1-\alpha)^4} \cdot \frac{\mathrm{x}_0^2}{\tau_1^4}\right] \frac{\mathrm{x}_0^2}{\tau_1^2} \tag{43}$$

The minimum condition of the equivalent power results from cancelling the derivative of the power with the restrictions: $v \le 1$; $\mu \le \mu_{max}$; $\mu_{ech} \le 1$:

$$\frac{\partial\, \mathrm{p}_{\mathrm{N}}^2}{\partial\, \alpha} = \frac{\mu_0^2}{x_0^2}\tau_1^4 \alpha^2 (1-\alpha)^2 - (1 - 5\alpha) = 0 \tag{44}$$

For the particular case of no-load operation ($\mu_0 = 0$), it results the optimum value of the power:

$$\alpha = \beta = 0,2 \tag{45}$$

The regime speed:

$$v = 1,25\frac{x_0}{\tau_l} \tag{46}$$

Minimum power:

$$P_{N\,\min} = 4,94\sqrt{\varepsilon}\,\frac{x_0^2}{\tau_l^3} \tag{47}$$

In case of load operation ($\mu_0 \neq 0$) it results:

$$0 < \alpha_{opt} \le 0,2$$

Analysing the above presented method, the main conclusion is that it is a practical, operative method but it is valid only for a linear variation of speed. There is no certainty that this type of variation is optimum for ensuring the minimum value of speed. Moreover, choosing the trapezoidal tachogram is not scientifically justified, being made only empirically based on experience. Therefore, there may be another form of the operational diagram to ensure the minimum value of the actuating power.

III. Establishing the Optimisation Functional for Single Cable Vertical Transport Installations Powered by an Asynchronous Motor

The actual peripheral force is:

$$F_{ef} = \sqrt{\frac{\int_0^T F^2 dt}{T_{ef}}} \approx \sqrt{\frac{\sum F_i^2 t_i}{T_{ef}}} \quad [N] \tag{48}$$

Because function $F(t)$ varies during different phases, the integral $\int_0^T F^2\, dt$ is solved separately for each phase:

$$\int_0^T F^2 dt = \sum_0^n \int_0^{ti} F_i^2 dt \tag{49}$$

According to the general equation of the dynamics of vertical transport installations, the force at periphery of the reeling organism is expressed using the following relation:

$$F = \left[kQ_u + (q - q_1)(H - 2x)\right] g \pm a \sum m \quad [N] \tag{50}$$

The functional based on which the electric energy consumption may be minimised during a race, may be established as follows:

$$\exists(x,a) = \int_0^T f(x,a)\, dt = \int_0^T F_i^2\, dt \tag{51}$$

The peripheral force, are:

$$F = kQ_u g + (q - q_1) g (H - 2x) + a\sum m$$

$$F = A + D(H - 2x) + a\sum m = A + DH - 2Dx + a\sum m \tag{52}$$

where $A = k \cdot Q_u \cdot g$; $D = (q - q_1) g$; only the positive sign has been considered for the acceleration.

Squaring up expression (52), it results:

$$F^2 = A^2 + 2ADH + D^2H^2 - 4ADx - 4D^2Hx + 4D^2x^2 + 2Aa$$
$$\sum m + 2DH\, a\sum m - 4Dx\, a\sum m + a^2\left(\sum m\right)^2$$

By replacing the expression of the force it results:

$$f(x,a) = (A + DH)^2 - 4ADx - 4D^2Hx + 4D^2x^2 +$$
$$+2Aa\sum m + 2DH\, a\sum m - 4Dx\, a\sum m + a^2\left(\sum m\right)^2$$

or:

$$\begin{aligned} f(x,a) &= (A + DH)^2 - 4D(A + DH)x + 4D^2x^2 + \\ &+2a(A + DH)\sum m - 4Dx\, a\sum m + a^2\left(\sum m\right)^2 \end{aligned} \tag{53}$$

Using the relation between the actual force and the quantity of heat developed within the reeling of the motor during a transportation cycle, the actual force expression (equivalent) may be used as an optimisation criterion. Therefore, the actual force there is the following relation:

$$F_{ef} = \sqrt{\frac{\int_0^T F^2 dt}{T_{ef}}} = \sqrt{\frac{\int_0^T f(x,a) dt}{T_{ef}}} \tag{54}$$

The beginning and the end of a transport cycle are characterised by the following conditions:

$$x(0) = 0;\ x(T) = H;\ v(0) = 0;\ x'(T) = v(T) = 0 \tag{55}$$

IV. Restrictions on the Transport Cycle

In optimising the parameters of the transport cycle the respect of a series of technical prescriptions is imposed in order to ensure the continuous operation in full safety conditions.

A. Kinematics restrictions

The variation of kinematics parameters (speed and acceleration) during a transport cycle is defined by the diagram of speed (tachogram) as well as by the diagram of the acceleration, characterised by the relations:

$$\begin{aligned} &\int_0^T x'(t)dt = \int_0^T v(t)dt = H \leftrightarrow a) \\ &x'(0) = v(0) = x'(T) = v(T) = 0 \leftrightarrow b) \\ &x'(t) = v(t) \le v_{adm} \leftrightarrow c) \\ &x''(t) = \frac{dv(t)}{dt} = a \le a_{adm} \leftrightarrow d) \\ &x'''(t) \le \rho_{adm} \leftrightarrow e) \\ &\frac{t_2}{T} \ge 0.6 \leftrightarrow f) \end{aligned} \tag{56}$$

Conditions (56, a) and (56, b) define the requirements regarding movement and speed: at the end of the cycle, the space undergone by the transport enclosures has to be equal to the length of the transport race; the speed, both at the beginning of the movement as well as at the end of the race has to be null.

Conditions (56, c) and (56, d) are defined by the technical prescriptions regarding the speed limit and acceleration with their maximum admissible values.

Condition (56, e) limits the maximum value of the variation of the force within the time unit, seldom used measured uring the automated control of vertical transport installations.

Condition (56, f) is imposed by the cooling off of the electric motors through their own ventilation. The duration of the movement with constant speed (maximum) it is recommended to be at least 60% from the movement of a transport race.

B. Restrictions regarding the actuating motor

The power of the actuating motor needs to satisfy the following criteria:

$$P_{ef} = \frac{F_{ef} \cdot v_{max}}{1000\,\eta_a} = \frac{v_{max}}{1000\eta_a}\sqrt{\frac{\int_0^T F^2 dt}{T_{ef}}} \leq P_M \tag{57}$$

$$\frac{P_{max}}{P_{ef}} = \frac{F_{max}}{F_{ef}} \leq \gamma \tag{58}$$

where:

γ is the overload admissible coefficient ($y = 1{,}6 \div 1{,}8$ for asynchronous motors; $y = 1{,}8 \div 2{,}0$ for continuous current motors);

F_{max} is the maximum value of the peripheral force appearing during the transport race;

P_{max} is the power corresponding to the maximum force.

Two models based on relations (54) may be used for the optimisation:

- The optimisation model with the limit conditions (56, a) and (56, b);
- The optimisation model with all the kinematics restrictions imposed by the motor given by relations (54) and (55).

The first model covers criterion (48) and the limit conditions (55). A practical model needs therefore to consider all the restrictions, such as the second one foresees.

Therefore the amendment of functional (48) and the optimisation criterion (52) needs to be made, dividing the transport cycle in several according to the expression:

$$F_{ef} = \sqrt{\frac{\sum_{i=1}^{n}\int_{t_{ii}}^{t_{fi}} F^2 dt}{T_{ef}}} = \sqrt{\frac{\sum_{i=1}^{n}\int_{t_{ii}}^{t_{fi}} f(x,a)dt}{T_{ef}}} \quad [N] \tag{59}$$

where:

- n is the number of phases of the extraction cycle;
- t_{ii} is the beginning of all n phases;
- t_{fi} is the ending of all n phases.

V. The Extremes of the Optimisation Functional; Euler-Poisson Equations of the Functional

The establishment of the function characterising the law of variation of space $x(t)$, considering that the integral $\exists = \int_a^b f\left(x,y,y'\right) dx$ represents a superior order function related to the first derivative, may be made using the Euler-Poisson equation. The equation (59) adapted for the present case is:

$$\frac{\partial f}{\partial x} - \frac{d}{dt}\left(\frac{\partial f}{\partial x'}\right) + \frac{d^2}{dt^2}\left(\frac{\partial f}{\partial x''}\right) = 0 \tag{60}$$

Obtaining therefore:

$$\frac{d^4x}{dt^4} + \frac{4D}{\Sigma m}\frac{d^2x}{dt^2} + \frac{16D}{(\Sigma m)^2}x = \frac{2D(A\text{-}DH)}{(\Sigma m)^2} \tag{61}$$

or:

$$\frac{d^4x}{dt^4} + \lambda\frac{d^2x}{dt^2} + \frac{\lambda^2}{4}x = \psi \tag{62}$$

where $\lambda = \dfrac{4D}{\Sigma M}$ and $\Psi = \dfrac{2D(A\text{-}DH)}{(\Sigma m)^2}$.

Considering that the difference in weight between the transport cable and the balance one is characterised by D, three cases may be distinguished in solving the above presented equation

a) $D = g\left(q - q_1\right) > 0$ - unbalanced installation; the roots of equation (62) are real;

b) $D = g\left(q - q_1\right) < 0$ - dynamically balanced installation; the roots of equation (62) are imaginary;

c) $D = g\left(q - q_1\right) = 0$ - statically balanced equation.

For $D = 0$, based on expressions (54) and (61) the following are obtained:

$$\frac{d^4x}{dt^4} = 0 \tag{63}$$

Unbalanced installation (D > 0)

For this case, the solutions of equation (63), space, speed, acceleration and the third derivative of space in relation to time are the following:

$$\left.\begin{aligned}
&x = e^{\alpha t}\left(C_1 + C_2 t\right) + e^{-\alpha t}\left(C_3 + C_4 t\right) + \beta \\
&x' = v = C_1\alpha e^{\alpha t} + C_2 e^{\alpha t}\left(1 + \alpha t\right) - C_3\alpha e^{-\alpha t} + C_4 e^{-\alpha t}\left(1 - \alpha t\right) \\
&x'' = a = C_1\alpha^2 e^{\alpha t} + C_2\alpha e^{\alpha t}\left(2 + \alpha t\right) + C_3\alpha^2 e^{-\alpha t} + C_4\alpha e^{-\alpha t}\left(\alpha t - 2\right) \\
&x''' = \rho = \alpha^3 e^{\alpha t}\left(C_1 + C_2 t\right) + 3C_2\alpha^2 e^{\alpha t} - \alpha^3 e^{-\alpha t}\left(C_3 + C_4 t\right) + \\
&\qquad + 3C_4\alpha^2 e^{-\alpha t}
\end{aligned}\right\} \tag{64}$$

where: $\alpha = \sqrt{\left|-\dfrac{\lambda}{2}\right|}$; $\beta = \dfrac{4\psi}{\lambda^2}$

C_i – integration constancies, i = 1; 2; 3; 4.

Statically balanced installation (D = 0)

For this case, the solutions of equation (63) are:

$$\left.\begin{aligned} x &= C_1 + C_2 t + C_3 t^2 + C_4 t^3 \quad x'' = 2C_3 + 6C_4 t \\ x' &= C_2 + 2C_3 t + 3C_4 t^2 \quad x''' = 6C_4 \end{aligned}\right\} \tag{65}$$

where C_i are integration constancies, i = 1; 2; 3; 4.

Dynamically balanced equation (D < 0)

in this case, the solutions of equation (63) may have the following form:

$$\left.\begin{aligned} &x = \cos\alpha t\left(C_1 + C_2 t\right) + \sin\alpha t\left(C_3 + C_4 t\right) + \beta \\ &x' = v = \sin\alpha t\left(-C_1\alpha - C_2\alpha t + C_4\right) + \cos\alpha t\left(C_2 + C_3\alpha + C_4\alpha t\right) \\ &x'' = a = \cos\alpha t\left(-C_1\alpha^2 - C_2\alpha^2 t + 2C_4\alpha\right) - \sin\alpha t\left(2C_2\alpha + C_3\alpha^2 + C_4\alpha^2 t\right) \\ &x''' = \alpha^3\left(C_1 + C_2 t\right)\sin\alpha t - \alpha^3\left(C_3 + C_4 t\right)\cos\alpha t - 3C_2\alpha^2\cos\alpha t - \\ &-3C_4\alpha^2\sin\alpha t \end{aligned}\right\} \tag{66}$$

where: $\alpha = \sqrt{\left|-\frac{\lambda}{2}\right|}$;

C_i – integration constancies, i = 1; 2; 3; 4.

A. Optimum transport cycle with limit conditions

Mathematically speaking, the optimisation of the transport cycle consists in founding the function $x(t)$, the law of movement, ensuring the minimum of the integral:

$$F_{ef} = \sqrt{\frac{\int_0^T f(x,a)dt}{T_{ef}}} = min$$

The case of unbalanced installations (D > 0)

Based on the solution of the equation given by expression (52) and the initial conditions, a four equation system is formed in order to determine the integration constancies. Following the solution of this equation system, the integration constancies are:

$$\left.\begin{aligned} C_1 &= -C_3 - \beta \\ C_2 &= 2C_3\alpha - C_4 + \alpha\beta \\ C_3 &= \frac{a_3}{a_1} - \frac{a_2}{a_1}C_4 \\ C_4 &= \frac{a_1 b_3 - a_3 b_1}{a_1 b_2 - a_2 b_1} \end{aligned}\right\} \tag{67}$$

$$\left.\begin{aligned} a_1 &= e^{-\alpha T} - e^{\alpha T} + 2\alpha T e^{\alpha T} \\ a_2 &= Te^{-\alpha T} - Te^{\alpha T} \\ a_3 &= H - \beta + \beta e^{\alpha T} - \alpha\beta T e^{\alpha T} \\ b_1 &= \alpha e^{-\alpha T} + 2\alpha^2 T e^{\alpha T} + \alpha e^{-\alpha T} \\ b_2 &= e^{-\alpha T} - \alpha T e^{-\alpha T} - e^{-\alpha T} - \alpha T e^{\alpha T} \\ b_3 &= -\alpha^2\beta T e^{\alpha T} \end{aligned}\right\} \tag{68}$$

The case of statically balanced installations (D = 0)

Proceeding analogically, based on the solution of the equation given by expression (67), the integration constancies are:

$$C_1 = C_2 = 0; C_3 = \frac{3H}{T^2}; C_4 = -\frac{2H}{T^3} \tag{69}$$

The case of dynamically balanced installations (D < 0)

Considering the relations (54), the values of the integration constancies are:

$$C_4 = \frac{a_1 b_3 - a_3 b_1}{a_1 b_2 - a_2 b_1}; C_3 = \frac{a_3}{a_1} - \frac{a_2}{a_1}C_4; C_2 = -C_3\alpha; C_1 = -\beta \tag{70}$$

where:

$$\begin{aligned} a_1 &= \sin\alpha T - \alpha T\cos\alpha T; \; a_2 = T\sin\alpha T \\ a_3 &= H - \beta + \beta\cos\alpha T; \\ b_1 &= \alpha^2 T\sin\alpha T + \alpha T\cos\alpha T \\ b_2 &= \sin\alpha T + \alpha T\cos\alpha T; \; b_3 = -\alpha\beta\sin\alpha T \end{aligned} \tag{71}$$

B. The optimum transport cycle with all technological restrictions

The functional $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) + \frac{d^2}{dx^2}\left(\frac{\partial f}{\partial y''}\right) = 0$ will be adjusted with all the restrictions imposed by the kinematics installation. Considering a three period transport cycle, where the second period is characterised by constant speed, the limit conditions for each period may be explained as follows:

During the first period (the acceleration period)

$$x(0) = 0; x(t_1) = h_1; x'(0) = v(0) = 0; x'(t_1) = v(t_1) = v_{max} \tag{72}$$

During the second period (constant speed operation)

$$\begin{aligned} x(t_1) &= h_1; \quad x(t_1 + t_2) = h_1 + h_2 \\ x'(t_1) &= v(t_1) = x'(t_1 + t_2) = v(t_1 + t_2) = v_{max} \end{aligned} \tag{73}$$

During the third period (the deceleration period)

$$\begin{gathered} x(t_1+t_2)=h_1+h_2; \quad x(t_1+t_2+t_3)=H \\ x'(t_1+t_2)=v(t_1+t_2)=v_{max}; x'(t_1+t_2+t_3)=v(t_1+t_2+t_3)=0 \end{gathered} \tag{74}$$

where:

- t_i - represents the duration of the corresponding periods;
- h_i – is the distances undergone by the extraction containers during different periods.

In the same time, relations (72), (73) and (74) also need to comply with the following requirements:

$$t_1+t_2+t_3=T; \; h_1+h_2+h_3=H; \; \frac{t_2}{T}\geq 0,6$$

VI. Adopted optimisation model

According to the expression (59), the following optimisation model based on the equivalent force is adopted:

$$F_{ef}=\sqrt{\frac{\sum_{i=1}^{3}\int_{t_{ii}}^{t_{fi}} f(x,a)dt}{T_{ef}}}=min(!) \tag{75}$$

The conditions from the start and the end of the cycle:

$$\begin{gathered} x(t_{1,1}=0)=0; \quad x(t_{f3}=T)=H \\ x'(0)=v(0)=0; \quad x'(T)=v(T)=0 \end{gathered} \tag{76}$$

The requirements the actuating motor needs to comply with:

$$P_{ef}=\frac{v_{max}}{1000\eta_a}\sqrt{\frac{\sum_{i=1}^{3}\int_{t_{ii}}^{t_{fi}} f(x,a)dt}{T_{ef}}}\leq P_M; \frac{P_{max}}{P_{ef}}\leq\gamma \tag{77}$$

Restrictions imposed on periods:

For the starting period:

$$\begin{gathered} 0\leq t\leq t_1; 0\leq x(t)\leq h_1; 0\leq x'(t)\leq v_{max}; a_{1min}\leq x''(t)\leq a_{1max} \\ x'''(t)\leq|\rho_{1max}|; \quad x(0)=0; \quad x(t_1)=h_1; x'(0)=v(0)=0; \\ x'(t_1)=v(t_1)=v_{max} \end{gathered} \tag{78}$$

For the second period of constant speed operation:

$$\begin{gathered} t_1\leq t\leq t_1+t_2; h_1\leq x(t)\leq h_1+h_2 \\ x'(t)=v_{max}=const.; x''(t)=0 \\ x(t_1)=h_1; \quad x(t_1+t_2)=h_1+h_2 \end{gathered} \tag{79}$$

For the deceleration period:

$$\begin{gathered} t_1+t_2\leq t\leq T; h_1+h_2\leq x(t)\leq H \\ 0\leq x'(t)\leq v_{max}; |a_{3min}|\leq x''(t)\leq|a_{3max}| \\ x'''(t)\leq|\rho_{3max}| \\ x(t_1+t_2)=h_1+h_2; \quad x(T)=H \\ x'(t_1+t_2)=v(t_1+t_2)=v_{max}; \\ x'(T)=v(T)=0 \end{gathered} \tag{80}$$

Considering the expressions of x, x' and x'', the aspect of functional (53) for different balance degrees will be:

<u>Unbalanced installation (D > 0)</u>

$$\begin{gathered} f(x,a)=(A+DH)^2- \\ -4D(A+DH)\left[e^{\alpha t}(C_1+C_2t)+e^{-\alpha t}(C_3+C_4t)+\beta\right]+ \\ +4D^2\left[e^{\alpha t}(C_1+C_2t)+e^{-\alpha t}(C_3+C_4t)+\beta\right]^2+2(A+DH)\cdot \\ \cdot\sum m\left[C_1\alpha^2e^{\alpha t}+C_2\alpha e^{\alpha t}(2+\alpha t)+C_3\alpha^2e^{-\alpha t}+C_4\alpha e^{-\alpha t}(\alpha t-2)\right]- \\ -4D\sum m\left[e^{\alpha t}(C_1+C_2t)+e^{-\alpha t}(C_3+C_4t)+\beta\right]\cdot \\ \cdot\left[C_1\alpha^2e^{\alpha t}+C_2\alpha e^{\alpha t}(2+\alpha t)+C_3\alpha^2e^{-\alpha t}+C_4\alpha e^{-\alpha t}(\alpha t-2)\right]+ \\ +\left(\sum m\right)^2\left[C_1\alpha^2e^{\alpha t}+C_2\alpha e^{\alpha t}(2+\alpha t)+C_3\alpha^2e^{-\alpha t}+C_4\alpha e^{-\alpha t}(\alpha t-2)\right]^2 \end{gathered} \tag{81}$$

The C_i integration constancies are determined using relations (67) and (68).

<u>Statically balanced installation (D = 0)</u>

$$f(x,a)=A^2+A\sum m(4C_3+12C_4)+(2C_3+6C_4)^2\left(\sum m\right)^2 \tag{82}$$

The C_i integration constancies are determined using relation (69).

<u>Dynamically balanced installation (D < 0)</u>

$$\begin{gathered} f(x,a)=(A+DH)^2- \\ -4D(A+DH)\left[e^{\alpha t}(C_1+C_2t)+e^{-\alpha t}(C_3+C_4t)+\beta\right]+ \\ +4D^2\left[e^{\alpha t}(C_1+C_2t)+e^{-\alpha t}(C_3+C_4t)+\beta\right]^2+2(A+DH)\cdot \\ \cdot\sum m\left[C_1\alpha^2e^{\alpha t}+C_2\alpha e^{\alpha t}(2+\alpha t)+C_3\alpha^2e^{-\alpha t}+C_4\alpha e^{-\alpha t}(\alpha t-2)\right]- \\ -4D\sum m\left[e^{\alpha t}(C_1+C_2t)+e^{-\alpha t}(C_3+C_4t)+\beta\right]\cdot \\ \cdot\left[C_1\alpha^2e^{\alpha t}+C_2\alpha e^{\alpha t}(2+\alpha t)+C_3\alpha^2e^{-\alpha t}+C_4\alpha e^{-\alpha t}(\alpha t-2)\right]+ \\ +\left(\sum m\right)^2\left[C_1\alpha^2e^{\alpha t}+C_2\alpha e^{\alpha t}(2+\alpha t)+C_3\alpha^2e^{-\alpha t}+C_4\alpha e^{-\alpha t}(\alpha t-2)\right]^2 \end{gathered} \tag{83}$$

The C_i integration constancies are determined using relations (69) and (70).

Therefore, considering the three phases of the transport cycle, the numerator of expression (75) of the equivalent force may be written as follows:

$$\sum_{i=1}^{3}\int_{t_{ii}}^{t_{fi}} f(x,a)dt=\int_0^{t_1} f(x,a)dt+\int_{t_1}^{t_1+t_2} f(x,a)dt+\int_{t_1+t_2}^{T} f(x,a)dt \tag{84}$$

Considering the large volume of calculations, the digital integration of the components of expression (83) is imposed.

VII. Example 1

Based on the proposed method a, C language software has been developed. Software which was tested for an extraction installation with cages with the following parameters:

– practical load extracted during a race:
Q_u = 6000 kg;
– extraction depth:
H = 480 m;
– the sum of reduced masses:
Σm = 66368 kg;
– specific weight of the extraction cable:
q = 5,77 kg/m;
– specific weight of the balance cable:
q_1 = 6,72 kg/m;
– maximum acceleration at starting:
$a_{1\ max}$ = 0,8 m/s^2;
– maximum acceleration in breaking:
$a_{3\ max}$ = 1 m/s^2;
– maximum extraction speed:
v_{max} = 9,35 m/s;
– operational period of extraction containers:
T = 62 s;
– pause period between races:
t_p = 20 s;
– transmission efficiency:
η = 0,92.

In order to obtain a maximum efficiency, the following have been considered:

$$\frac{t_2}{T} = 0{,}6 \text{ and } t_1 = t_3 = 0{,}2 \cdot T$$

Eliminating q_1 for the unbalanced case and considering $q_1 = q$ for the statically balanced one, minimum values of the equivalent force and the actuating power have resulted with approximately 10% smaller than the classic method.

Figure 5 presents a print screen of the results obtained.

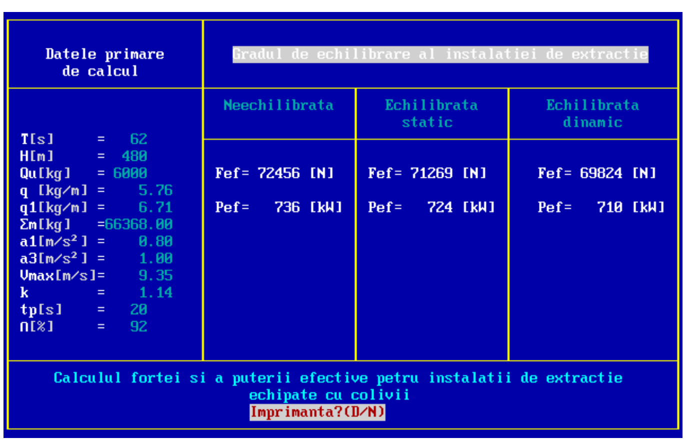


Fig.5 Print screen of obtained results for an extraction installation with cages

VIII. The Case of Multicable Vertical Transport Installations Powered by a Continuous Current Motor

For this kind of installation, the general equation of dynamics is:

$$F = kQ_u g \pm \left[\alpha_c Q_{sch} + \left[(nq - n_1 q_1)(H - 2x)\right] g \pm a\sum m - \beta_c Q_u g\right] \quad (85)$$

It is the case of several extraction installations with tilting buckets and cages, due to the fact that in the beginning and the end of the transport cycle one of the transport containers is found on the interior of the guiding rails of the tower, some of the weight is taken by it, therefore the tension in the cable decreases on the mentioned branch. Moreover, due to the beginning of the evacuation process before its complete stop, the useful load varies as well during this period.

Putting together the terms from relation (84), the following form is obtained:

$$F = (kQ_u + \alpha_c Q_{sch} - \beta_c Q_u) g + (nq - n_1 q_1)(H - 2x) g + a\sum m \quad (86)$$

Considering:

$$A_1 = (kQ_u + \alpha_c Q_{sch}) g$$

$$A_2 = (kQ_u + \alpha_c Q_{sch} - \beta_c Q_u) g$$

$$A = kQ_u g$$

$$D = (nq - n_1 q_1) g$$

$$H = H_e + 2h_b$$

where:
- H_e - is the level difference between the transport horizons;
- h_b - is the height of the silo.

For:
$t = 0$; $\alpha_c \geq 0$; and $\beta_c = 0$
$t \geq t_0$; $\alpha_c = 0$ (t_0 is the movement period of the empty container within the guiding rail)
$t < T$; $\beta_c = 0$
$t = T$; $\beta_c > 0$ (0,3 - 0,75)

Squaring up expression (85) it results:

$$F^2 = A_i^2 + 2A_i DH + D^2H^2 - 4A_i Dx - aD^2Hx + 4D^2x^2 + \\ +2A_i a\sum m + 2DHa\sum m - 4Dxa\sum m + a^2\left(\sum m\right)^2 \quad (87)$$

Replacing the expression of the force in functional (50), it results a relation similar to (52) where the value of A_i may either be A_1, A_2 or A depending on the transport phase:

$$f(x,a) = (A_i + DH)^2 - 4D(A_i + DH)x + 4D^2x^2 + \\ +2a(A_i + DH)\sum m - 4Dxa\sum m + a^2\left(\sum m\right)^2 \quad (88)$$

The ex tremis of f unctional (88) ar e d etermined w ith t he same r elations as i n t he cas e p resented ab ove f or different balancing degrees, but n ew r estrictions ap pear d uring t he starting and the ending period of a transport cycle.

The case of a f ive p hase ex traction cy cle u sing t he s ame constancies:

– During the movement o f t he e mpty t ransport c ontainer within the guiding rails:

$$x(0)=0\,; \quad x(t)=h_0$$

$$x'(0)=v(0)=0\,; \quad x'(t_0)=v(t_0)=v_0 \tag{89}$$

– During the second period (acceleration):

$$x(t_1)=h_1\,; \quad x(t_0+t_1)=h_0+h_1$$

$$x'(t_1)=v(t_1)=v_1\,; \quad x'(t_0+t_1)=v(t_0+t_1)=v_{max} \tag{90}$$

– During the third period (constant speed operation):

$$x(t_0+t_1)=h_0+h_1\,; \quad x(t_0+t_1+t_2)=h_0+h_1+h_2$$

$$\begin{gathered} x'(t_0+t_1)=v(t_0+t_1)=v_{max} \\ x'(t_0+t_1+t_2)=v(t_0+t_1+t_2)=v_{max} \end{gathered} \tag{91}$$

– During the fourth period (acceleration):

$$\begin{gathered} x(t_0+t_1+t_2)=h_0+h_1+h_2 \\ x(t_0+t_1+t_2+t_3)=h_0+h_1+h_2+h_3 \end{gathered}$$

$$\begin{gathered} x'(t_0+t_1+t_2)=v(t_0+t_1+t_2)=v_{max} \\ x'(t_0+t_1+t_2+t_3)=v(t_0+t_1+t_2+t_3)=v_4 \end{gathered} \tag{92}$$

– During the movement period of t he f ull b ucket i n t he guiding rail:

$$\begin{gathered} x(t_0+t_1+t_2+t_3)=h_0+h_1+h_2+h_3 \\ x'(t_0+t_1+t_2+t_3)=v(t_0+t_1+t_2+t_3)=v_4 \\ x(t_0+t_1+t_2+t_3+t_4)=H \\ x'(t_0+t_1+t_2+t_3+t_4)=v(T)=0 \end{gathered} \tag{93}$$

where:

$$t_0+t_1+t_2+t_3+t_4=T$$

$$h_0+h_1+h_2+h_3+h_4=H$$

The used optimisation model is an extension of expression (75):

$$F_{ef}=\sqrt{\frac{\sum_{i=1}^{5}\int_{t_{ii}}^{t_{fi}} f(x,a)dt}{T_{ef}}}=min(!) \tag{94}$$

Imposed restrictions during the periods:

– For the starting period:

$$\left.\begin{array}{l} 0\le t\le t_0 \\ 0\le x(t)\le h_0 \\ 0\le x'(t)\le v_0\,; \quad v_0\le 2{,}5\ m/s \\ a_{0\,min}\le x''(t)\le a_{0\,max}\,; \quad a_{0\,max}=0{,}5\ m/s^2 \\ x'''(t)\le\left|\rho_{0\,max}\right|; \quad \rho_{0\,max}=3\ m/s^3 \\ x(0)=0\,; \quad x(t_0)=h_0 \\ x'(0)=0\,; \quad x'(t_0)=v(t_0)=v_0 \end{array}\right\} \tag{95}$$

– For the acceleration period:

$$\left.\begin{array}{l} t_0\le t\le t_0+t_1 \\ h_0\le x(t)\le h_0+h_1 \\ v_0\le x'(t)\le v_{max} \\ a_{1\,min}\le x''(t)\le a_{1\,max}\,; \quad a_{1\,max}=0{,}5\div 0{,}8\ m/s^2 \\ x'''(t)\le\left|\rho_{1\,max}\right|; \quad \rho_{1\,max}=5\ m/s^3 \end{array}\right\} \tag{96}$$

– For the constant speed operation period:

$$\left.\begin{array}{l} t_0+t_1\le t\le t_0+t_1+t_2 \\ h_0+h_1\le x(t)\le h_0+h_1+h_2 \\ x'(t)\le v_{max}=const \\ x''(t)=0 \\ x(t_0+t_1)=h_0+h_1\,; \quad x(t_0+t_1+t_2)=h_0+h_1+h_2 \end{array}\right\} \tag{97}$$

– For the deceleration period:

$$\left.\begin{array}{l} t_0+t_1+t_2\le t\le t_0+t_1+t_2+t_3 \\ h_0+h_1+h_2\le x(t)\le h_0+h_1+h_2+h_3 \\ v_4\le x'(t)\le v_{max} \\ \left|a_{3\,min}\right|\le x''(t)\le\left|a_{3\,max}\right|; \quad a_{3\,max}=0{,}5\div 1{,}0\ m/s^2 \\ x'''(t)\le\left|\rho_{3\,max}\right|; \quad \rho_{3\,max}=5\ m/s^3 \end{array}\right\} \tag{98}$$

– For the period o f th e m ovement o f th e f ull c ontainer within the guiding rail:

$$\left.\begin{array}{l} t_0+t_1+t_2+t_3\le t\le T \\ h_0+h_1+h_2+h_3\le x(t)\le H \\ 0\le x'(t)\le v_4 \\ a_{4\,min}\le x''(t)\le a_{4\,max}\,; \quad a_{4\,max}=0{,}5\ m/s^2 \\ x'''(t)\le\left|\rho_{4\,max}\right|; \quad \rho_{4\,max}=3\ m/s^3 \end{array}\right\} \tag{99}$$

Considering the 5 phases of the transport, the numerator of expression (93) of the equivalent force becomes:

$$\sum_{i=1}^{5}\int_{t_{ii}}^{t_{fi}} f(x,a)dt = \int_{0}^{t_0} f(x,a)dt + \int_{t_0}^{t_0+t_1} f(x,a)dt + \int_{t_0+t_1}^{t_0+t_1+t_2} f(x,a)dt + \int_{t_0+t_1+t_2}^{t_0+t_1+t_2+t_3} f(x,a)dt + \int_{t_0+t_1+t_2+t_3}^{T} f(x,a)dt \tag{100}$$

in the expression of the functional $f(x,a)$ measure A_i will have the following values depending on the phase of transport:

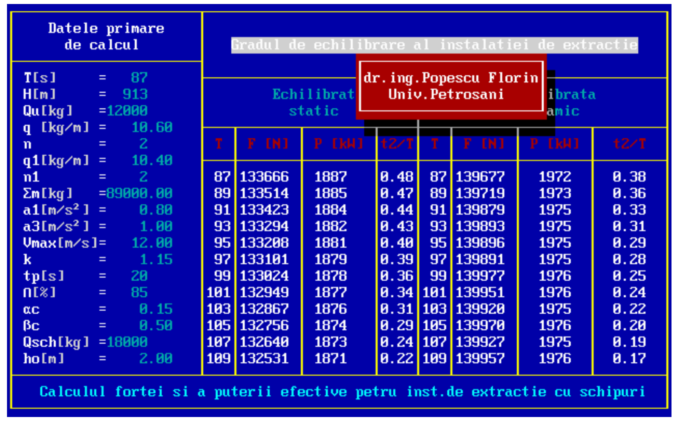
– in moment t = 0: $A_i = A_1 = (kQ_u + \alpha_c Q_{sch})g$
– in phase $0 < t < t_0$: $A_i = A = kQ_u g$
– in phase $t = (t_0) \div (t_0 + t_1)$: $A_i = A = kQ_u g$
– in phase $t = (t_0 + t_1) \div (t_0 + t_1 + t_2)$: $A_i = A = kQ_u g$
– in phase $t = (t_0 + t_1 + t_2) \div (t_0 + t_1 + t_2 + t_3)$: $A_i = A = kQ_u g$
– in $(t_0 + t_1 + t_2 + t_3) < t < T$: $A_i = A = kQ_u g$
– in moment t =T: $A_i = A_2 = (kQ_u + \alpha_c Q_{sch} - \beta_c Q_u)g$

IX. EXAMPLE 2

Based on the proposed method a software in C language has been developed, software which has been tested for a multicable extraction installation with the following parameters:

- practical load extracted during a race:
  $Q_u = 12.000$ kg;
- the weight of the tilting bucket:
  $Q_{sch} = 18.000$ kg;
- extraction depth:
  $H = 913$ m;
- the sum of reduced masses:
  $\Sigma m = 89.000$ kg;
- the specific weight of an extraction cable:
  $q = 10.6$ kg/m;
- the number of extraction cables:
  $n = 2$;
- the specific weight of a balance cable:
  $q_1 = 10.4$ kg/m;
- the number of balance cables:
  $n_1 = 2$;
- maximum extraction speed:
  $v_{max} = 12$ m/s;
- the length of the discharge guiding rails:
  $h_0 = h_4 = 2$ m;
- maximum acceleration during the starting period:
  $a_1 = 0,8$ m/s^2;
- maximum deceleration during breaking:
  $a_3 = 1,0$ m/s^2;
- the exit speed from the guiding rail of the empty tilting bucket:
  $v_0 = 2,5$ m/s;
- the entering speed in the guiding rail of the full tilting bucket:
  $v_4 = 1,5$ m/s;
- the movement period of the extraction containers:
  $T = 87$ s;
- pause period between the races:
  $t_p = 20$ s;
- transmission efficiency:
  $\eta = 0,85$;
- the coefficients characterising the extraction container:
  $k = 1,15$; $\alpha_c = 0,15$; $\beta_c = 0,5$.

Figure 6 presents a print screen of the results obtained.



T	F [N]	P [kW]	t2/T	T	F [N]	P [kW]	t2/T
87	133666	1887	0.48	87	139677	1972	0.38
89	133514	1885	0.47	89	139719	1973	0.36
91	133423	1884	0.44	91	139879	1975	0.33
93	133294	1882	0.43	93	139893	1975	0.31
95	133208	1881	0.40	95	139896	1975	0.29
97	133101	1879	0.39	97	139891	1975	0.28
99	133024	1878	0.36	99	139977	1976	0.25
101	132949	1877	0.34	101	139951	1976	0.24
103	132867	1876	0.31	103	139920	1975	0.22
105	132756	1874	0.29	105	139970	1976	0.20
107	132640	1873	0.24	107	139927	1975	0.19
109	132531	1871	0.22	109	139957	1976	0.17

Fig.6 Results obtained for the multicable extraction installation

Figures 7...18 present the variation diagrams of the speed and acceleration for $T \in [87,109]$ s.

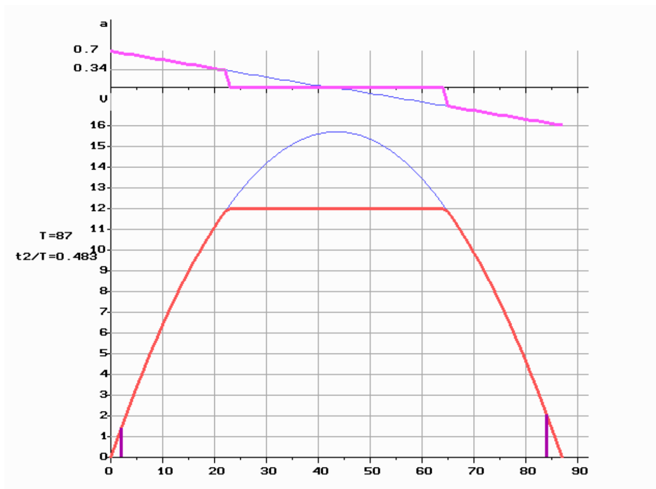


Fig.7 The variation of speed and acceleration for T=87 s

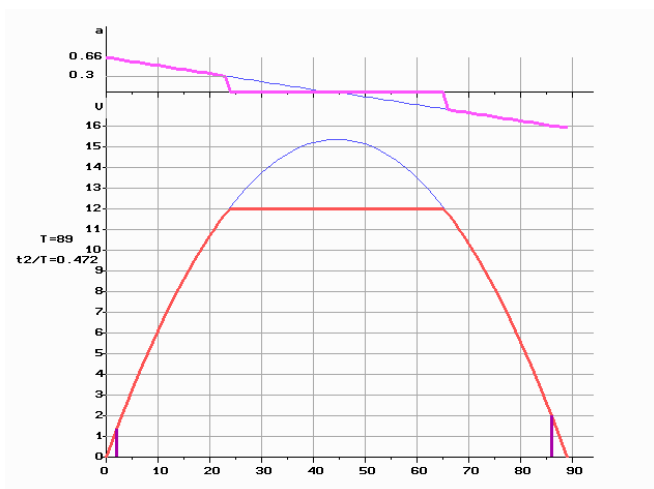


Fig.8 The variation of speed and acceleration for T=89 s

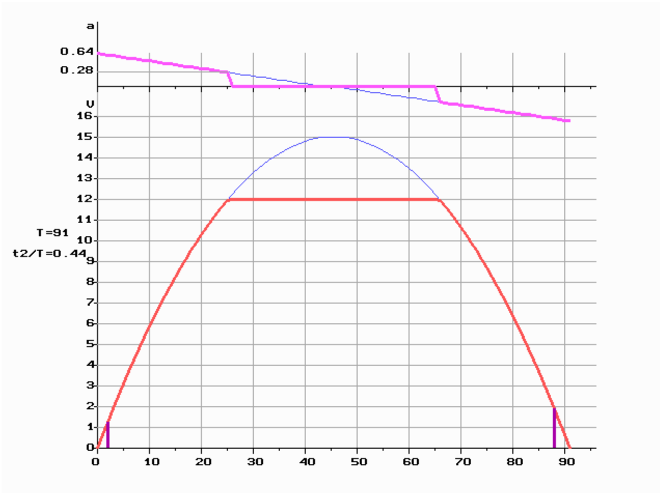


Fig.9 The variation of speed and acceleration for T=91 s

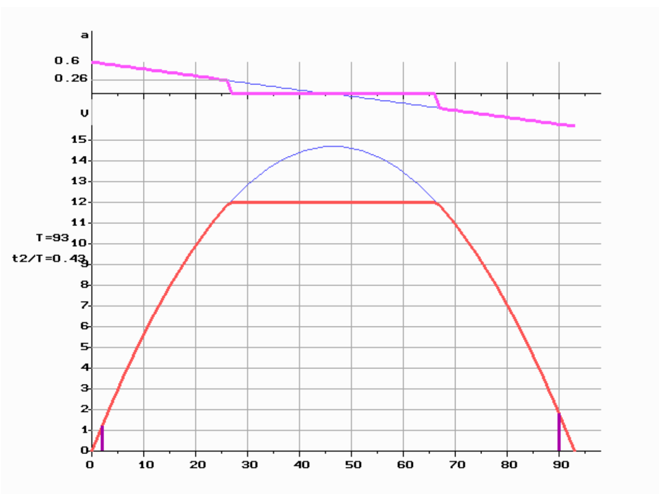


Fig.10The variation of speed and acceleration for T=93 s

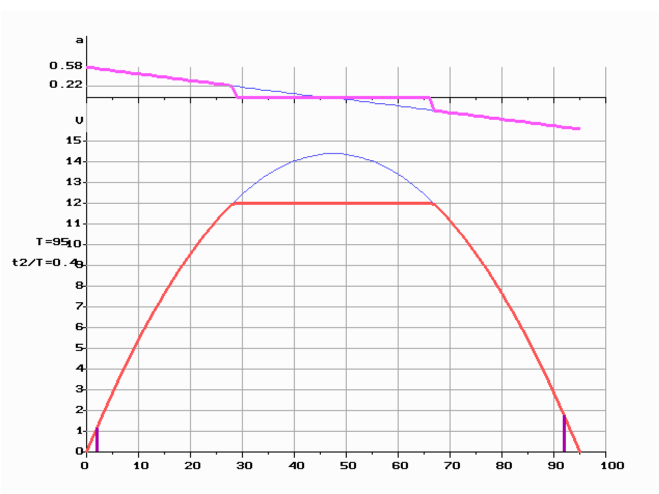


Fig.11 The variation of speed and acceleration for T=95 s

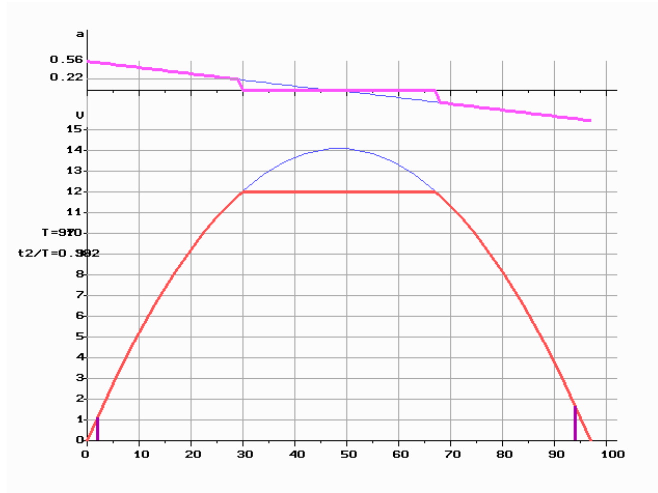


Fig.12 The variation of speed and acceleration for T=97 s

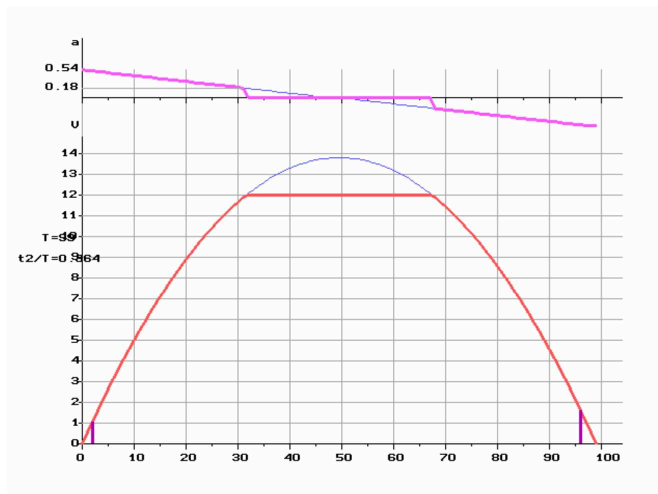


Fig.13 The variation of speed and acceleration for T=99 s

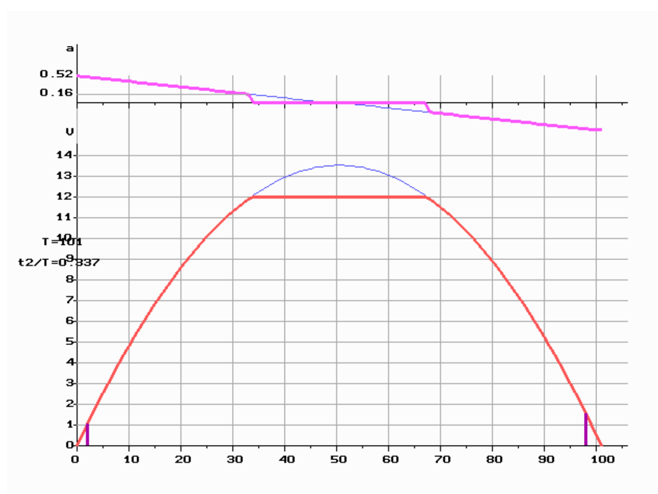


Fig.14 The variation of speed and acceleration for T=101 s

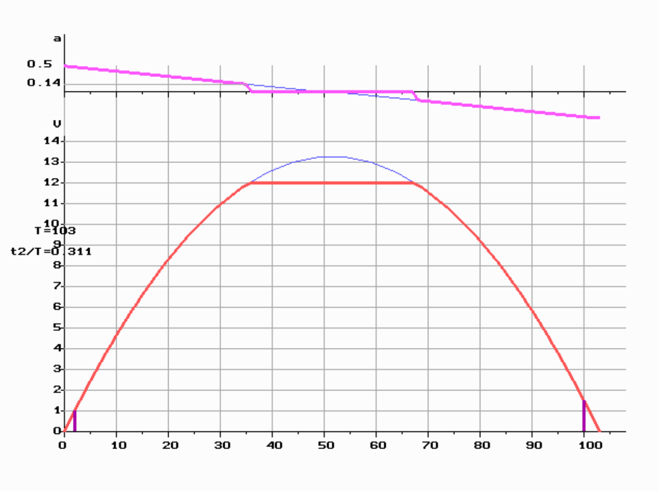


Fig.15 The variation of speed and acceleration for T=103 s

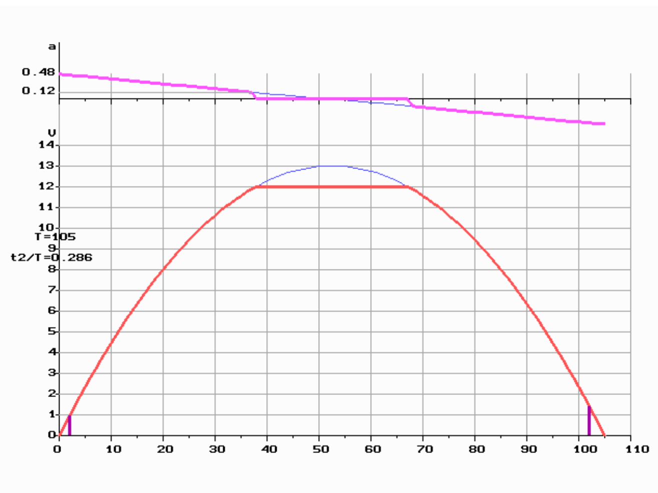


Fig.16 The variation of speed and acceleration for T=105 s

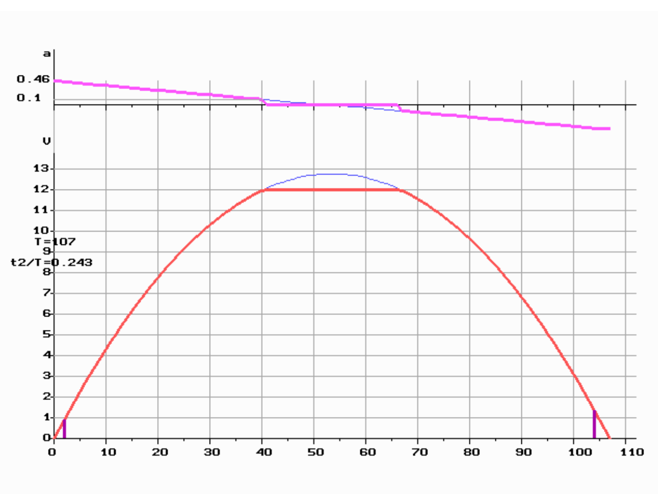


Fig.17 The variation of speed and acceleration for T=107 s

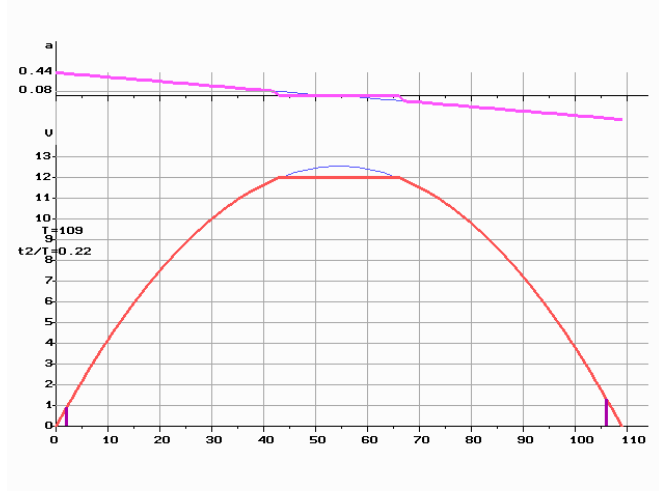


Fig.18 The variation of speed and acceleration for T=109 s

X. Conclusions

- Analysing the optimisation trials of electric operation of hoisting installations, presented in the speciality literature, it is observed that these are valid only for trapezoid tachograms (with constant accelerations and linear variation of speed in extreme periods). There is no certainty that this type of variation is optimum for ensuring the value of the minimum power. Imposing from the beginning a trapezoid form of the tachogram does not have any scientific justification, being made empirically;
- In order to minimise the actuating power of the extraction installations, the method of the calculus of variations is used, establishing an adequate mathematical model;
- In order to use the proposed optimisation method, the definition of the optimisation and restriction functional was imposed. The optimisation functional is based on the peripheral force of the cable actuating organism results from the general equation of dynamics;
- The solutions of Euler-Poisson equations of the optimisation functional differ depending the degree of balance of the installation;
- The digital integration of the functional of the equivalent force has to be made separately, for each phase of the extraction, considering the difference between the restrictions characterising the distinct phases;
- Using the third degree quadrate formula for the digital integration of the functional corresponds completely to the precision required by the calculations;
- The important determination volume for integrating the optimisation functional implies the use of computers. The software developed in C language and also experimented proved itself to be a fast tool for practical calculations;
- The developed calculation software allow the fast determination of the minimum actuating power for any mono or multi cable, with tilting containers or cage extraction installation (unbalanced, statically or dynamically balanced);
- Following the use of the developed software for the extraction installations with cages or tilting containers, considering the real characteristic parameters, values of the

actual power resulted with 50 - 70 kW smaller than when classical methods were used, representing therefore a relative decrease of power and consequently of the consumption of energy with approximately 10%;

- The proposed method is an operative and precise one and may serve to verify and design the extraction installations, determining the optimum functional parameters.

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