Effect of a multiscale factor on the axisymmetric vibrations of composite and layered cylindrical shells with cracks

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Abstract: Vibrations of circular cylindrical shells made of layered composite materials are considered. The shells are weakened by circumferential cracks. The influence of circumferential cracks with constant depth on the vibration of the shell is prescribed with the aid of a matrix of local flexibility coupled with the coefficient of the stress intensity known in the linear elastic fracture mechanics. Effect of a multiscale factor on the axisymmetric vibrations of composite and layered cylindrical shells with cracks is considered. Dependence of the Young's modulus from quantity of molecular layers of a cylindrical shell is investigated.

Numerical results are presented for the case of the shell with one circular crack.

KeyWords: **-**composite, layered shell, axisymmetric vibration, crack, multiscale factor

I. INTRODUCTION

UNTILthe fracture takes place the large class of composites behave as elastic bodies, following the Hooke's law. Usually the facture of bodies from such material is brittle, for example, destruction of bodies from fiber-glass. Therefore for investigations of small deformations in bodies from such materials, methods of the classical theory of elasticity of an anisotropic body can be used.

Circular cylindrical shells, made of composite materials, are widely used in many fields of engineering, especially in civil, mechanical, aerospace, marine and chemical industry. Vibration of circular cylindrical shells from composite materials is of interest in a number of different fields.

The problem of the theory of elasticity of an anisotropic body is similar to that of an isotropic body. The equations of balance and the geometrical equations of the theory of elasticity do not depend on a choice of a material of a body. Only physical equations are different because laws of deformation for anisotropic and isotropic bodies are different. For investigations of laws of deformation of anisotropic materials like fiber-glasses or other composite materials there are two approaches. The first approach, so-called phenomenological, considers a composite material, as a homogeneous monolithic anisotropic material.

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In this case mechanical parametersof a material (limits of elasticity,durability, etc.) are considered as some integral characteristics.

Exact definition of modulus of elasticity of such materials is an intricate problem. Such approach assumes direct usage of the theory of elasticity of the anisotropic continuums. The second approach – structural – assumes that a composite material is considered as an inhomogeneous reinforced continuum. In the case of this approach the mechanical characteristics of the reinforced material are defined through mechanical characteristics of initial components - matrix and fibre. For example, the rule of mixture is used widely [16], where the modulus of elasticity of composite is defined as

$$E_k = \alpha E_b V_b + E_m V_m .$$

Here $E_b$and$E_m$are moduli of elasticity of the fibre and matrix, respectively; E_k - modulus of elasticity of the composite; α - the coefficient depending on the layout of a fibre [14] (for unidirectional lay out α =1, for perpendicular α =0,5, for casual α =3/8); V_b and V_m –volumes of fibre and matrix in the composite. Both these approaches supplement each other mutually at development. As show experiments, elastic composites are anisotropic.

Substance transition from macro- and micro- to the nanosizes involves high-quality changes in their physical, mechanical, physical and chemical and other properties. These changes are so perspective in the practical therefore before scientists is an urgent task to study and understand the mechanism of their emergence.

Than the bodies having the nanosizes, differ from usual bodies? The most obvious distinction - growth of a role of near-surface area. Interaction between molecules (atoms) on a surface differs from volume as they have no neighbors from outer side. In volume, block, bodies the contribution of this layer with macroscopic properties is smallest, and it usually neglect. However when the sizes of a body become small, commensurable with molecular sizes (nanodimensional), influence of near-surface area becomes considerable, and properties of substance qualitatively change.

Since the crack- like defects are practically unavoidable during the manufacturing and operation of structural elements there exists the need for the information about the sensitivity of vibrational parameters of the shell with respect to defects. Vibration and stability of notched beams was investigated by Dimarogonas [1], Chondros and Dimarogonas [2,3], Rizos et al [4], Liang et al [5], Kisa et al [6], Lellep and Sakkov [7], Krawczuk, Ostachowich [8,9] making use of the weightless rotational spring model. In the paper [12-14, 17] Lellep and Roots investigated axisymmetric vibrations of cylindrical shells with circumferential cracks.

According to this concept a beam with a crack can be treated as a structure consisting of two segments. These segments are connected each other with a rotational spring which stiffness is coupled with the stress intensity coefficient of the structure with the crack.

This idea was extended to composite structures and to buckling of composite columns by Nikpour and Dimarogonas [10,11].

In this paper we will study free axisymmetric vibrations of composite andlayered cylindrical shells with cracks and effect of a multiscale factor on them.

II. FORMULATION OF THE PROBLEM FOR LAYERED SHELLS

Consider a layered, circular cylindrical shell with length *l*and radius *R*(see Fig. 1). The shell can be divided into n ring segments. The symbol n denotes the number of total ring segments separated from the rest cylindrical shell by the sections where the thickness variations take place. Every *j*th ring segment of shell has *q* layers. Each layer is isotropic with thickness h_{ij}, Young's modulus E_{ij}, Poisson'sratio ν_{ij}, and mass density ρ_{ij} as show in Fig.1.

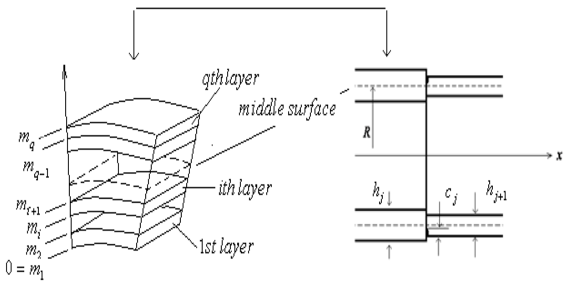


Fig.1: Geometry of a layered shell.

Let's denote

$$\rho_{ij} = \rho_{1j} d_{ij}, \qquad (1)$$

where$d_{ij}$is a constant of proportionality, d_{1j}=1and similarly Young's modulus for each layer

$$E_{ij} = E_{1j} e_{ij}, \qquad (2)$$

where$e_{ij}$is a constant of proportionality, e_{1j}=1.

We will denote the thickness of each layer $h_{ij}$by

$$h_{ij} = (z_{i+1j} - z_{ij}) h_{1j}, \qquad (3)$$

where$z_{ij}$is a local coordinate of a layer with the thickness $h_j$and z_{1j}=0.

The mass of *j*th ring segment will be equal

$$\rho_{1j} h_{1j} \sum_{i=1}^{q} d_{ij} (z_{i+1j} - z_{ij}). \qquad (4)$$

For the *j*th ring segment, the free axisymmetric vibration motion can be described by the equations [12]

$$\frac{\partial N_{1j}}{\partial x} = 0, \qquad (5)$$

$$\frac{\partial^2 M_j}{\partial x^2} - \frac{N_j}{R} - \rho_j h_j \frac{\partial^2 w}{\partial t^2} = 0,$$

where for calculation of thin shells often use following formulas [15]:

$$N_{1j} = \int_0^{h_j} \sigma_{1j} dz,$$

$$N_j = \int_0^{h_j} \sigma_j dz, \qquad (6)$$

$$M_j = \int_0^{h_j} \sigma_{1j} z dz,$$

where

$$\sigma_{1j} = \left[E_j / (1 - \nu_j^{\,2})\right]\left[(\varepsilon_1 + z\chi) + \nu_j \varepsilon\right],$$

$$\sigma_j = \left[E_j / (1 - \nu_j^{\,2})\right]\left[\varepsilon + \nu_j (\varepsilon_1 + z\chi)\right].$$

Let's donate $u_j(x,t)$ as the axial displacement and $w_j(x,t)$ as the radial displacement of the *j*th ring segment, where *t* is time.

If for all ring segments $\nu_{ij} = \nu$,by using (1) - (3) and [12]

$$\varepsilon_{1j} = \frac{\partial u_j}{\partial x},$$

$$\varepsilon_j = \frac{w_j}{R}, \qquad (7)$$

$$\chi_j = -\frac{\partial^2 w_j}{\partial x^2}$$

theforce N_j and bending moment M_j(6) can be written as

$$N_j = \int_0^{h_{1j}} \sigma_j dz + \int_{h_{1j}}^{h_{2j}} \sigma_j dz + ... + \int_{h_{ij}}^{h_{i+1j}} \sigma_j dz + ...$$

$$... + \int_{h_{q-1j}}^{h_{qj}} \sigma_j dz = b_j E_{1j} h_j \frac{w}{R}, \qquad (8)$$

$$M_j = -\frac{E_{1j} h_j^{\,3}}{12(1-\nu^2)} \bar{a}_j \frac{\partial^2 w}{\partial x^2},$$

where

$$\bar{a}_j = 4(\sum_{i=1}^{q} e_{ij}(z_{i+1j}^3 - z_{ij}^3)) - \frac{-3(\sum_{i=1}^{q} e_{ij}(z_{i+1j}^2 - z_{ij}^2))^2}{\sum_{i=1}^{q} e_{ij}(z_{i+1j} - z_{ij})}, \qquad (9)$$

and

$$b_j = \sum_{i=1}^{q} e_{ij}(z_{i+1j} - z_{ij}). \qquad (10)$$

III. DETERMINATION OF ELASTIC CHARACTERISTICS OF UNIDIRECTIONAL FIBROUS COMPOSITES

Composite materials have high specific durability and stiffness. Such materials allow in present-day machines and designs to lower the weight and to raise corrosion stability. It opens essentially new possibilities in the optimum design of structures and creation of new designs. Engineering approaches to calculation of composites allow to find approximate results, analytical approaches yield exact results only for periodic structures with enough simple geometry.

Let us consider small deflections of axisymmetric circular cylindrical shells similarly with that done in chapter 2. In this case the material of shells is a unidirectional fibrous composite. Let $E_f$and E_m be the Young's modulus of fiber material and matrix material, respectively; υ_f is the volume fraction of fibres, d is the diameter of a fibre. Let's denote the ratio of Young's moduli

$$E_f / E_m = f.$$

In this case the basic equations of static equilibrium of the shell are of the following form [12]

$$\frac{\partial N_1}{\partial x} = 0, \qquad \frac{\partial M}{\partial x} = \mathrm{Q},$$

$$\frac{\partial^2 M}{\partial x^2} - \frac{N}{R} - q = 0,$$

where

$$N_1 = \frac{E}{1-\nu^2}\int_0^h (\varepsilon_1 + z\chi + \nu\varepsilon)dz, \quad (N_1 = 0),$$

$$N = \frac{E}{1-\nu^2}\int_0^h (\varepsilon + \nu(\varepsilon_1 + z\chi)dz, \qquad (11)$$

$$M = \frac{E}{1-\nu^2}\int_0^h (\varepsilon_1 + z\chi + \nu\varepsilon)zdz,$$

Note that relations (16) hold good, if E=*const* and ν=*const*. However, $E=E(z)$ and $\nu=\nu(z)$,in a layered or laminate shell. In a composite structure

$$E(z) = \begin{cases} E_m, & \text{for matrix layer,} \\ E_f, & \text{for fibre layer,} \end{cases}$$

and

$$\nu(z) = \begin{cases} \nu_m, & \text{for matrix layer,} \\ \nu_f, & \text{for fibre layer,} \end{cases}$$

Thus the generalized stresses in a non-homogeneous shell should be calculated as

$$N_1 = \int_0^h \frac{E(z)}{1-\nu(z)^2}(\varepsilon_1 + z\chi + \nu\varepsilon)dz,$$

$$N = \int_0^h \frac{E(z)}{1-\nu(z)^2}(\varepsilon + \nu(\varepsilon_1 + z\chi)dz, \qquad (12)$$

$$M = \int_0^h \frac{E(z)}{1-\nu(z)^2}(\varepsilon_1 + z\chi + \nu\varepsilon)zdz.$$

Comparing (11) with (12), one can evaluate the mean values of elastic moduli as

$$Eh = E_m(f\upsilon_f + (1\text{-}\upsilon_f))h,$$

$$\frac{Eh^3}{12(1-\nu^2)} = \int_0^h \frac{E(z)}{(1-\nu(z)^2)} z^2 dz - \frac{(\int_0^h \frac{E(z)}{(1-\nu(z)^2)} zdz)^2}{\int_0^h \frac{E(z)}{(1-\nu(z)^2)} dz}.$$

This results in

$$E = E_m(f\upsilon_f + (1\text{-}\upsilon_f)),$$

$$\frac{E}{1-\nu^2} = 4\left[(\frac{E_m}{1-\nu_m^2} - \frac{E_f}{1-\nu_f^2})\sum_{i=1}^{n} z_i^3 + \frac{E_f}{1-\nu_f^2}\right] + 3\frac{\left[(\frac{E_m}{1-\nu_m^2} - \frac{E_f}{1-\nu_f^2})\sum_{i=1}^{n} z_i^2 + \frac{E_f}{1-\nu_f^2}\right]^2}{\frac{E_m}{1-\nu_m^2}(1-\upsilon_f) + \frac{E_f}{1-\nu_f^2}\upsilon_f},$$

where

$$E_i = \begin{cases} E_m, & \text{for matrix layer,} \\ E_f, & \text{for fibre layer} \end{cases}$$

and

$$\nu_i = \begin{cases} \nu_m, & \text{for matrix layer,} \\ \nu_f, & \text{for fibre layer.} \end{cases}$$

IV. EFFECT OF A MULTISCALE FACTOR ON THE YOUNG'S MODULUS OF A CYLINDRICAL SHELL

The most obvious distinction between bodies having the nanosizes and usual bodies - growth of a role of near-surface area. Interaction between molecules (atoms) on a surface differs from volume as they have no neighbors from outer side. It is reason of reorganization of surface area. In volume, block, bodies the contribution of this layer with macroscopic properties is smallest, and it usually neglect. However when the sizes of a body become small, commensurable with molecular sizes (nanodimensional), influence of near-surface area becomes considerable, and properties of substance qualitatively change.

The following example shows it. Consider a layered, circular cylindrical shell with length l and radius R, like in p.2. We will denote 2 types of layers: internal and external with molecular sizes. So we will have 2 surface layers and n-2 internal layers. All layers have thickness h_i. Let's denote

$$E_2=\alpha E_1,$$

where α is a constant of proportionality between Young's modulus of surface layer E_2 and Young's modulus of internal layer E_1 and each layer has Poisson's ratioν.

In this case bending moment M(8) can be written as

$$M = -\frac{E_1 h_i^3}{12(1-\nu^2)} \bar{a} \frac{\partial^2 w}{\partial x^2}, \qquad (13)$$

where

$$\bar{a} = 4(\alpha(1+n^3-(n-1)^3)+(n-1)^3-1) - \frac{3(2\alpha n+(n-1)^2-1)^2}{2\alpha+n-2}.$$

Usually in the theory of shells bending moment looks like [15]

$$M = -\frac{E_\infty h_i^3 n^3}{12(1-\nu^2)} \frac{\partial^2 w}{\partial x^2}, \qquad (14)$$

whereE_∞ is Young's modulus of shell.

By comparing formulas (13) and (14), we find

$$E_\infty / E_1 = \bar{a} / n^3. \qquad (15)$$

The results of calculations by (15) regarding to the tuba with different constant of proportionality between Young's modulus of surface layer E_2 and Young's modulus of internal layer E_1 are presented in Figs.2-3.

The influence number of layers n in the tubeon the $E_\infty/E_1$for different values of $\alpha$is depicted in Fig. 2. Here

Young's modulus of surface layer E_2larger than Young's modulus of internal layer E_1,thus here isα>1.

In Fig. 3 different curves corresponding to different values of α<1. Here Young's modulus of surface layer E_2less than Young's modulus of internal layer E_1.

From Figs. 2-3 we can see when n<200, Young's modulus of shell E_∞ significantly depends on Young's modulus of surface layer E_2 and$E_\infty/E_1 \to 1$ if $n \to \infty$.

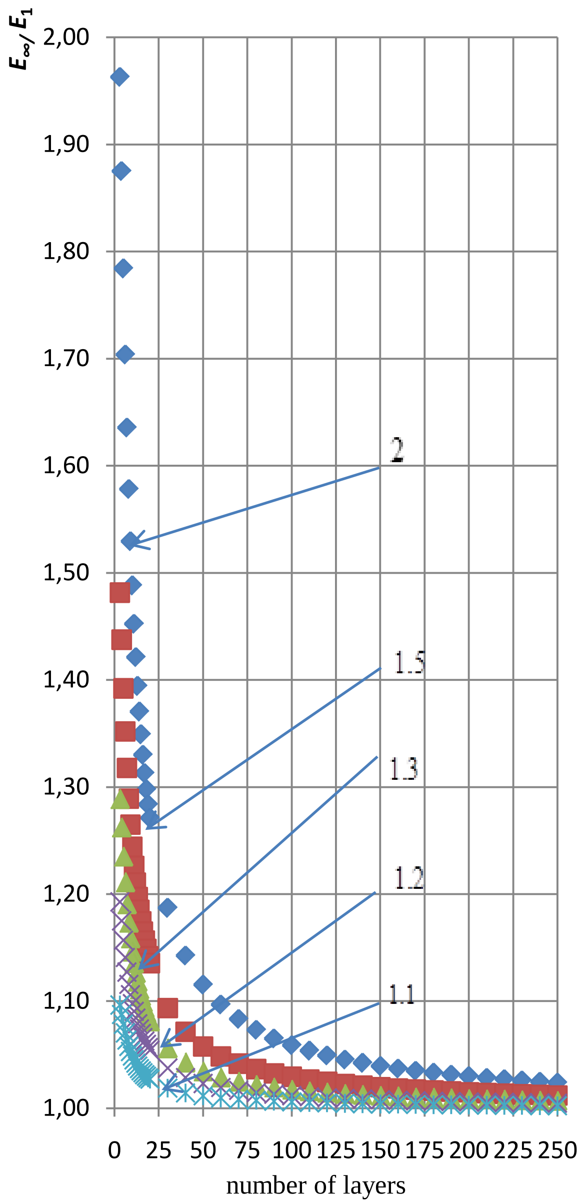


Fig.2. Dependence E_∞/E_1 from number of layers n in the tube for the cases α=2; 1.5; 1.3; 1.2; 1.1.

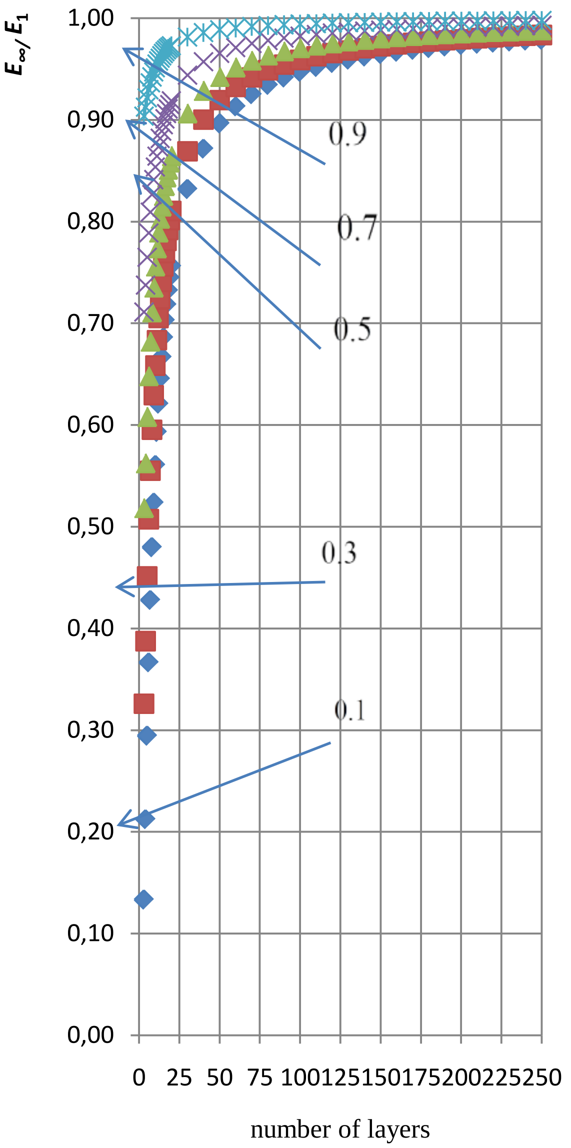


Fig.3. Dependence E_{∞}/E_1 from number of layers n in the tube for the cases α=0.9; 0.7; 0.5; 0.3; 0.1.

V. THE CRACK DISTURBANCE FUNCTION

The presence of flaws or cracks in a structural member involves considerable local flexibility. Additional local flexibility due to a crack depends on the crack geometry as well as on the geometry of the structural element and its loading. Probably the first attempt to prescribe the local flexibility of a cracked beam was undertaken by Irwin [18]who recognized the relationship between the compliance C of the beam and stress intensity factor K. Later on, Rizos, Aspragathos, Dimarogonas[19]; Dimarogonas[1]; Chondros, Dimarogonas, Yao [20]; Kukla [21] introduced so called massless rotating spring model which reveals the relationship between the stress intensity factor and local compliance of the beam. In the present study we attempt to extend this approach to axisymmetric vibrations of circular cylindrical nanoshells with circular cracks of constant depth.

Let us consider the crack located at the cross section $x=a_j$ and let the segments adjacent to the crack have thicknesses h_{j-1} and h_j, respectively. According to the current approach it is assumed that the slope of deflection w' is discontinuous, e.g.

$$w'(a_j+0,t)\text{-}w'(a_j-0,t)=\theta_j, \qquad (16)$$

whereθ_j=0.

The quantity θ_j can be treated as an additional angle caused by the crack at $x=a_j$. Ifθ_j is a generalized coordinate then corresponding generalized force is $M(a_j)$ whereas

$$\theta_j = C_j M(a_j), \qquad (17)$$

whereC_j stands for the additional compliance of the shell at $x=a_j$. Note that the compliance C is a quantity reverse to the stiffness K_T of the shell. On the other hand,

$$\theta_j = \frac{\partial U_T}{\partial M(a_j)}, \qquad (18)$$

providedU_T is the extra strain energy due to the crack. It immediately follows from (17) and (18) that

$$C_j = \frac{\partial \theta_j}{\partial M(a_j)} \qquad (19)$$

or

$$C_j = \frac{\partial^2 U_T}{\partial M^2(a_j)}. \qquad (20)$$

Note that equalities (17) - (20) are well known in the linear elastic fracture mechanics in the case when the generalized displacement and generalized force are u_j and P_j, respectively (see [22-24]).

Generalized stresses, energy release rate G and the stress intensity factor K are related to each other as

$$G = \frac{M^2}{2}\frac{dC}{dA} \qquad (21)$$

and

$$G = \frac{K^2}{E'}, \qquad (22)$$

where$A$ stands for the crack surface area and $E'=E$ for the plane stress state and $E'=E/(1-\nu^2)$for the plane deformation state.

The stress intensity factor is defined as

$$K = \sigma\sqrt{\pi c}\, F(\frac{c}{h}) \qquad (23)$$

(see[25]). Here c is the crack depth and $\sigma=6M/h^2$whereas F stands for a function to be determined experimentally. When applying (20) – (23) for the cross section $x=a_j$ with crack depth c_j one has

$$\frac{M_j^2}{2}\frac{dC_j}{dc_j} = \frac{36M_j^2}{E'h_j^4}\pi c_j F^2\left(\frac{c_j}{h_j}\right) \qquad (24)$$

provided $h_j = h_{j-1}$ and $M_j = M(a_j)$. We introduce the notation $s_j = c_j/h_j$.

Thus it follows from (24) that

$$\frac{dC_j}{ds_j} = \frac{72\pi}{E'h_j^2} s_j F^2(s_j) \qquad (25)$$

and after integration one obtains

$$C_j = \frac{72\pi}{E'h_j^2}\int_0^{s_j} s_j F^2(s_j) ds_j \qquad (26)$$

for the plane stress state. It is assumed herein that C_j=0 when s_j=0.

The function $F(s_j)$ in (23) – (26) is called shape function as it is different for experimental specimens of different shape. Many authors have investigated the problem of determination of the stress intensity factor for various specimens (among others Brown and Srawley, 1967; Freund and Hermann, 1976; Irwin, 1960; Tada, Paris, Irwin, 2000).

In the present study we are resorting to the data of experiments conducted by Brown and Srawley which can be approximated as (see [25])

$$F(s_j) = 1{,}93 - 3{,}07 s_j + 14{,}53 s_j^2 - 25{,}11 s_j^3 + 25{,}8 s_j^4. \qquad (27)$$

Combining (26) and (27) one can obtain

$$C_j = \frac{72\pi}{E'h_j^2} f(s_j), \qquad (28)$$

where

$$\begin{aligned} f(s_j) = {} & 1{,}862 s_j^2 - 3{,}95 s_j^3 + 16{,}375 s_j^4 - \\ & - 37{,}226 s_j^5 + 76{,}81 s_j^6 - 126{,}9 s_j^7 + \\ & + 172{,}5 s_j^8 - 143{,}97 s_j^9 + 66{,}56 s_j^{10}. \end{aligned} \qquad (29)$$

The function (29) is employed also in papers by Dimarogonas[1];,Chondros, Dimarogonas, Yao [20];, Kukla[21];.

According to the concept of massless rotating spring one can equalize $K_{Tj}=1/C_j$ and thus

$$K_{Tj} = \frac{E'h_j^2}{72\pi \cdot f(s_j)}, \qquad (30)$$

where

$$f(s_j) = \int_0^{s_j} \xi F^2(\xi) d\xi. \qquad (31)$$

From (17) with (7), (8) one obtains

$$\theta_j = -\frac{Eh_j^3}{12K_{Tj}(1-\nu^2)} \cdot w''(a_j+,t). \qquad (32)$$

VI. SOLUTION OF GOVERNING EQUATIONS FOR LAYERED SHELLS

By using (4) - (8) the equation of motion (5) of jth ring segment can be described by the equation

$$\overline{D}_j \overline{a}_j \frac{\partial^4 w}{\partial x^4} + \frac{E_{1j}h_j}{R^2} b_j w = -\rho_{1j} h_j c_j \ddot{w}, \qquad (33)$$

where $\overline{D}_j = E_{1j}h_j^3/12(1-\nu^2)$, ν=const and

$$c_j = \sum_{i=1}^{q} d_{ij}(z_{i+1j} - z_{ij}). \qquad (34)$$

Evidently, it is reasonable to look for the general solution of the equation (33) in the form

$$w(x,t) = X_j(x)T(t) \qquad (35)$$

It follows from (33) with (35) that

$$X_j^{IY} - r_j^4 X_j = 0 \qquad (36)$$

where

$$r_j^4 = \omega^2 \cdot \frac{12\rho_1(1-\nu^2)}{E_1 h_j^2}\frac{c_j}{\overline{a}_j} - \frac{12(1-\nu^2)}{R^2 h_j^2}\frac{b_j}{\overline{a}_j}, \qquad (37)$$

Frequency of free vibrations of a layered shell will be equal

$$\omega = \sqrt{\frac{E_1}{\rho_1}}\frac{1}{R}\sqrt{\frac{k^4R^2}{12(1-\nu^2)}\frac{\overline{a}_j}{c_j} + \frac{b_j}{c_j}},$$

where $r_j = k/\sqrt{h_j}$. Here k is characteristic number.

The general solution of the linear fourth order equation (36) can be presented as

$$\begin{aligned} X_j(x) = {} & A_j \sin(r_j x) + B_j \cos(r_j x) + \\ & + C_j \sinh(r_j x) + D_j \cosh(r_j x) \end{aligned} \qquad (38)$$

Assume that the ends of the shell are simply supported. We arrive at the boundary conditions at the points x=0 and $x=l$

w=0, $\qquad M_x$=0.

It is known in the linear elastic fracture mechanics that repeated loading and stress concentration at sharp corners entails cracks. Thus it is reasonable to assume that at the re-entrant corners of steps e.g. at $x=a_j$ (j=0,..., n) cracks of depth c_j are located. For the simplicity sake we assume that these flaws are stable circular surface cracks. In the present study like in [1-4] no attention will be paid to the crack extension during operation of the structure. The concontinuity and jump conditions at x=a are (see [12])

$$w(a+0)-w(a-0)=0,$$
$$w'(a+0)-w'(a-0)=\frac{72\pi}{E'h_1^2}f(s)M_x(a\text{-}0),$$
$$M_x(a\text{-}0)=M_x(a+0),$$
$$M_x'(a\text{-}0)=M_x'(a+0).$$

Here

$$f(s)=1{,}862s^2-3{,}95s^3+16{,}375s^4-$$
$$-37{,}226s^5+76{,}81s^6-126{,}9s^7+$$
$$+172{,}5s^8-143{,}97s^9+66{,}56s^{10}$$

is the stress correction function and is employed in papers [1, 2]. We introduce the notation $s=c/h$.

Let us consider now the case when n=2. By using equation (8) we can rewrite the equation for definition of characteristic number k (see [12]) as

$$((1-g_1)\cos kl_1\cosh kl_4+((g_2-g_4)\sin kl_1+$$
$$+g_3\cos kl_1)\sinh kl_4)((1-g_1)\cosh kl_1\cos kl_4-$$
$$-((g_2-g_4)\sinh kl_1+g_3\cosh kl_1)\sin kl_4)-$$
$$-((1+g_1)\cos kl_1\cos kl_4+((g_2+g_4)\sin kl_1+$$
$$+g_3\cos kl_1)\sin kl_4)((1+g_1)\cosh kl_1\cosh kl_4-$$
$$-((g_2+g_4)\sinh kl_1+g_3\cosh kl_1)\sinh kl_4)=0,$$

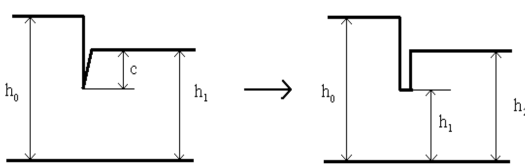
where

$$g_1=(h_0\bar{a}_0/h_1\bar{a}_1)^2,$$
$$g_2=(h_1/h_0)^{1/2},$$
$$g_4=g_1g_2,$$
$$g_3=6\pi f(s_1)\sqrt{h_1}\,g_1k$$

and

$$l_1=a/\sqrt{h_0},\qquad l_4=(l-a)/\sqrt{h_1}.$$

Here $\bar{a}_0$ and $\bar{a}_1$ –are the values of $\bar{a}_j$ for j=0 and j=1, respectively.



a) Crack model I b) Crack model II

Fig.4. Two models of a crack in shell.

In the presented solution of governing equations for layered shells we used the crack model I as shown in Fig.4 a).

The crack model IIas shown in Fig.4 b) is more preferable to use on the molecular (atomic) level of body. In this case the shell with a crack is simulated geometrically as a two-stepped shell with small length of the intermediate section. This intermediate section has a size as ainteratomic distance. In angular points of this intermediate section$x=a_1$and $x=a_2$we will have the following continuity conditions

$$w(a_{1,2}+0)-w(a_{1,2}-0)=0,$$
$$w'(a_{1,2}+0)-w'(a_{1,2}-0)=0,$$
$$M_x(a_{1,2}\text{-}0)=M_x(a_{1,2}+0),$$
$$M_x'(a_{1,2}\text{-}0)=M_x'(a_{1,2}+0),$$

where

$$M_x(x)=-\frac{\mathrm{E}_1h_i^3}{12(1-\nu^2)}\bar{a}\,\frac{\partial^2w}{\partial x^2}.$$

VII. NUMERICAL RESULTS

For an illustration of the method offered in that article the simply supported shell has been considered (See Fig.5).

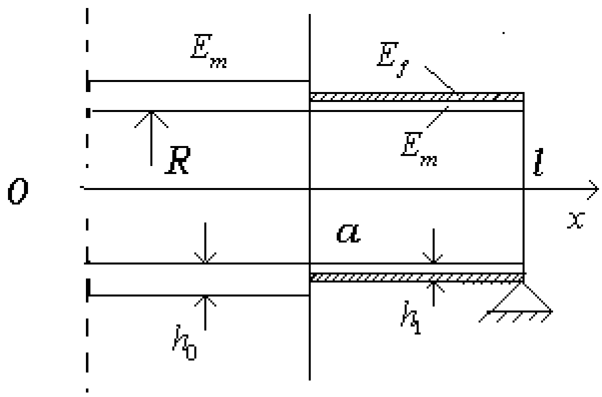


Fig. 5.Cylindrical shell with fiber-glass layer.

The shell under consideration has a uniform shell wall with Young's modulus $E_m$for $x\in(0,a)$whereas it consists of two layers with Young's moduli $E_m$and $E_s$respectively, and thickness h_1 for $x\in(a,l)$, as shown in Fig. 5. Geometrical parameters for the one-stepped shell are: l=0,6; h_0=0,006; $h_1=\gamma h_0$; γ=0,7. It is assumed herein that the material of the shell segment with $h_0$is a homogeneous elastic material - aluminium-lithium alloy with E_m=76GPa. The shell segment with h_1 has two layers. The inner layer is made of the same material as the another segment and the material of the top layer is a fiber-glass. In the segment with h_1, v is the volume fraction of fibres. We will consider four kinds of fiber-glasses with E_s= 20GPa, 35GPa, 50GPa, 152GPa (s= E_s/ E_m), respectively.In this case the equation for definition of characteristic number k is

$$((1-g_1)\cos kl_1\cosh kl_4+((g_2-g_4)\sin kl_1+$$
$$+g_3\cos kl_1)\sinh kl_4)((1-g_1)\cosh kl_1\cos kl_4-$$
$$-((g_2-g_4)\sinh kl_1+g_3\cosh kl_1)\sin kl_4)-$$
$$-((1+g_1)\cos kl_1\cos kl_4+((g_2+g_4)\sin kl_1+$$
$$+g_3\cos kl_1)\sin kl_4)((1+g_1)\cosh kl_1\cosh kl_4-$$
$$-((g_2+g_4)\sinh kl_1+g_3\cosh kl_1)\sinh kl_4)=0,$$

where

$$g_1 = (h_0/h_1(f\nu \; + (1-\nu\;))^2,$$
$$g_2 = (h_1/h_0)^{1/2},$$
$$g_4 = g_1 g_2,$$
$$g_3 = 6\pi f(s_1)\sqrt{h_1}\, g_1 k$$

and

$$l_1 = a/\sqrt{h_0}, \qquad l_4 = (l-a)/\sqrt{h_1}.$$

The results of calculations regarding to the shell with simply supported ends are presented in Figs. 6-7. The influence of the crack *c/h* on the characteristic number *k* for the fixed values ν=0,2; β=0,2; γ=0,7 and different values of *s* is depicted in Fig. 6. In Fig. 7 different curves corresponding to different values of $\nu$are presented in the case *s*=2; β=0,2; γ=0,7. Here $\beta=a_1/l$,$\gamma=h_1/h_0$, as in previous sections of the study.

Calculations carried out showed that the characteristic number *k* of the shell decreases when the crack depth increases as might be expected.

Calculations were made by means of the package Mathcad.

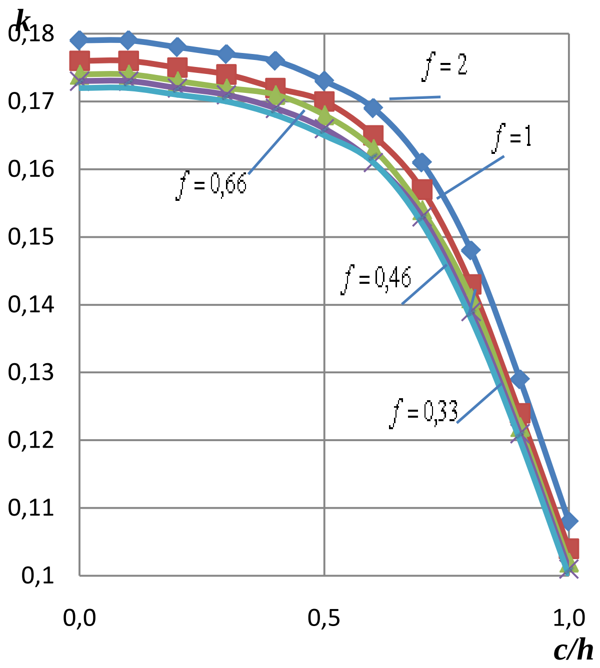


Fig. 6. Frequency parameters *k* for simply supported shells with one-step thickness variation and creck, the case ν=0,2; β=0,2; γ=0,7.

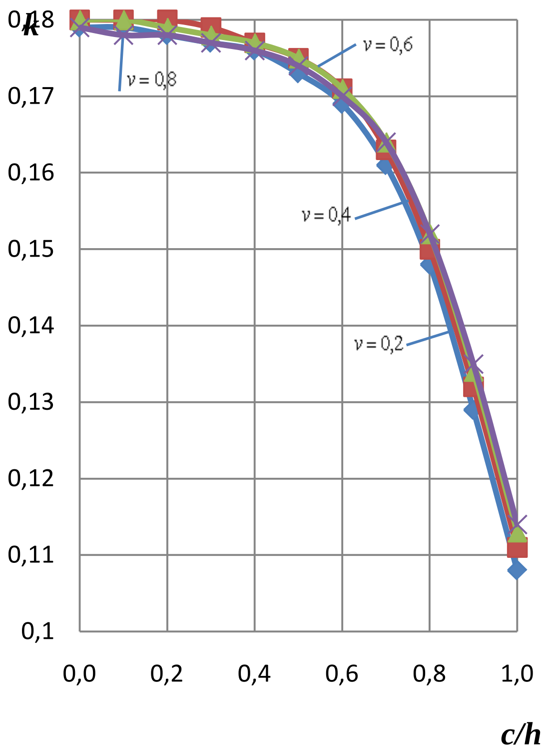


Fig.7. Frequency parameters *k* for simply supported shells with one-step thickness variation and crack, the case *f*=2; β=0,2; γ=0,7.

VIII. CONCLUDING REMARKS

The natural frequency of vibrations is determined for various non-homogeneous materials.

Calculations carried out showed that the crack location and its dimensions have strong influence on the natural frequency of vibrations. Results of calculations showed that when the crack depth increases then the frequency of natural vibrations decreases.

When the sizes of a body become small, commensurable with molecular sizes (nanodimensional), influence of near-surface area becomes considerable, and properties of substance qualitatively change. Therefore at nano level it is more preferable to consider layered model of a material.

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