Computer Modeling of Heat Balance in Counterflow Tube Heat Exchanger

H. Charvátová, D. Janáčová, R. Drga, O. Líška, V. Vašek, M. Zálešák

***Abstract*—** The paper deals with mathematical modeling of heating and cooling of fluids in heat exchanger by use software MAPLE. It describes computer teaching application programmed in MAPLE for computation of temperature curves derived from mathematical model of heat balance in the thermal isolated counterflow tube heat exchanger. The accuracy of data computed by MAPLE were verified by simulation with use commercial software COMSOL MULTIPHYSICS. The obtained results are also presented.

***Keywords*—**Heat balance, counterflow tube heat exchanger, MAPLE software, COMSOL MULTIPHYSICS software, computer teaching aid

I. Introduction

Heat transfer problem is a part of study subject Process engineering that is taught at Faculty of Technology and at Faculty of Applied Informatics of the Tomas Bata University in Zlín. But study and calculation relating to these problems are in many cases relatively complicated and also time-consuming. Moreover, using mathematical software is often required to obtain sufficiently accurate calculations. Therefore we make software applications which help students to study and solve selected technological problems.

In this paper we present software application that is designed for determination of the heat balance in thermal insulated counterflow tube exchanger. We made this application by use computer algebraic system MAPLE. The application can calculate and visualize the temperature courses of heated and cooled fluids flowing inside thermal insulated heat exchanger. Mathematical model, The computing principle, user interface, use of the application and verification of its accuracy we describe in the following sections.

H. Charvátová, Tomas Bata University in Zlín, Faculty of Applied Informatics, Department of Automation and Control Eigineering, nám. T. G. Masaryka 5555, 760 01 Zlín, Czech Republic (e-mail: charvatova@fai.utb.cz)

D. Janáčová, Tomas Bata University in Zlín, Faculty of Applied Informatics, Department of Automation and Control Eigineering, nám. T. G. Masaryka 5555, 760 01 Zlín, Czech Republic; phone: +420 576 035 274; fax: +420 576 032 716; e-mail: janacova@fai. utb.cz

R. Drga, Tomas Bata University in Zlín, Faculty of Applied Informatics, Department of Security Eigineering, nám. T. G. Masaryka 5555, 760 01 Zlín, Czech Republic (e-mail: rdrga@fai.utb.cz)

O. Líška, Technical University of Košice, Mechanical Engineering Faculty, Department of Automation, Control and Human Machine Interactions, Letná 9, 042 00 Košice, Slovak Republic (e-mail: ondrej.liska@tuke.sk)

V. Vašek, Tomas Bata University in Zlín, Faculty of Applied Informatics, Department of Automation and Control Eigineering, nám. T. G. Masaryka 5555, 760 01 Zlín, Czech Republic (e-mail: vasek@fai. utb.cz)

M. Zálešák, Tomas Bata University in Zlín, Faculty of Applied Informatics, Department of Automation and Control Eigineering, nám. T. G. Masaryka 5555, 760 01 Zlín, Czech Republic (e-mail: zalesak@fai. utb.cz)

II. Heat Balance of Counterflow Tube Heat Exchanger

The studied heat exchanger is engineered as a cylindrical vessel with a deeply arched bottom and lid, which is thermally isolated from the surroundings. Inside the container is stored tube sheet with tens to hundreds of straight tubes. One fluid flows inside the inner tubes and a other fluid runarounds the inner tubes of the heat exchanger [1], [2].

Thermal energy transfers between heated and cooled fluid through the inner tube wall. In consideration counterflow construction, heated and cooled fluids flow to each other in opposite directions.

The heat flow between fluids inside the exchanger can be described by equation (1)

$$\dot{m}_1 \cdot c_{p1} \cdot \frac{dt_1(x)}{dx} = \dot{m}_2 \cdot c_{p2} \cdot \frac{dt_2(x)}{dx} \tag{1}$$

The heat transfer through the walls of the inner tubes describes equation (2)

$$\dot{m}_2 \cdot c_{p2} \cdot \frac{dt_2(x)}{dx} = -N \cdot k_L \cdot L \cdot \left(t_1(x) - t_2(x)\right). \tag{2}$$

Temperatures of liquids incoming to the heat exchanger are given by conditions (3) - (4)

$$t_1(0) = t_{11} \tag{3}$$

$$t_2(L) = t_{22}. \tag{4}$$

The heat flow through wall of the inner tubes is described by equation (5)

$$\dot{Q} = k_L \cdot N \cdot L \cdot \Delta t_{LS}, \tag{5}$$

N is number of inner tubes.

Δt_{LS} is log mean temperature difference

.

$$\Delta t_{LS} = \frac{(t_{11} - t_{21}) - (t_{12} - t_{22})}{\ln \frac{t_{11} - t_{21}}{t_{12} - t_{22}}}. \quad (6)$$

k_L is the heat passage coefficient. It can be computed according equation (7)

$$k_L = \frac{\pi}{\frac{1}{\alpha_A d_1} + \frac{\ln \frac{d_2}{d_1}}{2\lambda_{mater}} + \frac{1}{\alpha_B d_2}}, \quad (7)$$

where α is heat transfer coefficient

$$\alpha = \frac{Nu \cdot \lambda}{d}. \quad (8)$$

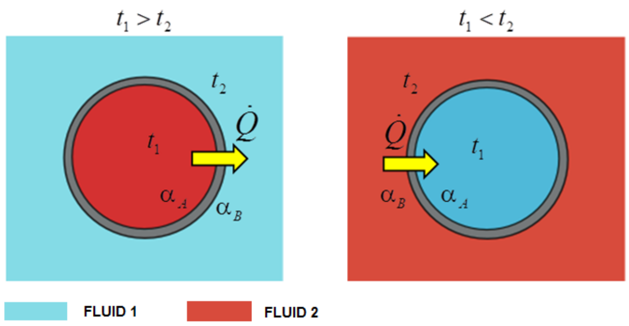

Fig. 1 Sketch of the heat flow through the wall of inner tube inside the heat exchanger

Inside the heat exchanger, the heat transfer coefficient can be computed by dimensionless criterions generally used for forced convection of fluids inside tube systems. Therefore Reynolds criterion *Re*, Grashof criterion *Gr*, Prandtl criterion *Pr* and Nusselt criterion *Nu* at average temperature are computed according to following relations (8 -16) :

$$t_m = \frac{t_1 + t_2}{2} \quad (9)$$

Reynolds criterion *Re*:

$$Re = \frac{v \cdot d \cdot \rho}{\eta} \quad (10)$$

Nusselt criterion *Nu*:

for $Re < 2{,}3 \cdot 10^3$:

$$Nu = 0{,}15 Re^{0{,}32} Pr^{0{,}33} (Gr \cdot Pr)^{0{,}1} \left(\frac{Pr}{Pr_w} \right)^{0{,}25} \cdot \varepsilon_f \quad (11)$$

Tab. 1: Values of constants ε_f of equation (11)

L/d	1	2	5	10	15	20	30	40	≥ 50
ε_f	1,90	1,70	1,44	1,28	1,18	1,13	1,05	1,02	1,00

for $2{,}3 \cdot 10^3 \leq Re \leq 1 \cdot 10^4$:

$$Nu = 0{,}116 \left(Re^{2/3} - 125 \right) Pr^{1/3} \left[1 + (d/L)^{2/3} \right] \left(\frac{\eta}{\eta_w} \right)^{0{,}14} \quad (12)$$

for $Re \geq 1 \cdot 10^4$

$$Nu = 0{,}023 Re^{0{,}8} Pr^{0{,}4} \quad (13)$$

Prandtl criterion *Pr*:

$$Pr = \frac{c_p \cdot \eta}{\lambda} \quad (14)$$

Grashof criterion *Gr*:

$$Gr = \frac{g d^3 \alpha_V \Delta t \rho^2}{\eta^2} \quad (15)$$

If the section of tube is not circular, the characteristic size is computed according equation (16)

$$d_{ekv} = \frac{4S}{o}. \quad (16)$$

III. Computer Modeling of Heat Transfer Inside the Heat Exchanger

Analytical solution of the model described in previous section we have solved by MAPLE software. For this purpose we have entered system of equations (1), (2) with conditions (3) and (4) by MAPLE source code:

```
> function1:=m1*cp1*diff(t1(x),x)=
m2*cp2*diff(t2(x),x);
```

$$function1 := m1\, cp1 \left(\frac{d}{dx} t1(x) \right) = m2\, cp2 \left(\frac{d}{dx} t2(x) \right)$$

```
> function2:=m2*cp2*diff(t2(x),x)=
-N*kL*L*(t1(x)-t2(x));
```

$$function2 := m2\, cp2 \left(\frac{d}{dx} t2(x) \right) = -N\, kL\, L\, (t1(x) - t2(x))$$

```
> sys_ode := function1,function2;
```

$$sys\_ode := m1\, cp1 \left(\frac{d}{dx} t1(x) \right) = m2\, cp2 \left(\frac{d}{dx} t2(x) \right),$$
$$m2\, cp2 \left(\frac{d}{dx} t2(x) \right) = -N\, kL\, L\, (t1(x) - t2(x))$$

```
> dsolve([sys_ode]);
```

$$\left\{ t1(x) = \_C1 + \_C2\, e^{\frac{N\, kL\, L\, (m1\, cp1 - m2\, cp2)\, x}{m2\, cp2\, m1\, cp1}},\ t2(x) = \frac{m2\, cp2\, \_C1 + \_C2\, e^{\frac{N\, kL\, L\, (m1\, cp1 - m2\, cp2)\, x}{m2\, cp2\, m1\, cp1}}\, m1\, cp1}{m2\, cp2} \right\}$$

```
> ics := t1(0) = t11, t2(L) = t22;
```

$$ics := t1(0) = t11,\ t2(L) = t22$$

Analytical solution of the model we computed by dsolve command designed for solving of ordinary differential equations:

```
> solution:=dsolve([sys_ode, ics]);
```

$$solution := \left\{ t1(x) = \frac{-t22\, m2\, cp2\, e^{\frac{L^2 N\, kL}{m1\, cp1}} + t11\, e^{\frac{L^2 N\, kL}{m2\, cp2}}\, m1\, cp1}{-m2\, cp2\, e^{\frac{L^2 N\, kL}{m1\, cp1}} + e^{\frac{L^2 N\, kL}{m2\, cp2}}\, m1\, cp1} - \frac{m2\, cp2\, e^{\frac{L^2 N\, kL}{m1\, cp1}}\, (t11 - t22)\, e^{\frac{N\, kL\, L\, (m1\, cp1 - m2\, cp2)\, x}{m2\, cp2\, m1\, cp1}}}{-m2\, cp2\, e^{\frac{L^2 N\, kL}{m1\, cp1}} + e^{\frac{L^2 N\, kL}{m2\, cp2}}\, m1\, cp1},\ t2(x) = \frac{1}{m2\, cp2} \left(\left(m2\, cp2 \left(-t22\, m2\, cp2\, e^{\frac{L^2 N\, kL}{m1\, cp1}} + t11\, e^{\frac{L^2 N\, kL}{m2\, cp2}}\, m1\, cp1 \right) \right) \Big/ \left(-m2\, cp2\, e^{\frac{L^2 N\, kL}{m1\, cp1}} + e^{\frac{L^2 N\, kL}{m2\, cp2}}\, m1\, cp1 \right) - \left(m2\, cp2\, e^{\frac{L^2 N\, kL}{m1\, cp1}}\, (t11 - t22)\, e^{\frac{N\, kL\, L\, (m1\, cp1 - m2\, cp2)\, x}{m2\, cp2\, m1\, cp1}}\, m1\, cp1 \right) \Big/ \left(-m2\, cp2\, e^{\frac{L^2 N\, kL}{m1\, cp1}} + e^{\frac{L^2 N\, kL}{m2\, cp2}}\, m1\, cp1 \right) \right) \right\}$$

```
> op(1,solution);
```

$$t1(x) = \frac{-t22\, m2\, cp2\, e^{\frac{L^2 N\, kL}{m1\, cp1}} + t11\, e^{\frac{L^2 N\, kL}{m2\, cp2}}\, m1\, cp1}{-m2\, cp2\, e^{\frac{L^2 N\, kL}{m1\, cp1}} + e^{\frac{L^2 N\, kL}{m2\, cp2}}\, m1\, cp1} - \frac{m2\, cp2\, e^{\frac{L^2 N\, kL}{m1\, cp1}}\, (t11 - t22)\, e^{\frac{N\, kL\, L\, (m1\, cp1 - m2\, cp2)\, x}{m2\, cp2\, m1\, cp1}}}{-m2\, cp2\, e^{\frac{L^2 N\, kL}{m1\, cp1}} + e^{\frac{L^2 N\, kL}{m2\, cp2}}\, m1\, cp1}$$

```
> op(2,solution);
```

$$t2(x) = \frac{1}{m2\, cp2} \left(\left(m2\, cp2 \left(-t22\, m2\, cp2\, e^{\frac{L^2 N\, kL}{m1\, cp1}} + t11\, e^{\frac{L^2 N\, kL}{m2\, cp2}}\, m1\, cp1 \right) \right) \Big/ \left(-m2\, cp2\, e^{\frac{L^2 N\, kL}{m1\, cp1}} + e^{\frac{L^2 N\, kL}{m2\, cp2}}\, m1\, cp1 \right) - \left(m2\, cp2\, e^{\frac{L^2 N\, kL}{m1\, cp1}}\, (t11 - t22)\, e^{\frac{N\, kL\, L\, (m1\, cp1 - m2\, cp2)\, x}{m2\, cp2\, m1\, cp1}}\, m1\, cp1 \right) \Big/ \left(-m2\, cp2\, e^{\frac{L^2 N\, kL}{m1\, cp1}} + e^{\frac{L^2 N\, kL}{m2\, cp2}}\, m1\, cp1 \right) \right)$$

For exchanger with only one inner tube, the equation (2) simplifies into equation (16)

$$\dot{m}_2 \cdot c_{p2} \cdot \frac{dt_2(x)}{dx} = -k_L \cdot L \cdot \left(t_1(x) - t_2(x)\right) \qquad (16)$$

and computed analytical solutions are:

```
> op(1,solution);
```

$$t1(x) = \frac{-t22\, m2\, cp2\, e^{\frac{L^2 kL}{m1\, cp1}} + t11\, e^{\frac{L^2 kL}{m2\, cp2}}\, m1\, cp1}{-m2\, cp2\, e^{\frac{L^2 kL}{m1\, cp1}} + e^{\frac{L^2 kL}{m2\, cp2}}\, m1\, cp1} - \frac{m2\, cp2\, e^{\frac{L^2 kL}{m1\, cp1}}\, (t11 - t22)\, e^{\frac{kL\, L\, (m1\, cp1 - m2\, cp2)\, x}{m2\, cp2\, m1\, cp1}}}{-m2\, cp2\, e^{\frac{L^2 kL}{m1\, cp1}} + e^{\frac{L^2 kL}{m2\, cp2}}\, m1\, cp1}$$

```
> op(2,solution);
```

$$t2(x) = \frac{1}{m2\,cp2}\left(\left(m2\,cp2\left(-t22\,m2\,cp2\,e^{\frac{L^2\,kL}{m1\,cp1}} + t11\,e^{\frac{L^2\,kL}{m2\,cp2}}\,m1\,cp1\right)\right)\Big/\left(-m2\,cp2\,e^{\frac{L^2\,kL}{m1\,cp1}} + e^{\frac{L^2\,kL}{m2\,cp2}}\,m1\,cp1\right) - \frac{m2\,cp2\,e^{\frac{L^2\,kL}{m1\,cp1}}\,(t11 - t22)\,e^{\frac{kL\,L\,(m1\,cp1 - m2\,cp2)\,x}{m2\,cp2\,m1\,cp1}}\,m1\,cp1}{-m2\,cp2\,e^{\frac{L^2\,kL}{m1\,cp1}} + e^{\frac{L^2\,kL}{m2\,cp2}}\,m1\,cp1}\right)$$

The computed temperature curves are shown in Fig. 2.

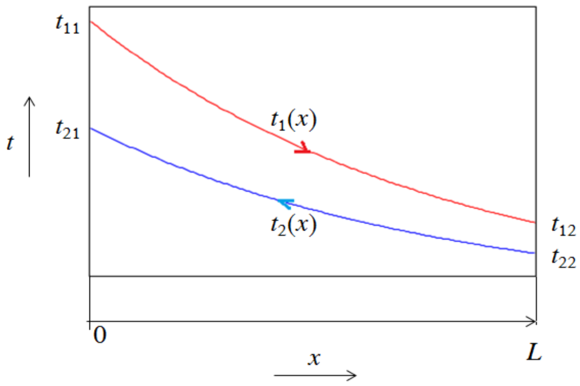

Fig. 2. Temperature courses in the counterflow heat exchanger

We have used the computed analytical solutions for simulation of fluids heating and cooling. For this purpose we programmed special application which can compute and display temperature curves $t_1(x)$ and $t_2(x)$ under required conditions. The application will be especially used as a teaching tool. Therefore we programmed it as a Maplet which is composed from windows with text fields, buttons and other tools for comfortable and easy control. Fig. 2 shows window of application for insert input parameters and computing of the temperature courses. The other windows (fig. 3 - 5) are designed for needed calculation of the heat passage coefficient inside the exchanger according equations (6 -14).

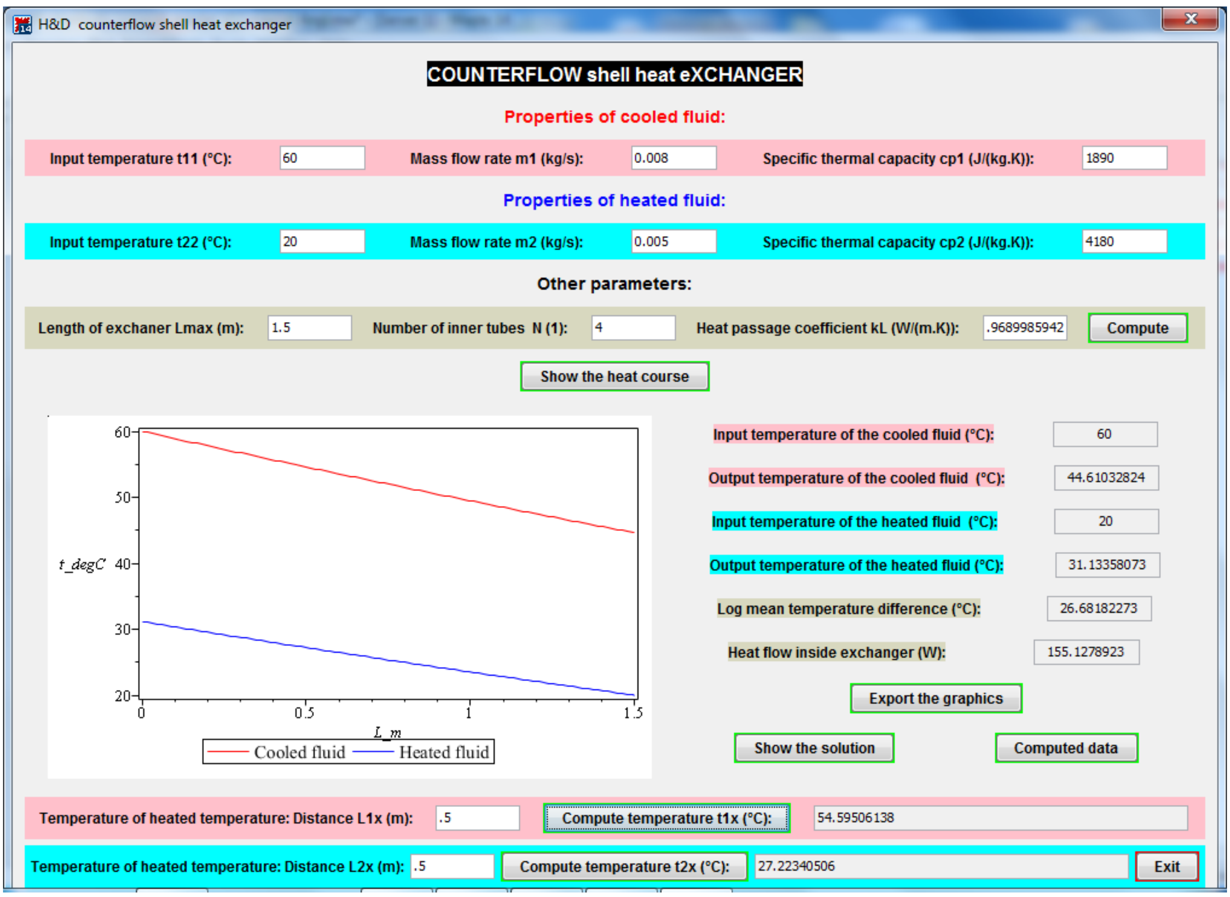

Fig. 3 Main window of the software application for simulation of fluids heating and cooling in the thermal isolated tube heat exchanger

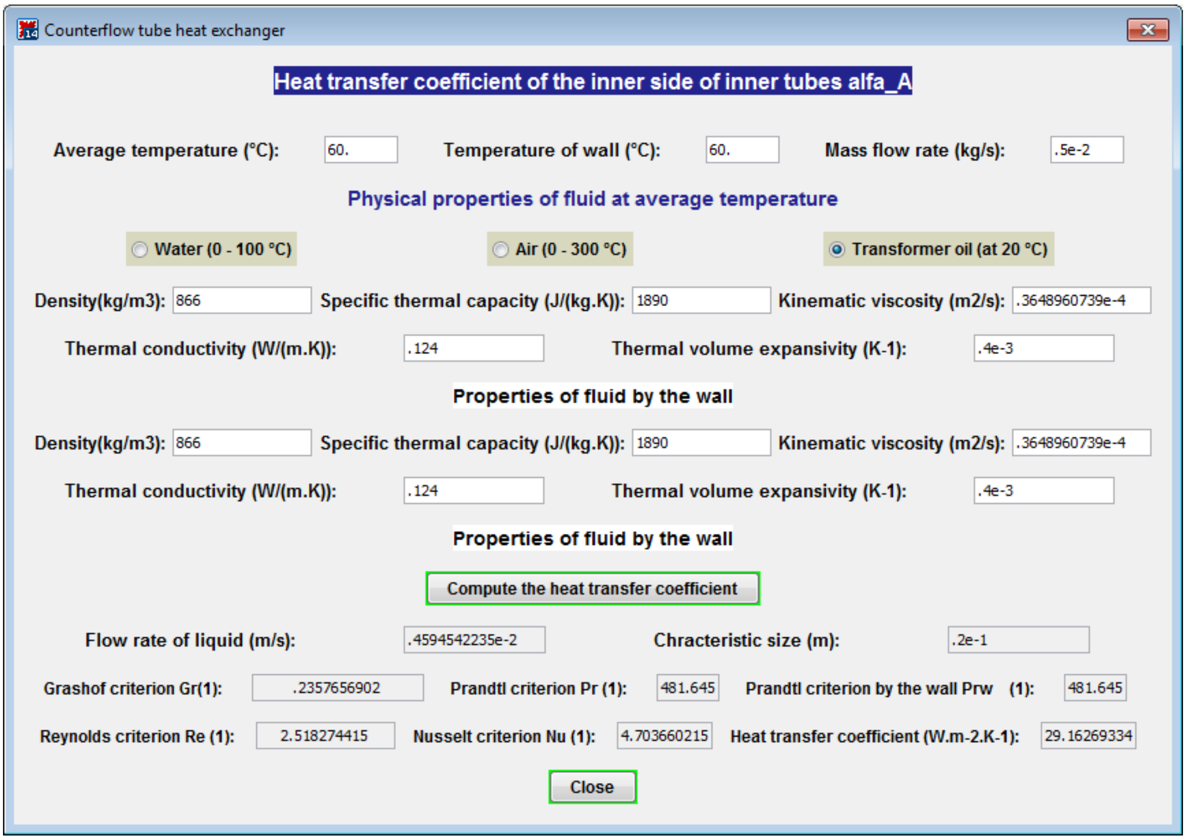

Fig. 4 Window for computing of heat transfer coefficient

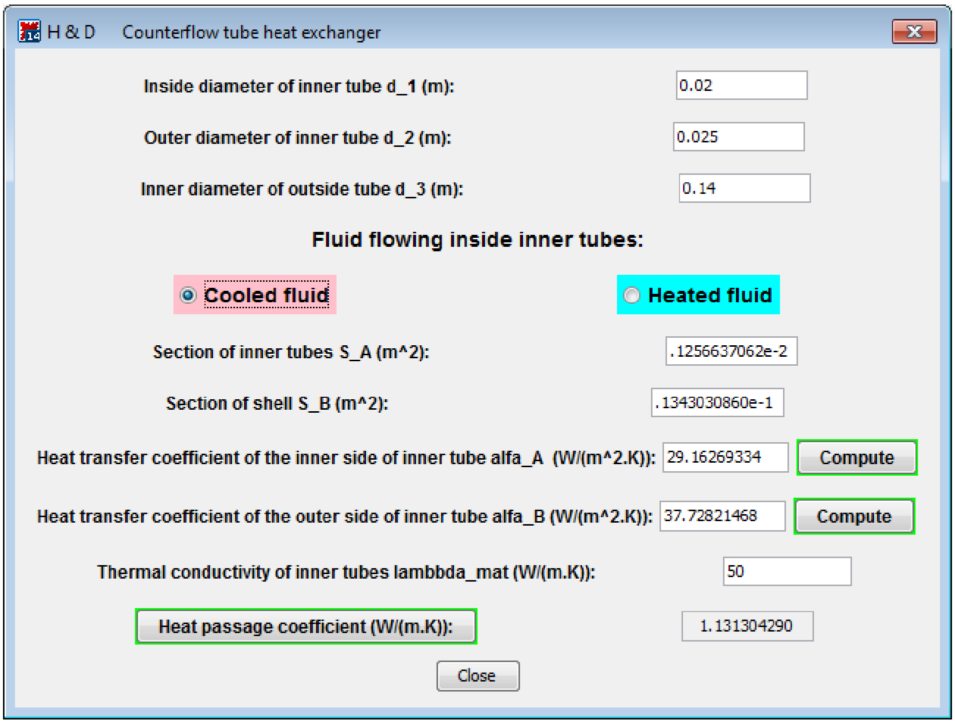

Fig. 5 Window for computing of heat passage coefficient

IV. Verification of the Software Application Accuracy

We verified accuracy of data computed by programmed software application by simulation of thermal balance by heat transfer module of commercial software COMSOL MULTIPHYSICS.

For testing we used these parameters:

Heated fluid: water
Cooled fluid: transformer oil
Initial temperature of water: 20 °C
Initial temperature of oil: 60 °C
Mass flow of water: 0.008 kg/s

Mass flow of oil: 0.005 kg/s
Inner diameter of inner tubes: 2 cm
Outer diameter of inner tubes: 2.5 cm
Number of inner tubes: 4
Length of tubes: 1.5 m
Inner diameter of shell: 14 cm
Material of tubes: steel

Data computed by MAPLE application are shown in fig. 6 - 8. fig. 6 shows computed temperatures of both liquids along the length of the heat exchanger. Red values are temperatures of oil and blue values are temperatures of water. In the fig. 7 are computed values of heat passage coefficient and heat transfer coefficients. The fig. 8 shows computed temperature curves and heat flow inside exchanger.

Distribution of temperature in the slices of exchanger computed by COMSOL MULTIPHYSICS are shown in fig. 9 - 10. Fig. 9 depicts temperature distribution in the cross section of heat exchanger. In the fig. 10 is temperature distribution in the longitudinal section of heat exchanger. It is evident a good accordance of both methods of the heat transfer computation.

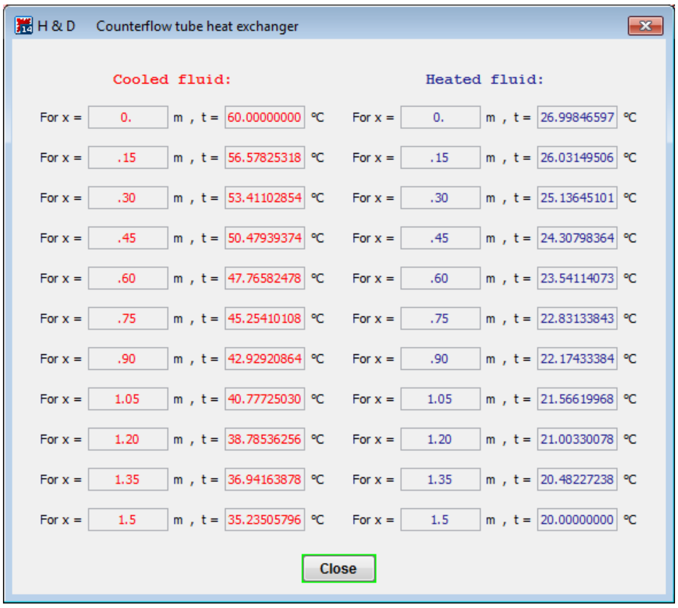

Fig. 6 Solving of the heat transfer by MAPLE - temperatures of both liquids along the length of the heat exchanger

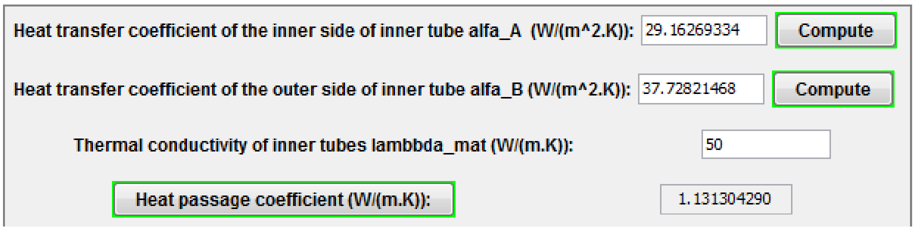

Fig. 7 Solving of the heat transfer by MAPLE - computed values of heat passage coefficient and heat transfer coefficients

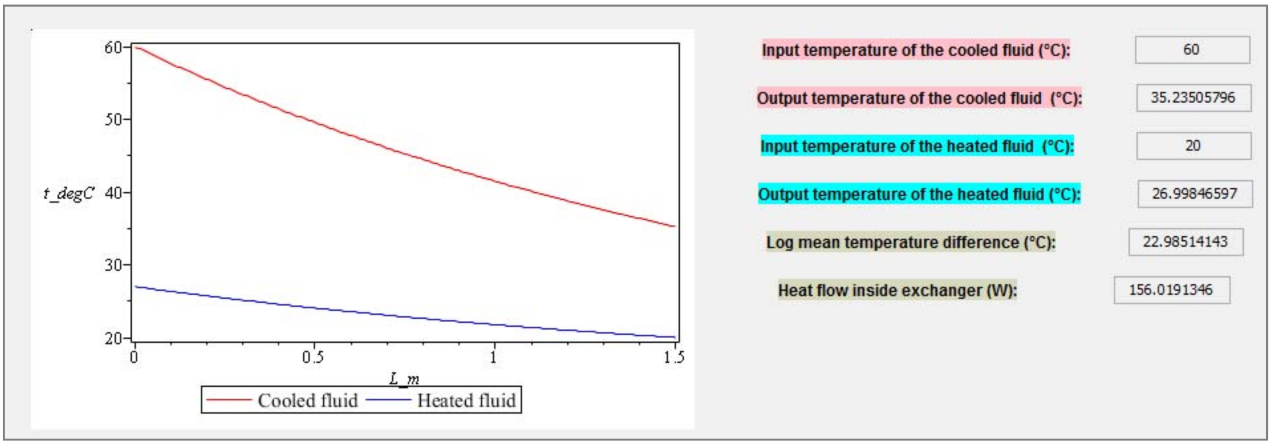

Fig. 8 Solving of the heat transfer by MAPLE - temperature curves and heat flow inside exchanger

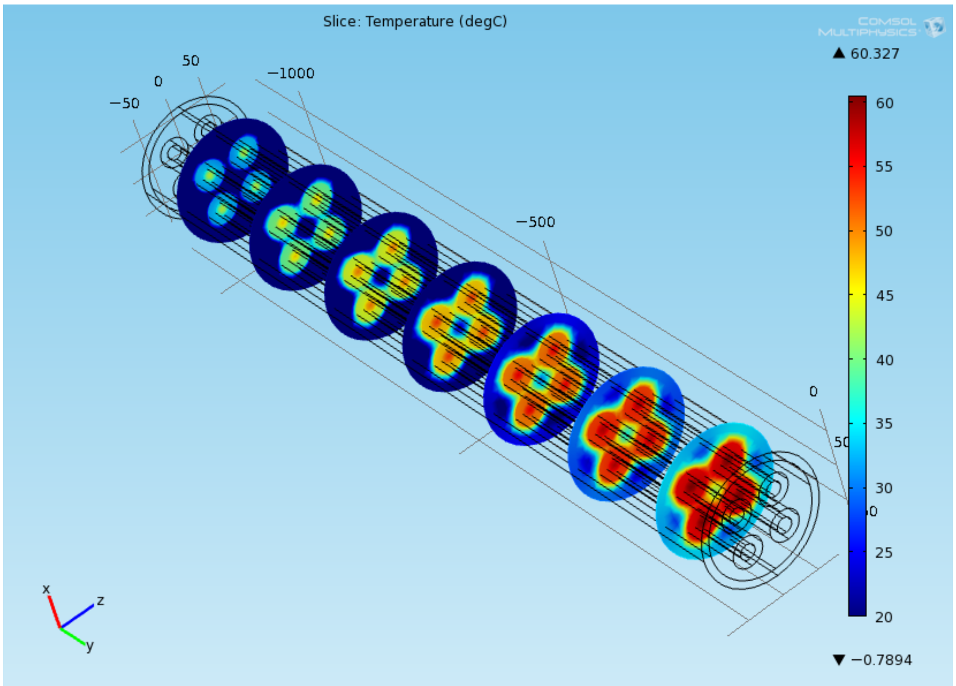

Fig. 9 Simulation of the heat transfer by Comsol Multiphysics - temperature distribution in the cross section of heat exchanger

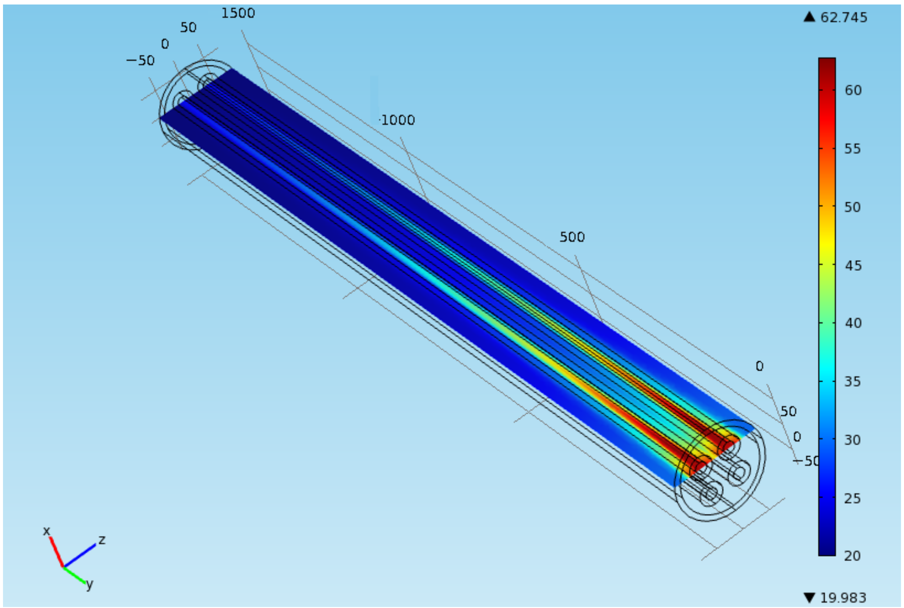

Fig. 10 Simulation of the heat transfer by COMSOL MULTIPHYSICS - temperature distribution in the longitudinal section of heat exchanger

V. Conclusion

Analytical solution of above described mathematical model enabled us to made application for study of the temperature courses in the heating and cooling fluid in counterflow heat tube heat exchanger. The application we made by use of software MAPLE as a teaching aid for the study subject Process engineering taught at the Tomas Bata University in Zlín. The application performs the calculation of the heat passage coefficient and heat transfer coefficients on the basis of criteria relations generally used in the fluid flow inside the tube systems where there is no change of state. All the functions and calculations programmed into the application have been verified by an independent calculation and by simulation in COMSOL MULTIPHYSICS. The computed temperature curves were in accordance with results of simulation.

For the calculation of temperature in parallel flow heat exchanger is necessary to adjust the mathematical model described by equations (1) - (4) in relation to the direction of fluid flow in the heat exchanger. Similarly, in the case of phase change some of the fluid (by condensation or boiling), it would be necessary to adjust the balance equations to include condensing and characterizing the latent heat of vaporization of liquids conversion. It would also be necessary to adjust the generally used dimensionless criterions for calculating the heat transfer coefficient.

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LIST OF SYMBOLS

t	- temperature, [°C];
t_m	- average temperature, [°C];
Δt_{LS}	- log mean temperature, [°C];
$\dot{m}$	- mass flow, [$kg \cdot s^{-1}$];
c_p	- scpecific thermal capacity, [$J \cdot kg^{-1} \cdot K^{-1}$];
L	- length of tube, [m];
k_L	- heat passage coefficient, [$W \cdot m^{-1} \cdot K^{-1}$];
N	- number of inner tubes, [1];
x	- space coordinate, [m];
α	- heat transfer coefficient, [$W \cdot m^{-2} \cdot K^{-1}$];
λ	- thermal conductivity, [$W \cdot m^{-1} \cdot K^{-1}$];
ρ	- density, [$kg \cdot m^{-3}$];
ν	- kinematic viscosity, [$m^{-2} \cdot s^{-1}$];
d	- diameter , [m];
d_{ekv}	- characteristic size, [m];
S	- section, [m^2];
o	- dipped circumference, [m];
Gr	- Grashof criterion, [1];
Pr	- Prandtl criterion, [1];
Nu	- Nusselt criterion, [1];
Re	- Reynolds criterion, [1];
v	- flow rate, [$m \cdot s^{-1}$];
$\dot{Q}$	- heat flow, [W].

Indexes:

mater.	- physical properties of tubes material;
1,2	- signification of fluid.

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Hana Charvátová is a Research worker at the Department of Automation and Control Engineering, Faculty of Applied Informatics, of Tomas Bata University in Zlín. Her research activities include recycling technology and modeling of natural and synthetic polymers treatment.

Dagmar Janáčová is a Professor in the Department of Automation and Control Engineering, Faculty of Applied Informatics, of Tomas Bata University in Zlín. Her research activities include: modeling of treatment processes of natural polymers, transport processes, recycling of tannery wastes, and optimization and ecological approach of tannery processes. She has received the following honors: Diploma of England, XXIII IULTCS Congress, London, 11–14 September, 1997; Gold Medal - EUREKA EU Brussels 1997; Special Prize, Ministry of Agriculture, Belgium, 1997.

Rudolf Drga is an Associate Professor in the Department of Security Engineering, Faculty of Applied Informatics of Tomas Bata University in Zlín. His research activities include electronic security systems.

Ondrej Líška is an Associate Professor in the Department of Automation, Control and Human Machine Interactions of Technical University of Košice. His research activities include automatic control of machines and processes, monitoring and visualization of processes, sensor systems for automation and control, control with the use of advanced management methods.

Vladimír Vašek is a Professor in the Department of Automation and Control Engineering,Faculty of Applied Informatics, of Tomas Bata University in Zlín. His research activities include: microcomputer applications in technology processes, computer monitoring and control systems, and discrete deterministic controllers approach of tannery processes. He has received the following honors: Diploma of England, XXIII IULTCS Congress, London, 11–14 September, 1997; Gold Medal - EUREKA EU Brussels 1997; Special Prize, Ministry of Agriculture, Belgium, 1997.

Martin Zálešák is an an Associate Professor in the Department of Automation and Control Engineering, Faculty of Applied Informatics, of Tomas Bata University in Zlín. His research activities include energy audits, technology of energy systems, ventilation, heating and cooling systems, energy management, Building Energetics, energy conception.