Infinitesimal Equivalence between Linear and Curved Sources in Newtonian Fields: Application to Acoustics

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Abstract: - The equivalence between the vectorial field produced by a scalar linear source and the one produced by a scalar curved source is proved in the centre of the circumference which contains the curved source, if this circumference is tangent to the line of the linear source. Since this equivalence is true already at the infinitesimal level, many consequences come out. Applications of this result can be highlighted in electrostatics and in acoustics, since these fields fulfil the above requirement. In particular, in acoustics, the problem of isolated receivers along a high velocity rail line can be approached by means of studying the equivalent curved source.

Keywords— Acoustical Field, Isotropic sources, Linear and curved sources, Vectorial and Central Field.

I. Introduction

In this paper we want to pinpoint a very interesting relation between two infinitesimal sources and their common receiver. In particular we want to show that the signal produced at the receiver by an infinitesimal linear source is equivalent to the one generated by a curved source (arc of a circumference centred in the receiver), if they have the relative position as shown in Fig. 1.

The condition is that the line *dx* belongs to, has to be tangent to the circumference which contains the arc *ds*. In other words we say that the signals generated by *dx* and *ds* are equal when measured at the receiver's position *P*, centre of the circumference.

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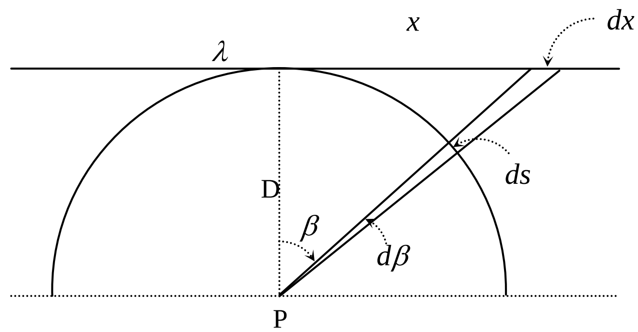

Fig. 1

This equivalence is true for every isotropic scalar source which produces a vectorial field proportional to $1/r^2$. In particular this result is applicable to acoustical and electrostatic fields and, since it is verified already at the infinitesimal level, many consequences can be derived.

In this paper we focus in particular on the acoustic field. This calculation has to be done as if there is no air attenuation of the sound. As a rule of thumb, for standard weather condition, we know this is true if the maximum distance is less than 100 *m*. Otherwise the attenuation of sound due to the air has to be taken into account. [4, 5]

Obviously we work with the signal pressure in its vectorial form (before calculating its level).

The need of characterization of acoustic noise sources is peculiar for every environmental impact analysis. In particular, noise coming from railways, highways and other typical linear sources may be a primary reason of critical life condition in proximity of these sources. Therefore, during last years, interest on noise pollution had a significant growth in Europe. Many models have been developed in order to offer a solid basis to the development of noise control rules. Furthermore, compared to other environmental problems, noise's one must be still deeply investigated.

Some studies showed that 17–22% of the population of the European Union (EU) are daily exposed to traffic noise exceeding the tolerance limit of 65 dBA [1, 2, 3]. Another 170 million of citizens live in so-called "grey zones" where noise levels within the range 55–65 dBA cause serious disturbance, especially at night-time.

This means that a big effort in reducing noise level in these zones must be performed. Several European countries decided to establish domestic control procedures on noise emissions in order to correctly monitor the acoustical impact.

After this control phase, an operative phase must follow, with the aim of reducing the noise in the hazardous zones. Therefore the need of characterization of sources will be impellent in the next years, when countries will start the preparation of projects aimed at the screening and the reduction of the exceeding noise.

II. Linear source

Let's now consider the infinitesimal linear source *dx* having an acoustical power dW_{linear}.

The resulting vectorial acoustical intensity at distance *r* is:

$$d\vec{I}_{linear} = \frac{1}{4\pi}\frac{dW_{linear}}{r^2}\hat{r} \qquad (1)$$

If λ is the linear acoustical power density, we have:

$$dW_{linear} = \lambda\, dx$$

Looking at Fig. 2, we see that

$$x = D\, tg\beta$$

and

$$dx = \frac{D}{\cos^2\beta}d\beta$$

Moreover, $r = \frac{D}{\cos\beta}$, so by substituting in formula (1), we have:

$$d\vec{I}_{linear} = \frac{1}{4\pi}\frac{\lambda D\, d\beta}{\cos^2\beta}\frac{\cos^2\beta}{D^2}\hat{r} = \frac{1}{4\pi}\frac{\lambda}{D}d\beta\,\hat{r} \qquad (2)$$

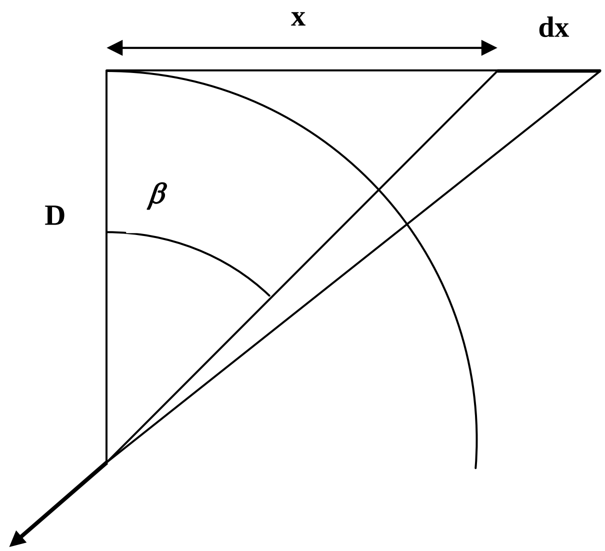

Fig. 2

III. Curved source

In this case we simply have:

$$dW_{curved} = \lambda\, ds$$

Looking at the geometry of the problem, we find:

$$r = D$$

and

$$ds = D d\beta$$

So, by substituting, we obtain:

$$d\vec{I}_{curved} = \frac{1}{4\pi}\frac{dW_{curved}}{r^2}\hat{r} = \frac{1}{4\pi}\frac{\lambda D\, d\beta}{D^2}\hat{r} = \frac{1}{4\pi}\frac{\lambda}{D}d\beta\,\hat{r} \quad (3)$$

which is equal to (2).

IV. Application in acoustical sources studies

Since this equivalence is true already at the infinitesimal level, we can deduce that ideally a linear acoustical noise source can be substituted by a curved one placed in the position discussed above.

Therefore when we are dealing with any linear sources, such as straight railway or highway, we are allowed to face the problem such as the source is a curved one, with some relevant facilitations.

If we consider, for example, a bunch of buildings placed along a straight railway, in principle we could study the acoustical impact on the buildings by means of considering an equivalent curved source placed as in Fig. 3, according to the above demonstrated result.

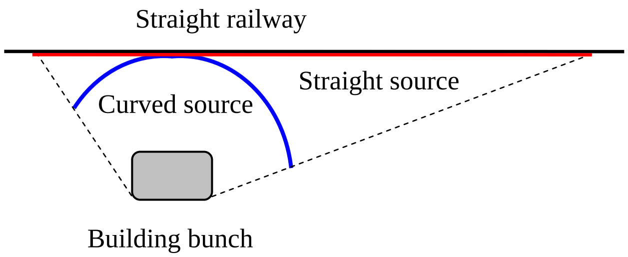

Fig. 3

Actually there are many critical zones in which this configuration is well suited and in which this equivalence can be applied.

During last years the design of high velocity railways in EU has led to the birth of long and straight rail lines devoted to high speed trains. These rail lines represent a critical noise source for which the environmental acoustical impact not always has been taken into account during the construction.

We postpone to a future work a quantitative analysis of this problem, by means of application of the above equivalence to the high velocity rail line.

V. Conclusions

In this paper we showed the equivalence between a linear source and a curved one if they fulfil the geometry shown in Fig. 1. This equivalence, which is true already at the infinitesimal level, is verified for every vectorial field generated by scalar sources, which reduces itself according to the inverse of the square of the distance.

This means that the range of applications of this development is very wide in term of acoustical, electrostatic and other fields.

In particular in the acoustical field we plan to extend these studies in more forthcoming papers by means of comparison of theoretical calculations with experimental measurements [6, 7] and software predictions, in different scenarios such as high speed rail lines, straight railways and similar noise sources.

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