Taylor-Goertler Viscous Instability in a Supersonic Axisymmetric Jet

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Abstract: Numerical modeling of the characteristics of Taylor-Goertler disturbances in a supersonic axisymmetric jet in the viscous approximation of hydrodynamic stability theory was performed. The basic equations for small disturbances in a curved cylindrical coordinate system were obtained. The regularities and peculiarities of typical relations of various-scale vortices under changed mean flow parameters were studied. The critical Reynolds numbers of stability loss were found. It was defined that large-scale vortices with low increments as compared with small-scale ones loose stability at low Reynolds numbers. Some experimental results were interpreted.

Key-Words: supersonic axisymmetric jet, hydrodynamic stability, Taylor-Goertler disturbances.

1. Introduction

Large-scale organized motion, an essential component of turbulent flow, is now known occur in shear zones of fully developed turbulent jets. Coherent structures in the jets may assume the forms of the various types of streets, lines, rings, toruses, simple and double spirals breaking down the initially homogeneous flow structure [1-3]. This secondary large-scale motion may be result from instability of the initial flow.

In recent years some researchers experimentally found that under certain conditions stationary and quasi-stationary azimuthal inhomogeneities of the averaged flow fields appear in a mixing layer of a supersonic axisymmetric nonisobaric jet [4-9]. They are registered in the form of longitudinal black and white bands on the photographs with long exposure (Fig. 1) [4] and under visualization of cross-sections of the first barrel (Fig. 2,3) a wave-shaped or saw-shaped boundary of a supersonic mixing layer is seen [5,6].

The measurements showed [4, 8, 9] that in coldambient turbulent air jets at high Reynolds numbers of exhaustion $\mathrm{Re}_d \sim 10^6$ (d is the nozzle diameter) and small nozzle pressure ratios N great deviations from the mean values of excessive total pressure appear, which means that there are azimuthal defects of the longitudinal velocity.

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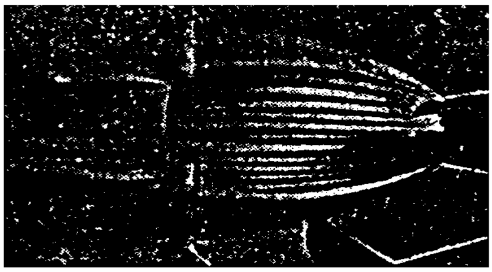

Fig. 1. The longitudinal black and white bands on the photographs of nonisobaric jet with long exposure [4]

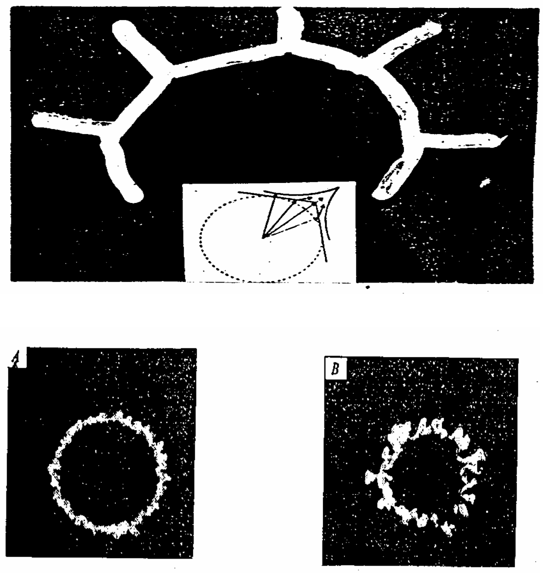

Fig. 2,3 Under visualization of cross-sections of the first barrel a wave-shaped or saw-shaped boundary of a supersonic mixing layer is seen [5,6]

In [7] the same density variation for nitrogen jets exhausting in vacuum space at higher (about 50) degrees of expansion with $\mathrm{Re}_d > 3000$ was found (Fig. 4).

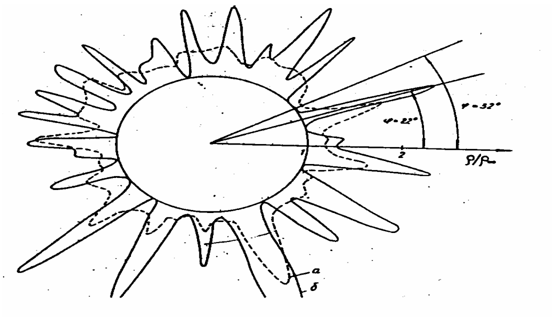

Fig. 4. The density variation for nitrogen jets exhausting in vacuum space at $\mathrm{Re}_d > 3000$ [7]

A question about these azimuthal inhomogeneities comes into existence. The most realistic hypothesis is that fluctuations may appear in a mixing layer of the first cell, which may be connected with rotating or centrifugal instability. Let us consider the action of forces in a compressed layer of the first barrel of an underexpanded jet (Fig. 5). The gas moves in it along the curved trajectories flowing around the suspended shock wave (SSW) and decelerating in a mixing layer δ. Unstable stratification may appear in it as the gas particles in the inner layers try to come out into the fields of small velocities under the effect of the centrifugal force [10,11].

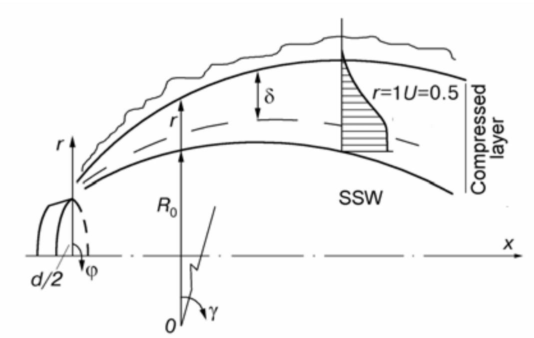

Fig. 5. Scheme of the flow in the initial part of an underexpanded jet

The centrifugal force directed along the normal to a motion trajectory from the curvature center (hypothetical point O in the figure) provides this transition. If this motion is not balanced by a normal gradient of pressure, a transverse overflow may appear which is analogous to that in a flow over a concave surface (Goertler flow) or between co-axial cylinders, one of which (internal) rotates and another (external) does not move. Rayleigh [11] formulated a theorem about appearing of centrifugal or rotating instability, a variant of which may be presented here. Instability appears if the square of absolute circulation decreases while the cylindrical radius increases $(r_1 u_1)^2 > (r_2 u_2)^2, r_1 > r_2$, which is possible if the gas velocity decreases more than $1/r$ with an increase in the cross distance. As is shown further, this condition is provided in a mixing layer.

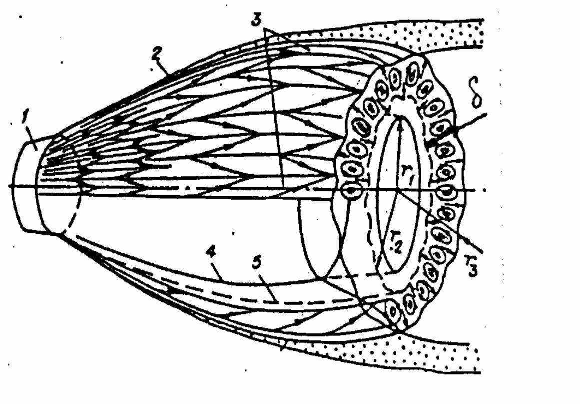

Fig. 6. The flow in the initial part of an underexpanded jet with the longitudinal quasi-stationary vortices

These disturbances of the flows are presented as longitudinal quasi-stationary vortices at a linear stage (Fig. 6). At the nonlinear stage, helical secondary vortices may appear as well as more complicated structures whose shape is defined by particular conditions. There is no inconsistency in applying these considerations to free shear layer and one may expect that such instability causing turbulent or other type of laminar motion may be realized at free supersonic exhaustion.

A certain interest to this problem has arisen, which is proved by a great number of experimental researches, some of them have been mentioned above. It may be explained by the appearance of instability in a form of stationary longitudinal vortices in a jet, which is uncommon. The acknowledged idea is that a surface with a curvature or an inflection is a result of rotating instability even in the case when the vortices do not touch the surface directly and are at a boundary of stagnation zones as in separated flows. In nonisobaric jets, this trajectory curvature of gas motion is created by exhaustion conditions and may occur in a region of flow mixing with external ambient or subsonic external jet zone contracted by the shock wave.

This hypothesis may be confirmed or rejected by either direct methods of discovering three-dimensional velocity fields, which should be the first task of experimental research or by indirect methods i. e., mathematical modeling. At present time, there is a positive answer to this question under a condition when the equations describing instability in a mixing layer of a supersonic jet with active centrifugal forces allow the presence of stationary unstable disturbances.

Centrifugal or rotating instability called Taylor - Goertler (T - G) instability was theoretically well investigated [8, 12-16]. Mathematical modeling is limited by inviscid frame. This fact has good grounds. First, it is known that a mean profile of the longitudinal velocity with a bend is realized in a mixing layer of a jet. It means that instability of such flows is defined mainly by inviscid convective effects. Secondly, the appearance of T - G instability is owed to centrifugal forces, which are taken into account in an inviscid analysis; hence, we suppose that the influence of other forces is secondary. Modeling a mixing layer stability of a nonisobaric jet with curvatures in the inviscid analysis showed that quasi-stationary waves defined by these forces may occur in it. Thus it was found that T - G instability and longitudinal vortices may appear in a jet.

The question about which characteristics obtained by calculation may be compared with experimental data has arisen. The coefficients of longitudinal fluctuation increase (increments) are supposed to be the most informative at this stage. The analysis of investigations [4-9] showed that the measurements define irregular diagrams according to azimuthal angle which manifest a polymode structure of the disturbances. That is why it is necessary to obtain reliable information about the spectral structure of a signal in order to get transverse pictures of fluctuations by calculation and compare them with experimental ones. At present time, this spectral analysis was done only in [4, 8], which gives small grounds for the hypothesis considered. All other investigations may be used as informative qualitative data presenting additional results. This fact complicates the problem as all measurements [4, 8] for jets exhausting at high turbulence and noisiness have peculiarities which laminar and pre-turbulent jets do not have [7]. Unfortunately, the researchers of [7] did not make such spectral analysis of data.

Complication of the signal structure with an increase of the Reynolds number of exhaustion is a common factor for all papers considered. The photographs [5, 6] show that at low Re_d the number of azimuthal ”saw teeth’ is small and increases with a rise of Re_d. The measurements [7] prove the same fact. In experiments [4, 8], the signal has a complicated structure.

The data about the place of azimuthal inhomogeneities are contradictory. In [4,8,9] they were registered only in a mixing layer and were not discovered in a compressed layer in the region from the SSW to the inner boundary of the layer (at any high values of Reynolds numbers of exhaustion. At the same time, the authors [7] stated that this region and the SSW may be deformed with Red increase. This difference may be connected with peculiarities [7] of exhaustion into vacuum space at high nozzle pressure ratios, a fact which may intensify the process.

The evolution of the signal after the normal shock (Mach disc) has not been clear yet. In [4,8] it was shown that after passing a triple point only the traces of previous intensive fluctuations are present, whereas photographs [5] show that neither the number of “saw teeth” nor their intensity change. It may be explained by different regimes. In [4,8] the jets are turbulent, in [5] they are laminar.

Thus after analyzing experimental results we begin to discuss mathematical description. Within an inviscid analysis [8,12] a qualitative correspondence of the increment order of low azimuthal modes realized in [4, 8] and reproduction of the azimuthal distribution of a signal of excessive total pressure are shown. At the same time, a number of factors have been discovered which are not adequate to the experiments.

The main one is the observed quick destruction of high mode components in a spreading jet while calculations within the inviscid frame show that the increments of small-scale components increase with a rise of the mode number. So according to theory, these components should increase intensively.

If we take into consideration a great decrease of increments and a reconstruction of wave configurations of these modes with an increase of thickness of a mixing layer [14-16], we can better describe the real dynamics of the disturbances. However, it does not settle the theoretical description as there is a number of physical processes which should be taken into account to describe the spectral characteristics of the disturbances in the right way. We distinguish three main factors: dissipative influence of viscosity, non-parallelism of the mean fields of velocity in the first barrel, and non-linearity leading to secondary instability of longitudinal vortices and changing their increments. Consideration of these factors defines the research strategy. It is impossible to evaluate preliminarily their importance and take them all simultaneously into account in calculation. That is why in the present paper one of these factors - influence of viscosity - is considered and viscous Taylor - Goertler instability is studied.

We consider a limit variant of introduction of viscosity into analysis, namely for an incompressed fluid, since it is impossible to overcome difficulties in calculations having axisymmetry (cylindrical coordinates with longitudinal curvature). As a result of theoretical research we shall have two marginal solutions: the first one is within the inviscid frame and the second one is with constant viscosity. It should be expected that the true solution will be between these two marginal approaches.

Since we have no reliable experimental calculation data now, we shall consider first the qualitative influence of viscous forces on stability of a mixing jet layer and find general regularities of its functioning. A detailed analysis of experimental research was done for this purpose.

Let us evaluate the quantitative influence of factors that are ignored. The change of the first physical (shear) viscosity across the mixing layer is not considered. Moderate Mach numbers $1< M_0 <2$ were considered. Regimes with a small change of the average static temperature for cold jets are performed. For example, for M_0=1,5 the difference of static temperatures across the layer proportional to $1/\rho_0$ causes a change of dynamic viscosity by 20 %, it may be neglected under qualitative consideration. Secondly, the secondary (volume) viscosity is not considered. The reason is a comparison of two approaches to calculation of the spectral characteristics of the disturbances for a supersonic boundary layer with and without consideration of the secondary viscosity [17]. It appeared that, other conditions being equal, the difference of the increments of the disturbances for low Mach numbers makes up some percent and does not affect phase velocities.

Probably, the influence of the secondary viscosity of stability problems for air is not decisive. At last, viscous dissipation of heat is ignored and the equation of entropy conservation at the streamline is considered instead of the equation of energy as it is in the inviscid case of. This simplification may be justified by a choice of regimes considered in the research. At low Mach numbers, the relative difference of average velocities in a mixing layer is by two times more than the relative difference of temperatures, which means that viscous dissipation of the dynamic characteristics is much greater than viscous dissipation of the heat ones. Let us accept it as a true fact for small disturbances.

In spite of a great number of simplifications, such a statement of the problem is well grounded as the main factor - shear viscosity - is considered. Factors, which are not considered, vary. Methods of mathematical modeling should be used to find and reflect the main features of a true process avoiding difficulties.

It should be mentioned that within the inviscid analysis several families or branches of eigenvalues satisfying the boundary-value problem have been obtained. In other words, its polysemantism has been shown [8,12,13]. It was found that the increments of the disturbances in the main branch are proportional to the azimuthal wavenumber and the increments of other (additional) branches do not depend on it. In recent calculations [14] similar results were obtained for inviscid running waves in a plane shear layer with curvature. A more accurate consideration of this problem may help to make clear these additional solutions.

2 Basic formulas and methods

A compressed layer of the first barrel of an axisymmetric ambient nonisobaric jet (see Fig. 5) has been considered. Its normal spread is from the SSW to the boundary of a mixing layer and the longitudinal one is from the near region of the nozzle exit to the Mach disc. Shock waves and a change of the mean parameters in them are not considered but it is assumed that the SSW position determines the values of the radius of curvature R_0 and the centrifugal forces proportional to U^2/R_0. The longitudinal spread of the calculated domain is expressed through the mixing layer thickness δ, which is one of the main parameters. The considered range $0.1<\delta<0.65$ corresponds to real thicknesses of the mixing layer. Dependences of the longitudinal coordinate x on δ determine the longitudinal conjunctions $x = x(\delta)$ connected with the type of exhaustion. The values of the nozzle pressure ratio N are not specified. A wide range of R_0 which may be connected with N was taken. The values $5< R_0 <25$ are most realistic. The ambient cold air jet with $k = 1.4, k = c_P / c_V$ was considered.

A compressed layer consists of two sub-regions which differ in excessive total pressure. The first one spreading from the external SSW region up to the maximum value of excessive total pressure (dashed line in Fig. 5) is called the “inviscid” sub-region. There are the recovery pressure and a small increase of the mean velocity up to the maximum value in it. The flow parameters are determined from the equations for an ideal gas. In the second sub-region, there is a smooth transition from the parameters of the external boundary of the compressed layer to the parameters in the ambient space. This is a mixing layer.

The profiles of the longitudinal velocity and density in the first region are taken as constant and equal to their maximum value. It is based on data [18], which experimentally show that an increase of the mean velocity at small nozzle pressure ratios is not great as well as it is in the problem of stability. The theory of hydrodynamic stability states that, in the regions with small acceleration, the flow is as stable to small disturbances as it is in a homogeneous flow without shear. This simplification makes it possible to formulate the boundary conditions for the disturbances. In averaging the equations, the values of the mean velocity $\overline{U}$ and density $\overline{\rho}$ in this analogue of the potential nucleus are taken as typical.

In mixing layers δ the dimensionless profiles of the longitudinal mean velocity are prescribed by the relation

$$U(r) = \exp(-0.693\eta^2), \quad \eta = 2(r - r_1)/\delta, r_1 = 1-\delta/2. \tag{1}$$

This problem is devoted much attention to in experimental work [18]. It shows that in a mixing layer the distributions of longitudinal average velocities are well described by a universal function, the so-called reverse Schlichting profile. Relation (1) derived for a supersonic isobaric jet coincides with this universal function and has an advantage because it describes better the conjugation with the ambient zone near the external boundary. As is seen, a change of velocity across the layer is greater than $1/r$.

The profile of mean density ρ_0 is connected with U by dependence $\rho_0 = \left[1+(k-1)M_0^2(1-U^2)/2\right]^{-1}$.

Profile (1) has an inflection $\left((\rho_0 U')/r\right)' = 0$.

There value $\bar{r}$ is taken as a typical linear scale on the line of half velocity, that is why at $U_{r=1} = 0.5$. This value coincides with half thickness of the mixing layer whose spread is $r_1 < r < 1+\delta/2$.

The Mach number M_0 is also determined by the line of maximum velocity. It may be connected with the Mach number of exhaustion at the nozzle exit Ma by the known isoentropic relation

$$\mathrm{M}_0^2 = 2\left[1+(k-1)\mathrm{M}_a^2/2\right]N^{(k-1)/k} - 1/(k-1).$$

Thus, all necessary relations have been determined. The scheme of the flow in the first barrel of an underexpanded jet is shown above in Fig. 5. Here $R = R_0 + r$ where r is a changing radial variable and R_0 is the radius of curvature ($R_0 >> r$) and angular variables φ and γ are taken as curved orthogonal coordinates. The velocity components v, w, u correspond to them.

The metrical form in this coordinates system is $dS^2 = H_1^2 + H_2^2 \cos d\varphi^2 + H_3^2 \cos d\gamma^2$, and the Lame coefficients are $H_1 = 1,\ \ H_2 = R\cos\gamma - R_0,\ \ H_3 = R$. It is natural to assume that in a region we are interested in $R_0 = const$ and the parameters of the mean field depend on the mixing layer thickness δ (plane-parallel approximation). Then $\cos\gamma \approx 1$ and $H_1 = 1, H_2 = r, H_3 = R_0$ and the longitudinal coordinate is introduced as $dx = R_0 d\gamma$.

The viscous terms of equations of moments are written as for an incompressed fluid and the equation of energy is reduced to the equation of entropy conservation on the streamline as the in inviscid approximation.

In [12] a complete inviscid system in these coordinates is given. Let us add viscous terms using formula [19]. Since the conclusions are complicated, we present only the initial system and the final form of the linearized equations for the disturbances

$$\begin{aligned}
&\bar{u}_t + \mathrm{grad}\bar{u}^2/2 + \overline{\Omega}\times\bar{u} = F - 1/\rho\,\mathrm{grad}p + \left[\mathrm{graddiv}\bar{u} - rot\overline{\Omega}\right]\\
&\rho_t + \bar{u}\,\mathrm{grad}\rho + \rho\,\mathrm{div}\bar{u} = 0\\
&s_t + \bar{u}\,\mathrm{grad}s = 0\\
&\bar{u} = \left|v, w, u\right|, \overline{\Omega} = \mathrm{rot}\bar{u}.
\end{aligned}$$

To exclude the entropy, the adiabat relation is used: $s = \ln(p/\rho^k)^{c_V}$.

For a one-dimensional mean flow the velocity field is written as $\bar{u} = | v', w', U(r) + u' |$, where the wave components have the form $u' = u(r)\exp[i(\alpha x - \omega t + n\varphi)]$. Here $\alpha = \alpha^r + i\alpha^i, \alpha^r$ and n are the longitudinal and azimuthal wavenumbers, α^i is the coefficient of longitudinal increase, and the circular frequency ω is real. For waves T - G α^r and $\omega \cong 0$ that is why $v', u' = (v, u)(r)e^{-\alpha^i x}\cos n\varphi$ and $w' = iw(r)e^{-\alpha^i x}\sin n\varphi$. The values n determine the number of vortices or vortex pairs over the jet diameter, small n correspond to large-scale vortices and big ones correspond to small-scale vortices.

For T - G disturbances it is impossible to simplify the equations similar to those for the near-wall boundary layer and neglect viscous dissipation for the normal component of the wave velocity as the inviscid approximation shows that all disturbances, especially for low n, are of the same order.

The linear system of equations of motion and conservation in the dimensionless form in a curved cylindrical system of coordinates is written as

$$\begin{aligned}
&iFv + p'/\rho_0 - 2Uu/R_0 = V_1/\mathrm{Re},\\
&iFw + inp/(\rho_0 r) = V_2/\mathrm{Re},\\
&iFu + U'v + i\alpha p/\rho_0 + Uv/R_0 = V_3/\mathrm{Re},\\
&iFM_0^2 p + v' + v/r + inw/r + i\alpha u + v/R_0 = 0, \qquad (2)\\
&V_1 = Dv - v/r^2 - 2inw/r^2 + (v' - v/R_0 - 2i\alpha u)/R_0,\\
&V_2 = Dw - w/r^2 + 2inv/r^2 + w'/R_0,\\
&V_3 = Du + (u' - u/R_0 + 2i\alpha v)/R_0,\\
&Dv = v'' + v'/r - (\alpha^2 + n^2/r^2)v,\\
&F = \alpha U - \omega, \mathrm{Re} = \overline{U}\bar{r}/\nu
\end{aligned}$$

with the boundary conditions $v, w, u, p \to 0$ at $r \to 0$ and $r \to \infty$. Here and further the prime denotes a derivative of r. As is seen, besides the additional convective terms $\sim 1/R_0$ the senior of which is Uu/R_0, there appeared terms in viscous parts of the equations connected with geometrical effects. The Reynolds number introduced above may be easily transformed to the parameters at the nozzle exit, to the frequently mentioned $\mathrm{Re}_L = \mathrm{Re}_d/\sqrt{N}$ and Re_x defining the dependences $x = x(\delta)$.

System (2) for the variables u, u', w, w', v, p was solved by the orthogonalization method [20,21]. A problem of constructing three linearly independent vectors to bring the boundary-value problem to its eigenvalues in a cylindrical system of coordinates is a difficult one. As usual analytical solutions in the domains of constant parameters of the mean flow - in the potential nucleus $r \to 0$ and in the far field of the jet $r \to \infty$ are applied in which the absence of additional centrifugal terms in (2) is postulated. According to Morris' method [17] we obtain:

$$u = C_1 Z_n(\lambda_1 r) + C_2 Z_n(\lambda_2 r),$$
$$v = -C_1 i/\alpha Z_n'(\lambda_1 r) - C_2 \alpha/\lambda_2 Z_{n+1}(\lambda_2 r) - C_3 in/r Z_n(\lambda_2 r),$$
$$w = C_1 n/(\alpha r) Z_n(\lambda_1 r) + C_2 i\alpha/\lambda_2 Z_{n+1}(\lambda_2 r) + C_3 in/r Z_n^i(\lambda_2 r),$$
$$p = -C_1 \rho_0 F \operatorname{Re} Z_n(\lambda_1 r) / \left(\alpha\left(\operatorname{Re} + iM_0^2 F\right)\right),$$ where
$$\lambda_1^2 = \left(\alpha^2 \operatorname{Re} + i\lambda_2^2 M_0^2 F\right) / \left(\operatorname{Re} + iM_0^2 F\right),$$
$$\lambda_2^2 = \left(\alpha^2 + i\rho_0 F \operatorname{Re}\right).$$

Here Z are modified Bessel functions of order n, $Z_n = I_n$ (of the first kind) at $r \to 0$ and $Z_n = 2K_n/(\pi i^{n+1})$ (of the second kind) at $r \to \infty$. The first vectors in this expression correspond to the inviscid approximation and $\lambda_1 = \lambda_{inv}$ at $\operatorname{Re} \to \infty$. The formulated boundary-value problem of eigenvalues for the determinant (6×6) makes it possible to study polyfunctional relations $\alpha^i = \alpha^i(\operatorname{Re}, M_0, R_0, \delta, n)$ and find the critical Reynolds numbers separating instability domains ($\alpha^i < 0$) from domains of stability ($\alpha^i > 0$) for T - G waves.

3 Results and discussion

Discussing the results we should begin with some general considerations.

The eigenvalues of α^r and α^i, vortex configuretions for T – G waves are shown in Fig. 11 in inviscid frame. Consider now sequentially the results of numerical simulation of this regime with respect to R_0 value variation [15,16]:

I. High limiting values of R_0 ($R_0 > 10^4$). In the limit $R_0 \to \infty$ this case may be correlated with the isobaric flow regime when no trajectory curvature is observed. It is seen from figure that the values of increments α^i are rather small, they decrease slightly with R_0 growing. The weak variation of the disturbance parameters gives grounds to claim that such quasi-steady waves will be typical of a free optimum axisymmetric M_0=1,5.

Only one vortex in the interval $0 \le n\varphi \le 2\pi$ is shown in the figure. The second vortex of this pair is symmetric to that shown in the figure and counter-rotating.

At low values a vortex pair with approximately equal ratio of azimuthal and radial motion is formed near the nozzle exit in the near-root region. The vortex center is close to coordinate $|U'|_{MAX}$. Advection swirls the vortices in this pair, thus, right- and left- rotating vortices, overlapping, change their places to form a vortex core. The azimuthal overflow enhances, and the radial one decreases considerably. Further downstream the vortex orientation does not change.

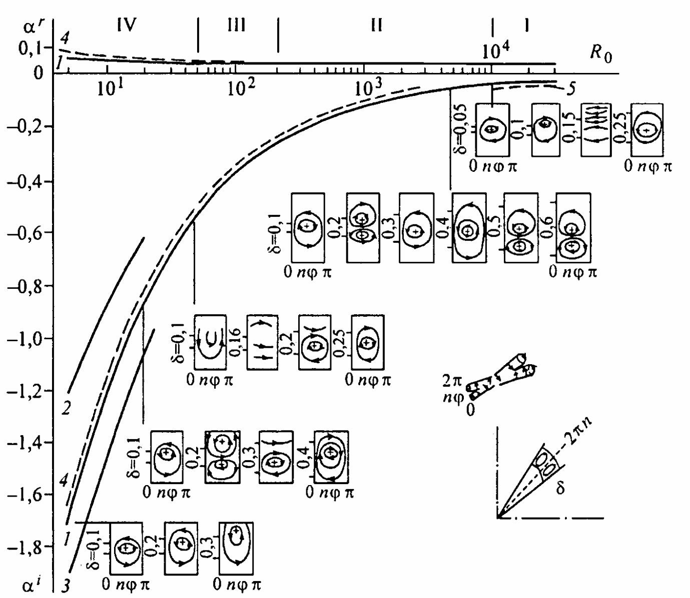

Fig. 7. The wavenumbers α^r and increments α^i in inviscid frame within a wide range R_0 ($\delta = 0{,}2$; n=16 (*1*); n=8 (*2*); n=24 (*3*)). Solid lines M=1,5; dashed – M=3.

IV. Low values of R_0 ($1 < R_0 < 50$). There is the regime of Taylor-Goertler waves. The longitudinal increments α^i increase considerably for disturbances with $R_0 < 50$.

Precisely these R_0 values are observed in nonisobaric underexpanded jets at N >1. The features of Taylor-Goertler waves have been substantially studied in [12-16].

It was found that the increase of centrifugal forces (reduction R_0) results in vortex localization in the mixing layer at small R_0 and the downstream dynamics demonstrates the gradual expulsion of the vortex into the external region, the process being attenuated by R_0 reduction.

The downstream reconstruction of the vortex pattern at R_0 shown that in the initial root vortex with the same order of *v* and *w* the radial component is considerably reduced and azimuthal one increases. Thus the gas particles trajectories in external regions become nearly circular as if they flow around the second counter-rotating vortex originated near the internal boundaries with a considerable radial motion. The center of the second vortex shifts gradually from internal boundaries to the center of the layer δ and further to the external boundary.

At R_0=5 the single vortex is formed in the mixing layer, its center shifts gradually from the layer middle (r~1) towards the external boundary. The ratios of v and w are approximately equal there the both components grow downstream as well as the axial component u.

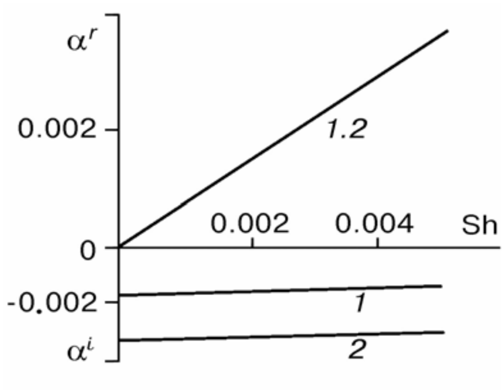

Fig. 8. Search of stationary T-G waves with $\omega = 0$ at $R_0 = 25$. 1 – Re=1765, $n = 25$, 2 - Re=2365, $n = 30$.

Inviscid parameters of the main branch of disturbances [8, 12] obtained at small and finite frequencies expressed by the Strouhal number $\mathrm{Sh} = 2\pi\omega\bar{r} / a_0$, where a_0 is the speed of sound on the line of the maximum velocity were taken as initial basic eigenvalues. It was necessary to make sure that they were really close to the parameters of stationary disturbances. Figure 8 confirms it.

At $\mathrm{Sh} \to 0$ $\alpha^r \to 0$ and α^i does not considerably change. It proves that results of the inviscid analysis are correct.

The figure 9 shows the dependences $\alpha^i(\mathrm{Re})$ for one of the typical variants of calculation $R_0 = 25, \delta = 0.15$ and $M_0 = 1.5$.

It is seen (the critical Reynolds numbers Re_c are on the axis $\alpha^i = 0$) that large-scale vortices (small n) lose their stability at lower Re and in a certain range of Re they have greater increments in comparison to small-scale vortices. The asymptote $\alpha^i = const$ shows the limits of α^i when the increments do not depend on Re (as in the case of the inviscid approximation). It happens quickly enough for small n but with an increase of the mode number the effect of viscosity increases and for $n = 30$ the limit corresponds to a high Reynolds number: $\mathrm{Re} \approx 10^5$. This fact may be used to explain experimental data. The results of rough estimates of the effect of viscosity in [12] have been proved.

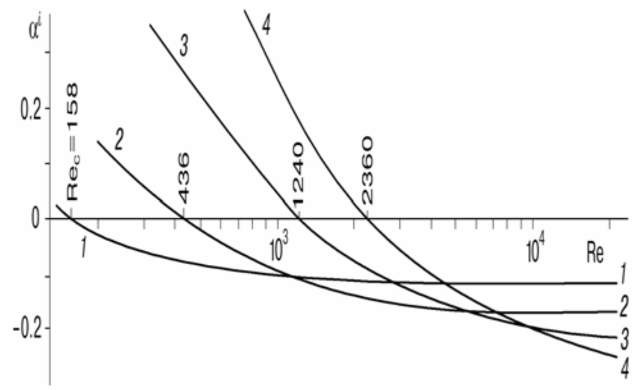

Fig. 9. Dependences of α^i (Re) for T - G vortices with $R_0 = 25, \delta = 0.15, M_0 = 1.5, n = 5$, 10, 20, 30 (1-4).

A more detailed interpretation of this conclusion is shown in Fig. 10. The coefficients α^i for different mode numbers n in a wide range of numbers Re are shown. The limiting dashed line shows the inviscid increments.

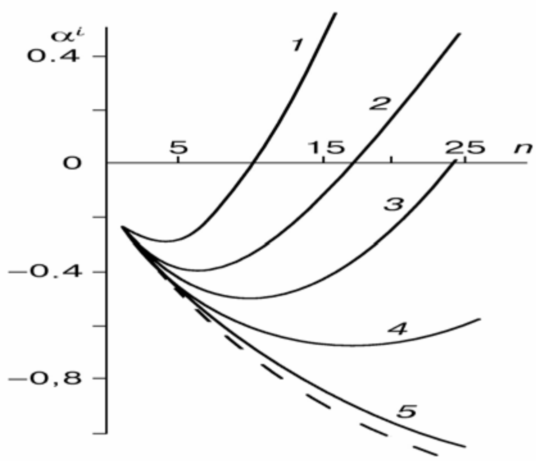

Fig. 10. Coefficients α^i of different modes for the same (see Fig. 9) parameters at Reynolds numbers: Re=443, 1000, 2000, 5000, 6000 (*1-5*).

It is seen what values of Re make it possible to use the inviscid values. Thus it has been found that viscosity leads to a decrease of the increments of T - G disturbances at moderate Re and it influences small scale waves.

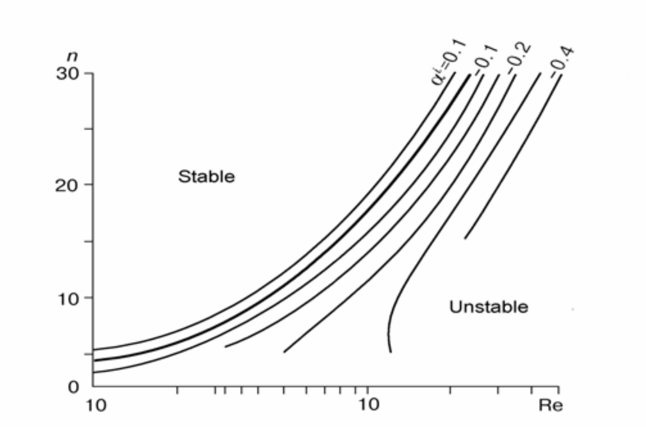

Fig. 11. Curves neutral stability and lines of equal increase ($\alpha^i = const$).

The neutral curve and the line of equal increase are shown in Fig. 11. The stability region is above the neutral curve α^i=0, the instability region is below. It has been found that the mode $n = 3$ is marginal and waves with $n = 1$ and 2 are unstable at any parameters of exhaustion for $R_0 > 5$.

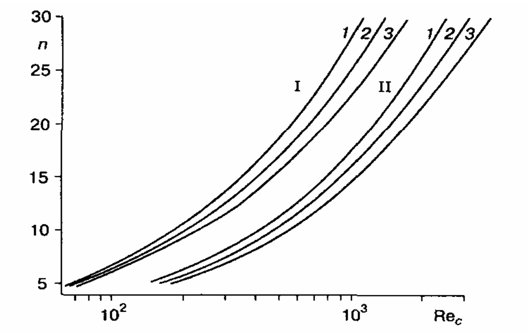

Fig. 12. Critical Reynolds number at different δ and R_0. R_0=5 (family *I*) and R_0=25 (family *II*); $\delta = 0.2;0.4;0.6$ (*1-3*).

In Fig. 12 the neutral curves determining the dependences of the critical Reynolds numbers on thicknesses δ and radiuses of curvature R_0 are shown. The figure demonstrates that with an increase of thickness of the mixing layer in the process of jet spreading, the waves become steady, it is noticeable at high modes. It is clear that with an increase of R_0 the value of centrifugal forces decreases and their effect become smaller. This fact agrees with the conclusions of the inviscid analysis.

The data of dependences of the increments on the Mach number of exhaustion shown in Fig. 13 agree with the inviscid analysis. Increments of disturbances decrease with an increase of M_0 and soon become the decrements (nearcritical Re for M_0=1.5 were investigated).

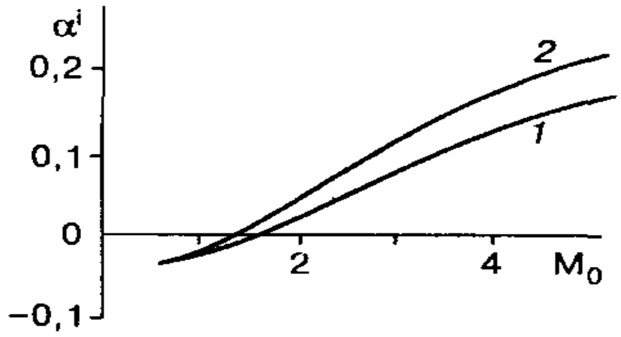

Fig. 13. Effects of the Mach number of exhaustion α^i; *n*=5 (*1*) and 10 (*2*).

There is a question about constructing a generalized picture in which all considered polyfunctional relations should be presented. One variant is given in Fig. 14. It is known that considering T - G instability the Gertler parameter (or number) $G^2 = \mathrm{Re}/R_0$ is introduced which describes the relation of viscous and centrifugal effects. The curves $\mathrm{Re}_c(G)$ for $n = const$ separate stability regions (to the left below Re_c lines) and instability regions (to the right above them). These curves calculated at small thicknesses δ for typical values of the radiuses of curvature give basic critical values for all possible parameters of the jets.

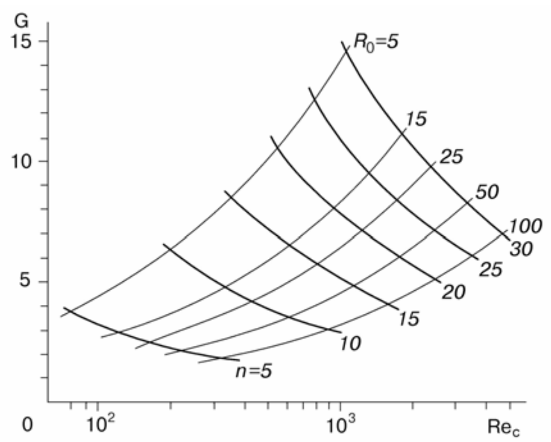

Fig. 14. Regions of Taylor-Goertler instability in a jet

The results of research make it possible to consider the mechanism of distinguishing modes in the spectrum depending on the Reynolds number. We may assume that under conditions when there is no entrainment of inner nozzle vortices into the free flow or when there is roughness of the edges inducing initial plume of waves with a certain composition, modes with the maximum increments will be released in the mixing layer of the jet.

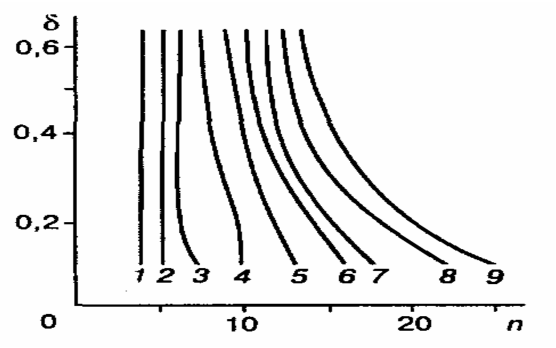

Fig. 15. Modes with the maximum increment α^i on various thicknesses of the mixing layer at $R_0 = 25$.
Re = 500, 750, 1000, 2000, 3000, 4000, 5000, 7500, 10000 (*curves 1-9*).

Let us consider Fig. 15 which shows these modes in a range of 500 <Re< 10^4 for one radius of curvature R_0= 25. It becomes clear that at low Re a dominant mode appears which has the maximum increment for all thicknesses of the layer δ: these are the mode *n* = 4 for Re = 500 and the mode *n* = 6 for Re = 10^3. With an increase of Re, the number of this dominant mode increases too at small δ but with an increase of the mixing layer thickness its value shifts to a domain of small azimuthal numbers.

In Fig. 16 a more detailed interpretation of this research is presented which shows the relation of the increments of the dominant and neighbouring modes at various thicknesses δ. It is seen that at Re=10^3 (Fig. 16 *a*) the mode $n = 6$ is maximum increasing in comparison to the neighbouring ones and its increment exceeds the increments of the neighbouring modes. Another situation takes place at high Reynolds number Re=10^4 (Fig. 16 *b*). In addition to the shift of the mode of the maximum increment by small n (points on the curves), the absence of dominance of such a mode over the neighbouring ones may be noticed. It proves that at high Re the evolution of the spectrum of disturbances may be more complicated with evolving of a group of modes.

Experiments [5-7] in laminar and preturbulent regimes confirm this fact. Direct measurements and visualization of flow cross-sections showed that, at moderate Reynolds numbers of exhaustion, low-mode wave components are realized. We know now that they should have great increments in comparison to high-mode ones and this process is determined by viscosity. Numerous photographs [5, 6] illustrate that with an increase of Re the scales of vortices decrease. This is shown in [7] by direct measurements, structures with 11 “saw teeth” at Re_d =1950 and with 22 “saw teeth” at Re_d =4100 were discovered.

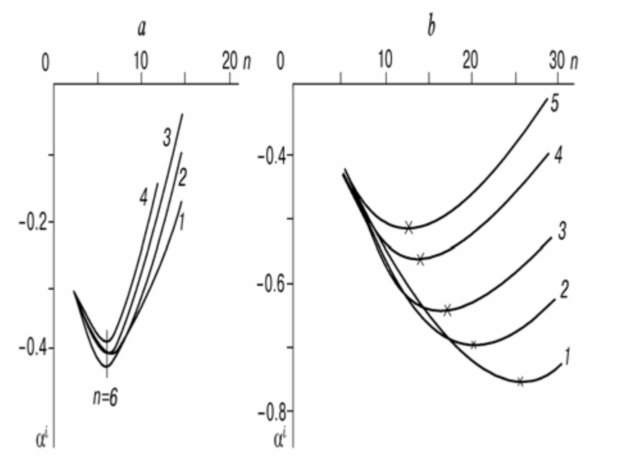

Fig. 16. Increments of various modes for $R_0 = 25$.
a (Re=10^3, δ =0.1, 0.3, 0.5, 0.65 (*1-4*);
b (Re=10^4, δ =0.1, 0.2, 0.3, 0.5, 0.65 (*1-5*).

Taking into consideration this fact let us explain data [4, 8]. Because of initial turbulent regime of a flow in calculations of T - G stability of these jets Reynolds numbers based on turbulent rather than physical viscosity should be introduced. It is impossible to do now because there is no reliable model describing turbulence in a free flow but we may evaluate Re_T which can be realized in mixing layers. We may use simple algebraic Prandtl model with a correction for compressibility [22]:

$$\mathrm{Re}_T = \left[\beta^2 \delta^2 \mid dU/dr \mid_{MAX}\right]^{-1}, \beta = 0.09 - \sqrt{M_0 - 1.2/42.}$$

Then, for example, for $\delta = 0.1$ turbulent Reynolds number will be $\mathrm{Re}_T = 1200$, this is a range in which the effect of viscosity is appreciable. Small-scale fluctuations of high azimuthal numbers brought into the flow by a high prime background should have smaller increments than those predicted by inviscid theory. In addition, the increments of these modes decrease faster than others do during the spread of the jet and an increase of δ which leads to damping of the corresponding components of the spectrum.

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