Optimal design of two-stage speed reducer using two-phase evolutionary algorithm

L. Tudose, O. Buiga, D. Jucan and C. Ştefanache

***Abstract*—**In this paper an optimal design of two-stage speed reducer is presented. The novelty of this work consists in the complex and complete approach of the optimal design of gearings. The chosen objective function was the volume bounded by the inner surface of the reducer housing. For this example of optimal design, eleven genes were taken into consideration and a set of thirty six constraints were formulated. In solving the optimization problem we used an original two-phase evolutionary algorithm (2PhEA) inspired from the evolutionary concept of "punctuated equilibrium". 2PhEA is implemented in Cambrian v.3.2 which is in operation at the Optimal Design Centre of the Technical University of Cluj-Napoca, Romania.

***Keywords*—**Evolutionary algorithms, helical gears, optimal design.

I. Introduction

THE main goal of this paper lies in emphasizing once again the advantages of the optimal design of all sorts of products as compared to the classical design. In this particular case, we deal with the optimal design of a two-stage speed reducer, which is an important mechanical part widely used in aerospace industry, automobile industry, lathe, etc. A two-stage speed reducer optimization problem induces a number of challenges especially when the design problem involves gear kinematics, geometry and strength. The resulting optimization problem involves design variables which can be integer (number of teeth), discrete (normal module), and real (gear width). Many researchers have reported solutions to this problem.

In the chronological order the first who dealt with this problem was Osman [21], in 1978. He made a design synthesis of a nine-speed gear drive. The objective of the synthesis was to minimize the size of all gears from the mesh and speed ratio equations so that the size of the largest gears is kept to a minimum. Due to the mesh and speed ratio equations, it is found that only the following three independent parameters need to be selected: x_1, x_2 and x_3 (x_1– gear ratio, x_2, x_3 –constraints). Because of practical considerations, the minimization of $|x_2 - x_3|$ was found to result in the reduction of the cost of manufacturing the gear drive.

In [1], Aberšek described an expert system (STATEX) to design and manufacture a gear box. In the first stage of the process, genetic algorithms were used to determine the optimal dimensions of a gear box (with special requirements) then, the expert system took technological requirements into account, related to the selection of cutting tool and cutting conditions, the special sequence of machining, the tolerances etc.

Other researchers that reported solutions to optimal design of one stage speed reducer problem were Kuang et al. in [11] and Liand in [16]. However, the solutions reported in [11] and [16] are non-feasible.

Li and Symmons in [15] performed the optimized design of helical gear reducer using the minimum centre distance as the objective function.

In [17], [22] and [23] an optimal design problem of a one stage speed reducer is presented. In these papers the objective was to minimize the weight of the reducer. In [17], [22] and [23] the authors used a set of seven variables as follows: face width, module of teeth, number on pinion teeth, length of input shaft between the bearings, length of output shaft between bearings, diameter of shaft 1 and diameter of shaft 2. The objective function was subjected to a simply formulated (from a mechanical point of view) set of 11 constraints. Mezura in [17] utilized this problem only to test one of previous version of his Simple Multimembered Evolution Strategy (SMES) software.

Vu in [18], [19] and [20] presented a study of the optimization and regression techniques for optimum determination of partial ratios of two-step, three-step and four-step helical gearboxes in order get different objectives including the minimum gearbox length, the minimum gearbox cross section dimension and the minimum mass of gears. In order to reach the above mentioned objectives, Vu started from the moment equilibrium condition of the mechanic system, which includes all the gear units and their regular resistance condition. However the author made a couple of simplifications which could affect the optimization results. One of these simplifications was made in the calculus of the

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reducer gearing mass. In this calculus the gears were considered as simply cylinders with the base diameter equals to the pitch diameter of the gear and with the height equals to the gear's width. In fact the mass calculus is more complicated and the result is significant different from the simplified variant.

In [14], Li et al presented an optimal design problem of a two-stage speed reducer. The purpose of the paper is to obtain the multi-objective optimization design scheme of a gear reducer with a Fuzzy Genetic Algorithm (FGA). The authors used fuzzy technique to adjust the weights of objective functions, crossover probability, mutation probability, crossover positions and mutation positions in the process of running the genetic algorithm utilized for their paper. The design variables used in the optimization problem are: normal module, helix angle, tooth width, number of teeth – variables corresponding of both stages of the reducer, and transmission ratio, in total a set of 9 variables. However they use a set of only 7 constraints too little related to the design realities.

The approach chosen by the authors of this paper for the design problem makes it one of the most round works from all above mentioned papers. As opposed to the researchers mentioned above who were only interested in the programming and mathematical aspects of the problem, we also took into consideration the strength component utilizing in the constraints calculi DIN 3990 [24] norms. With DIN 3990 we obtained a design problem with a higher level of complexity. We dealt separately with the gearing (which in this case represents a two pairs of gears – a two-stage speed reducer) and the shafts. The separate treatment is related to our experience (when we could find a reducer gearing optimal design solution it was easy to reach the matching shafts solution). The reducer shafts will be the topic of another paper. Thus, the objective consists in minimizing the volume defined by the reducer housing inner surface. It is known that a reducer is a part of an industrial application that has to occupy the smallest space possible; its weight is given by its housing, which is by far the heaviest part of the assembly. In the following subsections, we outline the formulation of the optimal reducer design problem in a systematic manner.

Note that for our design problem we used a set of 36 constraints and for the shafts problem a set of 61 constraints (i.e. a total of 97 constraints), while in [17], [22] and [23] there were used only 11 simplified ones. Even in the work [14], which is the most complete from all above presented, they used only a number of 7 restrictions.

II. Two-phase enhanced evolutionary algorithm

Optimization problems with a very large number of constraints can be very difficult to solve. In order to remove this shortcoming, a two-phase enhanced evolutionary algorithm (2PhEA) inspired from the evolutionary concept of *punctuated equilibrium* is presented in this paper.

Punctuated equilibrium [12] is a theory about how new species evolve that was first advances by paleontologists Niles Eldridge and Stephan Jay Gould in 1972 [5]. Before punctuated equilibrium, most scientists assumed that evolutionary change occur slowly and continuously in almost all species, and that new species originate either by slow divergence of small, isolated groups or by slow evolutionary transformation of whole species. But studies of the fossil records have shown that the biological evolution is a strong non-equilibrium process with long periods of stasis interrupted by avalanches of large changes in biosphere. According to the proponents of punctuated equilibrium [5] [7], for the majority of time species are in evolutionary stasis, with little or no change occurring and hence little or no increase in adaptation to their environments. Occasionally, often due to some environmental catastrophe (or planetwide climatic change [8]), there will be punctuations, periods of rapid evolutionary change during which speciations occur. So, punctuationists claim that (i) except when speciation occurs, species are in stasis and do not become increasingly adapted to their environments, and (ii) gradual natural selection alone is insufficient for speciation, which requires a punctuation event. Therefore the biological evolution can be considered as a kind of self-organized criticality (SOC) dynamics [6] and, therefore, SOC gives an insight into emergent complexity in nature. Bak [2] contended that the critical state was the *most efficient state that can actually be reached dynamically*, and in this state, a population in an apparent equilibrium evolves episodically in spurts. Local change may affect any other element in the system, and this delicate balance arises without any external, organizing force.

In other words, in terms of evolutionary computation, evolution of a species consists of exaptations of jumping from one hilltop to another nearby in some fitness landscape. Naturally such jumps will be rare, separated by large time intervals where species are located at a fitness peak, and the resulting evolutionary pattern will show punctuations as indeed seen in the fossil record [3].

Probably punctuated equilibrium is the best known example of evolutionary metastability [4]. From the beginning, the theory of punctuated equilibrium has inspired many computational approaches. Hereinafter we presented some significant results.

Bornholdt and Sneppen [3] have studied evolution of a single genetic network, ideally representing a single species, in the absence of any competition. The evolution is driven by a noisy environment and the evolutionary step consists of random mutations combined with selection of mutants preserving the phenotype with respect to a given environment. Thus, the only requirement in this minimalistic model is continuity in phenotype. This simplification allowed them to discuss how the requirement of evolving robust networks in itself may lead to an evolution which exhibits punctuated equilibrium.

Jonnal and Chemero [10] describe experiments in artificial life in which a neural network is artificially evolved to control a virtual creature. With the evolutionary algorithm employed in the artificial evolution, it was possible to simulate punctuated equilibrium: in some trials, instead of keeping the overall rate of mutations μ constant for the entire trial, they introduced a probability *p* that *μ* increased by some factor *m* over the course of a trial, so that for an individual generation, there is probability *p* that the mutation rate is set to *m*μ. In all but one case, the trials that included occasional punctuations had final fitness scores that were better than the scores of the trials that had no punctuations.

Lewis et al [13] utilized the punctuated equilibrium concept in developing a new Evolutionary Programming algorithm that, in addition to the conventional mutation and selection operations, implements a further selection operator to encourage the development of a SOC system. The algorithm is evaluated on a range of test cases drawn from real-world problems, and compared against a selection of algorithms, including gradient descent, direct search methods and a genetic algorithm. The results were very encouraging.

Martz et al [9] used genetic algorithms in order to design reliability experiments. Genetic algorithms were executed in batches of 100 generations in order to allow for punctuated equilibrium. The best 10 solutions after a given batch has been completed become the initial set of designs for the next batch of 100 generations. After several batches of 100 generations of solutions have been obtained in this way, we finally report the design having the highest utility as our desired near-optimal Bayesian experimental design.

The authors of the present paper have a totally different point of view on implementing the concept of punctuated equilibrium in an evolutionary optimization algorithm. We think that the high level of stress in the population (which determines sudden and massive changes of the species) is comparable to the effect of constrains of an optimization problem. Therefore, the main idea behind our 2PhEA algorithm is its operation in two phases. In each phase, the individual's fitness is determined by another factor. In *Phase 1*, the individual’s fitness depends only on the way in which an individual is more suitable (or not) in terms of constraints. In this phase, the population “fight for survival” and there is no interest for the best individual. For this reason, the number and level of mutations is high, respectively very high. We thought this phase as some kind of “feasible individual generator”. The algorithm moves into the second phase when the number of feasible individuals of the population exceeds a preset threshold. *Phase 2* is a common evolutionary algorithm (sometimes a simple genetic algorithm).

In the following we present, in short, how to determine an individual’s fitness in both phases of the algorithm. The optimization problem consists of an objective function *f* accompanied by certain number of constraints. The search space is considered the space of the *n* – dimensional decision vectors:

$$\bar{x} = \left(x^{(1)}, x^{(2)}, \ldots, x^{(n)}\right)$$

where:

n – number of genes (variables);

The constraints of the problem are:

n_u – inequality type constraints:

$$g_i(\bar{x}) \le 0, \quad i = \overline{1, n_u}$$

n_s – strict inequality type constraints:

$$g_i(\bar{x}) < 0, \quad i = \overline{n_u + 1, n_u + n_s}$$

n_e – equality type constraints:

$$g_i(\bar{x}) = 0, \quad i = \overline{n_u + n_s + 1, n_u + n_s + n_e}$$

In order to use the use this constraints in our algorithm we needed to aggregate them in the following form:

$$G_i(\bar{x}) = \begin{cases} \left.\begin{array}{l} 0, \; g_i(\bar{x}) \le 0 \\ g_i(\bar{x}), \; g_i(\bar{x}) > 0 \end{array}\right\} i = \overline{1, n_u} \\ \left.\begin{array}{l} 0, \; g_i(\bar{x}) < 0 \\ g_i(\bar{x}) + \varepsilon, \; g_i(\bar{x}) \ge 0 \end{array}\right\} i = \overline{n_u + 1, n_u + n_s} \\ \left.\begin{array}{l} 0, \; g_i(\bar{x}) = 0 \\ \left|g_i(\bar{x})\right|, \; g_i(\bar{x}) \ne 0 \end{array}\right\} i = \overline{n_u + n_s + 1, n_u + n_s + n_e} \end{cases} \quad (1)$$

where:

ε – very small positive quantity.

In each phase, for each individual a so-called *score* is computed. The *partial score* of an individual (from those *N* individuals of the population) $\bar{x}_j$, $j = \overline{1, N}$, regarding to the constraint *i*, $i = \overline{1, n_u + n_s + n_e}$ is calculated as follows:

$$PS_i(\bar{x}_j) = \frac{G_i(\bar{x}_j)}{\sum_{k=1}^{N} G_i(\bar{x}_k)} \quad (2)$$

Eventually, the (*individual*) *score* of each individual $\bar{x}_j$, $j = \overline{1, N}$ of the population is:

$$S(\bar{x}_j) = \sum_{i=1}^{n_u + n_s + n_e} PS_i(\bar{x}_j) \quad (3)$$

Obviously, any feasible individual has null score. During *Phase 1* the population is sorted by the *score*, and during *Phase 2*, the population is sorted by *score* and *objective value*. In both phases the fitness of an individual is set according to its rank. 2PhEA is implemented in our Cambrian software.

In Table I, an example of a population of 7 individuals is presented, and it is explained how the rank of an individual is established according to its capacity to meet the constraints of the optimization problem. Not that the rank of feasible individuals (#2 and # 6 in this example) are randomly set.

Table I Example of population ranking in Phase 1

Ind	**Constraint 1**		**Constraint 2**		**Constraint 3**		**Ind. Score**	**Rank**
#	Value	Partial Score	Value	Partial Score	Value	Partial Score		phase 1

1	52.3	0.17	22	0.07	512	0.31	0.55	4
2	31.2	0.10	37	0.12	831	0.50	0.72	5
3	0	0.00	0	0.00	0	0.00	**0.00**	**1**
4	211.0	0.69	0	0.00	294	0.18	0.87	7
5	0	0.00	8	0.03	0	0.00	0.03	3
6	0	0.00	0	0.00	0	0.00	**0.00**	**2**
7	11.8	0.04	253	0.79	18	0.01	0.84	6
∑	306.3		320		1655			

In order to implement such two-phase algorithm it is necessary to design some appropriate adjustable evolutionary features, even more because the two phases of the evolution require different tuning. This should not be considered a shortcoming, but an opportunity for fine control of evolution. The original evolutionary features discussed here are the fitness function and the genetic operators.

In the design of the fitness function we consider two aims: (i) the function should be as simple as possible, even it is adjustable, and (ii) the adjustment should be made by meaning of a single parameter. With these in mind we propose here a *linear adjustable fitness function* with a single tuning parameter, the *selection pressure*.

As it was already mentioned, we assume that the fitness $\Phi(j)$ of an individual j, $j = \overline{1, N}$ is set according to its rank in the sorted population and represents its probability of selection. We consider the *selection pressure SP* as the ratio of the best individual's *selection* probability to the average *selection* probability of all individuals in the *selection* pool (the whole population here). Since:

$$\sum_{j=1}^{N} \Phi(j) = 1$$

it results that:

$$\Phi(1) = \frac{SP}{N}.$$

The proposed *linear adjustable fitness function* has the following form:

$$\Phi(k) = \begin{cases} a_1 \cdot j + b_1, & \text{if } j = \overline{1, p} \\ a_2 \cdot j + b_2, & \text{if } j = \overline{p+1, N} \end{cases}$$

where:

$$a_1 = \frac{2N - SP \cdot (N + p - 1)}{N(N-1)(p-1)}$$

$$b_1 = \frac{p \cdot SP - 2}{(N-1)(p-1)}$$

$$a_2 = \frac{p \cdot SP - 2N}{N(N-p)(p-1)}$$

$$b_2 = \frac{2N - p \cdot SP}{N(N-p)(N-1)}$$

The value of the threshold p should be in the range [$p_{\min}$, $p_{\max}$], where:

$$p_{\min} = \frac{2N}{SP} - N + 1$$

$$p_{\max} = \frac{2N}{SP}$$

Note that if $p_{\min}$ or $p_{\max}$ exceeds the bounds of the range, it will be set to the appropriate bound. In order to reduce the number of parameters, that could make difficult the tuning of the selection operator, we set the value of the threshold at:

$$p = \text{round}\left(\frac{p_{\min} + p_{\max}}{2}\right)$$

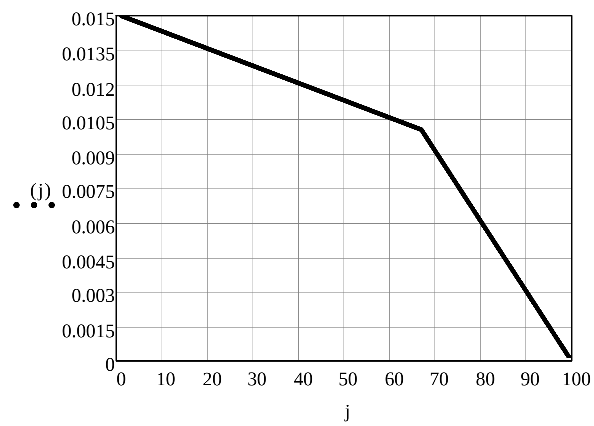

Fig. 1 Fitness function (N = 100, SP = 1.5, p = 67)

In Fig. 1 the graph of fitness function is presented in the case of a selection pressure of 1.5. A similar or smaller value is used in *Phase 1*, when it is mandatory to not to prioritize none/any of the feasible individuals (in order to preserve the diversity of population). In *Phase 2* the selection pressure has to have a larger value. The more an elitist evolution is desirable, the larger the value of the selection pressure should be set.

In Fig. 2 the graph of a strong elitist fitness function is plotted for a selection pressure of 3.0.

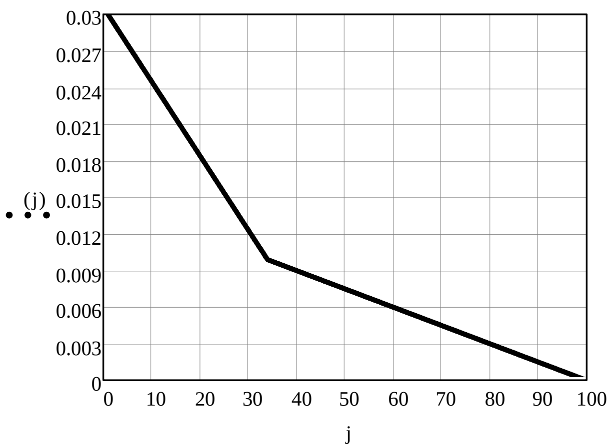

Fig. 2 Fitness function (N = 100, SP = 3.0, p = 34)

The above presented 2PhEA requests adjustable genetic operators for recombination and mutation, respectively. These two genetic operators are inspired by the *Monte Carlo random number generators* which generate normally distributed (statistically independent) numbers.

In order to present the *normal recombination operator* let us consider two individuals $\bar{x}_1$ and $\bar{x}_2$ that will be mated, and assume that their k-th gene will suffer the recombination: $x_1^{(k)}, x_2^{(k)} \in \left[x_{lo}^{(k)}, x_{up}^{(k)}\right]$. Let also assume that the parents satisfy the relationship $x_1^{(k)} \le x_2^{(k)}$. The obtained off-springs are:

$$y_1^{(k)} = x_1^{(k)} + \sigma_1 \cdot \sqrt{-2\ln u_2} \cdot \eta(u_1)$$

$$y_2^{(k)} = x_2^{(k)} + \sigma_2 \cdot \sqrt{-2\ln u_2} \cdot \eta(u_2)$$

where:

u_1, u_2 – random uniform distributed number on (0, 1), and

$$\eta(u) = \begin{cases} \sin(2\pi u_1), & \text{if } u \le 0.5 \\ \cos(2\pi u_1), & \text{if } u > 0.5 \end{cases}$$

$$\sigma_1 = q_c \cdot \min\left(x_1^{(k)} - x_{lo}^{(k)}, \frac{x_2^{(k)} - x_1^{(k)}}{2} \right)$$

$$\sigma_2 = q_c \cdot \min\left(x_{up}^{(k)} - x_2^{(k)}, \frac{x_2^{(k)} - x_1^{(k)}}{2} \right)$$

It is very easy to understand that $y_1^{(k)} \in N\left(x_1^{(k)}, \sigma_1\right)$ and $y_2^{(k)} \in N\left(x_2^{(k)}, \sigma_2\right)$ (where $N(\mu, \sigma)$ is a normal distribution with mean μ and variance σ^2). That means that each off-spring is part of a normal distribution with one of the parents as mean. The standard deviation is adjustable through the value of the parameter q_c. If the parameter q_c has a small value then the off-springs will be generated in the very neighborhood of the parents, and if q_c has higher value then the off-springs will be produced far away from their parents. For this reason in *Phase 1* we set the q_c parameter at higher values (closed to 1), and in *Phase 2* we used almost always q_c = 1/3.

The two off-springs $y_1^{(k)}, y_2^{(k)}$ should be in the range $\left[x_{lo}^{(k)}, x_{up}^{(k)}\right]$. If they exceed these bounds then their values will be trimmed.

Regarding to the *natural mutation operator* we constructed it in the same manner as the recombination operator. If $y^{(k)} \in \left[x_{lo}^{(k)}, x_{up}^{(k)}\right]$ is one of the two off-springs obtained after recombination and which will suffer a mutation, then the mutant is given by the equation:

$$z^{(k)} = y^{(k)} + \sigma \cdot \sqrt{-2\ln u_2} \cdot \eta(u_1)$$

where:

$$\sigma = q_m \cdot \min\left(y^{(k)} - x_{lo}^{(k)}, x_{up}^{(k)} - y^{(k)}\right)$$

Obviously $z^{(k)} \in N\left(y^{(k)}, \sigma\right)$ and the mutant $z^{(k)}$ should be in the range $\left[x_{lo}^{(k)}, x_{up}^{(k)}\right]$. If the mutant $z^{(k)}$ exceeds these bounds then its value will be trimmed. The strategy of setting the parameter q_m is the same as those of the setting of q_c, and the used value were q_m = 1/6 … 1/3.

I. DESIGN PROBLEM FORMULATION

The aim of our problem is to obtain a two-stage speed reducer (Fig. 3) as compact as possible in the following input data:

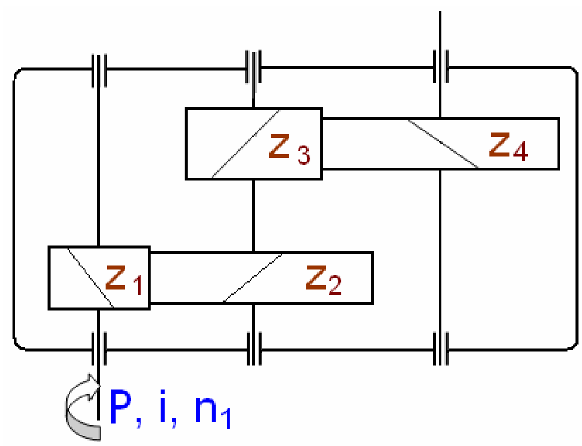

Fig. 3 The reducer sketch

Electrical engine horsepower:
$P = 2.9$ kW;
Overall transmission ratio:
$i = 7.6$;
Rotational speed of shaft 1:
$n_1 = 925$ rpm
Gear necessary life:
$L_{h1,2,3,4} = 8000$ hours;
Basic metric rack profile – ISO 53;
The materials chosen for the pinions are:
42CrMo4 quenched and tempered at $HB_1 = 3000$ MPa;
Allowable Hertzian stress: $\sigma_{H\,\mathrm{lim}1,3} = 760$ MPa;
Allowable bending stress: $\sigma_{F\,\mathrm{lim}1,3} = 580$ MPa;

And for wheels are:
41Cr4 quenched and tempered at $HB_2 = 2700$ MPa;
Allowable Hertzian stress: $\sigma_{H\,\mathrm{lim}\,2,4} = 720$ MPa;
Allowable bending stress: $\sigma_{F\,\mathrm{lim}\,2,4} = 560$ MPa.
Safety factor for pitting and bending:
$S_{H\,\mathrm{min}} = 1.15$,
$S_{F\,\mathrm{min}} = 1.25$.
Tooth root surface factors:
$Y_{R1,2,3,4} = 1.02$.
Hardness ratio factor for the teeth flank:
$Z_w = 1$.

II. USEFUL RELATIONSHIPS

In order to aggregate the objective function and the constraints used in this optimization problem, some preliminary parameters have to be first determined.

In the following are presented the relationships, used in the reducer gearings calculi. Because the relationships are identical for both the reducer stages we present here only the relationship corresponding to the first stage of the reducer. In these relations the symbols have 2 types of indexes as follows:

– Indexes "$_1$" or "$_2$" referring to the gears (pinion and wheel) of the first stage (as standard pitch diameters d_1, base diameter d_{b1}, working pitch diameters d_{w1}, etc.);
– Index symbolized with "$_{\_1}$" representing the common elements for the gears of the first stage (as normal module m_{n_1}, centre distance $a_{\_1}$, preliminary blank width $b_{\_1}$ etc.).

By replacing "$_1$", "$_2$" with "$_3$" and "$_4$", and "$_{\_1}$" with "$_{\_2}$" the useful relationship for the second stage of the reducer could be obtained.

A. Geometrical values

Transmission ratio for the second stage:

$$i_{34} = \frac{i}{i_{12s}}$$

Number of teeth for wheel:

$$z_2 = \mathrm{round}(i_{12s} \cdot z_1)$$

Number of teeth ratio:

$$u_{12} = \frac{z_2}{z_1}$$

Number of revolutions for the 2nd and 3rd shaft, [rpm]:

$$n_2 = \frac{n_1}{u_{12}}$$

$$n_3 = \frac{n_2}{u_{12} \cdot u_{34}}$$

Horsepower on the 2nd and 3rd shaft, [kW]:

$$P_1 = P \cdot \eta_{rul}$$

$$P_2 = P \cdot \eta_{rul}{}^2$$

$$P_3 = P \cdot \eta_{rul}{}^3$$

Torques on the 2nd and 3rd shaft, [Nmm]:

$$T_1 = \frac{3 \cdot 10^7 \cdot P_1}{\pi \cdot n_1}$$

$$T_2 = \frac{3 \cdot 10^7 \cdot P_2}{\pi \cdot n_2}$$

$$T_3 = \frac{3 \cdot 10^7 \cdot P_3}{\pi \cdot n_3}$$

Preliminary module, [mm]:

$$m_{np\_1} = \frac{2 \cdot a_{w\_1} \cdot \cos\beta_{\_1}}{z_1 + z_2}$$

Centre distance, [mm]:

$$a_{\_1} = \frac{m_{n\_1} \cdot (z_1 + z_2)}{2 \cdot \cos\beta_{\_1}}$$

Preliminary blank width, [mm]:

$$b_{\_1} = \psi_{a\_1} \cdot a_{w\_1}$$

Radial pressure angle:

$$\alpha_{t\_1} = \mathrm{atan}\left(\frac{\tan\alpha_n}{\cos\beta_{\_1}}\right)$$

Working pressure angle:

$$\alpha_{wt\_1} = \mathrm{acos}\left(\frac{a_{\_1}}{a_{w\_1}} \cdot \cos\alpha_{t\_1}\right)$$

Normal addendum modification coefficient sum in normal direction:

$$x_{sn\_1} = \frac{(\mathrm{inv}\alpha_{wt\_1} - \mathrm{inv}\alpha_{t\_1}) \cdot (z_1 + z_2)}{2 \cdot \tan\alpha_n}$$

$$x_{n2} = x_{sn\_1} - x_{n1}$$

Addendum modification coefficient sum:

$$x_{st\_1} = x_{sn\_1} \cdot \cos\beta_{\_1}$$

Numbers of teeth of the equivalent spur gears:

$$z_{n1,2} = \frac{z_{1,2}}{\cos^3\beta_{\_1}}$$

Minimum values for the normal addendum modification coefficients:

$$x_{n1,2\,\mathrm{min}} = \frac{14 - z_{n1,2}}{17}$$

Addendum modification coefficients:

$$x_{t1,2} = x_{n1,2} \cdot \cos\beta_{\_1}$$

Standard pitch diameters, [mm]:

$$d_{1,2} = \frac{m_{n\_1} \cdot z_{1,2}}{\cos\beta_{\_1}}$$

Base diameters, [mm]:

$$d_{b1,2} = d_{1,2} \cdot \cos\alpha_{t\_1}$$

Working pitch diameters, [mm]:

$$d_{w1,2} = d_{1,2} \cdot \left(\frac{\cos\alpha_{t\_1}}{\cos\alpha_{wt\_1}}\right)$$

Root diameters, [mm]:

$$d_{f1,2} = m_{n\_1} \cdot \left[\frac{z_{1,2}}{\cos \beta_{\_1}} - 2 \cdot \left(h_{an} + c_{sa} - x_{n1,2} \right) \right]$$

Outside diameters, [mm]:

$$d_{a1,2} = 2 \cdot a_{w\_1} - m_{n\_1} \cdot \left[\frac{z_{2,1}}{\cos \beta_{\_1}} - 2 \cdot \left(h_{an} + x_{n2,1} \right) \right]$$

Radial outside cylinder pressure angle:

$$\alpha_{at1,2} = \text{acos}\left(\frac{d_{1,2}}{d_{a1,2}} \cdot \cos \alpha_{t\_1} \right)$$

Standard pitch circular tooth thickness in normal direction and in radial direction, [mm]:

$$s_{n1,2} = (0.5 \cdot \pi + 2 \cdot x_{n1,2} \cdot \tan \alpha_n) \cdot m_{n\_1}$$

$$s_{t1,2} = \frac{(0.5 \cdot \pi + 2 \cdot x_{t1,2} \cdot \tan \alpha_{t\_1}) \cdot m_{n\_1}}{\cos \beta_{\_1}}$$

Outside cylinder helix angles:

$$\beta_{a1,2} = \text{atan}\left(\frac{d_{a1,2}}{d_{1,2}} \cdot \tan \beta_{\_1} \right)$$

Outside cylinder circular tooth thickness in normal direction and in radial direction, [mm]:

$$s_{at1,2} = \left[\left(\text{inv}\alpha_{t\_1} - \text{inv}\alpha_{at1,2} \right) \cdot \frac{m_{n\_1} \cdot z_{1,2}}{\cos \beta_{\_1}} + s_{t1,2} \right] \cdot \frac{\cos \alpha_{t\_1}}{\cos \alpha_{at1,2}}$$

$$s_{an1,2} = s_{at1,2} \cdot \cos \beta_{a1,2}$$

Radial contact ratio:

$$\varepsilon_{\alpha\_1} = \left(\frac{\left(d_{a1}^2 - d_{b1}^2 \right)^{1/2} + \left(d_{a2}^2 - d_{b2}^2 \right)^{1/2}}{2 \cdot \pi \cdot m_{n\_1} \cdot \cos \alpha_{t\_1}} - \right.$$

$$\left. - \frac{2 \cdot a_{w\_1} \cdot \sin \alpha_{wt\_1}}{2 \cdot \pi \cdot m_{n\_1} \cdot \cos \alpha_{t\_1}} \right) \cdot \cos \beta_{\_1}$$

Overlap contact ratio:

$$\varepsilon_{\beta\_1} = \frac{b_{\_1} \cdot \sin \beta_{\_1}}{\pi \cdot m_{n\_1}}$$

Total contact ratio:

$$\varepsilon_{\gamma\_1} = \varepsilon_{\alpha\_1} + \varepsilon_{\beta\_1}$$

Base cylinder helix angle:

$$\beta_{b\_1} = \text{atan}\left(\frac{d_{b1}}{d_1} \cdot \tan \beta_{\_1} \right)$$

Working pitch cylinder helix angle:

$$\beta_{w\_1} = \text{atan}\left(\frac{d_{w1}}{d_1} \cdot \tan \beta_{\_1} \right)$$

B. *Elements of the equivalent spur gears*

Standard pitch diameter of the equivalent spur gears, [mm]:

$$d_{n1,2} = m_{n\_1} \cdot z_{n1,2}$$

Base diameter of the equivalent spur gears, [mm]:

$$d_{bn1,2} = d_{n1,2} \cdot \cos \alpha_n$$

Outside diameter of the equivalent spur gears, [mm]:

$$d_{an1,2} = d_{n1,2} + d_{a1,2} - d_{1,2}$$

Working pressure angle of the equivalent spur gearing:

$$\alpha_{wn\_1} = \text{acos}\left(\frac{\cos \alpha_{wt\_1} \cdot \cos \beta_{b\_1}}{\cos \beta_{w\_1}} \right)$$

Working centre distance of the equivalent spurs gearing, [mm]:

$$a_{wn\_1} = \frac{a_{\_1}}{\cos^2 \beta_{b\_1}} \cdot \frac{\cos \alpha_n}{\cos \alpha_{wn\_1}}$$

Radial contact ratio of the equivalent spurs gearing:

$$\varepsilon_{\alpha n\_1} = \left(\frac{\left(d_{an1}^2 - d_{bn1}^2 \right)^{1/2} + \left(d_{an2}^2 - d_{bn2}^2 \right)^{1/2}}{2 \cdot \pi \cdot m_{n\_1} \cdot \cos\alpha_{t\_1}} - \right.$$

$$\left. - \frac{2 \cdot a_{wn\_1} \cdot \sin\alpha_{wn\_1}}{2 \cdot \pi \cdot m_{n\_1} \cdot \cos\alpha_n} \right) \cdot \cos\beta_{\_1}$$

C. *Strength parameters*

The strength parameters (Z_R, Z_v, Y_{Fa}, Y_{Sa}, Y_δ, $Y_{\delta st}$, Z_x, and Y_x) used in this optimization problem are according to DIN 3990. To obtain the proper value for the above mentioned factors we used the corresponding diagram from DIN. Each curve from these diagrams was digitalized. After that, we developed a function *f* which interpolates between the curves from diagrams and return the right value of the parameters.

We developed this function because the difference between the values of the parameters obtained with the proper formula from DIN and the value given by the function *f* are significant.

Surface roughness factor for pressure contact and for bending, [μm]:

$$R_{zf1,2} = 4.4 \cdot R_{af1,2}^{0.97}$$

$$R_{z100\_1} = \frac{R_{zf1} + R_{zf2}}{2} \cdot \left(\frac{100}{a_{w\_1}} \right)^{1/2}$$

$$R_{zr1,2} = 4.4 \cdot R_{ar2,1}^{0.97}$$

Roughness factors:

$$Z_{R1,2} = \text{f}\,(R_{z100\_1}, \sigma_{H\lim1,2})$$

Sliding speed factor:

$$v_{\_1} = \frac{\pi \cdot d_1 \cdot n_1}{60000}$$

Velocity factors:

$$Z_{v1,2} = \text{f}\,(v_{\_1}, \sigma_{H\lim1,2})$$

Tooth profile factor:

$$Y_{Fa1,2} = \text{f}\,(z_{n1,2}, x_{n1,2})$$

Dimension factor of root stress:

$$Y_{Sa1,2} = \text{f}\,(z_{n1,2}, x_{n1,2})$$

Factor of material sensibility to stress concentration at the tooth root:

$$Y_{\delta1,2} = \text{f}\,(Y_{Sa1,2}, \sigma_{02})$$

Factor of material sensitiveness to stress concentration at the tooth root in static loading:

$$Y_{\delta st1,2} = \mathrm{f}\,(Y_{Sa1,2}, \varepsilon_{\alpha n\_1}, \sigma_{02})$$

Dimension factor for pitting and for bending:

$$Z_{x\_1} = \mathrm{f}\,(m_{n\_1}, \mathrm{H.\,treatment})$$

$$Y_{x1} = \mathrm{f}\,(m_{n\_1}, \mathrm{H.\,treatment})$$

$$Y_{x2} = \mathrm{f}\,(m_{n\_1}, \mathrm{H.\,treatment})$$

Number of loading cycles:

$$N_{L1,2} = 60 \cdot n_{1,2} \cdot L_{h1,2} \cdot \chi_{1,2}$$

$$m_{H1,2} = \frac{\log\left(N_{BH} / N_{stH}\right)}{\log\left(Z_{N\max} / \left(Z_{L1,2} \cdot Z_{R1,2} \cdot Z_{v1,2}\right)\right)}$$

$$m_{F1,2} = \frac{\log\left(N_{BF} / N_{stF}\right)}{\log\left(Y_{N\max} / \left(Y_{\delta 1,2} \cdot Y_{R1,2} \cdot Y_{x1,2}\right)\right)}$$

Life factors for pitting:

$$Z_{N1,2} = \begin{cases} Z_{N\max} \text{ if } N_{L1,2} \le N_{stH} \\ \left(\dfrac{N_{BH}}{N_{L1,2}}\right)^{1/m_{H1,2}} \text{ if } (N_{stH} < N_{L1,2}) \wedge (N_{L1,2} < N_{BH}) \\ 1 \quad \text{otherwise} \end{cases}$$

Life factors for bending:

$$Y_{N1,2} = \begin{cases} Y_{N\max} \text{ if } N_{L1,2} \le N_{stF} \\ \left(\dfrac{N_{BF}}{N_{L1,2}}\right)^{1/m_{F1,2}} \text{ if } (N_{stF} < N_{L1,2}) \wedge (N_{L1,2} < N_{BF}) \\ 1 \quad \text{otherwise} \end{cases}$$

Allowable pitting stresses, [MPa]:

$$\sigma_{HP1,2} = \frac{\sigma_{H\lim 1,2} \cdot Z_{N1,2} \cdot Z_{w\_1} \cdot Z_{L1,2} \cdot Z_{R1,2} \cdot Z_{v1,} \cdot Z_{x\_1}}{S_{F\min}}$$

$$\sigma_{HP\_1} = \begin{cases} \sigma_{HP1} & \text{if} \quad \sigma_{HP1} < \sigma_{HP2} \\ \sigma_{HP2} & \text{if} \quad \sigma_{HP1} > \sigma_{HP2} \end{cases}$$

Allowable bending stresses, [MPa]:

$$\sigma_{FP1,2} = \frac{\sigma_{F\lim 1,2} \cdot Y_{N1,2} \cdot Y_{\delta 1,2} \cdot Y_{R1,2} \cdot Y_{x1,2}}{S_{F\min}}$$

Contact ratio factors:

$$Z_{\varepsilon\_1} = \begin{cases} \left(\dfrac{4 - \varepsilon_{\beta\_1}}{3} \cdot (1 - \varepsilon_{\beta\_1}) \cdot \dfrac{\varepsilon_{\beta\_1}}{\varepsilon_{\alpha\_1}}\right)^{1/2} & \text{if} \quad \varepsilon_{\beta\_1} < 1 \\ \varepsilon_{\alpha\_1}^{-1/2} \quad \text{if} \quad \varepsilon_{\beta\_1} \ge 1 \end{cases}$$

Load distribution factor:

$$Y_{\varepsilon\_1} = 0.25 + \frac{0.75}{\varepsilon_{\alpha n\_1}}$$

Zone factor:

$$Z_{H\_1} = \left(\frac{2 \cdot \cos\beta_{b\_1}}{\sin\alpha_{wt\_1} \cdot \cos\alpha_{wt\_1}}\right)^{1/2}$$

Helix angle factor for pitting:

$$Z_{\beta\_1} = \left(\cos\beta_{\_1}\right)^{1/2}$$

Dynamic load factor:

$$K_{v\alpha\_1} = \mathrm{f}\,(v_{1,2} \cdot z_{1,2} / 100, \text{ precision class})$$

$$K_{v\beta\_1} = \mathrm{f}\,(v_{1,2} \cdot z_{1,2} / 100, \text{ precision class})$$

$$K_{v\_1} = \begin{cases} K_{v\beta\_1} - \varepsilon_{\beta\_1} \cdot (K_{v\beta\_1} - K_{v\alpha\_1}) \quad \text{if} \quad \varepsilon_{\beta\_1} < 1 \\ K_{v\beta\_1} \quad \text{if} \quad \varepsilon_{\beta\_1} \ge 1 \end{cases}$$

Tooth flank load distribution factor for pitting and for bending:

$$K_{H\beta\_1} = \mathrm{f}\,(\psi_{d\_1}, \text{ gear position})$$

$$K_{F\beta\_1} = \mathrm{f}\,(\psi_{d\_1}, \text{ gear position})$$

Tangential force, [N]:

$$F_{t\_1} = \frac{2 \cdot T_1}{d_1}$$

Auxiliary factor:

$$q_{\alpha\_1} = \begin{cases} 0.5 \text{ if } 4 \cdot \left(0.1 + \dfrac{f_{pbr} - 4}{\dfrac{F_{t\_1}}{b_{\_1}}}\right) \le 0.5 \\ 1 \quad \text{otherwise} \end{cases}$$

Dimension factor of root stress for pitting and for bending:

$$K_{H\alpha\_1} = 1 + 2 \cdot \left(q_{\alpha\_1} - 0.5\right) \cdot \left(\frac{1}{Z_{\varepsilon\_1}^2} - 1\right)$$

$$K_{F\alpha\_1} = q_{\alpha\_1} \cdot \varepsilon_{\alpha\_1}$$

Hertzian stress, [Mpa]:

$$\sigma_{H\_1} = \frac{Z_E \cdot Z_{\varepsilon\_1} \cdot Z_{H\_1} \cdot Z_{\beta\_1}}{a_{w\_1}} \cdot \frac{\cos\alpha_{t\_1}}{\cos\alpha_{wt\_1}} \cdot$$

$$\cdot \left(\frac{T_1 \cdot K_A \cdot K_{v\_1} \cdot K_{H\beta\_1} \cdot K_{H\alpha\_1}}{2 \cdot b_2} \cdot \frac{(u_{12} + 1)^3}{u_{12}}\right)^{1/2}$$

Load distribution factor:

$$Y_{\beta\min\_1} = \begin{cases} 1 - 0.25 \cdot \varepsilon_{\beta\_1} \quad \text{if} \quad \varepsilon_{\beta\_1} \le 1 \\ 0.75 \quad \text{if} \quad \varepsilon_{\beta\_1} > 1 \end{cases}$$

Bending stress, [MPa]:

$$\sigma_{F1} = \frac{T_1 \cdot z_1 \cdot \left(\dfrac{z_2}{z_1} + 1\right)^2 \cdot K_A \cdot K_{v\_1} \cdot K_{F\beta\_1}}{2 \cdot b_1 \cdot a_{w\_1}^2 \cdot \cos\beta_{\_1}} \cdot K_{F\alpha\_1} \cdot Y_{\varepsilon\_1} \cdot$$

$$\cdot Y_{\beta\_1} \cdot Y_{Fa1} \cdot Y_{Sa1} \cdot \left(\frac{\cos\alpha_{t\_1}}{\cos\alpha_{wt\_1}}\right)^2$$

$$\sigma_{F2} = \sigma_{F2} \cdot \frac{b_1}{b_2} \cdot \frac{Y_{Fa2}}{Y_{Fa1}} \cdot \frac{Y_{Sa2}}{Y_{Sa1}}$$

D. Control elements

$$\alpha_{Nt1,2} = \mathrm{acos}\left(\frac{z_{1,2} \cdot \cos\alpha_{t\_1}}{z_{1,2} + 2 \cdot x_{n1,2} \cdot \cos\beta_{\_1}}\right)$$

Teeth number for span measurement:

$$N_{1,2} = \text{round}\left[\frac{1}{2} + \frac{z_{1,2}}{\pi}\cdot\left(\frac{\tan\alpha_{Nt1,2}}{\cos^2\beta_{\_1}} - \frac{2\cdot x_{n1,2}\cdot\tan\alpha_n}{z_{1,2}} - \text{inv}\alpha_{t\_1}\right)\right]$$

Normal and radial span measurement for no backlash gearing, [mm]:

$$W_{Nn1,2} = 2\cdot x_{n1,2}\cdot m_{n\_1}\cdot\sin\alpha_n + m_{n\_1}\cdot\cos\alpha_n\cdot[(N_{1,2}-0.5)]\cdot\pi +$$
$$+ z_{1,2}\cdot\text{inv}\alpha_{t\_1}$$

$$W_{Nt1,2} = \frac{W_{Nn1,2}}{\cos\beta_{b\_1}}$$

Curvature profile radii in the measurement points of span in radial direction, [mm]:

$$\rho_{Nt1,2} = 0.5\cdot W_{Nt1,2}$$

Curvature profile radii in the first and in the last point of contact, [mm]:

$$\rho_{At1} = a_{w\_1}\cdot\sin\alpha_{wt\_1} - 0.5\cdot d_{b2}\cdot\tan\alpha_{at2}$$

$$\rho_{Et2} = a_{w\_1}\cdot\sin\alpha_{wt\_1} - 0.5\cdot d_{b1}\cdot\tan\alpha_{at1}$$

Curvature profile radii, [mm]:

$$\rho_{at1,2} = 0.5 d_{a1,2}\cdot\sin\alpha_{at1,2}$$

III. OPTIMAL DESIGN OF TWO-STAGE SPEED REDUCER

In order to perform the optimal design of the two-stage speed reducer it is necessary to set up: the variables (genes) that uniquely describe the problem, the objective function and the problem constraints.

Hereinafter, since the optimization will be performed using evolutive algorithms, instead of the term *variable* we will use the term *gene*.

A. Genes

The design problem genes are presented in Table II.

Table II Genes of the optimization problem

No.	Genes	Range
Genes corresponding to the first stage		
1	Transmission ratio, i_{12s}	1.12 – 40
2	Working centre distance, a_{w_1}	71 – 400
3	Normal addendum modification coefficient, x_{n1}	– 0.5 – 1
4	Length width coefficient, ψ_{a_1}	0.2 – 0.8
5	Standard pitch cylinder helix angle, $\beta_{\_1}$	7° – 15°
6	Number of teeth of pinion, z_1	25 – 56
Genes corresponding to the second stage		
7	Working centre distance, a_{w_2}	71 – 400
8	Normal addendum modification coefficient, x_{n3}	– 0.5 – 1
9	Length width coefficient, ψ_{a_2}	0.2 – 0.8
10	Standard pitch cylinder helix angle, $\beta_{\_2}$	7° – 15°
11	Number of teeth of pinion, z_3	25 – 56

Note that the genes with number 1,2,6,7 and 11 (see Table II) have only standardized values in the appropriate mentioned range.

B. Objective function

The expression of the volume defined by the inner surface of the reducer housing is:

$$V_r = A_{fr}\cdot L_r \tag{4}$$

where:

A_{fr} – frontal surface area of the reducer housing (Fig. 4), [mm^2];

L_r – width of the housing reducer, [mm].

C. Constraints

The solutions of the optimization program have to satisfy the following constraints listed bellow. All values of these constraints have to be negative or maximum zero.

C1–2. The relative error of the actual transmission ratio has to be between – 2.5% and +2.5%.

$$g_{1,2} = \begin{cases} \dfrac{|i_{12,34s} - i_{12,34}|}{i_{12,34s}}\cdot\dfrac{100}{2.5} - 1 & \text{if} \quad i_{12,34} < 4 \\[2ex] \dfrac{|i_{12,34s} - i_{12,34}|}{i_{12,34s}}\cdot\dfrac{100}{3} - 1 & \text{if} \quad i_{12,34} \geq 4 \end{cases} \tag{5}$$

C3–4. The Hertzian stress ($\sigma_{H\_1,2}$) should be less or equal to the allowable Hertzian stress ($\sigma_{HP\_1,2}$) for both gearings.

$$g_{3,4} = \frac{\sigma_{H\_1,2}}{\sigma_{HP\_1,2}} - 1 \tag{6}$$

C5–8. The bending stress ($\sigma_{F1,2,3,4}$) at the tooth base has to be less or equal to the allowable bending stress ($\sigma_{FP1,2,3,4}$) for all gears.

$$g_{5-8} = \frac{\sigma_{F1,2,3,4}}{\sigma_{FP1,2,3,4}} - 1 \tag{7}$$

C9–12. The normal addendum modification coefficient should be in such range that the undercutting of all gears teeth does not get worse.

$$g_{9-12} = \frac{14 - 17\cdot x_{n1,2,3,4}}{z_{n1,2,3,4}} - 1 \tag{8}$$

C13–16. The profile shift should be in such a range that the tooth thickness at the top of all gears does not decrease under a certain value.

$$g_{13-16} = \frac{c_{sa}\cdot m_{n\_1,2}}{s_{an1,2,3,4}} - 1 \tag{9}$$

C17–18. Radial contact ration should be greater than a certain imposed value.

$$g_{17,18} = \frac{\varepsilon_{\alpha\,\text{lim}}}{\varepsilon_{\alpha\_1,2}} - 1 \tag{10}$$

C19–20. The normal addendum modification coefficient, $x_{n2,4}$ should be in the range of [–0.5…1].

$$g_{19,20} = \frac{|x_{n2,4} - 0.25|}{0.75} - 1 \tag{11}$$

C21–32. For the span measurement, the following conditions should be satisfied:

$$g_{21-24} = \frac{W_{Nn1,2,3,4} \cdot \sin \beta_{b\_1,2} + 5}{b_{1,2,3,4}} - 1 \tag{12}$$

$$g_{25,26} = \frac{\rho_{At1,3}}{\rho_{Nt1,3}} - 1 \tag{13}$$

$$g_{27,28} = \frac{\rho_{Nt1,3}}{\rho_{at1,3}} - 1 \tag{14}$$

$$g_{29,30} = \frac{\rho_{Et2,4}}{\rho_{Nt2,4}} - 1 \tag{15}$$

$$g_{31,32} = \frac{\rho_{Nt2,4}}{\rho_{at2,4}} - 1 \tag{16}$$

C33–34. The number of the pinion teeth $z_{1,3}$ and the number of the wheel teeth $z_{2,4}$ should be co-prime numbers.

$$g_{33,34} = \begin{cases} -1 & \text{if} \quad z_{1,3} \text{ and } z_{2,4} \text{ are coprime} \\ 1 & \text{otherwise} \end{cases} \tag{17}$$

C35. The non – interference condition between the high-speed big gear and the output shaft.

$$g_{35} = 0.5 \cdot \frac{d_{a2}}{a_{w\_2} - 5} - 1 \tag{18}$$

C36. The level of oil inside the reducer housing should be in the range of [$H_{\min}$, $H_{\max}$].

$$g_{36} = \frac{10}{\Delta_H} - 1 \tag{19}$$

where:

Δ_H – difference between the upper and lower level of oil, inside the reducer case, [mm].

$$\Delta_H = H_{\max} - H_{\min} \tag{20}$$

where:

H_{max} – upper level of oil, [mm];

H_{min} – lower level of oil, [mm].

D. Results

In solving this optimization problem, our own *Cambrian v.3.2* software was used. Written in Java, *Cambrian* is a platform that allows the assembling of all sort of evolutive algorithms in an original manner.

The values found for all considered genes are presented in Table III.

Table III Gene values obtained after optimization

No.	Genes	Value
	Genes corresponding to the first stage	
1	Transmission ratio, i_{12s}	2.8
2	Working centre distance, a_{w_1}	80 mm
3	Normal addendum modification coefficient, x_{n1}	0.84
4	Length width coefficient, ψ_{a_1}	0.49
5	Standard pitch cylinder helix angle, $\beta_{\_1}$	13.5°
6	Number of teeth of pinion, z_1	27
	Genes corresponding to the second stage	
7	Working centre distance, a_{w_2}	100 mm
8	Normal addendum modification coefficient, x_{n3}	1
9	Length width coefficient, ψ_{a_2}	0.74
10	Standard pitch cylinder helix angle, $\beta_{\_2}$	12.75°
11	Number of teeth of pinion, z_3	34

In Table IV, the main characteristics of the reducer gearings (classical and optimal solutions) are shown side-by-side.

IV. Conclusions

The comparative study of the solutions shown in Table IV leads to the following conclusions:

The volume of the inner surface of the reducer housing calculated with the classical method is $12.269 \cdot 10^{-3}$ $m^{3,}$ while the optimal design solution offers a smaller volume, equal to $9.964 \cdot 10^{-3}$ m^3, i.e. a 18.787% reduction.

The optimal design solution has the first stage transmission ratio almost equal to the second stage. That confirms the recommendations found in literature.

In the optimal design solution the shape of the reducer housing is rather like a "cube" than a "parallelepiped", which means that the available space will be used more efficiently.

Table IV Classical and optimal design solutions

No.	Denotation	Classical solution	Optimal solution
	Main characteristic of the first stage		
1	Transmission ratio		
	i_{12}	1.605	2.814
2	Centre working distance		
	a_{w_1}, [mm]	100	80
3	Normal module		
	m_{n_1}, [mm]	2	1.5
4	Number of teeth of the pinion		
	z_1	38	27
5	Number of teeth of the wheel		
	z_2	61	76

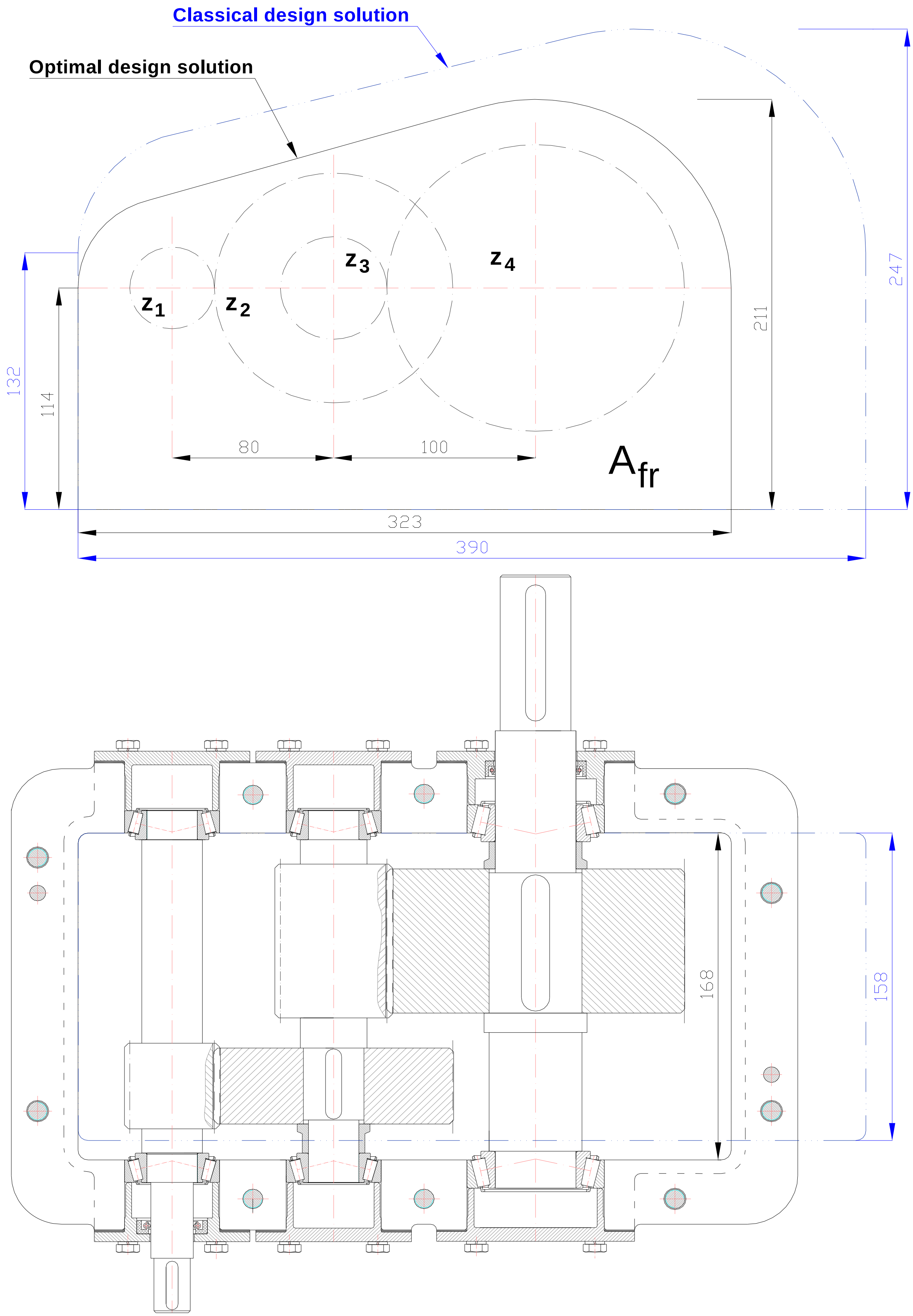

Fig. 4 Optimal and classical design solutions

6	Standard pitch cylinder helix angle		
	$\beta_{\_1}$, [deg]	15°	13.5°
7	Pinion width		
	b_1, [mm]	45	44
8	Wheel width		
	b_2, [mm]	40	39
9	Root diameters		
	d_{f1}, [mm]	71.681	40.42
	d_{f2}, [mm]	118.786	112.106
10	Outside diameters		
	d_{a1}, [mm]	80.214	47.143
	d_{a2}, [mm]	127.319	118.829
Main characteristic of the second stage			
11	Transmission ratio		
	i_{34}	4.516	2.794
12	Centre working distance		
	a_{w_2}, [mm]	112	100
13	Normal module		
	m_{n_2}, [mm]	1.25	1.5
14	Number of teeth of the pinion		
	z_3	31	34
15	Number of teeth of the wheel		
	z_4	140	95
16	Standard pitch cylinder helix angle		
	$\beta_{\_2}$, [deg]	15°	12.75°
17	Pinion width		
	b_3, [mm]	68	79
18	Wheel width		
	b_4, [mm]	63	74
19	Root diameters		
	d_{f3}, [mm]	39.492	51.539
	d_{f4}, [mm]	178.37	141.006
20	Outside diameters		
	d_{a3}, [mm]	45.005	58.243
	d_{a4}, [mm]	183.883	147.71
21	The volume of the reducer housing		
	$V_{reducer}$, [m^3]	$12.269 \cdot 10^{-3}$	$9.964 \cdot 10^{-3}$

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