Nonlinear system diagnosis: Bond graphs meets differential algebra

J.C. Cruz-Victoria, D.I. González-Sánchez

Abstract - **In this paper we state the properties of fault diagnosis through differential algebra and how it can be used in conjunction with the Bond graphs modeling to design fault tolerant controllers. This controller is applied to a DC motor because it is a well known system has a wide range of applications. The faults are estimated through a reduced order observer to reject their effect on the system. This paper represents the first phase in a Bond Graphs´ based approach to determine the diagnosability condition.**

Key words: **Diagnosis condition, observer, Bond graphs, differential algebra.**

I. INTRODUCTION

A physical system may fail when its observed behaviour is different from its expected behavior. Fault diagnosis consists on locating the physical fault in a structural or mathematical model of this system. Linear and nonlinear systems diagnosis has been studied for more than three decades, see for instance [1, 3, 8, 9, 27]. In this work we deal with nonlinear systems and some of its basic definitions may be found on [20, 26, 34]. In [4] is given a review of the different fault diagnosis approach for deterministic nonlinear dynamic systems. Another appealing approach is the one based on differential geometric methods, shown in [2, 3, 5, 6, 7, 8, 9, 10, 11]. Alternatively some authors have proposed solutions to the fault detection and identification problem for a nonlinear system class in a differential and algebraic setting, see [12, 13, 14, 15, 16, 17, 29]. For instance, in [12, 13] an approach has been considered to solve the diagnosis problem. It consists in translating the solvability of the problem in terms of the algebraic observability of the variable modeling the fault. In [16, 17, 18, 19, 20] the methodologies employed for the observer design only include full order observers without considering uncertainty estimation. The information provided about the fault dynamics by the algebraic observers is normally used by a controller, in this case a fault tolerant controller, whose work would be to negate the effects of the fault and maintain a system in nominal levels. There are different methodologies to obtain a fault tolerant control, see for instance [28, 31, 32]. In our paper, the fault dynamics is considered as an uncertainty. In the proposed procedure, the construction of a full order observer is not necessary, instead, a reduced-order uncertainty observer is constructed using differential algebraic techniques applied to the fault estimation in the diagnosis problem. Using these estimations, a trajectory tracking controller was constructed through algebraic methods. This controller´s objective is that given a system of the form

$$\dot{x}(t) = A(x, \bar{u}),$$
$$y(t) = h(x, \bar{u}),$$

and a reference output trajectory $y_d(t)$, $t \geq 0$, to find a dynamic output feedback such that the output *y(t)* of the closed loop system tracks $y_d(t)$ asymptotically.

The main achievement of this work is to fuse algebraic differential techniques of diagnosis with Bond Graphs modeling. This will be used to verify the diagnosability condition, this condition would imply that, the fault is algebraically observable with respect to a differential field $k\langle u, y\rangle$, and thus the coefficients of the differential polynomial are known, see [21]. This paper would be the initial phase for the purpose described.

The need for applying Bond Graphs modeling methodology to obtain the differential transcendence degree, comes from the difficulty that presents verifying the diagnosis condition in some systems, let us remind that the theorem found in [21] gives a non constructive proof of the diagnosis condition, which, sometimes makes more difficult to verify that this theorem is satisfied than to obtain the condition itself. The Bond Graph methodology allows a graphical construction of the model and to visually verify the relationships among outputs, known inputs and faults, thus the differential transcendence degree.

The system classes to which this approach can be applied include input dependant systems and its derivatives in polynomial form. In this work, we present an application to a DC motor, which is widely used in robotics where speed and positional control are of utmost importance. The proposed faults are a parasitic current in the field section and a nonlinear friction. A friction coefficient was not chosen because this would represent a motor under ideal conditions, instead, the LuGre´s mathematical friction model was used, see [23], to show the operation of the motor and of the approach under more realistic conditions. After that, with the same model of the motor, controller and observer we analyzed its behaviour when there was a Gaussian white noise at the measurement simulating a faulty sensor. Here we used the algebraic observer as a noise filter just by making some adjustments over the observer gains.

The rest of this paper is organized as follows: in section 2 the differential algebraic definitions are given, as well as faults diagnosability definitions and Bond graphs terminology. In section 3, the DC motor model is obtained, as well as the differential transcendence degree through Bond graphs of the output vector. In section 4 we obtain the reference tracking algebraic controller, and are shown the numerical simulations and the parameters used. In section 5 this paper is closed with concluding remarks and future work.

II. BASIC DEFINITIONS

2.1 DIFFERENTIAL ALGEBRA

We start by introducing some basic algebraic differential definitions; these can be found in [12, 13, 14, 18, 21].

Definition 1. A differential field extension L/k is given by two differential fields k and L, such that: (i) k is a subfield of L, (ii) the derivation of k is the restriction to k of the derivation of L.

Example. $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$ are trivial differential field extensions where $\mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$.

*Definition 2***.** Let L/k be a differential field extension. A differential trascendental family, which is the greatest with respect to the inclusion, is called a differential trascendental base of L/k. The cardinality of the base is called the differential trascendental degree of L/k and is denoted by

$$difftrd^{o}(L/k) \tag{1}$$

Example. Consider the following system:

$$\begin{aligned}\dot{x}_1 &= x_1 + x_2 + 2x_3\\ \dot{x}_2 &= x_2 + x_3 + u\\ \dot{x}_3 &= x_1\end{aligned} \tag{2}$$

Where u is an input variable which is by definition, differentially transcendental over R. From equation (2), it is not difficult to obtain the following relationships:

$$0 = \dot{x}_3 - x_1 \tag{3}$$
$$0 = -\ddot{x}_3 + \dot{x}_3 + x_2 + 2x_3 \tag{4}$$
$$0 = \dddot{x}_3 - 2\ddot{x}_3 - \dot{x}_3 + x_3 \tag{5}$$

Then, according to Definition 2 and from Equations (3) and (4), it can be concluded that x_1 and x_2 are both differentially algebraic over $\mathbb{R}\langle x_3\rangle$, since both x_1 and x_2 satisfy an algebraic polynomial with coefficients in the differential field $\mathbb{R}\langle x_3\rangle$. We can see that x_3 is differentially transcendental over $\mathbb{R}$, since x_3 satisfies an algebraic polynomial over $\mathbb{R}\langle u\rangle$ (see (5)), and not over $\mathbb{R}$. Then it is concluded that the cardinality of the transcendental base of the extension $\mathbb{R}\langle x_1, x_2, x_3\rangle/\mathbb{R}$ related to system (2) is equal to 1:

$$difftrd^{o}\, \mathbb{R}\langle x_1, x_2, x_3\rangle/\mathbb{R} = 1$$

Definition 3. The differential output rank ρ of a system is equal to the differential transcendence degree of the differential extension $k\langle y\rangle$ over the differential field k, i.e.

$$\rho = difftrd^{o} k\langle y\rangle/k$$

Property 1. The differential output rank ρ, of a system is smaller or equal to *min(m,p):*

$$\rho = difftrd^{o} k\langle y\rangle/k \;\leq\; min\,(m, p)$$

where m, p are the total number of inputs and outputs, respectively.

Definition 4. A system is left-invertible if, and only if, the differential output rank is equal to the total number of inputs, i.e.

$$\rho = m$$

Property 2. If a system is differentially left-invertible then the input u can be recovered from the output by means of a finite number of ordinary differential equations. This property was obtained in [33].

Definition 5. A dynamics is a finitely generated differential algebraic extension $G/k\langle u\rangle\,(k\langle u,\xi\rangle, \xi \in G)$. Any element of G satisfies an algebraic differential equation with coefficients being rational functions over k in the elements of u and a finite number of their time derivatives.

Example: Let consider the input-output system $\ddot{y} + \omega^2 sen(y) = u$, equivalent to the system:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -\omega^2 sen(x_1) + u\\ y &= x_1\end{aligned} \tag{6}$$

This system has a dynamics of the form $\mathbb{R}\langle u, y\rangle/\mathbb{R}\langle u\rangle$ where $G = \mathbb{R}\langle u, y\rangle$, $y \in G$ y $k = R$. Any solution of (6) satisfies the following differential algebraic equation:

$$\left(y^{(3)} - \dot{u}\right)^2 + \left(\dot{y}\left(y^{(2)} - u\right)\right)^2 = (\omega^2 \dot{y})^2$$

Definition 6. Let a subset $\{u,y\}$ of G in a dynamics $G/k\langle u\rangle$. An element in G is said to be algebraically observable with respect to $\{u,y\}$ if it is algebraically over $k\langle u,y\rangle$. Therefore, a state x is said to be algebraically observable if, and only if, it is algebraically observable with respect to $\{u,y\}$. A dynamics $G/k\langle u\rangle$, with output y in G is said to be algebraically observable if, and only if, the state is algebraically observable with respect to $\{u,y\}$.

Example. System (6) with output $y \in \mathbb{R}\langle u,y\rangle$ is algebraically observable, since x_1 and x_2 satisfy two differentially algebraic polynomials with coefficients in $\mathbb{R}\langle u,y\rangle$, i.e.

$$x_1 - y = 0$$
$$x_2 - \dot{y} = 0$$

Statement of the problem

Let us consider the class of nonlinear systems described by:

$$\dot{x}(t) = A(x,\bar{u}),$$
$$y(t) = h(x,u), \tag{7}$$

Where $x = (x_1,\dots,x_n)^T \in R^n$ is a state vector, $\bar{u} = (u,f) = (u_1,\dots,u_{m-\mu},f_1,\dots,f_\mu) \in R^{m-\mu} \times R^\mu$ where u is a known input vector and f is an unknown fault vector, $y = (y_1,\dots,y_p) \in R^p$ is the output, A and h are assumed to be analytical vector functions.

Definition 7. (*Algebraic observability*). An element $f \in k\langle\bar{u}\rangle$ is said to be algebraically observable if f satisfies a differential algebraic equation with coefficients over $k\langle u,y\rangle$

Definition 8. (*Diagnosability*). A nonlinear system is said to be diagnosable if it is possible to estimate the fault f from the system equations and the time histories of the data u and y, i.e. it is diagnosable if f is algebraically observable with respect to "u" and "y".

In other words it is required that each fault vector component be able to be written as a solution of a polynomial equation f_i and finitely many time derivatives of u and y with coefficients in k

$$H_i(f_i,u,\dot{u},\dots,y,\dot{y},\dots) = 0.$$

Theorem 1. If system (7) is observable then it is diagnosable, if and only if, f is observable with respect to u, y, and x. [13]

Remark 1. This is an immediate consequence of the general transitivity property of the observability condition [13].

The diagnosability conditions of f with respect to u, y, and x are generally expected to be simpler than those of f only in terms of u and y. In particular, if the system is observable then it is possible to reduce the number of time derivatives of the data in the fault differential algebraic equation.

A powerful tool to determine if a system is diagnosable is the following theorem.

Theorem 2. System (7) is diagnosable if, and only if, $difftrd^o k\langle u,y\rangle/k\langle u\rangle = \mu$ where μ is the number of components of the fault f.

For proof of this theorem see [21]

This theorem can be viewed as a generalization of left invertibility as stated in the following corollary.

Corollary . The *definition 4* of left invertibility could be easily obtained redefining u as an empty set and f as $\bar{u}$, then we have
$difftrd^o k\langle y\rangle/k = \mu = \rho$

With μ as the total number of inputs.

2.2 BOND GRAPHS

Now we would turn to some basic definitions of the Bond graph modeling method, for the full explanation see [22, 25].

A bond graph is a description of the physical dynamical system using lines and arrows. It is an energy-based graphical technique for building mathematical models of dynamic systems. It shows the energy´s flow among components to create a model of a system. Bond graphs is better suited to analyze physical systems than the conventional block diagram because they are designed to work on the principle of conservation of energy and any addition of energy or change would be easily noted. Through this technique, almost any type of physical system can be modeled, even the very difficult as long as they exchange energy.

Each bond represents a bi-directional flow, systems which produce a "back force" on the input are easily modeled without introducing extra feedback loops.

Bond graphs use also the principle of power continuity, this is for the case when a system dynamics operate on widely varying time scales. If this happens, fast continuous time behaviors can be modeled as instantaneous phenomena by using a special hybrid bond graph.

A bond is a connector that simultaneously connects two variables, the effort e and the flow f. A bond is shown in Fig. 1

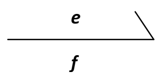


Fig.1 Bond for 1 port element

Where this bond graph is used to address 1-port elements, and at the port a single pair of effort and flow variables exists.

The junctions serve to interconnect other multiports into subsystem or system models. In the Bond Graph modeling there are the 0-junction and the 1-junction. These are important for the understanding of modeling through Bond Graphs because their objective is to display a physical system as a relation of series and parallel connections, see Fig. 2.

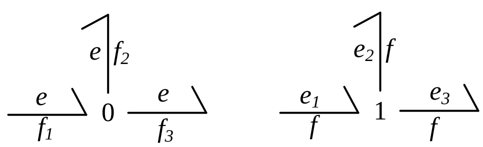


Fig. 2. 0-junction and 1-junction

On the 0-junction:

$f_1 - f_2 - f_3 = 0 \qquad and\ all\ efforts\ are\ equal$

On the 1-junction:

$e_1 - e_2 - e_3 = 0 \qquad and\ all\ flows\ are\ equal$

so the 0-junction and the 1-junction can be viewed as the Kirchoff´s currents law and voltage law respectively.

There are two representations for the two port elements, which work under the principle that power is conserved. These are useful when we need to express a change on energy type. See Fig.3.

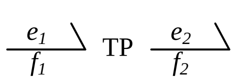


Fig.3 Bond for 2-port elements

For the 2-port elements the relations would be:

$e_1(t)f_1(t) = e_2(t)f_2(t)$

The electrical transformer and gyrator bonds are represented in Fig. 4.

Fig. 4 Bond for transformer and gyrator

The mathematical relation for these elements are:

- For the electrical transformer:
  $e_1 = me_2 \qquad mf_1 = f_2$

  where m is the transformer modulus
- For the gyrator:
  $e_1 = rf_2 \qquad rf_1 = e_2$

  where r is the gyrator modulus.

Our objective is to apply Bond graphs to obtain the differential transcendence degree in a DC motor and know if it complies with the diagnosability condition, so we need first to construct the Bond Graphs model of a DC motor.

In this process, see Fig. 5 a) ,we used a 1 junction to represent the constant current that flows through the armature components of the DC motor and the different voltages . In Fig. 5 b), the bond for the construction of the field subsystem was also a 1-junction because the current remains constant. After that, in Fig. 5 c) we added a Gyrator 2-port element representation for the translation of electrical energy to mechanical movement and how this effort must be used to overcome LuGre´s friction f_L and inertia J.

To finish the model we only need to add an active Bond from the field subsystem to the Gyrator subsystem to represent how small efforts and changes in the field subsystem of the motor affect in a greater way the Gyrator subsystem. The final Bond Graph model is shown in Fig. 6

III. DC MOTOR

Now, the reduced order observer design, which was applied to the DC motor model, will be described, see Fig. 7. The algebraic observer is of great importance since it can monitor the variables that the motor needs to work, particularly field and armature voltage and current, feeding information to the controller.

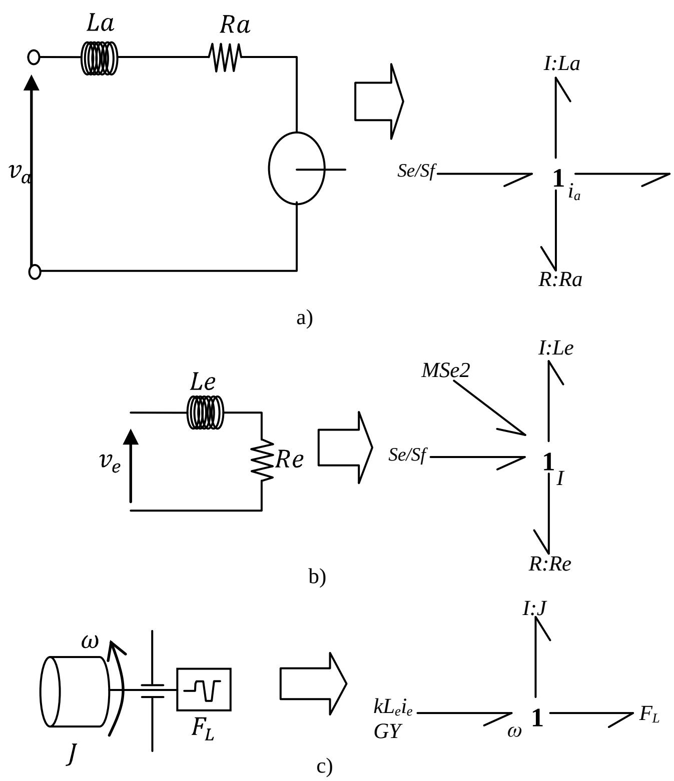
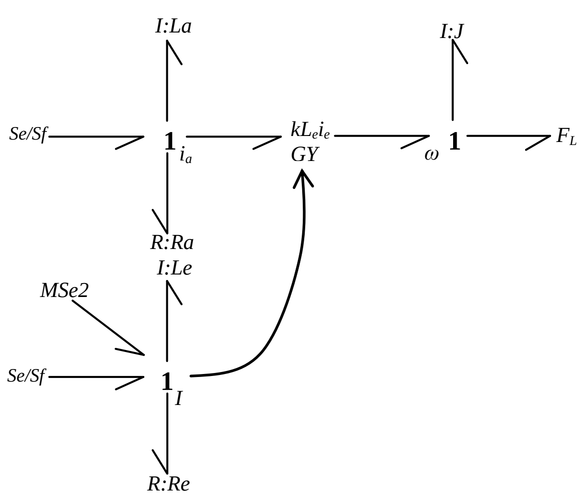


Fig. 5 Bond modeling of a DC motor, a)armature subsystem modeling, b) field subsystem modeling, c)shaft and friction subsystem modeling.

Let us define the DC motor model proposed by [22]



Motor diagram using Bond graphs

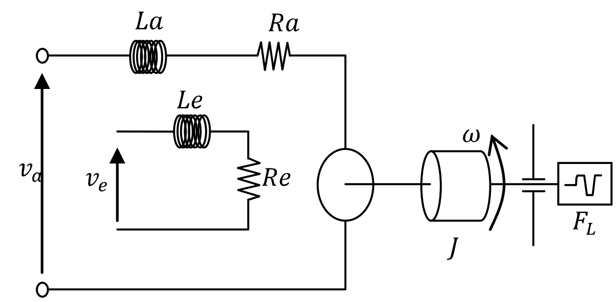


Fig. 7. DC motor diagram

$$
\begin{aligned}
\dot{x}_1 &= \frac{1}{Le}u_1 - \frac{Re}{Le}x_1 + F_1 \\
\dot{x}_2 &= \frac{1}{La}u_2 - \frac{Ra}{La}x_2 - \frac{k}{La}x_1x_3 \\
\dot{x}_3 &= \frac{k}{J}x_1x_2 + F_L
\end{aligned}
\tag{8}
$$

Where Le is the field inductance, Re is the field resistance, La is the armature inductance, Ra is the armature resistance. The state variables are $x_1 = i_e$, $x_2 = i_a$, y $x_3 = \omega$. It is considered that all the states are available; this means that the output vector is given as:

$y_i = x_i$, *for* $i = 1, 2, 3$.

In this model two faults are considered: F_1 is a parasitic current, to obtain F_L we consider the LuGre´s friction f_L and the inertia J as follows:

$$f_L = J\,F_L$$

The LuGre mathematical model, which has been previously validated through experimentation and represents a nonlinear friction (It must be noted that the motor load is included in the nonlinear friction), a more detailed description of this model is on [23, 24].

$$
\begin{aligned}
F_L &= \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{q} \\
\dot{z} &= -\sigma_0 a(\dot{q})z + \dot{q} \\
a(\dot{q}) &\triangleq |\dot{q}|/\alpha_0 + \alpha_1 e^{-\left(\frac{\dot{q}}{\alpha_2}\right)^2}
\end{aligned}
\tag{9}
$$

On table 1, can be seen the parameters used for model (9).

According to *Theorem 2* it is enough to find an algebraic relation fault-output; in the proposed model this relation is simple to see, in Fig. 8 and 9 the fault appears explicitly on the output node.

Once confirmed that the system is diagnosable, the following step is to obtain a reduced order observer to

TABLE 1.
PARAMETERS USED IN THE LuGre FRICTION MODEL

Parameter	Value	Unit
σ_0	5000	[s^{-2}]
σ_1	$\sqrt{5000}$	[s^{-1}]
σ_2	0.4	[s^{-1}]
α_0	5000	[s^{-1}]
α_1	$\sqrt{5000}$	[s^{-1}]
α_2	0.4	[s^{-1}]

estimate the faults. The convergence proof for this observer can be found in [21] so we will not show it here.

The general form of the reduced order observer is:

$$\dot{f}_e = \beta(f - f_e) \tag{10}$$

Where f_e denotes the fault estimate f and $\beta \in \mathbb{R}^+$ determines the convergence ratio desired by the observer.

It must be noted that f is replaced on the observer because of its diagnosis condition, this means, an algebraic differential equation whose coefficients are in the differential field $k\langle u, y\rangle$.

Note. Sometimes the output time derivatives (which are unknown), appear in the fault algebraic equation, and then an auxiliary variable is needed. The system dynamics (10) together with

$$\dot{\gamma} = \psi(x, \bar{u}, \gamma) \text{ with } \gamma_0 = \gamma(0) \text{ and } \gamma \in C^1 \tag{11}$$

constitute a proportional asymptotic reduced-order fault observer for system (10), where γ is a change of variable to which depends on the estimated fault f, and the state variables.

The diagnosis condition for fault F_1 is shown next:

$$F_1 = \dot{x}_1 - \frac{1}{Le}u_1 + \frac{Re}{Le}x_1$$

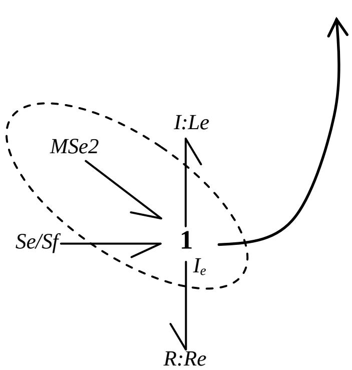


Fig. 8 Differential transcendence of output 1

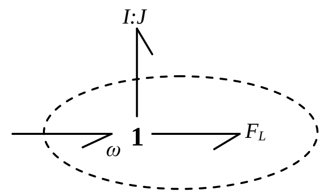


Fig. 9 Differential transcendence of output 3

And the observer of F_1 is given next:

$$\dot{\gamma}_1 = \beta_1\left(-\frac{1}{Le}u_1 + \frac{Re}{Le}x_1 - \gamma_1 - \beta_1 x_1\right)$$

Applying the change of variable:

$$\gamma_1 = \hat{F}_1 - \beta_1 x_1$$

It is evident that $\hat{F}_1$recovers from

$$\hat{F}_1 = \gamma_1 + \beta_1 x_1$$

In the same way for F_L, we must have the diagnosis condition.

$$F_L = \dot{x}_3 - \frac{k}{J}x_1 x_2$$

For F_L the observer is:

$$\dot{\gamma}_2 = \beta_2\left[-\frac{k}{J}x_1 x_2 - \beta_2 x_3 - \gamma_2\right]$$
$$\gamma_2 = \hat{F}_L - \beta_2 x_3$$

Where the observer gains are $\beta_1 = 2500$ and $\beta_2 = 100$.

Obtaining a fault tolerant control

Now the algebraic controller construction will be shown, This controller is used to compensate for the faults of a parasitic current F_1, and the nonlinear friction F_L. The algorithm used in this controller design can be seen on [20].

Let us define the error dynamics as:

$$\dot{e}_i = -\lambda_i e_i$$

If $\lambda_i > 0$ then the error dynamics is asymptotically stable. Then we can propose the following reference tracking control for x_1:

With

$e_1 = y_{d1} - x_1$ where $y_{d1} = const.$

Then

$$u_1 = -L_e\left[-\lambda_1(y_{d1} - x_1) - \frac{R_e}{L_e}x_1 + F_1\right] \quad (12)$$

Substituting u_1 on $\dot{x}_1$ we obtain:

$$\dot{x}_1 = \frac{1}{L_e}(-L_e)\left[-\lambda_1(y_{d1} - x_1) - \frac{R_e}{L_e}x_1 + F_1\right] - \frac{R_e}{L_e}x_1 + F_1$$

Similar terms can be reduced:

$$\dot{x}_1 = \lambda_1(y_{d1} - x_1)$$

From where, it is clear to see that if $\lambda_1 > 0$ then the system is stable and (12) allow us to have a good tracking of the reference signal.

Now let us suppose that it is desired that x_3 follows a time variant reference $y_{d2}(t)$, then:

$$\dot{e}_2 = -\lambda_2 e_2$$
$$e_2 = y_{d2}(t) - x_3$$

We repeat the procedure for the next controller, proposing the following tracking reference signal controller u_2.

$$u_2 = -L_a\left\{\frac{-JR_a}{kL_a x_1}[\lambda_2(y_{d2}(t) - x_3) - F_L] - \dot{x}_2 - \frac{k}{L_a}x_1x_3\right\} \quad (13)$$

Substituting u_2 we obtain:

$$\dot{x}_3 = \lambda_2(y_{d2}(t) - x_3) \quad (14)$$

With $\lambda_2 > 0$, the controller designed for u_2 will be good as a tracking reference control.

For the case shown here λ_1=10 and λ_2=250.

IV. NUMERICAL SIMULATIONS

The parameters used for the first part of the experiments [1] are shown on Table 2.

In this section, firstly are shown the results of applying the controller and observer to the system and analyzing the results from the perspective of the currents to maintain a close following of the reference signal. On Fig. 10 and 11 the field and armature current are shown. The objective of the controller is to force the field current to follow a constant reference signal even though the parasitic current fault is present, because the control of a DC motor is through the armature current. On Fig. 12 can be seen the graphic of the LuGre´s friction against its estimated, which closely follows the reference friction.

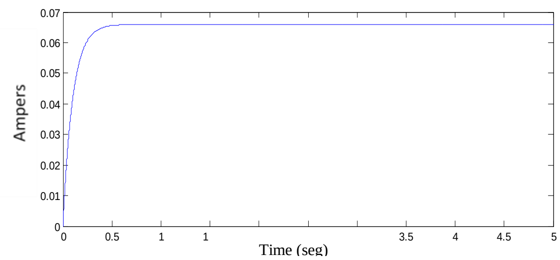


Fig. 10. Field current

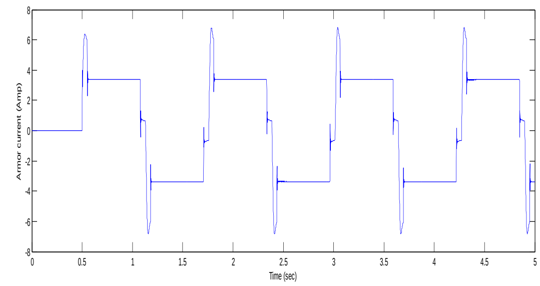


Fig. 11. Armature current

On Fig. 13 it is shown the reference signal´s simulation for velocity, for which a sine function was chosen, which has an amplitude of 200, representing a top angular velocity of 200 rad/sec, on the Figure 13 are shown the motor velocities using the controller and considering the fault effects.

And on Fig. 14 are shown the field and armor voltages that represent the system control vector. This part of the simulations has been useful for showing that this control is able to follow the reference with a minimal error as can be appreciated on the figures.

Now let us change the conditions for a new test of this controller and its reduced order observer. Let us suppose now that the element in charge of obtaining the measurements is faulty and it is adding Gaussian noise. This would be represented as zero mean Gaussian noise, and is present at the field current, see Fig. 15.

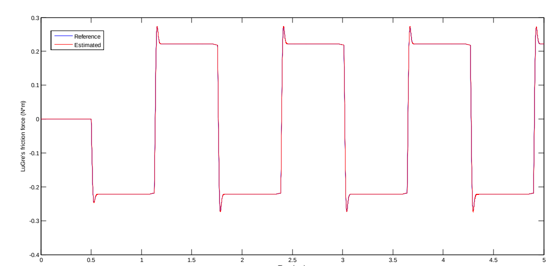


Fig. 12. LuGre´s friction

TABLE 2
DC MOTOR PARAMETERS

Parameter	Value	Unit
Le	20.5245	[H]
Re	320.6955	[Ω]
La	0.00436	[H]
Ra	4.12844	[Ω]
k	1	[dimensionless]
J	0.000043665	[kg·m^2]

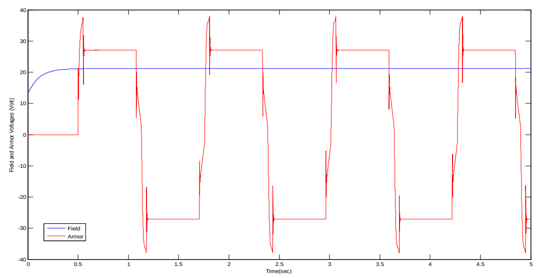


Fig. 13. Field and armature voltages comparison

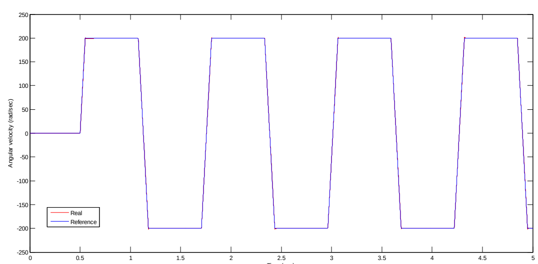


Fig. 14. Comparison of velocities

In this part the reduced order observer was applied, but not to follow the noise but to filter it, with its gains adjusted as β_1 = 10 and β_2 = 50, This observer was able to discard a good part of the noise and to follow the real signal with a good estimated signal, as Fig. 16 shows. After that, a controller was applied with the goal of following a reference even though the noise and the two original faults.

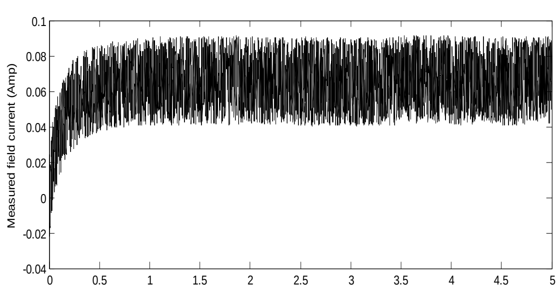


Fig. 15. Field current with Gaussian noise.

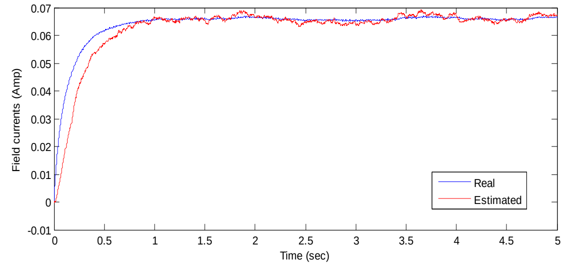


Fig. 16. Field current filtered.

The LuGre´s friction model is still used to represent the nonlinear friction and as can be appreciated in Fig. 17, the added noise causes some oscillations and the observer gains are adjusted at lower levels to avoid noise amplification. On Fig. 18 and 19 are shown the armature current and voltage respectively, they also struggle but are able to follow the change of speed given by the reference.

On Fig. 20 is shown a comparison of the velocities, the reference and the real one. The remarkable property is that the reference is a sine saturated to represent constant parts, and that even though the noise at the measurement, the parasitic current and the nonlinear friction, the real behavior follows very closely the reference only with little problems following the constant part, with an error in steady state of 0.5% of the desired velocity (< 1 rad/sec). The controller was adjusted with $\lambda_1 = 10$ and $\lambda_2 = 250$.

A last test run of the simulation was performed where we saw how the system behaved without consideration for the fault. In Fig. 21, is shown that the angular velocity of the motor is affected and not able to reach the constant part desired, the motor can not compensate the friction and the noise in the measurement is not eliminated. The controller´s performance improves when utilizing the faults estimates.

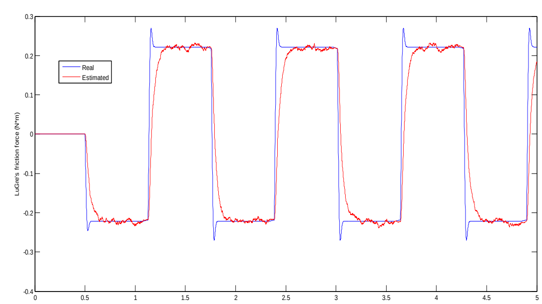


Fig. 17. LuGre´s friction.

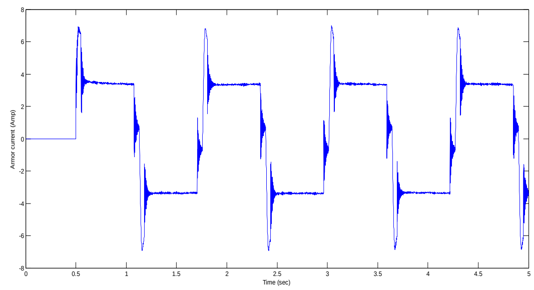


Fig. 18.Armature current .

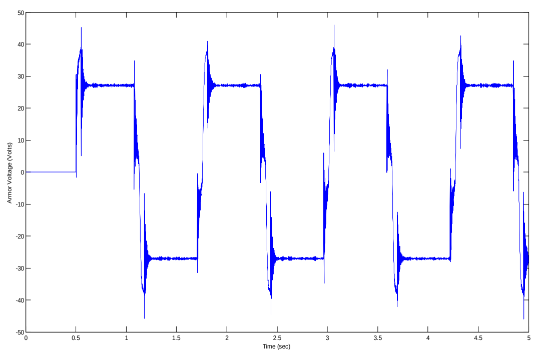


Fig. 19. Armature voltage

V. CONCLUDING REMARKS

On this article, we have introduced a first approach to the relation between fault diagnosis using differential algebra and Bond graphs. Also, a tracking reference controller was designed that is able to use the fault estimates to negate their effects on a DC motor angular velocity. A second test was developed where the observer was used as a noise filter in a scenario simulating Gaussian noise at the input. As future work, it is planned to develop a systematical method to obtain the differential transcendence degree of $k\langle u,y\rangle/k\langle u\rangle$ through Bond graphs for more complex systems.

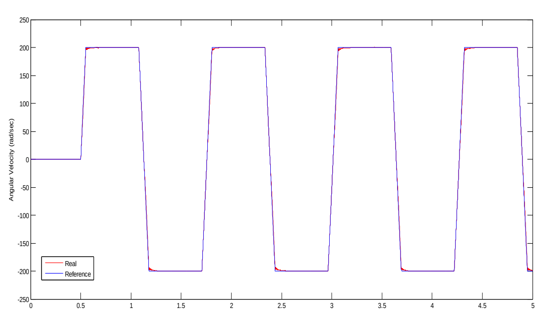


Fig. 20. Velocities comparison.

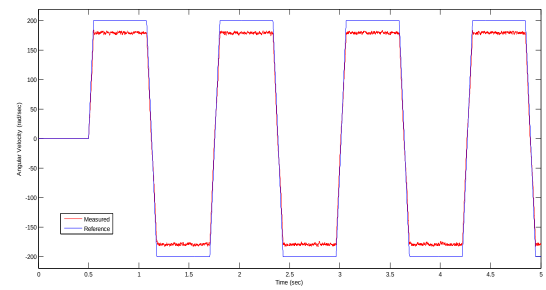


Fig. 21. Velocity without considering faults effects.

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