Recent progress in identification methods for the elastic characterization of materials

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***Abstract*—**A large number of methodologies have been suggested over time for determining the elastic properties of either isotropic or anisotropic materials. Currently, there is still great interest in this topic, mostly toward the development of new methodologies for the characterization of composite materials for which the traditional tests are generally expensive and time-consuming. In the present paper, the feasibility of using the so-called "mixed numerical/experimental technique", a promising and recently introduced methodology, is investigated. In particular, the paper reviews the recent progress made at the University of Calabria where approaches based on both static and dynamic tests have been developed.

Keywords—Elastic Constants Measurement; Mixed Numerical-Experimental Techniques (MNET); Finite Element Method (FEM); Non-Destructive Testing (NDT).

I. Introduction

A knowledge of the elastic properties of materials is important for many reasons. It is essential for both structural design and experimental mechanics but it also allows the assessment of the performance of newly developed materials and can provide information on the quality achieved during the manufacturing process and can serve for structural health monitoring. Great effort has been devoted to developing methodologies for the elastic characterization of materials. Such methodologies are commonly classified into approaches based on static tests and approaches based on dynamic tests (Fig. 1).

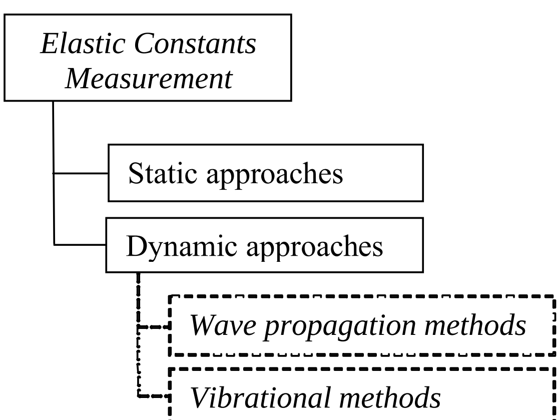

Fig. 1 Classification of the methodologies employed to determine the elastic properties of solids.

The static approaches are mostly based on the direct measurement of stresses and strains undergone by suitable specimens, during certain mechanical tests (tensile, compression, bending, torsion, etc). ASTM [1] and ISO [2] provide many standards for determining the elastic properties of isotropic as well as composite materials. These norms recommend the use of standard sized and shaped specimens. In the case of composite materials, they involve the analysis of a large number of specimens and consequently tedious and time-consuming procedures [3]. Nevertheless, these tests often yield poor results for properties such as shear modulus encouraging the development of specific methods to improve the accuracy in the identification of such a modulus (rail shear, picture-frame shear, off-axis tensile shear and Iosipescu shear tests are, just a few examples of these).

The above-mentioned methodologies usually require a significant range of stress-strain data to determine useful averaged values of the moduli. This necessarily involves destructive tests, as the deformation of the specimen must be measured until it fails, that is, until it deforms plastically or fractures. In either event, the sample is destroyed, and then is unavailable for further testing or other purposes.

In comparison with static approaches, dynamic approaches have the advantage of allowing the use of specimens with a greater variety of shapes and dimensions, and supplying, non-destructively, very precise measurements at a wide range of temperatures. Dynamic approaches can be classified into two groups: wave propagation based methods and modal vibration (or resonant) testing.

Much research has been dedicated to evaluating the possibility of measuring the material elastic properties by the methods belonging to the first group. Among these, the more commonly used is based on the measurement of the ultrasonic speed of wave propagation through the material or, in particular, the measurement of the transit time; i.e. the time that an ultrasonic impulse takes to cross a sample from the emitting transducer to the receiving transducer. Knowing the dimensions and the density of the specimen and the transit

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time of the transversal and longitudinal waves it is possible to calculate the Young modulus and the shear modulus of the material. Although these techniques are robust and quick to perform, they suffer the disadvantage of being sensitive to possible local inhomogeneities of the material between the transducers. In spite of the availability of ASTM standards [4] and patents [5, 6, 7], such methods are still not so largely used.

Ease of use and inexpensive equipment have recently increased the use of modal vibration testing in both research laboratories and industrial contexts. Such tests consist of making a specimen vibrate mechanically, in the sonic and/or ultrasonic frequency field, at one or more vibration resonant modes. A knowledge of the resonant modal shapes and/or the values of the associated frequencies together with the sizes and mass of the sample allow the determination of the elastic constants of the material. ASTM had provided standardized procedures for testing isotropic materials [8-9], while no standards have, till now, been provided for anisotropic materials.

Independently of the approach followed (static or dynamic) an ideal methodology for determining the material elastic properties is a methodology which allows the simultaneous measure of all the unknown elastic parameters by testing a single specimen non-destructively. It would be even better if the methodology is also suitable for characterizing specimens of various shapes. This would be very useful when the production of proper bulk specimens is not feasible or when, the object to be analysed must not be damaged and reduced in a conventional testing geometry and, therefore, should be tested as it is.

However, the elastic constants can be determined directly and quickly if the relationships relating the elastic properties to the measured quantities are known in the form of analytical equations. In such a case, the elastic problem becomes a simple inverse problem and the unknown properties can singly be backed out of each equation or can be simultaneously determined by solving a system of equations. Unfortunately, often, the determination of the analytical equations depends in a complicated way on the boundary conditions and the shape of the specimen and so they can be obtained for simple geometries and boundary conditions only. Nevertheless, when these equations are not available, the inverse elastic problem can be solved by recurring to non-direct methodologies using iterative optimization procedures.

Recently, a promising non-direct methodology for identifying all the material elastic constants of either isotropic or orthotropic any-shaped plates simultaneously, with a single experiment and without damaging the plates was proposed. Such a methodology is suitable for both static and dynamic approaches and was investigated by the author and co-workers in several separate papers [10-20]. The static approach involves the measurement, by an optical technique, of the superficial displacement field of a suitably loaded specimen, while the dynamic approach requires the measurement of the natural frequencies of the first modes of vibration of the free specimen. Such a methodology is based on a process that minimizes an error function based on the difference between the dynamic or static response of the real plate (measured response) and the response of the same plate predicted by a numerical model (calculated response). This method updates the values of the elastic constants of material in the model iteratively, until the calculated response matches the measured response. The values of the constants used in the last iteration are the elastic properties of the material.

The present paper synthesizes the current state of the art in the field of elastic characterization of materials with a particular emphasis on the procedures and equipment developed at the University of Calabria.

II. The Numerical-Experimental Method

The identification of the elastic constants of a material is an inverse problem that can be formulated and resolved as an optimization problem [21].

The flow-chart of a solution procedure is illustrated in Fig. 2. In synthesis, starting from an arbitrary initial trial set (initial solution), the set of elastic constants is updated iteratively (new solution) and this is given as input to an analytical or numerical model simulating the behaviour of the structure until its output (calculated response) fits the experimental data (measured response). The last set of constants (the best solution) identifies the elastic properties of the material. In particular, the process attempts to minimize an error function based on the differences between the calculated and measured response of the structure under examination and stops when this is less than a fixed value. The optimizing technique to be used to generate new guess solutions must be selected on the basis of the number of unknown parameters and the shape of the error function.

The structure must be excited with a suitable static or dynamic input and the response chosen so that, for fixed geometry, dimension and material density, it depends, exclusively, on the elastic constants of the material. Independently of the nature of the response, the numerical operations described above can only be done after a set of experimental measurements including the measurement of the size, shape and mass of the structure and obviously the measurement of its response to the input excitation are carried out.

In the following sections, the numerical and the experimental procedures and equipment for characterizing isotropic or anisotropic plates via static or dynamic tests will be illustrated. In each case, the FEM (Finite Element Method) is used to simulate the structural response. The numerical stage requires a series of operations on a computer. First, a CAD model of the plate must be created with the same geometry and size as the real plate. Then the density of the material is computed as the ratio of the volume of the model to the mass of the plate. Then the CAD model is meshed, and finally, the topology of the mesh, the material density and the trial values of the elastic constants are used as input, for the

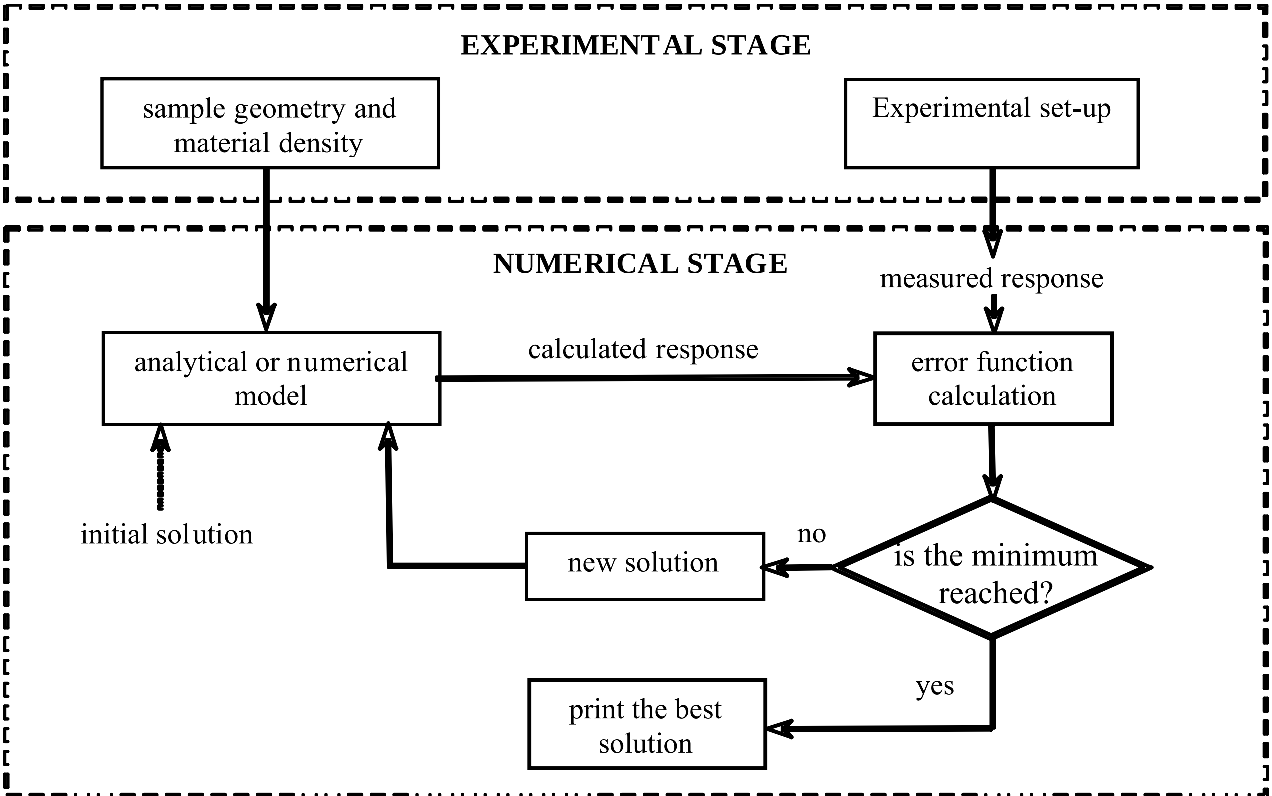

Fig. 2 Schema of a procedure for the elastic characterization of materials.

FEM code to calculate the structural response.

III. Approaches based on static tests

The idea of determining the elastic constants of a material from the surface displacement fields of specimens subjected to a static load has been exploited by many researchers. Following the approach first proposed in [22], several different methodologies have been developed over time [23-39]. Recently, inverse procedures based on finite element model updating have been proposed for both pointwise [40] and full-field measurements. In the last case, experimental data (strain or displacement fields) are usually measured on the surface of the specimen with an optical method, for example, an interferometric technique that enables the full-field surface displacement of an object to be determined with a very high resolution without any contact with the investigated surface [41].

Different tests and optical techniques have been used: in-plane loaded rectangular plate with speckle interferometry [42], open-hole uniaxial tensile tests with Moirè interferometry [43], and cruciform specimens under biaxial tests with digital image correlation technique [44].

In addition, a method that combines finite element analysis and generic algorithms in order to identify the elastic constants of materials from the full-field measurement of the surface displacements of plates under flexural loads was developed and presented in [10] by the author of the present paper. The method was tested on a thin square plate subjected to an out-of-plane loading condition but it is also suitable for characterizing both thin or moderately thick any-shaped anisotropic plates subjected to in-plane or out-of-plane loading and constraining configurations. In the paper, the feasibility of using the displacement component normal to the surface detected by speckle interferometry was investigated, but the methodology could be coupled with any optical technique, whatever the displacement components detected. Theoretical aspects of the methodology, numerical simulations for testing the accuracy and sensitivity of the method and an application to characterize metallic plates were presented.

It is worth pointing out that the amount of data provided by an optical whole-field technique is generally in excess of the data strictly necessary for identifying all the unknown elastic properties. It follows that any material characterization using this approach becomes an over-posed inverse problem. Obviously, an accurate solution can only be obtained if the problem is well-posed. For this reason, great care needs to be taken in choosing the geometry and the way of loading and constraining the specimen in order to obtain displacement fields containing sufficient information for determining all the unknown parameters quickly and unambiguously. In addition, to reduce the effect of the measurement uncertainties on the solution, the displacement must also be sufficiently sensitive to the variation in each elastic parameter.

The author has proposed in [10] a numerical procedure for optimising the loading and constraining conditions of the specimen. The procedure consists in determining the conditions which minimize the ‘‘correlation index’’. This index represents the degree of statistical correlation between

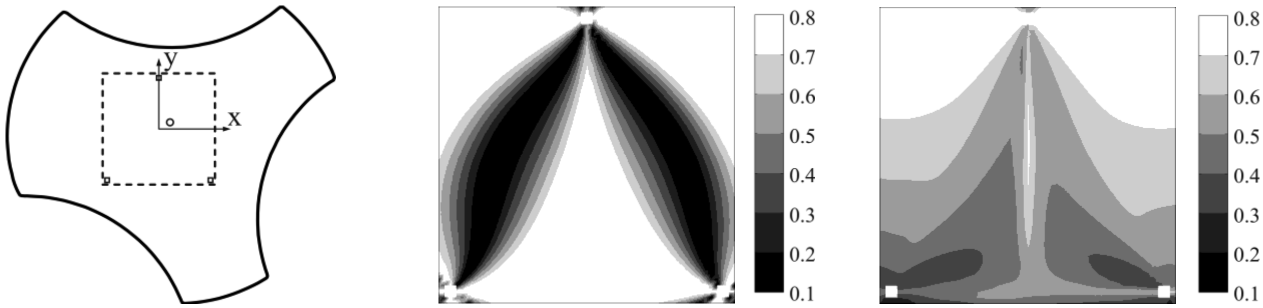

Fig. 3 Correlation index maps of isotropic (centre) and unidirectional (right) laminates.

the variation in the displacement fields due to a variation in the elastic constants and its absolute value is, by definition, less than or equal to unity. In the case of isotropic plates the correlation index is the same as the well-known correlation coefficient, while for orthotropic plates the correlation index is the mean of the absolute values of the correlation coefficients.

Such a procedure can be used to identify loading and constraining configurations that are practical and simple to replicate in the laboratory and also optimized with a view to obtaining faster and more stable solutions. Applications of the procedure can be found in [11] in which it was used to finding a suitable configuration for testing a square plate, and in [12] and [13] for testing any-shaped plates. In Fig. 3 the mean maps of the correlation coefficients (obtained by considering a sufficient number of either isotropic or orthotropic materials) of an irregularly-shaped specimen are reported. The fields of the out-of-plane components (normal to the surface) of the displacements, undergone by the upper surface of the plate, were considered. On the left of the figure the shape of the specimen is schematically represented: the small squares indicate the location of the support points (in these optimized configurations the plate was simply supported on three points lying on the corners of an isosceles triangle), while the small circle represents the point of the application of the load, which is varied in the numerical simulation in order to evaluate the correlation maps. The maps for isotropic and orthotropic materials (with the fibers parallel to the y axis) are reported on the centre and on the right of the figure, respectively. More dark is the area of the correlation maps, more low is the degree of correlation and better will be the choice for the loading point. By observing the maps of several differently shaped isotropic and orthotropic plates, it has been noticed [13] that in the first case a common loading area with a very low value of the correlation index is always found, while, in the second case, a loading area with a suitable correlation index is obtained only for the square shaped plates. As a consequence accurate and fast solutions are insured for both any-shaped isotropic plates and square orthotropic plates. In [13], an experimental set up and the feasibility of testing isotropic plates of generic form and orthotropic square plates was investigated and the results of the experimental assessment are reported and discussed.

In Fig. 4, the two main steps of the identification procedure are illustrated: the experimental measurements and the application of the genetic algorithm. The experimental step requires the measurements of the dimensions and mass of the plate, the applied load and the displacement field. Such data are necessary for the construction of the finite element model and for the numerical identification procedure starting at the second step.

The components of the experimental apparatus for measuring the displacements, which are described in detail in [13], are shown in the sketch of Fig. 3. The apparatus was assembled on an optical bench supported by pneumatic vibration isolators. The laser beam is filtered and expanded and the resulting spherical wavefront is divided by a beamsplitter into two equal intensity beams. The specimen and the reference surfaces are horizontal and are illuminated and observed by a 45° oriented mirror with the respect to the propagation direction of the beams. The scattered speckle wavefronts interfere at the image plane of the CCD of the TV camera. The camera is interfaced with a general purpose computer image processing system where the real time fringe patterns are generated by the subtraction of digitalized images. Essentially, the optical setup constitutes a speckle interferometer, based on the Michelson design, for measuring the out-of-plane component of displacements. The rigid body motion of the specimen due to the compliance of the fixture was compensated for by modifying the fitness of the genetic algorithm according to the least-squares method.

The applicability and the robustness of the procedure were proved with success on aluminium and unidirectional Graphite/PEEK laminate specimens. The results obtained for both the materials have shown a high repeatability and a good agreement with the reference values obtained with other measuring techniques.

A series of numerical simulations carried out considering different layer orientations and numbers of layers has shown the procedure to be unsuitable for the elastic identification of unidirectional laminates with the fibers oriented in the x-axis and for multidirectional laminates.

A very good robustness of the procedure with respect to the effects of measurement noise was demonstrated by adding

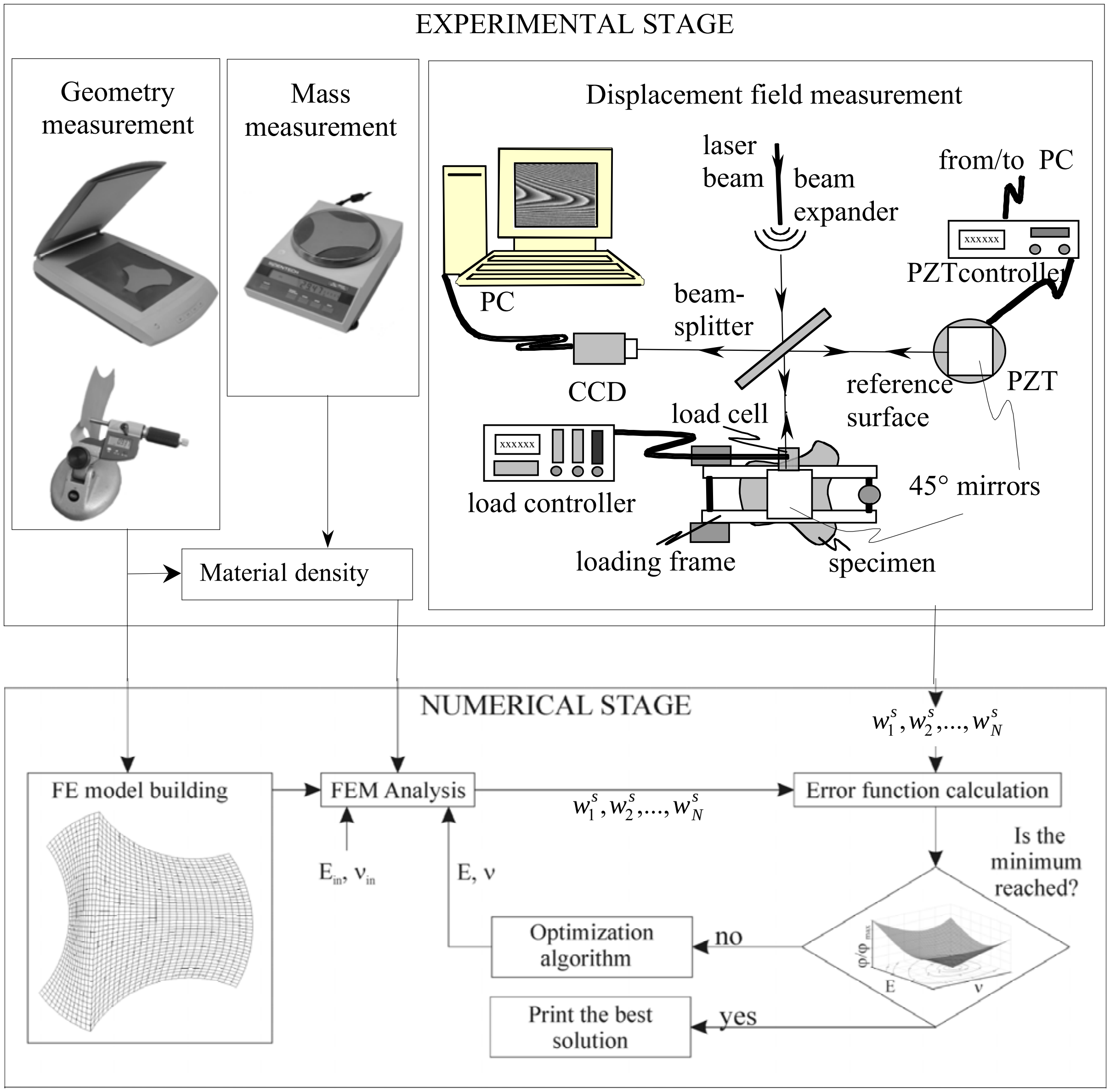

Fig. 4 Elastic identification via static test.

Gauss noise to the input displacement field.

IV. Approaches based on dynamic tests

The measurement of natural frequencies of specimens vibrating in a single mode has been used for many years and is still used today for determining the elastic properties of materials. A typical methodology consists in subjecting the specimen to a vibration test to measure a single modal frequency (usually, the first or fundamental mode frequency) and then substituting the measured value into a "frequency equation". Such equations relate the fundamental resonant frequency to the sizes and the mass of the specimen and generally to only one elastic constant of the material. The latter can then be calculated in a direct way if the other quantities are known. Unfortunately, the frequency equations are known only for some simple specimen geometries and boundary conditions. Free-edge specimens such as bars or rods [45] and cantilever beam [46] are commonly used for the characterization of homogenous and isotropic materials. ASTM E1875-00e1 [47] establishes the application procedures for determining Young's modulus, shear modulus and Poisson's ratio of free bars or rods from the fundamental flexural and torsional resonant vibrations. The test consists in forcing the specimen to vibrate at a single but variable frequency by means of a suitable exciter system, while the dynamic response is detected by a proper receiving transducer and transformed into an electrical signal, which is analysed with a suitable system for extracting the fundamental resonant frequencies. Finally, the elastic constants are calculated with some recommended numerical procedures based on the frequency equations. Even if the test procedures can be carried out automatically by computerized systems [48], they are typically slow and cumbersome. However, the advent of computers has made it possible to fast Fourier transform (FFT) a signal in real time and this has made impulsive excitation more attractive to use. This technique is fast and inexpensive and can be used on either small specimens or full-scale structural components. It is recommended in ASTM E1876-01 [49] for characterizing free-edge bars and rods following procedures similar to those indicated in ASTM E1875-00e1 [47].

It is worth noting that ASTM standards cover the determination of resonance frequencies and elastic properties of specific materials providing test methods that differ one from the other in several ways (for example; sample size, dimensional tolerances, sample preparation). Moreover, nowadays, systems for the elastic characterization of materials based on the standards mentioned above are available commercially (see, for example, [50] and [51]). These test methods are particularly appropriate for materials that are elastic, homogeneous, and isotropic and specimens or structures must have specific geometries. Sometimes, the measurement of elastic properties is carried out directly during manufacturing on end products constituted of various materials with geometries different from those mentioned above and in conditions different from the environmental conditions. As a result there have been a certain number of international patents [52-57].

All the direct methods mentioned above involve beam specimens and resonant frequency measurements. It is worth noting that some direct methods based on single modal testing involving both resonant frequencies measurements and mode shapes measurements have been developed for characterizing rectangular plates. In these methods, approximate analytical solutions providing explicit parameter dependencies are proposed (e.g., solutions obtained applying the Rayleigh method [55] or the concept of sinusoidal equivalent length [58, 59]). These methodologies have, unfortunately, the disadvantage of requiring sophisticated techniques to measure the mode shapes but, in compensation, they are also suitable for the elastic characterization of anisotropic plates.

As highlighted above, direct methods based on single modal testing are easy to apply to simple structures such as beams or rods, but generally it is difficult to apply them to more complex structures such as plates and shells (Fig. 5). More precisely, they are not applicable when the frequency equations are known but modal frequencies depend on more than one elastic constant or when the frequency equations are unknown in a closed form. In these cases, direct or indirect methods based on multiple mode testing must be used (the possible applications of the current available methodologies are summarized in Fig.5).

ASTM [47] and [49] provide procedures for isotropic circular thick plates with free edges by which Young's modulus and Poisson's ratio are obtained directly from the first two resonant frequencies of vibration excited by forced continuous wave and impulse, respectively. The shear modulus is then calculated exploiting the well-known

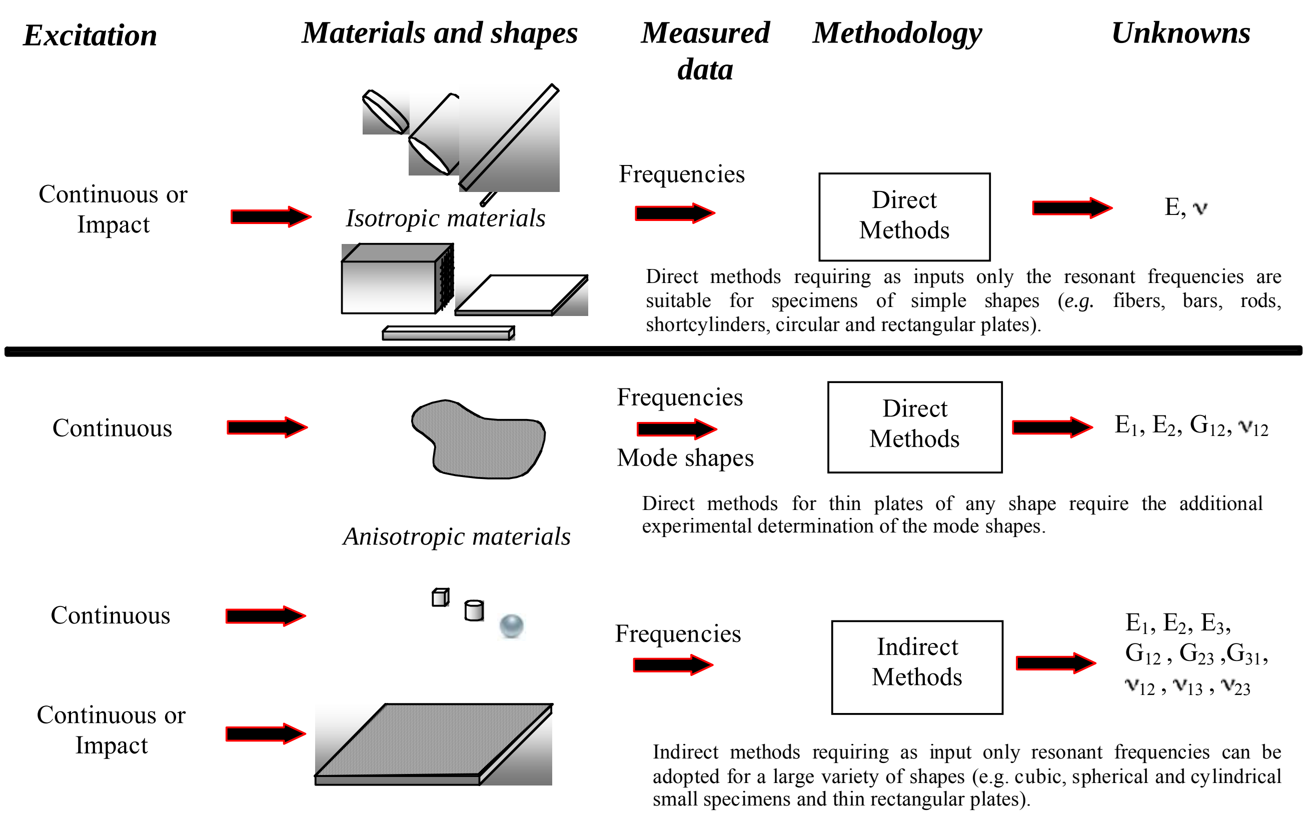

Fig. 5 Potentiality of currently available direct and indirect vibration based methods.

relationship for isotropic materials relating it to Young's modulus and Poisson's ratio. Recently, Nieves et al. [61] reported a direct methodology for characterizing free short cylindrical isotropic specimens by using only two of the first resonant frequencies. More recently, Alfano and Pagnotta [62] have proposed a direct method for testing thin isotropic rectangular plate with free edges. Such method is based on suitable approximated frequency equations obtained by correcting the Warburton formulas [68] with proper factors obtained from an accurate finite element analysis. The procedure requires the measurement of at least two of the first four natural frequencies in order to determine the Young modulus and Poisson ratio of the material sample. The experimental assessment of the method was carried out on square plates made with a variety of different materials [63, 66] and on aluminium rectangular plates [67]. In the papers tables and formulas for practical use are reported.

It is worth noting that Grediac and al. [102, 103] treated the more general case of anisotropic plate specimens of any shape. In these cases the frequency equations are not known but they proposed an original approach based on the Virtual Field Method. This method without requiring initial estimates of the stiffness or iterative computations allows the direct determination of the flexural stiffness from natural frequencies and mode shape measurements. Unfortunately, this methodology is not very practical to use because it has the disadvantage of requiring sophisticated and expensive techniques to measure the mode shapes.

When the frequency equations are not available, the inverse problem can be also solved indirectly using iterative optimization procedures.

Ohno [69] first introduced a technique using such a kind of identifying procedure. This technique, known now as RUS (acronym of Resonant Ultrasound Spectroscopy) [70-73], identified the elastic constants through a process minimizing the difference between the calculated and the measured frequency spectrum of parallelepiped-shaped samples. The values of the material elastic constants were updated iteratively in a numerical model able to calculate the resonance frequencies of the sample, until the calculated frequencies approximated as closely as possible the measured frequencies. The resonance measuring system for very low dissipation materials consisted in a little rectangular parallelepiped specimen lightly held between two piezoelectric transducers. One transducer was used to generate an elastic wave of constant amplitude and varying frequency, whereas the other was used to detect the resonances. The identification of all the elastic constants took place simultaneously without damaging the specimen. Consecutively, Migliori invented a resonance spectrometer which can also be used with high dissipation materials [74] and developed software dedicated to derive the elastic constants from natural resonant response data using the subroutine implemented by Ohno. Today, the RUS technique allows small anisotropic cubic, spherical or cylindrical specimens to be characterized and appropriate instrumentation packages are commercially available.

Successively, numerous different dynamic approaches, the so-called mixed numerical-experimental techniques, for characterizing square or rectangular plates of great dimensions have been introduced in the literature. These approaches require the measurement, in the sonic field, of a small number of natural frequencies of the free plate. De Wilde et al. [75, 76] and Deobald and Gibson [77] almost simultaneously proposed two similar approaches for determining the elastic constants of composite plates with free edges. Gibson and Ayorinde [78], in particular, obtained a patent for a method and apparatus that allow the determination of the four independent elastic constants (longitudinal and transverse Young's moduli, in plane shear modulus and major Poisson's ratio) of a composite material from the modal resonance data of freely-supported rectangular thin plate. The impulse excitation technique together with dedicated software for calculating properties from the vibration data, still today, constitute one of the most popular vibration systems for determining composite elastic constants. The analytical approaches based on the Rayleigh–Ritz [75-81] or Rayleigh [82-86] methods and the numerical approaches based on the finite element method [14-20, 87-95] have been adopted for determining the elastic constants of materials.

Thick rectangular plates were used to determine all five of the engineering elastic constants of transversely isotropic materials. In this case, the transverse properties, such as the through-the-thickness shear modulus, are determined by including not only the effects of bending, but also transverse shear and rotary inertia effects in describing the vibration behaviour of the plates [14, 90-93, 96-98].

It has been shown that Poisson's ratio and the transverse shear modulus are not as sensitive, with regard to the eigenfrequencies, as the other parameters. A way to circumvent this low sensitivity and identify the material properties accurately consists in using a specific size of the plate [58, 80, 98] and/or processing either the natural frequencies or the mode shapes of the plate [83-86]. The optimal design of the plate has to be determined in advance by preliminary tests. A method was presented in [99] whereby the Poisson's ratio and the other in-plane elastic properties were determined by matching the experimental modal testing results with theoretical modal analysis calculations for a set of plate bending modes and one in-plane compression mode. A great part of these techniques involves the measurement of natural frequencies of samples or structures while only a few of them also involve mode shape observation. Whatever the method employed, the apparatus for testing a material must always be constituted by the same components: a devise to induce the solid to vibrate, a device for detecting the vibration of the specimen and, a system for extracting the modal parameters from the vibration and calculating the elastic constants.

Continuous variable excitation (generally, forced sinusoidal or random stationary excitation) is commonly obtained by loud-speakers or piezoelectric actuators fed by a variable frequency oscillator, while impulse excitation is produced striking the object with a suitable impulser (e.g., a hammer).

Vibrations are generally detected by means of a signal pickup transducer that can be in direct contact with the specimen or not. Contact transducers are commonly accelerometers using piezoelectric or strain gauges, while non-contact transducers are commonly acoustic microphones, but laser, magnetic, or capacitance methods are also used. Pickup transducers transform the mechanical vibration into an electric signal that is successively analyzed in order to determine the resonant frequencies by a system consisting in a conditioner/amplifier, signal analyzer, and a frequency readout device. More inexpensively, the electrical signal can be addressed to an ordinary personal computer provided with a sound card and then analyzed and processed by a suitable virtual instrument. This latter must operate as a spectrum analyzer and then it must transform the sampled time function into a frequency spectrum by a fast Fourier transform algorithm and identify the values of the natural frequencies of vibration. Such procedures can be implemented in any environment (for example: LabView™, MATLAB™, etc.) using the proper library functions.

The observation of the mode shapes is generally more difficult than frequency measurements and requires specific and more complex equipment [58, 59, 83-86, 100, 103].

As regards the procedures for the elastic identification, it must be pointed out that many of these procedures involve iterative optimization processes requiring a starting point. Sometimes the solution depends on the starting point (especially when the error function presents more than one minimum) in such cases particular attention must be paid to the choice of the initial guess point. A suitable way to overcome this disadvantage is to use Genetic Algorithms (GAs). Due to the way the GA explores the region of interest, it avoids getting trapped at a particular local minimum and is able to locate the global optimum. GAs do not require initial estimates, but instead work within a suitable set of bounds which can often be rather broad. For these reasons, during the past few years, GAs have been used, by the author of the present paper and by many other researchers, for determining the elastic constants (and not only these) with static approaches [10-13], wave propagation based methods [104-107] and resonant tests [14, 93]. Reference [14], in particular, describes a method combining finite element analysis, genetic algorithms and vibration test data. The effectiveness of such a method was successfully verified on thin and thick laminate plates of materials such as carbon/epoxy, glass/epoxy and aluminium. One disadvantage of GAs is their high computational cost, but this drawback will certainly become less critical in the near future with advances in computer technology.

From the investigation of the behaviour of unidirectional orthotropic rectangular laminates a series of considerations arose. First of all, it was observed that the natural frequencies of unidirectional rectangular thin plates, are, generally very sensitive to E_1 and E_2 (longitudinal and transversal Young's moduli, respectively) and G_{12} (in-plane shear modulus). As a result, these constants can be predicted safely and without difficulty (no particular aspect ratios a/b, with a and b the plate sides are required). Major Poisson's ratio ν_{12}, on the contrary, does not significantly affect the natural frequencies and so, its accurate identification can prove to be troublesome (especially when measurement errors are present). A reliable estimate of ν_{12} can be obtained only at specific aspect ratios). It has been shown in references [58, 60, 80, 108] that Poisson's ratio is well-determined when $a/b = (E_1/E_2)^{1/4}$. If the aspect ratio is not near to the value provided by this formula the accuracy of Poisson's ratio can be very poor, even making the estimated values completely inaccurate [78]. The considerations reported above hold qualitatively also for moderately thick plates, but in this case, due to the transverse shear effects, the aspect ratio a/b obtained from the formula applies only approximately [108]. Moreover, it must be observed that, when dealing with relatively thin plates, the dynamic response of the material is rather insensitive to the transverse shear modulus G_{23}. It is well known that the transverse shear modulus can only be safely predicted from experiments with thick plates [98, 101]. It is important that the specimen be thick enough that the effects of transverse shear become significant. In contrast, it is also essential that the specimen is not so thick as to produce in-plane modes (at least for the number of natural frequencies required for solving the inverse problem) that are much more difficult to detect experimentally than out-of-plane modes. It was found that, in any case, plates with material axes parallel to the plate axes appear more advantageous compared to plates having other material directions.

For the sake of completeness, it must be mentioned that the mixed numerical-experimental method can also be used for determining the elastic properties of bi-layered materials. Applications of this kind are reported in [95], where the elastic properties of air-plasma sprayed thermal barrier coatings on a substrate of stainless steel have been determined.

Finally, with regard to the multi-directional laminated plates it must be emphasized that in this case the estimation of ply material constants can prove to be problematic. The parameters of such plates tend to be inaccurate and this shortcoming depends on the stacking sequence, which increases as the number of plies increases. In some cases, it may be found that some or most of the parameters are ill determined [108].

Although, the mixed numerical-experimental approaches have mainly been developed for the characterization of anisotropic materials, they can also be applied to the simplest isotropic materials. Applications of this kind are reported in most of the papers cited above in which they were always carried out with the aim of testing the methodologies proposed. The current greater availability of commercial finite element codes for carrying out, quickly and accurately, the dynamic analysis of complex structures, and the low-cost

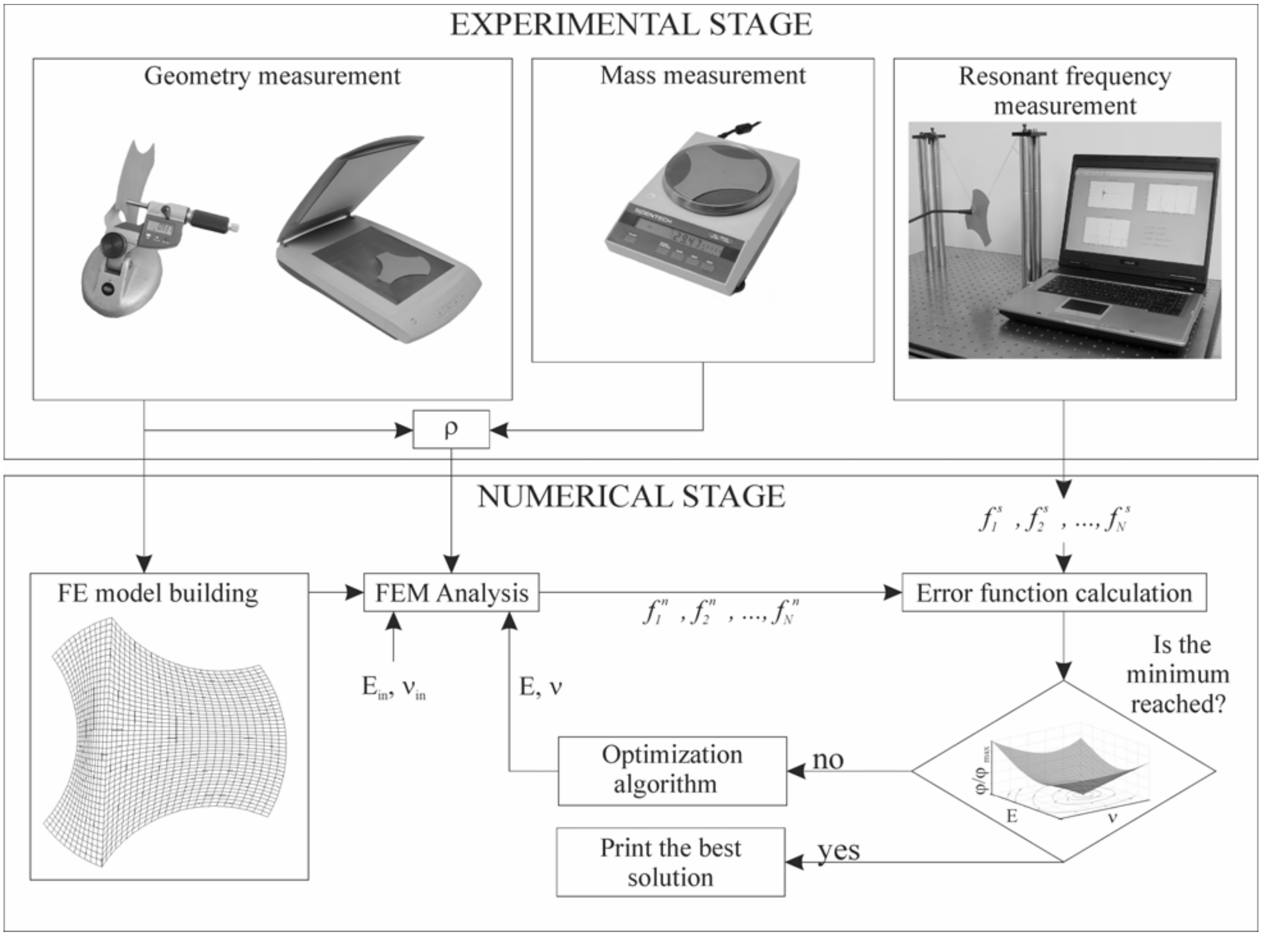

Fig. 6 Elastic identification via dynamic test [20].

accessibility to large calculation resources, have opened up the possibility of extending the application of the mixed numerical-experimental methods to specimens of various shapes.

The process for the identification of the elastic constants of isotropic materials developed in the Department of Mechanical Engineering of the University of Calabria [14-20] is summarized in Fig. 6. As usual it consists of two stages. The experimental stage includes the measurement of the sizes, shape, resonant frequencies and mass of the plate. The numerical stage consists of a series of procedures to be carried out on a computer. First, the plate is modelled using geometry and size data obtained in the first stage. Then, the model is meshed and the density of the material is computed as the ratio of the volume of the model to the mass of the plate. Mesh topology, material density and trial values of the elastic constants are required by the FEM (Finite Element Method) code for calculating the resonant frequencies.

The experimental equipment and procedures are illustrated in [19] and [20], in the latter paper the sources of error affecting the measurement process are also discussed. The components of the equipment are visible in Fig. 6. The plate is suspended in air with the two elastic bands fastened to the rigid frame. The exciting impulse is imparted repeatedly in various points by lightly hitting the plate with an impulser. The dynamic response of the plate is detected by the microphone and sent in the form of an electrical signal to the PC sound card. The signal is then analyzed and processed by dedicated software that identifies the values of the natural frequencies of vibration.

An optimizing procedure generates trial solutions (couples of E and ν) and identifies from among them the solution with the lowest error function value. It is worth noting that in the case of isotropic material, each resonant frequency only depends on two elastic properties and, as a consequence, the error function is a function of two variables. The minimization process is obviously simpler than the case of composite materials and will be very fast and accurate if the minimum of the error function is unique and easy to find inside its existence domain. The error function assumed influences the choice of the optimization procedure and, so it is crucial in terms of solution times and accuracy. Different optimization methods and error functions were compared in [18], in order to select the combination that provides the best compromise in terms of solution time, accuracy and stability of the results. The best performance was obtained combining the square root of the sum of the squares error function with the simplex method.

In the same paper, the effectiveness of the procedure has been shown by means of numerical simulations executed on a series of typical and atypical shaped plate models. Moreover, the robustness of the procedure with respect to the effects of measurement noise was assessed. It was observed that the shape of the plate negligibly affects the sensitivity to the experimental errors of the Young modulus, while the sensitivity of the Poisson ratio is highly dependent on it. Thus, to avoid incorrect estimations of the Poisson ratio of plates with particularly complex shapes, before starting the characterization process, a numerical check on the sensitivity of the error function to Poisson's ratio is always recommended. The experimental assessment of such a methodology was carried out with success on thin rectangular plate [15], thin and thick plates of various shapes [17, 19] and irregular drilled plates [20].

V. Conclusions & Future Developments

The goal of this research is to provide a methodology for simultaneously determining all the elastic constants of a material by testing a single, not-special-shaped specimen with an inexpensive and simple-to-use non-destructive technique. This potentials is fundamental in the cases in which the component made of the material under examination is unique and cannot be destroyed, or when the material to be tested is such that the production of proper bulk specimens is not feasible owing to the high cost of the material and/or its brittle nature.

Mixed numerical-experimental methods are among the more promising for achieving this aim. The increased availability of commercial finite element codes for performing, quickly and accurately, the static or dynamic analysis of complex structures and, the increasing economic accessibility of large calculation resources encourages studies on the applicability of these methods.

In the present paper, the feasibility of applying such methods to characterize plate-shaped specimens is discussed. Both static and dynamic approaches are considered.

As regard static approaches it has been shown that optics techniques, for measuring the whole displacement field undergone by the surface of the plate under static load, combined with finite element method can provide a potential alternative to the traditional static or quasi-static methodologies for determining the elastic constants of both isotropic and composite materials. Drawbacks of this approach are the need for expert operators and the high cost of experimental equipment but this latter disadvantage will certainly become less critical in the near future in virtue of the advances in low-cost optical techniques.

With regard to the dynamic approaches, it has been shown that the impulse excitation technique for measuring the resonant frequencies of the free plate combined with finite element method can be very competitive in comparison with the static approaches. The impulse technique is simple, inexpensive and fast and can be successfully carried out by a non-expert operator. In addition, the identification process could be easily integrated into any of the existing commercial systems for measuring elastic constants by the sonic resonance method.

Both the presented static and dynamic methodologies proved to be very effective for testing isotropic thin plates of any-shape. Some preliminary investigations carried out recently (the results of which must be assessed and so are not included in the present paper) seem to indicate the feasibility of extending the applicability of the dynamic methodology to thick isotropic plate and three-dimensional isotropic bodies as well.

Furthermore, both the methodologies proved to be suitable for unidirectional rectangular thin laminates, but more research is necessary to completely investigate the possibilities of using them for profitably analysing multidirectional rectangular laminates and, more generally, for analysing anysotropic plates of any shapes.

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