Euclidean & Geodesic Distance between a Facial Feature Points in Two-Dimensional Face Recognition System

Rachid AHDID[1], Khaddouj TAIFI[1], Said SAFI[1] and Bouzid MANAUT[2]

Abstract—**In this paper, we present two feature extraction methods for two-dimensional face recognition. Our approaches are based on facial feature points detection then compute the Euclidean Distance between all pairs of this points for a first method (ED-FFP) and Geodesic Distance in the second approach (GD-FFP). These measures are employed as inputs to a commonly used classification techniques such as Neural Networks (NN), k-Nearest Neighbor (KNN) and Support Vector Machines (SVM). To test the present methods and evaluate its performance, a series of experiments were performed on two-dimensional face image databases (ORL and Yale). The recognition rate across all trials was higher using Geodesic Distance (GD-FFP) than Euclidean Distance (ED-FFP). The experimental results also indicated that the extraction of image features is computationally more efficient using Geodesic Distance than Euclidean Distance.**

Keywords—**face recognition, Euclidean Distance, Geodesic Distance, Neural Networks, k-Nearest Neighbor, Support Vector Machines.**

I. Introduction

BIOMETRIC face recognition technology has received significant attention in the past several years due to its potential in different applications. Automated human face recognition was applied in different fields including automated secured access to machines and buildings, automatic surveillance, forensic analysis, fast retrieval of records from databases in police departments, automatic identification of patients in hospitals, checking for fraud or identity theft, and human-computer interaction [1-5].

In recent years face recognition has received substantial attention from both research communities and the market, but still remained very challenging in real applications. A lot of face recognition algorithms, along with their modifications, have been developed during the past decades. Automatic recognition of human faces based on the 2D images processing is well developed this last years, and several techniques have been proposed. We find several global, local and hybrids methods: The Principal Component Analysis PCA also known under the name eigenfaces [2, 3], two-dimensional version of PCA noted 2DPCA [4]. the Stochastic Approach in [6, 7]. In 1991 M. A. Turk et al. implemented The Principal Component Analysis (PCA) approach also known under the name Eigenfaces [27]. In 2001 G.D. Guo proposed to tackle multi-class classification problem for a K-class classification test, Optimal-Pairwise Coupling (O-PWC) SVM [28]. In 2003 J. Lu et al. implemented a method combines the strengths of the D-LDA and F-LDA approaches, while at the same time overcomes their shortcomings and limitations [31]. Two-dimensional version of Principal Component Analysis noted (2DPCA) was presented by J. Yang et al. in 2004 for image representation [4]. M. Visani et al. are proposed Two-Dimensional Linear Discriminant Analysis (2DO-LDA) in 2004, this approach is chosen to jointly maximize the mean variation between classes and minimize the mean of the variations inside each class [8]. H. Cevikalp et a.l proposed an approach called the Discriminative Common Vector method based on a variation of Fishers Linear Discriminant Analysis for the small sample size case in 2005 [29]. Linear Discriminant Analysis LDA also known under the name Fisherfaces was proposed by L. Bedoui et al in 2008 [5]. In 2010 M. Agarwalet al. implemented a method combines Principal Component Analysis (PCA) and Neural Network(NN) [32]. In 2012 V. More et al, used modified fisher face and fuzzy fisher face FFLD for a person is identified with face [33]. In 2014 W. Xu et al. proposed an integrated algorithm based on the respective advantages of wavelets transform (WT), 2D Principle Component Analysis (PCA) and Support Vector Machines (SVM) [34].

On the other hand, there are methods of 3D face recognition based on the use of three-dimensional information of the human face in the 3D space. Existing approaches that address the problem of 3D face recognition can be classified into several categories of approaches: geometric or local approaches 3D, Bronstein et al propose a representation based on the isometric nature of the facial surface [9, 10]. Samir et al use 2D and 3D facial curves for analyzing the facial surface [11, 12]. Holistic approaches, Heseltine et al have developed two approaches applying the representations ACP in Threedimensional face [13], Cook et al present a robust method for facial expressions based on Log Gabor models from images of deep [14]. There are some other approaches based on face Segmentation can be found in [15, 16, 17].

The objective of this paper is to achieve a two-dimensional face recognition system by the facial feature points detection and compute a distance between all this points using Euclidean

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Distance (ED) in Euclidean geometry and Geodesic Distance (GD) in Riemannian geometry. These measures are employed as inputs to a commonly used classification techniques such as Neural Networks (NN), k-Nearest Neighbor (KNN) and Support Vector Machines (SVM).

The rest of this paper is organized as follows: Section 2 describes the methodology of the proposed method with its stages: Face Feature Points (FFP) detection, Eclidean Distance between all pairs of Face Feature Points (ED FFP) and Geodesic Distance between all pairs of Face Feature Points (GD-FFP) and classification algorithms (NN, KPPV and SVM). Section 3 includes the simulation results with comparative study of the performance of our face authentication with respect to the performance obtained in other 2D face recognition systems. Section 4 draws the conclusion of this paper and possible points for future work.

II. METHODOLOGY

In order to overcome the limitations in the existing methods of 2D face recognition, we propose our geometric approach based on face feature points detection and compute a geodesic and euclidean distance between all this points. The figure 1 shows the proposed method as follows.

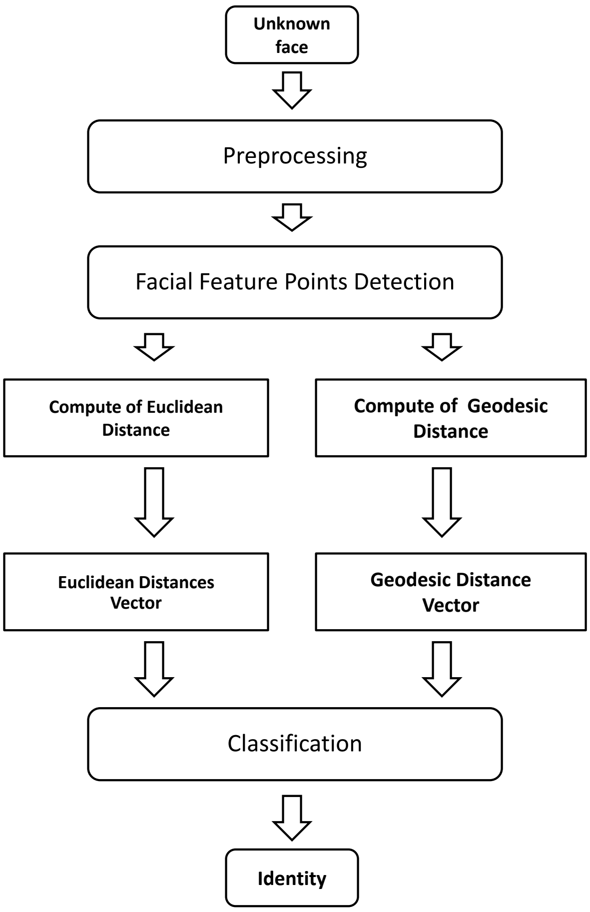

Fig. 1. Methodology Architecture

the presented method starts by detecting 20 facial feature points using Gabor Feature Based Boosted Classifiers algorithm [19]. Then, we use the Euclidean geometry for compute 190 Euclidean distances between all possible pairs the 20 fiducial points, for our first algorithm. The second algorithm is based on the computation of 190 geodesic distances between all 20 face feature points using Fast Marching algorithm for resolving a Eiconal equation in Riemannian geometry. The Euclidean and Geodesic distances computed respectively De and Dg presents the input vectors of classification algorithms used in our automatic 2D face recognition systems. In the classifying step, we use: Neural Networks (NN), K-Nearest Neighbor (KNN) and Support Vector Machines (SVM).

A. Facial Feature Points Detection

Automatic detection of facial feature points plays an important role in applications such as facial feature tracking, human-machine interaction, and face recognition [20]. A feature point is a point that constitutes an interesting part of an image. It can be either a corner, or an edge, or a blob etc. In this paper, we use 20 facial feature points such as: Outer corner of the left eye, Outer corner of the right eye, Inner corner of the left eye, Inner corner of the right eye, Bottom of the left eye, Bottom of the right eye, Top of the left eye, Top of the right eye, Inner corner of the left eyebrow, Inner corner of the right eyebrow, Outer corner of the left eyebrow, Outer corner of the right eyebrow, Left nose corner, Right nose corner, Top of the nose, Left mouth corner, Right mouth corner, Mouth top, Mouth bottom and Chin [19]. Figure 2 shows this 20 facial feature points on two images: (a) image from ORL database and (b) image from YaleB database.

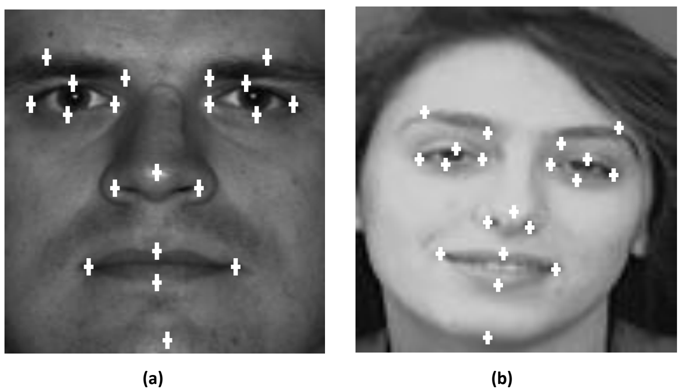

Fig. 2. Two examples of face feature points detection using ORL and YaleB 2D face database images: (a) ORL image; (b) YaleB image

Each year more and more efficient and accurate algorithms are being proposed. Mostly these algorithms are either for detecting some feature points in an image, or tracking movement in a videostream. Locating of 20 facial feature points in images of faces is an important stage for our automatic 2D face recognition approach. For this task, we use

Gabor Feature Based Boosted Classifiers algorithm proposed by Danijela Vukadinovic et al in [19]. The method consists of 4 steps: Face Detection using Haar feature based GentleBoost classifier [18], Region Of Interest (ROI) Detection, Feature Extraction based on Gabor filtering, and Feature Classification using Gentle Boost classifier. The figure 3 present the four steps for twenty facial feature points detection.

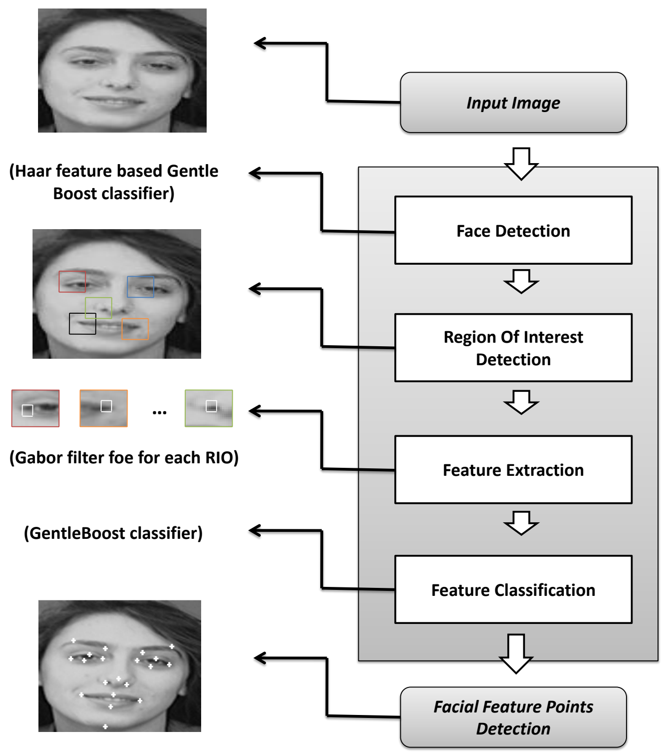

Fig. 3. Automatic 20 Facial Feature Points detection steps

B. Euclidean Distance

The Euclidean distance or Euclidean metric is the ordinary distance between two points that one would measure with a ruler, and is given by the Pythagorean formula. It is shown in Figure 4. This figure shows three Euclidean distance between facial feature points: d_1 is the euclidean distance between Top of the nose and Outer corner of the right eye, d_2 is the euclidean distance between Top of the nose and Inner corner of the left eye and d_3 is the euclidean distance between Top of the nose and Left mouth corner.

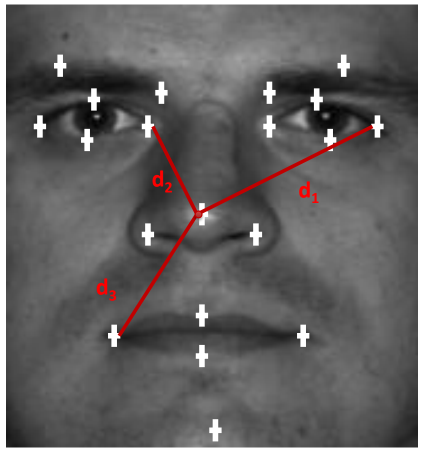

Fig. 4. Example of three Euclidean distances between facial feature points

By using this formula as distance, Euclidean space becomes a metric space. The Euclidean distance between points P and Q is the length of the line segment connecting them ($\overline{PQ}$). In Cartesian coordinates, if $P = (p_1, p_2, ..., p_n)$ and $Q = (q_1, q_2, ..., q_n)$ are two points in Euclidean n-space, then the distance from P to Q, or from Q to P is given by:

$$\begin{aligned} d(P,Q) &= \sqrt{(p_1-q_1)^2+(p_2-q_2)^2+\cdots+(p_n-q_n)^2} \\ &= \sqrt{\sum_{i=1}^{n}(p_i-q_i)^2} \end{aligned} \tag{1}$$

In three-dimensional Euclidean space, the distance is:

$$d(P,Q) = \sqrt{(p_1-q_1)^2+(p_2-q_2)^2+(p_3-q_3)^2} \tag{2}$$

The Euclidean distance between landmarks is used by most authors as a morphometric measure. Once facial feature points are obtained from a facial image or a two-dimensional face, they select some significant distances between them and compute the corresponding Euclidean distances. Then these distances are used to compare faces for face recognition systems. The 190 Euclidean distances computed between all possible pairs the 20 facial feature points constitute a vector D of 190 of element.

$$D = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_{190} \end{bmatrix} \tag{3}$$

This vector give the human face features of 2D image and used as input of classification algorithm for our automatic face recognition system.

C. Geodesic Distance

The geodesic distance between two points P and Q of 2D face surface is the shortest path between the two points while remaining on the facial surface. In the context of calculating the geodesic distance R. Kimmel and J. A. Sethian [21] propose the method of Fast Marching as a solution of the Eikonal equation.

The Eikonal equation given as:

$$\mid \bigtriangledown_u(x) \mid = F(x); \quad x \in \Omega \tag{4}$$

with: Ω is an open set in R^n housebroken limit; $\bigtriangledown$ denotes the gradient; $\mid . \mid$ is the Euclidean norm.

The Fast Marching method is a numerical method for solving boundary value problems of the Eikonal equation [21, 22]. The algorithm is similar to the Dijkstra's algorithm [23]. In this paper, we compute a geodesic distance on a facial surface, using the values of the surface gradient only [24].

The main step of the geodesic distance computing is the construction of the canonical form of a given surface (Facial surface). Let Img represent a 2D face image of ORL or YaleB database, we can represent mathematically Img as a plan $P = (p_1, p_2)$. To compute a geodesic distance, the facial surface can be thought of as a parametric manifold, represented by a mapping $F : R^2 \to R^3$ from the parameterization plan $P(p_1, p_2)$ to the manifold [24]:

$$F(P) = F(p_1, p_2) = (p_1, p_2, p_3(p_1, p_2)) \tag{5}$$

The metric tensor g_{ij} of the manifold is given by:

$$g_{ij} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} \tag{6}$$

The geodesic distance between two points on a surface is calculated as the length of the shortest path connecting the two points. Using the Fast Marching algorithm on the surface gradient, we can compute the geodesic distance between all possible pairs of the 20 facial feature points detected.

The geodesic distance $\delta_{P,Q}$ between two facial feature points P and Q is approximated by:

$$\delta_{P,Q} = min\gamma(\beta(P,Q)) \tag{7}$$

with: $\beta(P,Q)$ is the path between P and according to the facial surface S of the 2D face. $\gamma(\beta(P,Q))$ is the path length.

The distance element on the manifold is given by [25]:

$$\delta_{i,j} = \sqrt{g_{ij}\xi^i\xi^j} \tag{8}$$

with: g_{ij} is computed by (3); $i = 1$ or 2 and $j = 1$ or 2; $\xi^i = P$ and $\xi^j = Q$.

Figure 5 shown example of three geodesic distances between facial feature points using YaleB database image: δ_1 is the geodesic distance between Top of the nose and Outer corner of the right eye, δ_2 is the geodesic distance between Top of the nose and Inner corner of the left eye and δ_3 is the geodesic distance between Top of the nose and Left mouth corner.

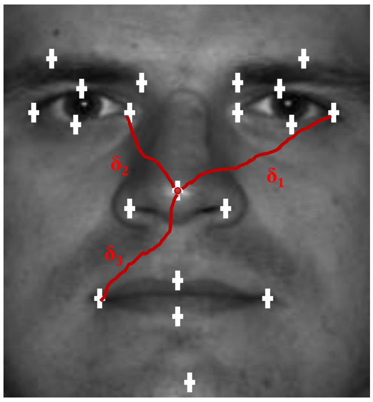

Fig. 5. Example of three Geodesic distances between facial feature points

We use the Geodesic distances between facial feature points to realize our automatic face recognition systems. Once facial feature points are obtained from a facial image or a two-dimensional face, they select some significant distances between them and compute the corresponding geodesic distances. Then these distances are used to compare faces for face recognition systems. The 190 geodesic distances computed between all possible pairs the 20 facial feature points constitute a vector δ of 190 of element.

$$\delta = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_{190} \end{bmatrix} \tag{9}$$

This vector give the human face features of 2D image and used as input of classification algorithm for our automatic face recognition system.

III. Simulation Results

Four experiments are performed to assess the effectiveness and robustness of our approaches (FFP-ED and FFP-GD) with respect to variations in lighting conditions, facial expression and head pose. And compare our two methods together and with the other methods that are applied in this area. Two face databases are used: ORL and YaleB. The first database was used to evaluate the performance of 2DPCA under conditions where the pose and sample size are varied. The second database was used to examine the system performance when both facial expressions and illumination are varied.

firstly, we starts our methods by detecting of 20 facial feature points using Gabor Feature Based Boosted Classifiers algorithm proposed by Danijela Vukadinovic. secondly, we compute 190 Euclidean distances between all possible pairs the 20 fiducial points, for our first algorithm (FFP-ED), using the equation (2). For our second approach (FFP-GD), we use a Fast Marching algorithm for resolving a Eiconal equation in Riemannian geometry. The190 euclidean and geodesic distances computed, we allow to obtain vectors of 190 elements. Each vector represent the human face features of ORL or YaleB database and each vector element represents a euclidean or geodetic distance between pairs of facial feature points. Therefore, we can represent each face by a vector of 190 elements. These vectors are used as inputs of the classification algorithms of our automatic 2D face recognition systems. Finally, we use three classification algorithms such as : Neural Networks (NN), K-Nearest Neighbor (KNN) and Support Vector Machines (SVM).

A. Databases

The ORL database contains 400 images of 40 individuals. For each person, we have 10 pictures in grayscale and standardized at a resolution of 112×92 pixels. The YaleB database contains 2432 images of 38 people in 64 different lighting conditions. Each image has been normalized at a resolution of 168×192 pixels. To realize our $2D$ face recognition systems, we use three classification algorithms such as: the Neural Networks (NN), K-Nearest Neighbor (KNN) and Support Vector Machines (SVM).

Twenty sample images of ORL and YaleB faces database are shown in Figure 6, the first and second lines shows ten sample images of deferent personnes of ORL database and the third and last lines gives ten images of deferent personnes of YaleB database.

Fig. 6. Twenty sample images of ORL and YaleB faces database. (First and second lines) Ten sample images of ORL database, (Third and last lines) Ten images of YaleB database.

B. Experiments on the ORL Database

The ORL database contained 400 2D facial models of 40 subjects.The experiments was performed using the first five image samples per class for training, and the remaining images for test. Thus, the total number of training samples and testing samples were both 200 images.

In this first experiment, we realize two 2D face recognition systems using ORL dabase images. Our systems are based on two algorithms such as: Eclidean Distance between all pairs of Face Feature Points (ED-FFP) and Geodesic Distance between all pairs of Face Feature Points (GD-FFP).

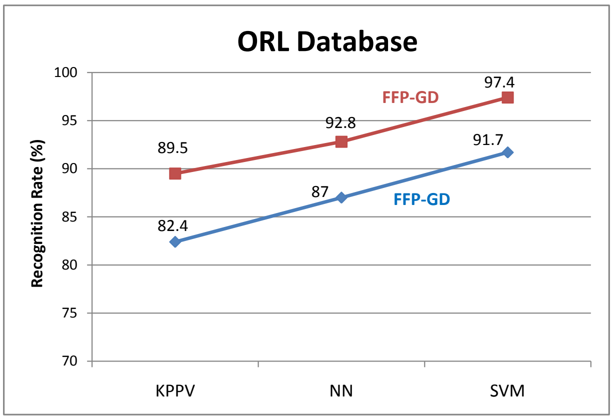

Fig. 7. Recognition Rate for ORL images using ED-FFP and GD-FFP for feature extraction and three classification algorithms (NN, KNN and SVM)

Fig. 6 shows the experiment results of recognition rate obtained for ORL images using a ED-FFP and GD-FFP for feature extraction step. In classification steps we applied three weak classifiers, namely, k- Nearest Neighbor (KNN), Neural Networks (NN) and Support Vector Machines (SVM). We achieved average recognition rates of 82.4%, 87% and 91.7% for our first method Eclidean Distance between all pairs of Face Feature Points (ED-FFP) using , respectively, KNN, NN and SVM as classificateurs. And 89.5%, 92.8% and 97.4% for our second method Geodesic Distance between all pairs of Face Feature Points (ED-FFP) using , respectively, KNN, NN and SVM.

C. Experiments on the YaleB Database

In this first experiment, we use the YaleB database images for our face recognition systems. As last experiment, we use Eclidean Distance between all pairs of Face Feature Points (ED-FFP) and Geodesic Distance between all pairs of Face Feature Points (GD-FFP) for feature extraction step. The experiments was performed using the first ten image samples per class for training, and ten images for test. Thus, the total number of training samples and testing samples were both 380 images.

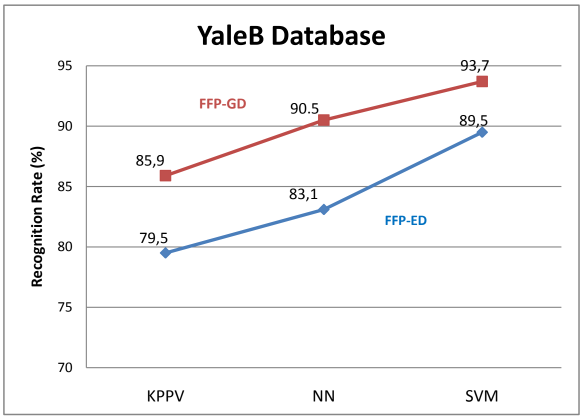

Fig. 8. Recognition Rate for YaleB images using ED-FFP and GD-FFP for feature extraction and three classification algorithms (NN, KNN and SVM)

Fig. 6 shows the experiment results of recognition rate obtained for YaleB images using a ED-FFP and GD-FFP for feature extraction step. In classification steps we applied three weak classifiers, namely, k- Nearest Neighbor (KNN), Neural Networks (NN) and Support Vector Machines (SVM). We achieved average recognition rates of 79.5%, 83.1% and 89.5% for our first method Eclidean Distance between all pairs of Face Feature Points (ED-FFP) using , respectively, KNN, NN and SVM as classificateurs. And 85.9%, 90.5% and 93.7% for our second method Geodesic Distance between all pairs of Face Feature Points (ED-FFP) using , respectively, KNN, NN and SVM.

D. Comparison of Experiment Results

In this paper, two face recognition algorithms ED-FFP and GD-FFP have been described . These methods have been verified on the ORL and YaleB dataset, and the testing protocols used in the experiments are almost the same, so that a direct comparison of the results reported in these works is possible. In Figure 8, we give a comparison of these face recognition algorithms.

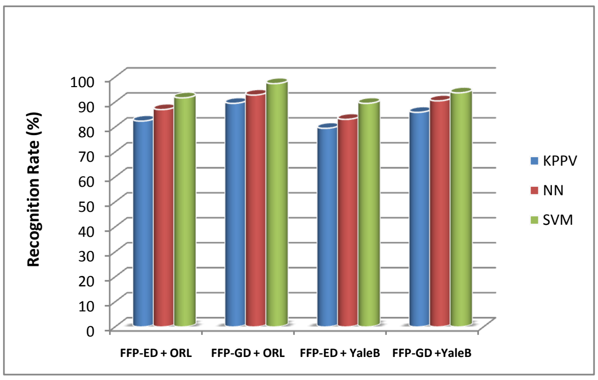

Fig. 9. Comparison of our 2D face recognition methods using ORL and YaleB Databases

Figure 8 gives a comparison recognition rate of these four features extraction algorithms ED-FFP and GD-FFP using ORL and YaleB database images. This comparison shows the best recognition rate (97.4%) was presented for GD-FFP using SVM classifier, then this method was also better than ED-FFP method.

In conclusion of this series of results, a summary table (Table I) compares the performance of our face authentication with respect to the performance obtained in other 2D face recognition systems.

We can notice that the performance of our automatic 2D face recognition system using Geodesic Distance between all pairs of Face Feature Points (GD-FFP) and Support Vector Machines (SVM), In addition our system (GD-FFP+SVM) is perfect in all assessment. Our goal was to improve 2D faces recognition system we affirm based on the results that our goal is achieved.

Date	Reference	Method	Database	Reported performance
1991	M. Turk et al [3]	Eigenface	ORL	90,00%
2001	G.D. Guo et al [15]	O-PWC +SVM	ORL	96,79%
2005	Cevikalp et al [8]	DCV	Yale	97,33%
2003	Lu et al[10]	DF-LDA	ORL	96.00%
2004	J. Yang et al [4]	2DPCA	ORL	96,00%
2004	M. Visani et al [7]	2DO-LDA	FERET	94,40%
2010	M. Agarwal et al [17]	PCA+NN	ORL	97.01%
2012	V. More et al [18]	FFLD	ORL	95.50%
2012	V. More et al [18]	FFLD	Yale	94.80%
2014	W. Xu et al [13]	WT+2DPCA +SVM	ORL	97.10%
2015	R. Ahdid et al [26]	GD+LDA	YaleB	92.00%
2015	R. Ahdid et al [26]	GD+LDA	ORL	96.20%
2015	R. Ahdid et al [26]	GIH	YaleB	93.70%
2015	R. Ahdid et al [26]	GIH	ORL	94.50%
2015	R. Ahdid et al [30]	I-GC	YaleB	91.70%
2015	R. Ahdid et al [26]	GD+PCA	YaleB	94.80%
2016	**Our System**	GD-FFP +SVM	ORL	**97.40%**

TABLE I
COMPARISON OF OUR METHODS WITH OTHER METHODS OBTAINED IN OTHER WORK SYSTEMS

IV. CONCLUSION

In this paper, we have presented two automatic 2D face recognition system using two feature extraction algorithms such as: Eclidean Distance between all pairs of Face Feature

Points (ED-FFP) and Geodesic Distance between all pairs of Face Feature Points (GD-FFP). For classification step we have used the k-Nearest Neighbor (KNN), Neural Networks (NN) and Support Vector Machines (SVM). The simulation results were performed on two face image databases: ORL and Yale face databases. The recognition rate across all trials was higher using ORL images than YaleB images. The simulation results also indicated that the extraction of image features is computationally more efficient using GD-FFP algorithm than ED-FFP algorithm. The Support Vector Machine (SVM) classifier is better than Neural Networks (NN) and k-Nearest Neighbor (KNN) classifiers in terms of recognition accuracy in all experiments. finally, a comparative study was conducted between our methods in this paper with other methods proposed in this field of 2D face recognition.

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