Fuzzy Semi – Alpha – Compactness and Fuzzy Semi – Alpha – Closed Spaces in Fuzzy Topological Spaces

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Abstract—This research paper deals with the concept of fuzzy semi – alpha compact space as well as fuzzy semi alpha – closed space setting of a fuzzy topological space. We also investigate the relationships between fuzzy semi alpha – almost compactness and fuzzy semi alpha – nearly compactness. We present a number of properties and characterizations of these notions of fuzzy semi – alpha compact space, fuzzy semi alpha – closed space, fuzzy semi alpha – almost compact space and fuzzy semi alpha – nearly compactness in fuzzy topological spaces.

Keywords—Fuzzy topological space, Fuzzy semi – alpha open set, Fuzzy semi – alpha closed set, Fuzzy semi – alpha compact space, Fuzzy semi – alpha closed space, Fuzzy semi alpha – almost compactness, Fuzzy semi alpha – nearly compactness

I. INTRODUCTION

In [12], Zadeh has introduced the important concept of fuzzy sets. In [Hakeem, 2009], Hakeem, etc. have introduced the concept of fuzzy semi $\alpha-$open sets in fuzzy topological space. The notion of fuzzy subsets naturally plays a significant role in the study of fuzzy topology which was introduced by Chang [Chang, 2004]. In [2009], Hakeem, etc. introduced the notion of fuzzy semi $\alpha-$ open sets in fuzzy topology. The purpose of this paper is devoted to introduce and study the concepts of semi $\alpha-$compactness and semi $\alpha-$closed spaces in fuzzy setting. Using fuzzy filterbases, we characterize fuzzy semi $\alpha-$compactness and fuzzy semi $\alpha-$closed spaces. We also explore some expected basic properties of these concepts.

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II. PRELIMINARIES

Throughout this paper (X,τ) and (Y,σ) or simply by X and Y respectively mean fuzzy topological spaces. A fuzzy point x_t in X is a fuzzy set having support $x\in X$ and value $t\in(0,1]$. If A is a fuzzy set then $Int(A)$, $Cl(A)$, A^c and $S(A)$ will denote respectively, the interior of A, the closure of A, complement of A and the support of A. For two fuzzy sets A and B, we shall write AqB $(A\tilde{q}B)$ to mean that A is quasi coincident (not quasi coincident) with B, *i.e.*, there exists $x\in X$ such that $A(x)+B(x)>1$ $(A(x)+B(x)\le 1)$.

DEFINITION 2.1. Let X be a non empty set and τ be a family of fuzzy subsets of X. Then τ is called a fuzzy topology on X if it satisfies the following conditions: (i) 0_X and 1_X belong to τ (ii) Any union of members of τ is in τ. (iii) Any finite intersection of members of τ is in τ. The pair (X,τ) is called a fuzzy topological space.

DEFINITION 2.2. A fuzzy set A in a fuzzy topological space (X,τ) is called a fuzzy $\alpha-$*open* set if $A\le Int\left[Cl\left(Int(A)\right)\right]$.

DEFINITION 2.3. A fuzzy subset A of a fuzzy topological space (X,τ) is called a "fuzzy semi $\alpha-$*open* set" if there exists a fuzzy $\alpha-$*open* set U in X such that $U\le A\le Cl(U)$. The family of all fuzzy semi $\alpha - open$ sets of X is denoted by $FS\alpha O(X,\tau)$ or simply by $FS\alpha O(X)$.

DEFINITION 2.4. The complement of fuzzy semi $\alpha - open$ set is called “fuzzy semi $\alpha - closed$ set”. The family of all fuzzy semi $\alpha - closed$ sets of X is denoted by $FS\alpha C(X,\tau)$ or simply by $FS\alpha C(X)$.

DEFINITION 2.5. Let U be a fuzzy set in a fuzzy topological space (X,τ). The fuzzy semi $\alpha - closure$ and fuzzy semi $\alpha - \text{int}\,erior$ of U are defined as follows:

$$fs\alpha Cl(U) = \bigcap\{A : U \le A,\ A \in FS\alpha C(X,\tau)\};$$

$$fs\alpha Int(U) = \bigcup\{A : U \ge A, A \in FS\alpha O(X,\tau)\}.$$

It is obvious that $fs\alpha Cl(U^c) = [fs\alpha Int(U)]^c$ and $fs\alpha Int(U^c) = [fs\alpha Cl(U)]^c$.

THEOREM 2.6. Let A be any fuzzy subset of a topological space (X,τ). Then the following statements are equivalent:

(i) $A \in FS\alpha O(X)$.

(ii) There exists a fuzzy open set say G such that $G \le A \le Cl[Int(Cl(G))]$.

(iii) $A \le Cl[Int(Cl(Int(A)))]$.

(iv) $Cl(A) = Cl[(Int(Cl(Int(A))))]$.

THEOREM 2.7. Let A be any fuzzy subset of a topological space (X,τ).

(i) Any union of fuzzy semi $\alpha - open$ sets is fuzzy semi $\alpha - open$ set.

(ii) Any intersection of fuzzy semi $\alpha - closed$ sets is fuzzy semi $\alpha - closed$ set.

REMARK 2.8. The Intersection (Union) of any two fuzzy semi $\alpha - open$ (fuzzy semi $\alpha - closed$) sets need not be fuzzy semi $\alpha - open$ (fuzzy semi $\alpha - closed$) set.

THEOREM 2.9. A fuzzy set A in a fuzzy topological space (X,τ) is fuzzy semi $\alpha - open$ set if and only if for every fuzzy point $p \in A$ there exists a fuzzy semi $\alpha - open$ set $M_p \le A$ such that $p \in M_p$.

THEOREM 2.10. Let A be a fuzzy semi $\alpha - open$ set in a fuzzy topological space (X,τ). Let B be a fuzzy set in X satisfying $A \le B \le Cl(A)$. Then B is fuzzy semi $\alpha - open$ set in X.

THEOREM 2.11. Let A be a fuzzy semi $\alpha - open$ set in a fuzzy topological space (X,τ). Let B be a fuzzy set in X such that $Int(A) \le B \le A$. Then B is fuzzy semi $\alpha - open$ set in X.

DEFINITION 2.12. [5] Let a function $f : (X,\tau) \longrightarrow (Y,\sigma)$ from a fuzzy topological space (X,τ) into a fuzzy topological space (Y,σ) is called a fuzzy semi $\alpha - continuous$ if and only if $f^{-1}(B)$ is fuzzy semi $\alpha - open$ (fuzzy semi $\alpha - closed$) set in X for each fuzzy open (fuzzy closed) set B in Y.

DEFINITION 2.13. [5] A function $f : (X,\tau) \longrightarrow (Y,\sigma)$ from a fuzzy topological space (X,τ) into a fuzzy topological space (Y,σ) is called a fuzzy semi $\alpha * - continuous$ if and only if $f^{-1}(B)$ is fuzzy semi $\alpha - open$ (fuzzy semi $\alpha - closed$) set in X for each fuzzy semi $\alpha - open$ (fuzzy semi $\alpha - closed$) set B in Y.

LEMMA 2.14. Let $f : (X,\tau) \longrightarrow (Y,\sigma)$ be a function. Then the following statements are equivalent:

(a) f is fuzzy semi $\alpha^* - continuous$.

(b) $f[fs\alpha Cl(U)] \le fs\alpha Cl[f(U)]$, for every fuzzy set U in X.

PROOF. $(a)\Rightarrow(b)$: Let U be a fuzzy set of X, then $fs\alpha Cl(U)$ is fuzzy semi $\alpha-closed$. By (a) $f^{-1}\left[fs\alpha Cl\left(f(U)\right)\right]$ is fuzzy semi $\alpha-closed$ and therefore it follows that $f^{-1}\left[fs\alpha Cl\left(f(U)\right)\right]=fs\alpha Cl\left(f^{-1}\left[fs\alpha Cl\left(f(U)\right)\right]\right)$. Since $U\leq f^{-1}\left[f(U)\right]$, we have $fs\alpha Cl(U)\leq fs\alpha Cl\left[f^{-1}\left(f(U)\right)\right]\leq$ $fs\alpha Cl\left[f^{-1}\left(fs\alpha Cl\left(f(U)\right)\right)\right]=f^{-1}\left[fs\alpha\left(f(U)\right)\right]$.

Hence $f\left[fs\alpha Cl(U)\right]\leq fs\alpha Cl\left[f(U)\right]$.

$(b)\Rightarrow(a)$: Let V be a fuzzy semi $\alpha-closed$ set in Y. By (b) if $U=f^{-1}(V)$, then $fs\alpha Cl\left[f^{-1}(V)\right]\leq f^{-1}\left[fs\alpha Cl\left(f\left(f^{-1}(V)\right)\right)\right]\leq$ $f^{-1}\left[fs\alpha Cl(V)\right]=f^{-1}(V)$. Since $f^{-1}(V)\leq fs\alpha Cl\left[f^{-1}V\right]$, then $f^{-1}(V)=fs\alpha Cl\left[f^{-1}(V)\right]$. Hence $f^{-1}(V)$ is fuzzy semi $\alpha-closed$ set in X. Hence f is fuzzy semi $\alpha^{*}-continuous$.

LEMMA 2.15. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be a function. Then the following statements are equivalent:

(a) f is fuzzy semi $\alpha-continuous$.

(b) $f\left[fs\alpha Cl(U)\right]\leq fs\alpha Cl\left[f(U)\right]$, for every fuzzy set U in X.

DEFINITION 2.16. A collection of fuzzy subsets ξ of a fuzzy topological space (X,τ) is said to form a fuzzy filterbase if and only if for every finite collection $\left\{A_j : j=1,2,...,n\right\}$, $\bigcap_{j=1}^{n}A_j\neq 0_X$.

DEFINITION 2.17. A collection μ of fuzzy sets in a fuzzy topological space (X,τ) is said to be cover of a fuzzy set U of X if and only if $\left(\bigcup_{A\in\mu}A\right)(x)=1_X$, for every $x\in S(U)$. A fuzzy cover μ of a fuzzy set U in a fuzzy topological space (X,τ) is said to have a finite subcover if and only if there exists a finite subcollection $\eta=\left\{A_j : j=1,2,...,n\right\}$ of μ such that $\left(\bigcup_{j=1}^{n}A_j\right)(x)\geq U(x)$, for every $x\in S(U)$.

DEFINITION 2.18. A fuzzy topological space (X,τ) is said to be almost compact if and only if every open cover of X has a finite subcollection whose closures cover X.

DEFINITION 2.19 Let (X,τ) and (Y,σ) be two fuzzy topological spaces. A mapping $f:(X,\tau)\longrightarrow(Y,\sigma)$ is said to be fuzzy strongly semi $\alpha-open$ if $f(V)$ is fuzzy semi $\alpha-open$ set of *Y* for every fuzzy semi $\alpha-open$ *V* of *X*.

III. FUZZY SEMI $\alpha-$COMPACT SPACE

DEFINITION 3.1. A fuzzy topological space (X,τ) is said to be fuzzy semi $\alpha-\mathbf{compact}$ if and only if for every family μ of fuzzy semi $\alpha-open$ sets such that $\bigcup_{A\in\mu}A=1_X$, there exists a finite subfamily $\eta\subseteq\mu$ such that $\bigcup_{A\in\eta}A=1_X$.

DEFINITION 3.2. A fuzzy set $\mathbf{U}$ in a fuzzy topological space (X,τ) is said to be fuzzy semi $\alpha-\mathbf{compact}$ relative to X if and only if every family μ of fuzzy semi $\alpha-open$ sets such that $\bigcup_{A\in\mu}A\geq U(x)$ there exists a finite subfamily $\eta\subseteq\mu$ such that $\bigcup_{A\in\eta}A\geq U(x)$ for every $x\in S(U)$.

THEOREM 3.3. A fuzzy topological space (X,τ) is fuzzy semi $\alpha-$**compact** if and only if for every collection $\{A_j : j\in J\}$ of fuzzy semi $\alpha-$**closed** subsets of $\mathbf{X}$ having the finite intersection property, $\bigcap_{j\in J} A_j \neq 0_X$.

PROOF. Let $\{A_j : j\in J\}$ be a collection of fuzzy semi $\alpha-$**closed** sets with the finite intersection property. Suppose that $\bigcap_{j\in J} A_j = 0_X$. Then $\bigcup_{j\in J} A_j^c = 1_X$. Since $\{A_j^c : j\in J\}$ is a collection of fuzzy semi $\alpha-$**open** cover of $\mathbf{X}$. Then from the fuzzy semi $\alpha-$**compactness** of $\mathbf{X}$ it follows that there exists a finite subset $\mathbf{F}\subseteq\mathbf{J}$ such that $\bigcup_{j\in F} A_j^c = 1_X$. Then $\bigcap_{j\in F} A_j = 0_X$ which gives a contradiction and therefore $\bigcap_{j\in J} A \neq 0_X$.

Conversely, let $\{A_j : j\in J\}$ be a collection of fuzzy semi $\alpha-$**open** cover of $\mathbf{X}$. Suppose that for every finite subset $\mathbf{F}\subseteq\mathbf{J}$, we have $\bigcup_{j\in F} A_j \neq 1_X$. Then $\bigcap_{j\in F} A_j^c \neq 0_X$. Hence $\{A_j^c : j\in J\}$ satisfies the finite intersection property. Then from the hypothesis we have $\bigcap_{j\in J} A_j^c \neq 0_X$ which implies $\bigcup_{j\in J} A_j \neq 1_X$ and this contradicts the fact that $\{A_j : j\in J\}$ is a fuzzy semi $\alpha-$**open** cover of $\mathbf{X}$. Thus $\mathbf{X}$ is fuzzy semi $\alpha-$**compact.**

Now, we give some results of fuzzy semi $\alpha-$**compactness** in terms of fuzzy filterbases.

THEOREM 3.4. A fuzzy topological space (X,τ) is fuzzy semi $\alpha-$**compact** if and only if for every filterbases ξ in $\mathbf{X}$, $\bigcap_{G\in\xi} G \neq 0_X$.

PROOF. Let μ be a fuzzy semi $\alpha-$**open** cover of $\mathbf{X}$ and μ has no finite subcover. Then for every finite subcollection $\{\mathbf{A}_1,\mathbf{A}_2,\ldots,\mathbf{A}_n\}$ of μ, there exists $\mathbf{x}\in\mathbf{X}$ such that $\mathbf{A}_j(\mathbf{x})<1$ for every $\mathbf{j}=1,2,\ldots,\mathbf{n}.$ Then $A_j^c(x)>0,$ so that $\bigcap_{j=1}^{n} A_j^c(x)\neq 0_X$. Thus $\{A_j^c(x) : A_j\in\mu\}$ forms a filterbases in X. Since μ is fuzzy semi $\alpha-$***open*** cover of X, then $\bigcup_{A_j\in\mu} A_j = 1_X$ for every $x\in X$ and therefore we obtain $\bigcap_{A_j\in\mu} fs\alpha Cl(A_j^c)(x) = \bigcap_{A_j\in\mu} A_j^c(x) = 0_X$, which is a contradiction. Therefore every fuzzy semi $\alpha-$**open** cover of X has a finite subcover and hence X is fuzzy semi $\alpha-$**compact.**

Conversely, suppose that there exists a filter bases ξ such that $\bigcap_{G\in\xi} fs\alpha Cl(G) = 0_X$, so that $\left(\bigcup_{G\in\xi}\left(fs\alpha Cl(G)\right)^c\right)(x) = 1_X$ for every $x\in X$ and hence $\mu=\left\{\left(fs\alpha Cl(G)\right)^c : G\in\xi\right\}$ is a fuzzy semi $\alpha-$**open** cover of X. Since X is fuzzy semi $\alpha-$**compact,** then μ has a finite subcover. Then $\left(\bigcup_{j=1}^{n}\left(fs\alpha Cl(G_j)\right)^c\right)(x) = 1_X$ and hence $\left(\bigcup_{j=1}^{n} G_j^c\right)(x) = 1_X$, so that $\bigcap_{j=1}^{n} G_j = 0_X$ which is a contradiction, since $G_1, G_2, \ldots, G_n$ are members of filterbases ξ. Therefore $\bigcap_{G\in\xi} G \neq 0_X$ for every filterbases ξ.

THEOREM 3.5. A fuzzy set $\mathbf{U}$ in a fuzzy topological space (X,τ) is fuzzy semi $\alpha-$**compact** relative to X if and only if for every filter bases ξ such that every finite of members of ξ is quasi coincident with $\mathbf{U}$, $\left(\bigcap_{G\in\xi} fs\alpha Cl(G)\right)\cap U \neq 0_X$.

PROOF. Let $\mathbf{U}$ not be fuzzy semi $\alpha-$**compact** relative to $\mathbf{X}$, then there exists a fuzzy semi $\alpha-$**open** cover μ of $\mathbf{U}$ such that μ has no finite subcover $\eta=\{A_1,A_2,\ldots,A_n\}$. Then

$\left(\bigcup_{k=1}^{n} A_k\right)(x) < U(x)$ for some $x \in S(U),$ so that $\left(\bigcap_{j=1}^{n} A_j^c\right)(x) > U^c(x) \geq 0$ and hence $\xi = \{A^c : A \in \mu\}$ forms a filter bases and $\left(\bigcap_{j=1}^{n} A_j^c\right) qU.$ By hypothesis $\left(\bigcap_{j=1}^{n} fs\alpha\left(Cl\left(A_j^c\right)\right)\right) \cap U \neq 0_X$ and hence $\left(\bigcap_{j=1}^{n} A_j^c\right) \cap U \neq 0_X.$ Then for some $x \in S(U),$ $\left(\bigcap_{A\in\mu} A^c\right)(x) > 0_X,$ that is $\left(\bigcup_{A\in\mu} A\right)(x) < 1_X,$ which is a contradiction. Hence **U** is fuzzy semi $\alpha-$**compact** relative to **X.**

Conversely, suppose that there exists a filter bases ξ such that every finite of members of ξ is quasi coincident with **U** and $\left(\bigcap_{G\in\xi} fs\alpha Cl(G)\right) \cap U \neq 0_X.$ Then for every $x \in S(U),$ $\left(\bigcap_{G\in\xi} fs\alpha Cl(G)\right)(x) = 0_X$ and hence $\left(\bigcup_{G\in\xi} \left(fs\alpha Cl(G)\right)^c\right)(x) = 1_X$ for every $x \in S(U).$ Thus $\mu = \left\{\left(fs\alpha Cl(G)\right)^c : G \in \xi\right\}$ is fuzzy semi $\alpha-$**open** cover of **U.** Since **U** is fuzzy semi $\alpha-$**compact** relative to **X,** then there exists a finite subcover, say $\left\{\left(fs\alpha Cl(G_k)\right)^c : k = 1,2,...,n\right\},$ such that $\left(\bigcup_{k=1}^{n} \left(fs\alpha Cl(G)\right)^c\right)(x) \geq U(x)$ for every $x \in S(U).$ Hence $\left(\bigcap_{k=1}^{n} fs\alpha Cl(G_k)\right)(x) \leq U^c(x)$ for every $x \in S(U),$ so that $\left(\bigcap_{k=1}^{n} fs\alpha Cl(G_k)\right)\tilde{q}(x) \leq U,$ which is a contradiction. Therefore for every filter bases ξ such that every finite of members of ξ is quasi coincident with **U,** $\left(\bigcap_{G\in\xi} fs\alpha Cl(G)\right) \cap U \neq 0_X.$

THEOREM 3.6. Every fuzzy semi $\alpha-$**closed** fuzzy subset of a fuzzy semi $\alpha-$**compact** space is fuzzy semi $\alpha-$**compact** relative to **X.**

PROOF. Let ξ be a fuzzy filter bases in X such that $Uq\left(\bigcap\{G : G \in \lambda\}\right)$ holds for every finite subcollection λ of ξ and a fuzzy semi $\alpha-$**closed** set **U.** Consider $\xi^* = \{U\} \bigcup \xi.$ For any finite subcollection λ^* of $\xi^*,$ if $U \notin \lambda^*,$ then $\bigcap \lambda^* \neq 0_X.$ If $U \in \lambda^*,$ and since $Uq\left(\bigcap\left\{G : G \in \lambda^* - \{U\}\right\}\right),$ then $\bigcap \lambda^* \neq 0_X.$ Hence λ^* is a fuzzy filter bases in X. Since X is fuzzy semi $\alpha-$**compact,** then $\bigcap_{G\in\xi^*} fs\alpha Cl(G) \neq 0_X,$ so that $\left(\bigcap_{G\in\xi} fsCl(G)\right) \cap U = \left(\bigcap_{G\in\xi} fsCl(G)\right) \cap fsCl(U) \neq 0_X.$ Hence by Theorem 3.5, we conclude that **U** is fuzzy semi $\alpha-$**compact** relative to **X.**

THEOREM 3.7. If a function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is fuzzy semi $\alpha*-$***continuous*** and **U** is fuzzy semi $\alpha-$**compact** relative to X, then so is $f(U).$

PROOF. Let $\{A_k : k \in K\}$ be a fuzzy semi $\alpha-$**open** cover of $S(f(U)).$ For $x \in S(U),$ $f(x) \in f(S(U)) = S(f(U)).$ Since f is fuzzy semi $\alpha*-$***continuous***, then $\left\{f^{-1}(A_k) : k \in K\right\}$ is fuzzy semi $\alpha-$**open** cover of $S(U).$ Since **U** is fuzzy semi $\alpha-$**compact** relative to X, there is a finite subfamily $\left\{f^{-1}(A_k) : k = 1,2,...,n\right\}$ such that $S(U) \leq \bigcup_{k=1}^{n} f^{-1}(A_k)$ which implies $S(U) \leq f^{-1}\left(\bigcup_{k=1}^{n} A_k\right)$ and then $S(f(U)) = f(S(U)) \leq f\left(f^{-1}\left(\bigcup_{k=1}^{n} A_k\right)\right) \leq \bigcup_{k=1}^{n} A_k.$

Therefore $f(U)$ is fuzzy semi $\alpha-\mathbf{compact}$ relative to Y.

LEMMA 3.8. If $f:(X,\tau)\longrightarrow(Y,\sigma)$ is fuzzy open and fuzzy continuous function, then f is fuzzy $\alpha*-\mathbf{continuous.}$

PROOF. Let V be a fuzzy semi $\alpha-\mathbf{open}$ set in Y, then

$$V\le Cl\Big(Int\Big(Cl\big(\big(Int(V)\big)\big)\Big)\Big)\le Cl\Big(Int\big(Cl(V)\big)\Big).$$

So $f^{-1}(V)\le f^{-1}\Big(Cl\Big(Int\big(Cl(V)\big)\Big)\Big)\le Cl\Big(f^{-1}\Big(Int\big(Cl(V)\big)\Big)\Big).$

Since f is fuzzy continuous, then $f^{-1}\big[Int\big(Cl(V)\big)\big]=Int\Big(f^{-1}\big(Cl(V)\big)\Big).$

Also by Theorem 2.6, $f^{-1}\big[Int\big(Cl(V)\big)\big]=Int\Big(f^{-1}\Big(Int\big(Cl(V)\big)\Big)\Big)\le Int\Big(f^{-1}\big(Cl(V)\big)\Big)\le Int\Big(Cl\big(f^{-1}(V)\big)\Big).$ Thus

$$f^{-1}(V)\le Cl\Big(f^{-1}\Big(Int\big(Cl(V)\big)\Big)\Big)\le Cl\Big(Int\big(Cl(V)\big)\Big).$$

Hence the result.

COROLLARY 3.9. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be fuzzy open and fuzzy continuous function and X is fuzzy semi $\alpha-\mathbf{compact},$ then $f(X)$ is fuzzy semi $\alpha-\mathbf{compact.}$

PROOF. . It follows directly from Lemma 3.8 and Theorem 3.7.

DEFINITION. 3.10. A function $f:(X,\tau)\longrightarrow(Y,\sigma)$ is said to be fuzzy semi $\alpha-\mathbf{open}$ if and only if the image of every fuzzy semi $\alpha-\mathbf{open}$ set in X is fuzzy semi $\alpha-\mathbf{open}$ set in Y.

Theorem 3.11. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be a fuzzy semi $\alpha-\mathbf{open}$ bijective function and Y is fuzzy semi $\alpha-\mathbf{compact},$ then X is fuzzy semi $\alpha-\mathbf{compact.}$

PROOF. Let $\{\mathbf{A}_j:\mathbf{j}\in\mathbf{J}\}$ be a collection of fuzzy semi $\alpha-\mathbf{open}$ cover of X, then $\{f(A_j):j\in J\}$ is fuzzy semi $\alpha-\mathbf{open}$ covering of Y. Since Y is fuzzy semi $\alpha-\mathbf{compact}$, there is a finite subset $F\subseteq J$ such that $\{f(A_j):j\in F\}$ is a cover of Y.

But $\mathbf{1}_X=f^{-1}(\mathbf{1}_Y)=f^{-1}\Big(f\Big(\bigcup_{j\in F}A_j\Big)\Big)=\bigcup_{j\in F}A_j$ and therefore X is fuzzy semi $\alpha-\mathbf{compact.}$

THEOREM 3.12. $f:(X,\tau)\longrightarrow(Y,\sigma)$ be a strongly semi $\alpha-\textbf{\textit{open}}$ function, bijective function and Y is fuzzy semi $\alpha-\textbf{\textit{compact}}$ space, then X is fuzzy semi $\alpha-\textbf{\textit{compact}}$ space.

PROOF. Let $\{\mathbf{A}_j:\mathbf{j}\in\mathbf{J}\}$ be fuzzy semi $\alpha-\mathbf{open}$ cover of X, and then $\{f(A_j):j\in J\}$ is fuzzy semi $\alpha-\textbf{\textit{open}}$ cover of Y. Since Y is fuzzy semi $\alpha-\textbf{\textit{compact}},$ there exists a finite subset J_0 of J such that finite family $\{f(A_j):j\in J_0\}$ covers Y.

But $\mathbf{1}_X=f^{-1}(\mathbf{1}_Y)=f^{-1}\Big[f\Big(\bigcup_{j\in J_0}A_j\Big)\Big]=\bigcup_{j\in J_0}A_j,$ and therefore X fuzzy semi $\alpha-\textbf{\textit{compact}}.$

IV. FUZZY SEMI $\alpha-\mathbf{CCLOSED}$ SPACES

DEFINITION 4.1. A fuzzy set $\mathbf{U}$ in a fuzzy topological space (X,τ) is said to be a fuzzy semi $\alpha q-nbd$ of a fuzzy point x_t in X if there exists a fuzzy semi $\alpha-\mathbf{open}$ set $\mathbf{A}\le\mathbf{U}$ such that $x_tqA.$

THEOREM 4.2. Let x_t be a fuzzy point in a fuzzy topological space (X,τ) and $\mathbf{U}$ be any fuzzy set of $X,$ then $x_t\in fs\alpha Cl(U)$ if and only if for every fuzzy semi $fs\alpha q-nbd$ H of $x_t,$ $HqU.$

PROOF. Let $x_t\in fs\alpha Cl(U)$ and there exists a $fs\alpha q-nbd$ H of x_t $H\tilde{q}U.$ Then there exists a fuzzy semi $\alpha-$**open** set $\mathbf{A \leq U}$ such that $x_t qA,$ which implies $A\tilde{q}U$ and hence $\mathbf{U \leq A^c}.$ Since $\mathbf{A^c}$ is fuzzy semi $\alpha-$**closed** set, then $fs\alpha Cl(U) \leq A^c.$ Since $x_t \notin A^c,$ then $x_t \notin fs\alpha Cl(U),$ which is a contradiction.

Conversely, suppose that $x_t \notin fs\alpha Cl(U) = \bigcap\{A : A\, is\ fsc\alpha - closed\ in\, X, A \geq U\}.$ Then there exists a fuzzy semi $\alpha-$**closed** set $A \geq U$ such that $x_t \notin A.$ Hence $x_t qA^c = H,$ where H is a fuzzy semi $\alpha-$**open** set in X and $H\tilde{q}U.$ Then there exists a $fs\alpha q - nbd\, H$ of x_t with $H\tilde{q}U.$ Hence the result.

DEFINITION 4.3 A fuzzy topological space (X,τ) is said to be fuzzy semi $\alpha-$**closed** space if and only if for every family μ of fuzzy semi $\alpha-$**open** sets such that $\bigcup_{A\in\mu} A = 1_X$ there is a finite subfamily $\eta \subseteq \mu$ such that $\left(\bigcup_{A\in\eta} fs\alpha Cl(A)\right)(x) = 1_X,$ for every $x \in X.$

THEOREM 4.4. A fuzzy topological space (X,τ) is said to be fuzzy semi $\alpha-$**closed** space if and only if for every fuzzy semi $\alpha-$**open** filter bases ξ in **X,** $\bigcap_{G\in\xi} fs\alpha Cl(G) \neq 0_X.$

PROOF. Let μ be a fuzzy semi $\alpha-$**open** cover of X and let for every finite subfamily η of $\mu,$ $\left(\bigcup_{A\in\eta} fs\alpha Cl(A)\right)(x) < 1_X$ for some $x \in X.$ Then $\left(\bigcap_{A\in\eta} fs\alpha Cl(A^c)\right)(x) > 0_X$ for some $x \in X.$ Thus $\xi = \left\{\left(fsc\alpha Cl(A)\right)^c : A \in \mu\right\}$ forms a fuzzy semi $\alpha-$**open** filter bases in X. Since μ is a fuzzy semi $\alpha-$**open** cover of X, then $\bigcap_{A\in\mu} A^c = 0_X$ which implies $\bigcap_{A\in\mu} fs\alpha Cl\left[\left(fs\alpha Cl(A)\right)^c\right] = 0_X,$ which is a contradiction. Then every fuzzy semi $\alpha-$**open** cover μ of X has a finite subfamily η such that $\left(\bigcup_{A\in\eta} fs\alpha Cl(A)\right)(x) = 1_X$ for every $x \in X,$ Hence X is fuzzy semi $\alpha-$**closed.**

Conversely, suppose there exists a fuzzy semi $\alpha-$**open** filter bases ξ in X such that $\bigcap_{G\in\xi} fs\alpha Cl(G) = 0_X,$ so that $\left(\bigcup_{G\in\xi}\left(fs\alpha Cl(G)\right)^c\right)(x) = 1_X$ for every $x \in X$ and hence $\mu = \left\{\left(fs\alpha Cl(G)\right)^c : G \in \mu\right\}$ is a fuzzy semi $\alpha-$**open** cover of X. Since X is fuzzy semi $\alpha-$**closed,** then μ has a finite subfamily η such that $\left(\bigcup_{G\in\eta} fs\alpha Cl\left(fs\alpha Cl(G)\right)^c\right)(x) = 1_X$ for every x∈ X, and hence $\bigcap_{G\in\eta}\left(fs\alpha Cl\left(fs\alpha Cl(G)\right)^c\right)^c = 0_X.$ Thus $\bigcap_{G\in\eta} G = 0_X$ which is a contradiction, since all the G are members of filter bases.

DEFINITION 4.5. A fuzzy set **U** in a fuzzy topological space (X,τ) is said to be a fuzzy semi $\alpha-closed$ relative to X if and only if for every family μ of fuzzy semi $\alpha-$**open** β-open sets such that $\bigcup_{A\in\mu} A = U,$ there is a finite subfamily $\eta \subseteq \mu$ such that $\left(\bigcup_{A\in\eta} fs\alpha Cl(A)\right)(x) \geq U(x)$ for every $x \in S(U).$

THEOREM 4.6. A fuzzy subset **U** in a fuzzy topological space (X,τ) is fuzzy semi $\alpha-closed$ relative to X if and only if every fuzzy semi $\alpha-open$ filter bases ξ in X, $\left(\bigcap_{G\in\xi} fsc\alpha(G)\right)\cap U = 0_X,$ there exists a finite subfamily λ of ξ such that $\left(\bigcap_{G\in\lambda} G\right)\tilde{q}U = 0_X.$

PROOF. Let U be a fuzzy semi $\alpha-closed$ relative to X. Suppose ξ is a fuzzy semi $\alpha-open$ filterbases in X such that for every finite subfamily λ of ξ, $\left(\bigcap_{G\in\lambda} G\right)qU,$ but $\left(\bigcap_{G\in\xi} fs\alpha(G)\right)\cap U = 0_X.$ Then for every $x \in S(U)$, $\left(\bigcap_{G\in\xi} fs\alpha(G)\right)(x) = 0_X$ and hence $\left(\bigcup_{G\in\xi}\left(fs\alpha(G)\right)^c\right)(x) = 1_X$ for every $x \in S(U)$. Then $\mu = \left\{\left(fs\alpha Cl(G)\right)^c : G \in \xi\right\}$ is a fuzzy semi $\alpha-$**open** cover of **U** and hence there exists a finite subfamily $\lambda \subseteq \xi$ such that $\bigcup_{G\in\lambda} fs\alpha Cl\left(\left(fs\alpha(G)\right)^c\right) \geq U$, so that $\bigcap_{G\in\lambda}\left(fs\alpha Cl\left(\left(fs\alpha Cl(G)\right)^c\right)\right)^c = \bigcap_{G\in\lambda} fs\alpha Cl\left(fs\alpha Cl(G)\right)$ $\leq U^c$ and hence $\bigcap_{G\in\lambda} G \leq U^c$. Then $\bigcap_{G\in\lambda} G\tilde{q}U$ which is a contradiction.

Conversely, let **U** not be a fuzzy semi $\alpha-$***closed*** set relative to X, then there exists a fuzzy semi $\alpha-$***open*** cover μ of **U** such that every finite subfamily $\eta \subseteq \mu$, $\left(\bigcup_{A\in\eta} fs\alpha Cl(A)\right)(x) \leq U(x)$ for some $x \in S(U)$ and hence $\left(\bigcap_{A\in\eta}\left(fs\alpha Cl(A)\right)^c\right)(x) > U^c(x) \geq 0$ for some $x \in S(U)$. Thus $\xi = \left\{\left(fs\alpha Cl(A)\right)^c : A \in \mu\right\}$ forms a fuzzy semi $\alpha-$***open*** β-open filterbases in X. Let there exists a finite subfamily $\left\{\left(fs\alpha Cl(A)\right)^c : A \in \eta\right\}$ such that $\left(\bigcap_{A\in\eta}\left(fs\alpha Cl(A)\right)^c\right)\tilde{q}U$. Then $U \leq \bigcup_{A\in\eta} fs\alpha Cl(A)$. So there exists a finite subfamily $\eta \subseteq \mu$ such that $\bigcup_{A\in\eta} fs\alpha Cl(A) \geq U$ which is a contradiction. Then for each finite subfamily $\lambda = \left\{\left(fs\alpha Cl(A)\right)^c : A \in \eta\right\}$ of ξ, we have $\left(\bigcap_{A\in\eta}\left(fs\alpha Cl(A)\right)^c\right)qU$. Hence by the given condition $\left(\bigcap_{A\in\mu} fs\alpha Cl\left(fs\alpha Cl(A)\right)^c\right) \cap U \neq 0_X$, so there exists $x \in S(U)$ such that $\left(\bigcap_{A\in\mu} fs\alpha Cl\left(fs\alpha Cl(A)\right)^c\right)(x) > 0_X$. Then $\left(\bigcup_{A\in\mu}\left(fs\alpha Cl\left(fs\alpha Cl(A)\right)^c\right)^c\right)(x) =$ $\left(\bigcup_{A\in\mu} fs\alpha Cl\left(fs\alpha Cl(A)\right)\right)(x) < 1_X$, and hence $\left(\bigcup_{A\in\mu} A\right)(x) < 1_X$ which contradicts the fact that μ is a fuzzy semi $\alpha-$***open*** cover of U. Therefore U is fuzzy semi $\alpha-$***closed*** relative to X.

THEOREM 4.7. Let $f : (X,\tau) \longrightarrow (Y,\sigma)$ be a fuzzy semi $\alpha-$***continuous*** surjection function. If X is fuzzy semi $\alpha-$***closed*** space, then Y is almost compact.

PROOF. Let $\left\{A_j : j \in J\right\}$ be a fuzzy open cover of Y. Then $\left\{f\left(A_j\right) : j \in J\right\}$ is fuzzy semi $\alpha-$***open*** cover of X. By hypothesis, there exists a finite subset $F \subseteq J$ such that $\left(\bigcup_{j\in F} fs\alpha Cl\left(f^{-1}\left(A_j\right)\right)\right)(x) = 1_X$. From the surjectivity of f and by Lemma 2.6, $1_Y = f(1_X) = f\left(\bigcup_{j\in F} fs\alpha Cl\left(f^{-1}\left(A_j\right)\right)\right)(x) \leq$ $\bigcup_{j\in F} Cl\left(f^{-1}\left(A_j\right)\right) = \bigcup_{j\in F} Cl\left(A_j\right)$. Hence Y is almost compact.

Using Lemma 2.14, we have also the following theorem which can be proved similarly to Theorem 4.7.

THEOREM 4.8. Let $f : (X,\tau) \longrightarrow (Y,\sigma)$ be a fuzzy semi $\alpha*-$***continuous*** surjection function. If X is fuzzy semi $\alpha-$***closed*** space, then Y is also fuzzy semi $\alpha-$***closed*** space.

V. FUZZY SEMI α – ALMOST COMPACTNESS and FUZZY SEMI α – NEARLY COMPACTNESS

In this section we investigate the relationships between fuzzy semi $\alpha-$***compactness***, fuzzy semi $\alpha-$***almost*** compactness, and fuzzy semi

$\alpha-\boldsymbol{nearly}$ compactness.

DEFINITION 5.1. A fuzzy topological space (X,τ) is said to be fuzzy semi $\alpha-\boldsymbol{almost}$ compact if and only if, for every family of fuzzy semi $\alpha-\boldsymbol{open}$ cover $\{A_j : j\in J\}$ of X, there exists a finite subset J_0 of J such that $\bigcup_{j\in J_0} fs\alpha Cl(A_j)=1_X$.

DEFINITION 5.2. A fuzzy topological space (X,τ) is said to be fuzzy semi $\alpha-\boldsymbol{almost}$ compact if and only if, for every family of fuzzy semi $\alpha-\boldsymbol{open}$ cover $\{A_j : j\in J\}$ of X, there exists a finite subset J_0 of J such that $\bigcup_{j\in J_0} fs\alpha Int\left[fs\alpha Cl(A_j)\right]=1_X$.

DEFINITION 5.3. A fuzzy topological space (X,τ) is said to be fuzzy semi $\alpha-\boldsymbol{regular}$ if, for each fuzzy semi $\alpha-\boldsymbol{open}$ subset A of X, $A=\bigcup\{A_j\in Fs\alpha O(X,\tau): fs\alpha Cl(A_j)\subseteq A\}$.

THEOREM 5.4. Let (X,τ) be a fuzzy topological space. Then fuzzy semi $\alpha-\boldsymbol{compactness}$ implies fuzzy semi $\alpha-\boldsymbol{nearly}$ compactness which implies fuzzy semi $\alpha-\boldsymbol{almost}$ compactness.

PROOF. Let (X,τ) be a fuzzy semi $\alpha-\boldsymbol{compact}$ space. Then for every fuzzy semi $\alpha-\boldsymbol{open}$ cover $\{A_j : j\in J\}$ of X, there exists a finite subset J_0 of J such that $\bigcup_{j\in J_0} A_j=1_X$. Since A_j is a fuzzy semi $\alpha-\boldsymbol{open}$n set, for each $j\in J$, $A_j = fs\alpha Int(A_j)$ for each $j\in J$. $A_j = fs\alpha Int(A_j)\subseteq fs\alpha Int\left[fs\alpha Cl(A_j)\right]$ for each $j\in J$. Therefore it follows that $\bigcup_{j\in J_0} A_j\subseteq \bigcup_{j\in J_0} fs\alpha Int(A_j)\subseteq \bigcup_{j\in J_0} fs\alpha Int(fs\alpha Cl(A_j))$. Thus $1_X=\bigcup_{j\in J_0} fs\alpha Int(fs\alpha Cl(A_j))$ which implies that (X,τ) is fuzzy semi $\alpha-\boldsymbol{nearly}$ compact. Now let (X,τ) be fuzzy semi $\alpha-\boldsymbol{nearly}$ nearly compact. Then for every fuzzy semi $\alpha-\boldsymbol{open}$ cover $\{A_j : j\in J\}$ of X, there exists a finite subset J_0 of J such that $\bigcup_{j\in J_0} fs\alpha Int(fs\alpha Cl(A_j))=1_X$. Since $fs\alpha Int(fs\alpha Cl(A_j))\subseteq fs\alpha Cl(A_j)$ for each $j\in J_0$, $1_X=\bigcup_{j\in J_0} fs\alpha Int(fs\alpha Cl(A_j))\subseteq\bigcup_{j\in J_0} fs\alpha Cl(A_j)$. Hence $\bigcup_{j\in J_0} fs\alpha Cl(A_j)=1_X$. Hence (X,τ) is fuzzy semi $\alpha-\boldsymbol{compact}$.

THEOREM 5.5. Let (X,τ) be a fuzzy semi $\alpha-\boldsymbol{almost}$ compact space and fuzzy semi $\alpha-\boldsymbol{regular}$. Then (X,τ) is fuzzy semi $\alpha-\boldsymbol{compact}$.

PROOF. Let $\{A_j : j\in J\}$ be fuzzy semi $\alpha-\boldsymbol{open}$ cover of X such that $\bigcup_{j\in J} A_j=1_X$. Since (X,τ) is fuzzy semi $\alpha-\boldsymbol{regular}$, $A_j=\bigcup\{B_j\in Fs\alpha O(X,\tau): fs\alpha Cl(B_j)\subseteq A_j\}$ for each $j\in J$. Since $1_X = 1_X=\bigcup_{j\in J} B_j$ and (X,τ) is fuzzy semi $\alpha-\boldsymbol{almost}$ compact, there exists a finite set $J_0\subseteq J$ such that $\bigcup_{j\in J_0} fs\alpha Cl(B_j)=1_X$. But $fs\alpha Cl(B_j)\subseteq A_j$ and $fs\alpha Int\left[fs\alpha Cl(B_j)\right]\subseteq fs\alpha Cl(B_j)$ for each $j\in J_0$. We have $\bigcup_{j\in J_0} A_j\supseteq\bigcup_{j\in J_0} fs\alpha Cl(B_j)=1_X$. Thus, $\bigcup_{j\in J_0} A_j=1_X$. Hence (X,τ) is fuzzy semi $\alpha-\boldsymbol{compact}$.

THEOREM 5.6. Let (X,τ) be a fuzzy semi $\alpha-\boldsymbol{nearly}$ compact space and fuzzy semi $\alpha-\boldsymbol{regular}$. Then (X,τ) is fuzzy semi

$\alpha - \textbf{\textit{compact}}$.

PROOF. Let $\{A_j : j \in J\}$ be fuzzy semi $\alpha - \textbf{\textit{open}}$ cover of X such that $\bigcup_{j\in J} A_j = 1_X$. Since (X,τ) is fuzzy semi $\alpha - \textbf{\textit{regular}}$, $A_j = \bigcup\{B_j \in Fs\alpha O(X,\tau) : fs\alpha Cl(B_j) \subseteq A_j\}$ for each $j \in J$. Since $1_X = \bigcup_{j\in J} B_j$ and (X,τ) is fuzzy semi $\alpha - \textbf{\textit{nearly}}$ compact, there exists a finite set $J_0 \subseteq J$ such that $\bigcup_{j\in J_0} fs\alpha Int[fs\alpha Cl(B_j)] = 1_X$. But $fs\alpha Int[fs\alpha Cl(B_j)] \subseteq fs\alpha Cl(B_j) \subseteq A_j$. We have $\bigcup_{j\in J_0} A_j \supseteq \bigcup_{j\in J_0} fs\alpha Int[fs\alpha Cl(B_j)] = 1_X$. Thus, $\bigcup_{j\in J_0} A_j = 1_X$. Hence (X,τ) is fuzzy semi $\alpha - \textbf{\textit{compact}}$.

THEOREM 5.7. A fuzzy topological space (X,τ) is fuzzy semi $\alpha - \textbf{\textit{almost}}$ compact, if and only if, for every family $\{A_j : j \in J\}$ of fuzzy semi $\alpha - \textbf{\textit{open}}$ sets having the FIP, $\bigcap_{j\in J} fs\alpha Cl(A_j) \neq 0_X$.

PROOF. Let $\{A_j : j \in J\}$ be a family of fuzzy semi $\alpha - \textbf{\textit{open}}$ sets having the FIP. Suppose that $\bigcap_{j\in J} fs\alpha Cl(A_j) = 0_X$. and then $\bigcup_{j\in J} [fs\alpha Cl(A_j)]^c = \bigcup_{j\in J} fs\alpha Int[(A_j)^c] = 1_X$. Since (X,τ) is fuzzy semi $\alpha - \textbf{\textit{almost}}$ compact, there exists a finite subset $J_0 \subseteq J$ such that $\bigcup_{j\in J_0} fs\alpha Cl[fs\alpha Int(A_j^c)] = 1_X$. This implies that $\bigcup_{j\in J_0} fs\alpha Cl(fs\alpha Int(A_j^c)) = \bigcup_{j\in J_0} fs\alpha Cl(fs\alpha Cl(A_j))^c = 1_X$. Thus, $\bigcap_{j\in J_0} fs\alpha Int(fs\alpha Cl(A_j)) = 0_X$. But $A_j = fs\alpha Int(A_j) \subseteq fs\alpha Int[fs\alpha Cl(A_j)]$.

This implies that $\bigcap_{j\in J_0} A_j = 0_X$ which is in contradiction with FIP of the family.

Conversely, let $\{A_j : j \in J\}$ be a family of fuzzy semi $\alpha - \textbf{\textit{open}}$ sets such that $\bigcup_{j\in J_0} A_j = 1_X$. Suppose that there does not exist a finite subset $J_0 \subseteq J$ such that $\bigcup_{j\in J_0} fs\alpha Cl(A_j) = 1_X$. Since $\{[fs\alpha Cl(A_j)]^c : j \in J\}$ has the FIP, then $\bigcap_{j\in J_0} fs\alpha Cl[fs\alpha Cl(A_j)]^c \neq 0_X$. This implies $\bigcup_{j\in J}\left[fs\alpha Cl(fs\alpha Cl(A_j))^c\right]^c \neq 1_X$. Hence $\bigcup_{j\in J} fs\alpha Int(fs\alpha Cl(A_j))^c \neq 1_X$. Since $A_j \subseteq fs\alpha Int[fs\alpha Cl(A_j)]$ for each $j \in J$, $\bigcup_{j\in J} A_j \neq 1_X$ which is in contradiction with $\bigcup_{j\in J} A_j = 1_X$.

THEOREM 5.8. Let (X,τ) and (Y,σ) be fuzzy topological spaces and let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be fuzzy semi $\alpha^* - \textbf{\textit{continuous}}$, surjective mapping. If (X,τ) is fuzzy semi $\alpha - \textbf{\textit{almost}}$ compact space then so is (Y,σ).

PROOF. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be fuzzy $\alpha^* - \textbf{\textit{continuous}}$ mapping of a fuzzy $\alpha - \textbf{\textit{almost}}$ compact space (X,τ) onto a fuzzy topological space (Y,σ). Let $\{A_j : j \in J\}$ be any fuzzy semi $\alpha - \textbf{\textit{open}}$ cover of (Y,σ). Then $\{f^{-1}(A_j) : j \in J\}$ is a fuzzy semi $\alpha - \textbf{\textit{open}}$ cover of X. Since X is fuzzy semi $\alpha - \textbf{\textit{almost}}$ compact, there exists a finite subset J_0 of J such that $\bigcup_{j\in J_0} fs\alpha Cl[f^{-1}(A_j)] = 1_X$. Now $f(1_X) = f\left[\bigcup_{j\in J_0} fs\alpha Cl[f^{-1}(A_j)]\right] = \bigcup_{j\in J_0} f\left[fs\alpha Cl[f^{-1}(A_j)]\right]$

$=\mathbf{1}_Y$. But $fs\alpha Cl\left[f^{-1}\left(A_j\right)\right]\subseteq f^{-1}\left[fs\alpha Cl\left(A_j\right)\right]$ and from the surjectivity of $f\left[fs\alpha Cl\left(f^{-1}\left(A_j\right)\right)\right]\subseteq f\left[f^{-1}\left(fs\alpha Cl\left(A_j\right)\right)\right]=fs\alpha Cl\left(A_j\right)$.

So $\bigcup_{j\in J_0} fs\alpha Cl\left(A_j\right)\supseteq\bigcup_{j\in J_0} f\left[fs\alpha Cl\left(f^{-1}\left(A_j\right)\right)\right]=\mathbf{1}_Y$.

Thus $\bigcup_{j\in J_0} fs\alpha Cl\left(A_j\right)=\mathbf{1}_Y$. Hence (Y,σ) is fuzzy semi $\alpha-\boldsymbol{almost}$ compact.

THEOREM 5.9. Let (X,τ) and (Y,σ) be fuzzy topological spaces and let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be fuzzy semi $\alpha-\boldsymbol{continuous}$, surjective mapping. If (X,τ) is fuzzy semi $\alpha-\boldsymbol{almost}$ compact space then (Y,σ) is fuzzy semi almost compact.

PROOF. Let $\left\{A_j : j\in J\right\}$ be any fuzzy open cover of (Y,σ). Then $\left\{f^{-1}\left(A_j\right): j\in J\right\}$ is a fuzzy semi $\alpha-\boldsymbol{open}$ cover of X. Since X is fuzzy semi $\alpha-\boldsymbol{almost}$ compact, there exists a finite subset J_0 of J such that $\bigcup\left\{fs\alpha Cl\left[f^{-1}\left(A_j\right)\right]: j\in J_0\right\}=\mathbf{1}_X$.

Now from the surjectivity of f,

$\mathbf{1}_Y=f\left(\mathbf{1}_X\right)=f\left[\bigcup_{j\in J_0} fs\alpha Cl\left(f^{-1}\left(A_j\right)\right)\right]\subseteq$

$\bigcup_{j\in J_0} f\left[fs\alpha Cl\left(f^{-1}\left(A_j\right)\right)\right]\subseteq\bigcup_{j\in J_0} fs\alpha Cl\left[f\left(f^{-1}(A)\right)\right]$

$\subseteq\left[\bigcup_{j\in J_0} Cl\left(f\left(f^{-1}\left(A_j\right)\right)\right)\right]\subseteq\bigcup_{j\in J_0} Cl\left(A_j\right)$, which implies that $\bigcup_{j\in J_0} Cl\left(A_i\right)=\mathbf{1}_Y$. Hence (Y,σ) is fuzzy almost compact.

DEFINITION 5.10. Let (X,τ) and (Y,σ) be fuzzy topological spaces. A function $f:(X,\tau)\longrightarrow(Y,\sigma)$ is said to be fuzzy semi $\alpha-\boldsymbol{weakly}$ continuous if, for each fuzzy semi $\alpha-\boldsymbol{open}$ set V in Y, $f^{-1}(V)\subseteq fs\alpha Int\left[f^{-1}\left(fs\alpha Cl(V)\right)\right]$.

THEOREM 5.11. A mapping $f:(X,\tau)\longrightarrow(Y,\sigma)$ from a fuzzy topological space (X,τ) to a fuzzy topological space (Y,σ) is fuzzy strongly semi $\alpha-\boldsymbol{open}$ if and only if $f\left[fs\alpha Int(V)\right]\subseteq fs\alpha Int\left[f(V)\right]$.

PROOF. If f is fuzzy strongly semi $\alpha-\boldsymbol{open}$ mapping then $f\left[fs\alpha Int(V)\right]$ is a fuzzy semi $\alpha-\boldsymbol{open}$ set in Y for fuzzy semi $\alpha-\boldsymbol{open}$ set V in X. Hence $f\left[fs\alpha Int(V)\right]=fs\alpha Int\left[f\left(fs\alpha\left(Int(V)\right)\right)\right]=fs\alpha Int\left[f(V)\right]$. Thus $f\left[fs\alpha Int(V)\right]\subseteq fs\alpha Int\left[f(V)\right]$.

Conversely, let V be a fuzzy semi $\alpha-\boldsymbol{open}$ set in X and then $V=fs\alpha Int(V)$. Then by hypothesis, $f(V)=f\left[fs\alpha Int(V)\right]\subseteq fs\alpha Int\left[f(V)\right]$. This implies that $f(V)$ is fuzzy semi $\alpha-\boldsymbol{open}$ set in Y.

THEOREM 5.12. Let (X,τ) and (Y,σ) be fuzzy topological spaces and let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be fuzzy semi $\alpha-\boldsymbol{weakly}$ continuous, surjective mapping. If (X,τ) is fuzzy semi $\alpha-\boldsymbol{compact}$ space, then (Y,σ) is fuzzy semi $\alpha-\boldsymbol{almost}$ compact.

PROOF. Let $\left\{A_j : j\in J\right\}$ be fuzzy semi $\alpha-\boldsymbol{open}$ cover of Y such that $\bigcup_{j\in J} A_j=\mathbf{1}_Y$. Then

$\bigcup_{j\in J} f^{-1}\left(A_j\right)=f^{-1}\left(\bigcup_{j\in J} A_j\right)=f^{-1}\left(\mathbf{1}_Y\right)=\mathbf{1}_X$. (X,τ) is fuzzy semi $\alpha-\boldsymbol{compact}$, and there exists a finite subset J_0 of J such that $\bigcup_{j\in J_0} f^{-1}\left(A_j\right)=\mathbf{1}_X$. Since f is fuzzy semi $\alpha-\boldsymbol{weakly}$ continuous.

$f^{-1}\left(A_j\right)\subseteq fs\alpha Int\left[f^{-1}\left(fs\alpha Cl\left(A_j\right)\right)\right]\subseteq f^{-1}\left[fs\alpha Cl\left(A_j\right)\right]$. This implies that

$\bigcup_{j\in J_0} f^{-1}\left[fs\alpha Cl\left(A_j\right)\right]\supseteq \bigcup_{j\in J_0} f^{-1}\left(A_j\right)=\mathbf{1}_X.$ Thus $\bigcup_{j\in J_0} f^{-1}\left[fs\alpha Cl\left(A_j\right)\right]=\mathbf{1}_X.$ Since f is surjective, $\mathbf{1}_Y = f\left(\mathbf{1}_X\right)= f\left[\bigcup_{j\in J_0} f^{-1}\left(fs\alpha Cl\left(A_j\right)\right)\right]= \bigcup_{j\in J_0} f\left[f^{-1}\left(fs\alpha Cl\left(A_j\right)\right)\right]=\bigcup_{j\in J_0} fs\alpha Cl\left(A_j\right).$ Hence $\bigcup_{j\in J_0} fs\alpha Cl\left(A_j\right)=\mathbf{1}_Y.$ It follows that (Y,σ) is fuzzy semi $\alpha-\boldsymbol{almost}$ compact.

THEOREM 5.13. Let (X,τ) and (Y,σ) be fuzzy topological spaces and let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be fuzzy semi $\alpha^*-\boldsymbol{continuous},$ surjective and strongly $\alpha-\boldsymbol{open}$ mapping. If (X,τ) is fuzzy semi $\alpha-\boldsymbol{nearly}$ compact space then so is $(Y,\sigma).$

PROOF. Let $\left\{A_j : j\in J\right\}$ be any fuzzy $\alpha-\boldsymbol{open}$ cover of $(Y,\sigma).$ Since f is fuzzy semi $\alpha^*-\boldsymbol{continuous},$ then $\left\{f^{-1}\left(A_j\right): j\in J\right\}$ is a fuzzy $\alpha-\boldsymbol{open}$ cover of *X*. Since (X,τ) is fuzzy semi $\alpha-\boldsymbol{nearly}$ compact, there exists a finite subset J_0 of J such that $\bigcup_{j\in J_0} fs\alpha Int\left[fs\alpha Cl\left(f^{-1}\left(A_j\right)\right)\right]=\mathbf{1}_X.$ Since f is surjective,

$$\mathbf{1}_Y = f\left(\mathbf{1}_X\right)= f\left[\bigcup_{j\in J_0} fs\alpha Int\left(fs\alpha Cl\left(f^{-1}\left(A_j\right)\right)\right)\right]= \bigcup_{j\in J_0} f\left[fs\alpha Int\left(fs\alpha Cl\left(f^{-1}\left(A_j\right)\right)\right)\right].$$

Since f is fuzzy semi strongly $\alpha-\boldsymbol{open},$

$$f\left[fs\alpha Int\left(fs\alpha Cl\left(f^{-1}\left(A_j\right)\right)\right)\right]\subseteq fs\alpha Int\left[f\left(fs\alpha Cl\left(f^{-1}\left(A_j\right)\right)\right)\right]$$

for each $j\in J.$ Since f is fuzzy semi $\alpha^*-\boldsymbol{continuous},$ then

$$f\left[fs\alpha Cl\left(f^{-1}\left(A_j\right)\right)\right]\subseteq fs\alpha Cl\left[f\left(f^{-1}\left(A_j\right)\right)\right].$$

Hence we have

$$\mathbf{1}_Y = \bigcup_{j\in J_0} f\left[fs\alpha Int\left(fs\alpha Cl\left[f^{-1}\left(A_j\right)\right]\right)\right]\subseteq \bigcup_{j\in J_0} fs\alpha Int\left(fs\alpha Cl\left[f^{-1}\left(A_j\right)\right]\right)\subseteq \bigcup_{j\in J_0} fs\alpha Int\left(fs\alpha Cl\left[f\left(f^{-1}\left(A_j\right)\right)\right]\right)= \bigcup_{j\in J_0} fs\alpha Int\left[fs\alpha Cl\left(A_j\right)\right].$$

Thus $\mathbf{1}_Y = \bigcup_{j\in J_0} fs\alpha Int\left[fs\alpha Cl\left(A_j\right)\right].$

Hence (Y,σ) is fuzzy semi $\alpha-\boldsymbol{nearly}$ compact.

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