Somewhat Supra Compactness and Somewhat Supra Connectedness

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Abstract— In 2016 Baker introduced the concept of a somewhat open set in topological space and used to characterize both somewhat continuity and contra – somewhat continuity. In the present paper we introduce somewhat supra open sets in supra topological spaces and investigate its basic properties. In this paper we originally originate the notions of somewhat supra compact spaces and interpret its several effects and characterizations. Also we newly originate and study the concepts of somewhat supra Lindelof spaces, countably somewhat supra compact spaces and somewhat supra connected spaces

Keywords— Supra Topological Space, Somewhat Supra Open Set, Somewhat Supra Compact Space, Somewhat Supra Lindelof Space, Countably Supra Compact Space, Somewhat Supra Connected space.

I. Introduction

Baker introduced the notion of somewhat open sets in topological space in 2016 and used to characterize somewhat continuity and contra – somewhat continuity. Throughout this paper, (X,τ) and (Y,σ) (or simply, X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a topological space (X,τ), $Cl(A)$, $Int(A)$ and $X-A$ denote the closure of $A,$ the interior of A and the complement of A in $X,$ respectively. Now we bring up with the new concepts of somewhat supra compact, somewhat supra Lindelof, Countably somewhat supra compact and somewhat supra connected spaces and investigate several properties and characterizations of these concepts.

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II. Preliminaries

DEFINITION 2.1. Let (X,τ) be a topological space. Then a subset U of a space X is said to be somewhat open if $U=\phi$ or if there exist $x\in U$ and an open subset V such that $x\in V\subseteq U.$ A set is called somewhat closed if its complement is somewhat open.

Obviously somewhat open sets are closed under arbitrary union but, as we see in the following example, not closed under intersection. The set of all somewhat open sets in a topological space (X,τ) is denoted by $SW(X,\tau).$ Clearly $\tau\subseteq SW(X,\tau).$ Also any set containing a nonempty somewhat open set is somewhat open. Semi-open implies somewhat open and the closure of a preopen set is somewhat open.

EXAMPLE 2.2. Let $X=\{a,b,c\}$ have the topology $\tau=\{X,\phi,\{a\},\{b\},\{a,b\}\}.$ The sets $\{a,c\}$ and $\{b,c\}$ are somewhat open, but, since $\{c\}$ is not somewhat open, somewhat open sets are not closed under intersection.

DEFINITION 2.3. Let A be a subset of a space $X.$ The somewhat closure of $A,$ denoted by $swCl(A),$ is given by $swCl(A)=\bigcap\{F : F \text{ is somewhat closed and } A\subseteq F\}$ and the somewhat interior of $A,$ denoted by $swInt(A),$ is given by $swInt(A)=\bigcup\{U : U \text{ is somewhat open and } U\subseteq A\}.$

The following properties of the closure and interior operators for somewhat open sets are stated for completeness. They are special cases of properties of operators defined for minimal structures by Popa and Noiri [36].

THEOREM 2.4. The following statements hold for a subset A of a space X:

(a) $swInt(X-A)=X-swCl(A).$

(b) $swCl(X-A)=X-swInt(A).$

(c) $swCl(A)$ is somewhat closed.

(d) A is somewhat closed if and only if $swCl(A)=A.$

(e) $\mathbf{swCl}(\mathbf{A}) = \left\{ \begin{matrix} \mathbf{x} \in \mathbf{X} : \textbf{for every somewhat open} \\ \textbf{subset } \mathbf{U} \textbf{ containing x, } \mathbf{U} \cap \mathbf{A} \neq \phi \end{matrix} \right\}.$

THEOREM 2.5. Let $\mathbf{A}$ be a subset of a space $\mathbf{X}.$ Then $\mathbf{swCl}(\mathbf{A}) = \mathbf{X}$ if $\mathbf{A}$ is dense in $\mathbf{X},$ and $\mathbf{swCl}(\mathbf{A}) = \mathbf{A}$ if $\mathbf{A}$ is not dense in $\mathbf{X}.$

PROOF: Assume $\mathbf{A}$ is dense in $\mathbf{X}.$ Let $\mathbf{x} \in \mathbf{X}$ and let $\mathbf{U}$ be a somewhat open set containing $\mathbf{x}.$ Then there exists a nonempty open set $\mathbf{V}$ such that $\mathbf{V} \subseteq \mathbf{U}.$ Since $\mathbf{A}$ is dense in $\mathbf{X},$ $\mathbf{A} \cap \mathbf{V} \neq \phi.$ Then $\mathbf{A} \cap \mathbf{U} \neq \phi$ and hence $\mathbf{x} \in \mathbf{swCl}(\mathbf{A}),$ which shows that $\mathbf{swCl}(\mathbf{A}) = \mathbf{X}.$ Assume $\mathbf{A}$ is not dense in $\mathbf{X}.$ Let $\mathbf{x} \in \mathbf{X}$ such that $\mathbf{x} \notin \mathbf{Cl}(\mathbf{A}).$ Then there exists an open set $\mathbf{U}$ such that $\mathbf{x} \in \mathbf{U}$ and $\mathbf{U} \cap \mathbf{A} = \phi.$ Thus $\mathbf{x} \in \mathbf{U} \subseteq (\mathbf{X} - \mathbf{A}),$ which proves that $\mathbf{X} - \mathbf{A}$ is somewhat open and that $\mathbf{A}$ is somewhat closed. Thus $\mathbf{swCl}(\mathbf{A}) = \mathbf{A}.$

COROLLARY 2.6. Let $\mathbf{A}$ be a subset of a space $\mathbf{X}.$ Then $\mathbf{swInt}(\mathbf{A}) = \mathbf{A}$ if $\mathbf{X} - \mathbf{A}$ is not dense in $\mathbf{X},$ and $\mathbf{swInt}(\mathbf{A}) = \phi$ if $\mathbf{X} - \mathbf{A}$ is dense in $\mathbf{X}.$

THEOREM 2.7 For a function $f : (X, \tau) \longrightarrow (Y, \sigma)$ the following conditions are equivalent:

$(\mathbf{a})$ f is somewhat continuous.

$(\mathbf{b})$ For every open subset $\mathbf{V}$ of $\mathbf{Y},$ $f^{-1}(V)$ is somewhat open.

$(\mathbf{c})$ For every closed subset $\mathbf{F}$ of $\mathbf{Y},$ $f^{-1}(F)$ is somewhat closed.

$(\mathbf{d})$ For every x ∈ X and every open subset $\mathbf{V}$ of $\mathbf{Y}$ containing $f(x)$ there exists a somewhat open set $\mathbf{U}$ of $\mathbf{X}$ containing $\mathbf{x}$ such that $f(U) \subseteq V.$

DEFINITION 2.8. Let (X, τ) and (Y, σ) be two topological spaces. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called somewhat irresolute if $f^{-1}(V)$ is somewhat closed in (X, τ) for every somewhat closed set V of $(Y, \sigma).$

DEFINITION 2.9. Let (X, τ) and (Y, σ) be two topological spaces. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called strongly somewhat continuous if the inverse image of every somewhat closed in $\mathbf{Y}$ is closed in $\mathbf{X}.$

DEFINITION 2.10. Let (X, τ) and (Y, σ) be two topological spaces. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called perfectly somewhat continuous if the inverse image of every somewhat closed set in $\mathbf{Y}$ is both closed and open set in $\mathbf{X}.$

DEFINITION 2.11. A family μ of subsets of $\mathbf{X}$ is said to be a supra topology on $\mathbf{X}$ if (i) $X, \phi \in \mu,$ (ii) if $A_i \in \mu$ for all $i \in I$ then $\cup \{A_i : i \in I\} \in \mu$. The pair (X, μ) is called supra topological space. The elements of μ are called supra open sets in (X, μ) and complement of a supra open set is called a supra closed set.

DEFINITION 2.12. (i) The supra closure of a set $\mathbf{A}$ is denoted by $sup\,Cl(A)$ and is defined as

$sup\,Cl(A) =$

$\cap \{B \subseteq X : B \textit{ is a supra closed set and } A \subseteq B\}.$

(ii) The supra interior of a set $\mathbf{A}$ is denoted by $sup\,Int(A)$ and is defined as

$sup\,Int(A) =$

$\cup \{B \subseteq X : B \textit{ is a supra open set and } B \subseteq A\}.$

DEFINITION 2.13. Let (X, τ) be a topological space and μ be a supra topology on $\mathbf{X}.$ We call μ a supra topology associated with τ if $\tau \subseteq \mu.$

III. SOMEWHAT SUPRA OPEN SETS

DEFINITION 3.1. Let (X, τ) be a supra topological space. Then A subset $\mathbf{U}$ of $\mathbf{X}$ is said to be somewhat supra open if $U = \phi$ or if there exist $\mathbf{x} \in \mathbf{U}$ and a supra open subset $\mathbf{V}$ in $\mathbf{X}$ such that $\mathbf{x} \in \mathbf{V} \subseteq \mathbf{U}.$ A set is called somewhat supra closed if its complement is somewhat supra open.

Obviously somewhat supra open sets are closed under arbitrary union but, as we see in the following example, not closed under intersection. The set of all somewhat supra open sets in a topological space $(\mathbf{X}, \tau)$ is denoted by $\mathbf{SW\,sup}(\mathbf{X}, \tau).$ Clearly $\tau \subseteq \mathbf{SW\,sup}(\mathbf{X}, \tau).$ Also any set containing a nonempty somewhat supra open set is somewhat supra open.

EXAMPLE 3.2. Suppose that $\mathbf{X} = \{1, 2, 3, 4, 5\}$ have the supra topology $\tau = \{\phi, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, X\}.$

The sets $\{1, 3\}$ and $\{2, 3\}$ are somewhat supra open, but, since $\{3\}$ is not somewhat supra open, somewhat supra open sets are not closed under intersection.

DEFINITION 3.3. Let (X, τ) be a supra topological space. Let A be a subset of $X.$ The somewhat supra closure of A, denoted by $SWsupraCl(A),$ is given by

$$SWsupraCl(A) = \bigcap \{F : F \text{ is somewhat supra closed in } X \text{ and } A \subseteq F\}$$

and the somewhat supra interior of A, denoted by $SWsupraInt(A),$ is given by

$$SWsupraInt(A) = \bigcup \{U : U \text{ is somewhat supra open in } X \text{ and } U \subseteq A\}.$$

The following properties of the closure and interior operators for somewhat supra open sets are stated for completeness.

THEOREM 3.4. Let (X, τ) be a supra topological space. The following statements hold for a subset A of a space $X:$

(a) $SWsupraInt(X - A) = X - SWsupraCl(A).$

(b) $SWsupraCl(X - A) = X - SWsupraInt(A).$

(c) $SWsupraCl(A)$ is somewhat supra closed.

(d) A is somewhat supra closed if and only if $SWsupraCl(A) = A.$

DEFINITION 3.5. Let (X, τ) be a supra topological space and let A be a subset of $X.$ We say that A is supra dense in X if for each $U \in \tau - \{\phi\}$ implies that $U \cap A \neq \phi.$ Clearly $SWsupraCl(A) = X$ if and only if A is supra dense in $X.$

THEOREM 3.6. Let (X, τ) be a supra topological space and let A be a subset of $X.$ Then $SWsupraCl(A) = X$ if A is supra dense in $X,$ and $SWsupraCl(A) = A$ if A is not supra dense in $X.$

PROOF: Assume A is supra dense in $X.$ Let $x \in X$ and let U be a somewhat supra open set containing $x.$ Then there exists a nonempty supra open set V such that $V \subseteq U.$ Since A is supra dense in $X,$ $A \cap V \neq \phi.$ Then $A \cap U \neq \phi$ and hence $x \in SWsupraCl(A),$ which shows that $SWsupCl(A) = X.$ Assume A is not supra dense in $X.$ Let $x \in X$ such that $x \notin supraCl(A).$ Then there exists a supra open set U such that $x \in U$ and $U \cap A = \phi.$ Thus $x \in U \subseteq (X - A),$ which proves that $X - A$ is somewhat supra open and that A is somewhat supra closed. Thus $SWsupraCl(A) = A.$

COROLLARY 3.7. Let A be a subset of a supra space $X.$ Then $SWsupraCl(A) = A$ if $X - A$ is not supra dense in $X,$ and $SWsupraCl(A) = \phi$ if $X - A$ is supra dense in $X.$

THEOREM 3.8 For a function $f : (X, \tau) \longrightarrow (Y, \sigma)$ the following conditions are equivalent:

(a) f is somewhat supra continuous.

(b) For every supra open subset V of $Y,$ $f^{-1}(V)$ is somewhat supra open.

(c) For every supra closed subset F of $Y,$ $f^{-1}(F)$ is somewhat supra closed.

(d) For every $x \in X$ and every supra open subset V of Y containing $f(x)$ there exists a somewhat supra open set U of X containing x such that $f(U) \subseteq V.$

DEFINITION 3.8. Let (X, τ) and (Y, σ) be two supra topological spaces. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called somewhat supra irresolute if $f^{-1}(V)$ is somewhat supra closed in (X, τ) for every somewhat supra closed set V of $(Y, \sigma).$

DEFINITION 3.9. Let (X, τ) and (Y, σ) be two supra topological spaces. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called strongly somewhat supra continuous if the inverse image of every somewhat supra closed in Y is supra closed in $X.$

DEFINITION 3.10. Let (X, τ) and (Y, σ) be two supra topological spaces. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called perfectly somewhat supra continuous if the inverse image of every somewhat supra closed set in Y is both supra closed and supra open set in $X.$

IV. SOMEWHAT SUPRA COMPACTNESS

DEFINITION 4.1. A collection $\{A_i : i \in I\}$ of somewhat supra open sets in a supra topological space (X, τ) is called a somewhat supra open cover of a subset B of X if $B \subseteq \bigcup \{A_i : i \in I\}$ holds.

DEFINITION 4.2. A topological space (X, τ) is called somewhat supra compact if every somewhat supra open cover of X has a finite subcover.

DEFINITION 4.3. A subset B of a supra topological space (X, τ) is said to be somewhat supra compact relative to

(X, τ) if, for every collection $\{A_i : i \in I\}$ of somewhat supra open subsets of X such that $B \subseteq U\{A_i : i \in I\}$ there exists a finite subset I_0 of I such that $B \subseteq U\{A_i : i \in I_0\}$.

DEFINITION 4.4. A subset B of a supra topological space (X,τ) is said to be somewhat supra compact if B is somewhat supra compact as a subspace of X.

THEOREM 4.5. Every somewhat supra compact space is supra compact.

PROOF. Let $\{A_i : i \in I\}$ be a supra open cover of (X, τ). Since $\tau \subseteq SW\sup(X,\tau)$. So $\{A_i : i \in I\}$ is a somewhat supra open cover of (X, τ). Since (X, τ) is somewhat supra compact. So somewhat supra open cover $\{A_i : i \in I\}$ of (X, τ) has a finite sub cover say $\{A_i : i = 1,2,\ldots,n\}$ for X. Hence (X, τ) is a supra compact space.

THEOREM 4.6. Every somewhat supra closed subset of a somewhat supra compact space is somewhat supra compact relative to X.

PROOF. Let A be a somewhat supra closed subset of a supra topological space (X, τ). Then $A^C = X - A$ is somewhat supra open in (X, τ). Let $\gamma = \{A_i : i \in I\}$ be a somewhat supra open cover of A by somewhat supra open subset in (X,τ). Let $\gamma^* = \{A_i : i \in I\} U \{A^C\}$ be a somewhat supra open cover of (X, τ). That is $X = U\gamma^* = (U\{A_i : i \in I\}) U\, A^c$. By hypothesis (X, τ) is somewhat supra compact and hence γ^* is reducible to a finite subcover of (X, τ) say $X = A_1 U A_2 U \ldots U A_n U A^c; A_k \in \gamma$ for $k = 1,2,\ldots,n$. But A and A^C are disjoint. Hence $A \subseteq A_1 U A_2 U \ldots U A_n;\ A_k \in \gamma$ for $k = 1,2,\ldots,n$. Thus a somewhat supra open cover γ of A contains a finite subcover. Hence A is somewhat supra compact relative to (X,τ).

THEOREM 4.7 A somewhat supra continuous image of a somewhat supra compact space is supra compact.

PROOF. Let (X,τ) and (Y,σ) be supra topological spaces. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a somewhat supra continuous map from a somewhat supra compact space X onto a supra topological space Y. Let $\{A_i : i \in I\}$ be a supra open cover of Y. Then $f^{-1}(\{A_i : i \in I\})$ is a somewhat supra open cover of X as f is somewhat supra continuous. Since X is somewhat supra compact, the somewhat supra open cover of X, $f^{-1}(\{A_i : i \in I\})$ has a finite sub cover say $\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$. Therefore $X = \bigcup\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$, which implies $f(X) = \bigcup\{A_i : i = 1,2,\ldots,n\}$, then $Y = \bigcup\{A_i : i = 1,2,\ldots,n\}$. That is $\{A_i : i = 1,2,\ldots,n\}$ is a finite subcover of $\{A_i : i \in I\}$ for Y. Hence Y is supra compact.

THEOREM 4.8. Let (X, τ) and (Y, σ) be supra topological spaces. If a map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is somewhat supra irresolute and a subset S of X is somewhat supra compact relative to (X, τ), then the image $f(S)$ is somewhat supra compact relative to (Y, σ).

PROOF. Let $\{A_i : i \in I\}$ be a collection of somewhat supra open cover of (Y, σ) such that $f(S) \subseteq U\{A_i : i \in I\}$. Then $S \subseteq U\{f^{-1}(A_i) : i \in I\}$, where $\{f^{-1}(A_i) : i \in I\} \subseteq SW(X,\tau)$. Since S is somewhat supra compact relative to (X, τ), there exists a finite subcollection $\{A_1, A_2, \ldots, A_n\}$ such that $S \subseteq U\{f^{-1}(A_i) : i = 1,2,\ldots,n\}$. That is $f(S) \subseteq U\{A_1, A_2, \ldots, A_n\}$. Hence $f(S)$ is somewhat supra compact relative to (Y, σ).

THEOREM 4.9. If a map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is strongly somewhat continuous map from a supra compact space (X, τ) onto a topological space (Y, σ), then (Y, σ) is somewhat supra compact.

PROOF. Let $\{A_i : i \in I\}$ be a somewhat supra open cover of (Y, σ). Since f is strongly somewhat supra continuous, $\{f^{-1}(A_i) : i \in I\}$ is a supra open cover of (X, τ). Again, since (X, τ) is supra compact, the supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, τ) has a finite subcover say $\{f^{-1}(A_i) : i = 1,2,\ldots,n\}$. Therefore $X = U\{f^{-1}(A_i) : i = 1,2,\ldots,n\}$, which implies

$f(X)=\cup\{A_i : i=1,2,\ldots,n\}$, so that $Y=\cup\{A_i : i=1,2,\ldots,n\}$. That is $A_1,A_2,\ldots,A_n$ is a finite subcover of $\{A_i : i\in I\}$ for (Y,σ). Hence (Y,σ) is somewhat supra compact.

THEOREM 4.10. If a map $f:(X,\tau)\longrightarrow(Y,\sigma)$ is perfectly somewhat supra continuous map from a supra compact space (X,τ) onto a supra topological space (Y,σ), then (Y,σ) is somewhat supra compact.

PROOF. Let $\{A_i : i\in I\}$ be a somewhat supra open cover of (Y,σ). Since f is perfectly somewhat supra continuous, $\{f^{-1}(A_i) : i\in I\}$ is a supra open cover of (X,τ). Again, since (X,τ) is supra compact, the supra open cover $\{f^{-1}(A_i) : i\in I\}$ of (X,τ) has a finite sub cover say $\{f^{-1}(A_i) : i=1,2,\ldots,n\}$. Therefore $X=\cup\{f^{-1}(A_i) : i=1,2,\ldots,n\}$, which implies $f(X)=\cup\{A_i : i=1,2,\ldots,n\}$, so that $Y=\cup\{A_i : i=1,2,\ldots,n\}$. That is $\{A_1,A_2,\ldots,A_n\}$ is a finite subcover of $\{A_i : i\in I\}$ for (Y,σ). Hence (Y,σ) is somewhat supra compact.

THEOREM 4.11. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be somewhat supra irresolute map from somewhat supra compact space (X,τ) onto supra topological space (Y,σ), then (Y,σ) is somewhat supra compact.

PROOF. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be somewhat supra irresolute map from a somewhat supra compact space (X,τ) onto a supra topological space (Y,σ). Let $\{A_i : i\in I\}$ be a somewhat supra open cover of (Y, σ). Then $\{f^{-1}(A_i) : i\in I\}$ is a somewhat supra open cover of (X,τ), since f is somewhat supra irresolute. As (X,τ) is somewhat supra compact, the somewhat supra open cover $\{f^{-1}(A_i) : i\in I\}$ of (X,τ) has a finite subcover say $\{f^{-1}(A_i) : i=1,2,\ldots,n\}$. Therefore $X=\cup\{f^{-1}(A_i) : i=1,2,\ldots,n\}$, which implies $f(X)=\cup\{A_i : i=1,2,\ldots,n\}$, so that $Y=\cup\{A_i : i=1,2,\ldots,n\}$. That is $\{A_1,A_2,\ldots,A_n\}$ is a finite sub cover of $\{A_i : i\in I\}$ for (Y,σ). Hence (Y,σ) is somewhat supra compact.

THEOREM 4.12. If (X,τ) is supra compact and every somewhat supra closed set in X is also supra closed in X, then (X,τ) is somewhat supra compact.

PROOF. Let $\{A_i : i\in I\}$ be a somewhat supra open cover of X. Since every somewhat supra closed set in X is also supra closed in X. Thus $\{X-A_i : i\in I\}$ is a supra closed cover of X and hence $\{A_i : i\in I\}$ is a supra open cover of X. Since (X,τ) is supra compact. So there exists a finite subcover $\{A_i : i=1,2,\ldots,n\}$ of $\{A_i : i\in I\}$ such that $X=\cup\{A_i : i=1,2,\ldots,n\}$. Hence (X,τ) is a somewhat supra compact space.

THEOREM 4.13. A supra topological space (X,τ) is somewhat supra compact if and only if every family of somewhat supra closed sets of (X,τ) having finite intersection property has a nonempty intersection.

PROOF. Suppose (X,τ) is somewhat supra compact, Let $\{A_i : i\in I\}$ be a family of somewhat supra closed sets with finite intersection property. Suppose $\cap\{A_i : i\in I\}=\phi$, then $X-(\cap\{A_i : i\in I\})=X$. This implies $\cup\{(X-A_i) : i\in I\}=X$. Thus the cover $\{(X-A_i) : i\in I\}$ is a somewhat supra open cover of (X,τ). Then, the somewhat supra open cover $\{(X-A_i) : i\in I\}$ has a finite subcover say $\{(X-A_i) : i=1,2,\ldots,n\}$. This implies $X=\cup\{(X-A_i) : i=1,2,\ldots,n\}$ which implies $X=X-\cap\{A_i : i=1,2,\ldots,n\}$, which implies $X-X=\cap\{A_i : i=1,2,\ldots,n\}$ which implies $\phi=\cap\{A_i : i=1,2,\ldots,n\}$. This disproves the assumption. Hence $\cap\{A_i : i=1,2,\ldots,n\}\neq\phi$.

Conversely suppose (X,τ) is not somewhat supra compact. Then there exists a somewhat supra open cover of (X,τ) say $\{G_i : i\in I\}$ having no finite subcover. This implies for any finite subfamily $\{G_i : i=1,2,\ldots,n\}$ of $\{G_i : i\in I\}$, we have $\cup\{G_i : i = 1,2,\ldots,n\} \neq X,$ which implies $X - (\cup\{G_i : i = 1,2,\ldots,n\}) \neq X - X,$ therefore $\cap\{G_i : i = 1,2,\ldots,n\} \neq \phi.$ Then the family $\{X - G_i : i \in I\}$ of somewhat supra closed sets has a finite intersection property. Also by assumption $\cap\{X - G_i : i = 1,2,\ldots,n\} \neq \phi$ which implies $X - (\cup\{G_i : i = 1,2,\ldots,n\}) \neq \phi,$ so that $\cup\{G_i : i = 1,2,\ldots,n\} \neq X.$ This implies $\{G_i : i \in I\}$ is not a cover of $(X, \tau).$ This disproves the fact that $\{G_i : i \in I\}$ is a cover for $(X, \tau).$ Therefore a somewhat supra open cover $\{G_i : i \in I\}$ of (X, τ) has a finite subcover $\{G_i : i = 1,2,\ldots,n\}.$ Hence (X, τ) is somewhat supra compact.

THEOREM 4.14. Let A be a somewhat supra compact set relative to a supra topological space X and B be a somewhat supra closed subset of $X.$ Then $A \cap B$ is somewhat supra compact relative to $X.$

PROOF. Let A be somewhat supra compact relative to $X.$ Let $\{A_i : i \in I\}$ be a cover of $A \cap B$ by somewhat supra open sets in $X.$ Then $\{A_i : i \in I\} \cup \{B^C\}$ is a cover of A by somewhat supra open sets in $X,$ but A is somewhat supra compact relative to $X,$ so there exist $i_1, i_2, \ldots, i_n \in I$ such that $A \subseteq (\cup\{A_i : i = 1,2,\ldots,n\}) \cup B^C.$ Then

$$A \cap B \subseteq \cup\{\cup A_i \cap B : i = 1,2,\ldots,n\} \subseteq \cup\{A_i : i = 1,2,\ldots,n\}.$$

Hence $A \cap B$ is somewhat supra compact relative to $X.$

THEOREM 4.15. If a function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is somewhat supra irresolute and a subset of X is somewhat supra compact relative to $X,$ then $f(B)$ is somewhat supra compact relative to $Y.$

PROOF. Let $\{A_i : i \in I\}$ be a cover of $f(B)$ by somewhat supra open subsets of $Y.$ Since f is somewhat supra irresolute. Then $\{f^{-1}(A_i) : i \in I\}$ is a cover of B by somewhat supra open subsets of $X.$ Since B is somewhat supra compact relative to $X.$ $\{f^{-1}(A_i) : i \in I\}$ has a finite subcover say $\{f^{-1}(A_1), f^{-1}(A_2), \ldots, f^{-1}(A_n)\}$ for $B.$ Now $\{A_i : i = 1,2,\ldots,n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for $f(B).$ So $f(B)$ is somewhat supra compact relative to $Y.$

V. COUNTABLY SOMEWHAT SUPRA COMPACTNESS

In this section, we concentrate on the concept of countably somewhat supra compactness and its properties.

DEFINITION 5.1. A supra topological space (X, τ) is said to be countably somewhat supra compact if every countable somewhat supra open cover of X has a finite subcover.

THEOREM 5.2. If (X, τ) is a countably somewhat supra compact space, then (X, τ) is countably supra compact.

PROOF. Let (X, τ) be countably somewhat supra compact space. Let $\{A_i : i \in I\}$ be a countable supra open cover of $(X, \tau).$ Since $\tau \subseteq SW\sup(X, \tau).$ So $\{A_i : i \in I\}$ is a countable somewhat supra open cover of $(X, \tau).$ Since (X, τ) is countably somewhat supra compact, countable somewhat supra open cover $\{A_i : i \in I\}$ of (X, τ) has a finite subcover say $\{A_i : i = 1,2,\ldots,n\}$ for X. Hence (X, τ) is a countably supra compact space.

THEOREM 5.3. If (X, τ) is countably supra compact and every somewhat supra closed subset of X is supra closed in $X,$ then (X, τ) is countably somewhat supra compact.

PROOF. Let (X, τ) be countably supra compact space. Let $\{A_i : i \in I\}$ be a countable somewhat supra open cover of $(X, \tau).$ Since every somewhat supra closed subset of X is supra closed in $X.$ Thus every somewhat supra open set in X is supra open in $X.$ Therefore $\{A_i : i \in I\}$ is a countable supra open cover of $(X, \tau).$ Since (X, τ) is countably supra compact, countable supra open cover $\{A_i : i \in I\}$ of (X, τ) has a finite subcover say $\{A_i : i = 1,2,\ldots,n\}$ for $X.$ Hence (X, τ) is a countably somewhat supra compact space.

THEOREM 5.4. Every somewhat supra compact space is countably somewhat supra compact.

PROOF. Let (X, τ) be somewhat supra compact space. Let $\{A_i : i \in I\}$ be a countable somewhat supra open cover of $(X, \tau).$ Since (X, τ) is somewhat supra compact, somewhat supra pen cover $\{A_i : i \in I\}$ of (X, τ) has a finite subcover say $\{A_i : i = 1, 2, \ldots, n\}$ for (X, τ). Hence (X, τ) is a countably somewhat supra compact space.

THEOREM 5.5. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a somewhat supra continuous injective mapping. If X is countably somewhat supra compact space, then (Y, σ) is countably supra compact.

PROOF. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a somewhat supra continuous map from a countably somewhat supra compact space (X, τ) onto a supra topological space (Y, σ). Let $\{A_i : i \in I\}$ be a countable supra open cover of Y. Then $\{f^{-1}(A_i) : i \in I\}$ is a countable somewhat supra open cover of X, as f is somewhat supra continuous. Since X is countably somewhat supra compact, the countable somewhat supra open cover $\{f^{-1}(A_i) : i \in I\}$ of X has a finite subcover say $\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$, which implies $f(X) = \cup\{A_i : i = 1, 2, \ldots, n\}$, then $Y = \cup\{A_i : i = 1, 2, \ldots, n\}$. That is $\{A_i : i = 1, 2, \ldots, n\}$ is a finite subcover of $\{A_i : i \in I\}$ for Y. Hence Y is countably supra compact.

THEOREM 5.6. If a map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is perfectly somewhat supra continuous map from a countably supra compact space (X, τ) onto a supra topological space (Y, σ), then (Y, σ) is countably somewhat supra compact.

PROOF. Let $\{A_i : i \in I\}$ be a countable somewhat supra open cover of (Y, σ). Since f is perfectly somewhat supra continuous, $\{f^{-1}(A_i) : i \in I\}$ is a countable supra open cover of (X, τ). Again, since (X, τ) is countably supra compact, the countable supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, τ) has a finite subcover say $\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$, which implies $f(X) = \cup\{A_i : i = 1, 2, \ldots, n\}$, so that $Y = \cup\{A_i : i = 1, 2, \ldots, n\}$. That is $\{A_1, A_2, \ldots, A_n\}$ is a finite subcover of $\{A_i : i \in I\}$ for (Y, σ). Hence (Y, σ) is countably somewhat supra compact.

THEOREM 5.7. If a map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is strongly somewhat supra continuous map from a countably supra compact space (X, τ) onto a supra topological space (Y, σ), then (Y, σ) is countably somewhat supra compact.

PROOF. Let $\{A_i : i \in I\}$ be a countable somewhat supra open cover of (Y, σ). Since f is strongly somewhat supra continuous, $\{f^{-1}(A_i) : i \in I\}$ is a countable supra open cover of (X, τ). Again, since (X, τ) is countably supra compact, the countable supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, τ) has a finite sub cover say $\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$, which implies $f(X) = \cup\{A_i : i = 1, 2, \ldots, n\}$, so that $Y = \cup\{A_i : i = 1, 2, \ldots, n\}$. That is $\{A_1, A_2, \ldots, A_n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y, σ). Hence (Y, σ) is countably somewhat supra compact.

THEOREM 5.8. The image of a countably somewhat supra compact space under a somewhat supra irresolute map is countably somewhat supra compact.

PROOF. If a map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is somewhat supra irresolute map from a countably somewhat supra compact space (X, τ) onto a supra topological space (Y, σ). Let $\{A_i : i \in I\}$ be a countable somewhat supra open cover of (Y, σ). Then $\{f^{-1}(A_i) : i \in I\}$ is a countable somewhat supra open cover of (X, τ), since f is somewhat supra irresolute. As (X, τ) is countably somewhat supra compact, the countable somewhat supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, τ) has a finite sub cover say $\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$, which implies $f(X) = \cup\{A_i : i = 1, 2, \ldots, n\}$, so that $Y = \cup\{A_i : i = 1, 2, \ldots, n\}$. That is $\{A_1, A_2, \ldots, A_n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y, σ). Hence (Y, σ) is countably somewhat supra compact.

VI. SOMEWHAT SUPRA LINDELOF SPACE

In this section, we concentrate on the concept of somewhat supra Lindelof space and their properties.

DEFINITION 6.1. A supra topological space $(\mathbf{X}, \tau)$ is said to be somewhat supra Lindelof space if every somewhat supra open cover of $\mathbf{X}$ has a countable subcover.

THEOREM 6.2. Every somewhat supra Lindelof space is supra Lindelof space.

PROOF. Let $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}\}$ be a supra open cover of $(\mathbf{X}, \tau)$. Since $\tau \subseteq \mathbf{SW}\sup(\mathbf{X}, \tau)$. Therefore $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}\}$ is a somewhat supra open cover of $(\mathbf{X}, \tau)$. Since $(\mathbf{X}, \tau)$ is somewhat supra Lindelof space, somewhat supra open cover $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}\}$ of $(\mathbf{X}, \tau)$ has a countable subcover say $\{\mathbf{A}_i : \mathbf{i} = 1, 2, \ldots, \mathbf{n}\}$ for $\mathbf{X}$. Hence $(\mathbf{X}, \tau)$ is a supra Lindelof space.

Theorem 6.3. If $(\mathbf{X}, \tau)$ is supra Lindelof space and every somewhat supra closed subset of $\mathbf{X}$ is supra closed in $\mathbf{X}$, then $(\mathbf{X}, \tau)$ is somewhat supra Lindelof space.

PROOF. Let $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}\}$ be a somewhat supra open cover of $(\mathbf{X}, \tau)$. Since every somewhat supra closed subset of $\mathbf{X}$ is supra closed in $\mathbf{X}$. So every somewhat supra open set in $\mathbf{X}$ is supra open in $\mathbf{X}$. Therefore $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}\}$ is a supra open cover of $(\mathbf{X}, \tau)$. Since $(\mathbf{X}, \tau)$ is supra compact, supra open cover $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}\}$ of $(\mathbf{X}, \tau)$ has a countable subcover say $\{\mathbf{A}_i : \mathbf{i} = 1, 2, \ldots, \mathbf{n}\}$ for X. Hence $(\mathbf{X}, \tau)$ is a somewhat supra Lindelof space.

THEOREM 6.4. Every somewhat supra compact space is somewhat supra Lindelof space.

PROOF. Let $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}\}$ be a somewhat supra open cover of $(\mathbf{X}, \tau)$. Since $(\mathbf{X}, \tau)$ is somewhat supra compact space. Then $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}\}$ has a finite subcover say $\{\mathbf{A}_i : \mathbf{i} = 1, 2, \ldots, \mathbf{n}\}$. Since every finite subcover is always countable subcover and therefore $\{\mathbf{A}_i : \mathbf{i} = 1, 2, \ldots, \mathbf{n}\}$ is countable subcover of $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}\}$. Hence $(\mathbf{X}, \tau)$ is somewhat supra Lindelof space.

THEOREM 6.5. A somewhat supra continuous image of a somewhat supra Lindelof space is supra Lindelof space.

PROOF. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a somewhat supra continuous map from a somewhat supra Lindelof space $\mathbf{X}$ onto a supra topological space $\mathbf{Y}$. Let $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}\}$ be a supra open cover of $\mathbf{Y}$. Then $\{f^{-1}(A_i) : i \in I\}$ is a somewhat supra open cover of $\mathbf{X}$, as f is somewhat supra continuous. Since $\mathbf{X}$ is somewhat supra Lindelof space, the somewhat supra open cover $\{f^{-1}(A_i) : i \in I\}$ of $\mathbf{X}$ has a countable subcover say $\{f^{-1}(A_i) : i \in I_0\}$ for some $I_0 \subseteq I$ and I_0 is countable. Therefore $X = \mathrm{U}\{f^{-1}(A_i) : i \in I_0\}$, which implies $f(X) = \mathrm{U}\{A_i : i \in I_0\}$, then $Y = \mathrm{U}\{A_i : i \in I_0\}$. That is $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}_0\}$ is a countable subcover of $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}\}$ for $\mathbf{Y}$. Hence $\mathbf{Y}$ is supra Lindelof space.

THEOREM 6.6. The image of a somewhat supra Lindelof space under a somewhat supra irresolute map is somewhat supra Lindelof space.

PROOF. Let a map $f : (X, \tau) \longrightarrow (Y, \sigma)$ be somewhat supra irresolute map from a somewhat supra Lindelof space $(\mathbf{X}, \tau)$ onto a supra topological space (Y, σ). Let $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}\}$ be a somewhat supra open cover of (Y, σ). Then $\{f^{-1}(A_i) : i \in I\}$ is a somewhat supra open cover of $(\mathbf{X}, \tau)$. Since f is somewhat supra irresolute. As $(\mathbf{X}, \tau)$ is somewhat supra Lindelof space, the somewhat supra open cover $\{f^{-1}(A_i) : i \in I\}$ of $(\mathbf{X}, \tau)$ has a countable subcover say $\{f^{-1}(A_i) : i \in I_0\}$ for some $I_0 \subseteq I$ and I_0 is countable. Therefore $X = \mathrm{U}\{f^{-1}(A_i) : i \in I_0\}$, which implies $f(X) = \mathrm{U}\{A_i : i \in I_0\}$, so that $Y = \mathrm{U}\{A_i : i \in I_0\}$. That is $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}_0\}$ is a countable subcover of $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}\}$ for Y. Hence (Y, σ) is somewhat supra Lindelof space.

THEOREM 6.7. If $(\mathbf{X}, \tau)$ is somewhat supra Lindelof space and countably somewhat supra compact space, then $(\mathbf{X}, \tau)$ is somewhat supra compact space.

PROOF. Suppose $(\mathbf{X}, \tau)$ is somewhat supra Lindelof space and countably somewhat supra compact space. Let $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}\}$ be a somewhat supra open cover of $(\mathbf{X}, \tau)$. Since $(\mathbf{X}, \tau)$ is somewhat supra Lindelof space, $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}\}$ has a countable subcover say $\{\mathbf{A}_i : \mathbf{i} \in \mathbf{I}_0\}$ for some $I_0 \subseteq I$ and I_0 is countable. Therefore

$\{A_i : i \in I_0\}$ is a countable somewhat supra open cover of (X, τ). Again, since (X, τ) is countably somewhat supra compact space, $\{A_i : i \in I_0\}$ has a finite subcover and say $\{A_i : i = 1, 2, ..., n\}$. Therefore $\{A_i : i = 1, 2, ..., n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (X, τ). Hence (X, τ) is somewhat supra compact space.

THEOREM 6.8. If a function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is somewhat supra irresolute and a subset of X is somewhat supra Lindelof relative to X, then $f(B)$ is somewhat supra Lindelof relative to Y.

PROOF. Let $\{A_i : i \in I\}$ be a cover of $f(B)$ by somewhat supra open subsets of Y. By hypothesis f is somewhat supra irresolute and so $\{f^{-1}(A_i) : i \in I\}$ is a cover of B by somewhat supra open subsets of X. Since B is somewhat supra Lindelof relative to X, $\{f^{-1}(A_i) : i \in I\}$ has a countable subcover say $\{f^{-1}(A_i) : i \in I_0\}$ for B, where I_0 is a countable subset of I. Now $\{A_i : i \in I_0\}$ is a countable subcover of $\{A_i : i \in I\}$ for $f(B)$. So $f(B)$ is somewhat supra Lindelof relative to Y.

VII. SOMEWHAT SUPRA CONNECTEDNESS

DEFINITION 7.1. A supra topological space (X, τ) is said to be supra connected if X cannot be written as a disjoint union of two nonempty supra open sets. A subset of (X, τ) is supra connected if it is supra connected as a subspace.

DEFINITION 7.2. A supra topological space (X, τ) is said to be somewhat supra connected if X cannot be written as a disjoint union of two nonempty somewhat supra open sets. A subset of (X, τ) is somewhat supra connected if it is somewhat supra connected as a subspace.

THEOREM 7.2. Every somewhat supra connected space is supra connected.

PROOF. Let A and B be two nonempty disjoint proper supra open sets in X. Since every supra open set is somewhat supra open set. Therefore A and B are nonempty disjoint proper somewhat supra open sets in X and X is somewhat supra connected space. Therefore $X \neq A \cup B$. Therefore X is connected.

The converse of the above theorem need not be true in general, which follows from the following example.

EXAMPLE 7.3. Let $X = \{1, 2, 3, 4\}$ and $\tau = \{\phi, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, X\}$. Then (X, τ) is a supra topological space. Since X cannot be written as a disjoint union of any two nonempty supra open sets. Therefore (X, τ) is a connected supra topological space. We notice $\{1\}$ and $\{2,3,4\}$ are nonempty disjoint somewhat supra open sets such $X = \{1\} \cup \{2,3,4\}$. Therefore (X, τ) is not a somewhat supra connected space.

THEOREM 7.4. For a supra topological space (X, τ) the following statements are equivalent

(1) (X, τ) is somewhat supra connected.

(2) The only subsets of (X, τ) which are both somewhat supra open and somewhat supra closed are the empty set X and ϕ.

(3) Each somewhat supra continuous map of (X, τ) into a discrete space (Y, σ) with at least two points is a constant map.

PROOF. $(1) \Rightarrow (2)$: Let G be a somewhat supra open and somewhat supra closed subset of (X, τ). Then $X - G$ is also both somewhat supra open and somewhat supra closed. Then $X = G \cup (X - G)$ a disjoint union of two nonempty somewhat supra open sets which contradicts the fact that (X, τ) is somewhat supra connected. Hence $G = \phi$ or $G = X$.

$(2) \Rightarrow (1)$: Suppose that $X = A \cup B$ where A and B are disjoint nonempty somewhat open subsets of (X, τ). Since $A = X - B$, then A is both somewhat supra open and somewhat supra closed. By assumption $A = \phi$ or $A = X$, which is a contradiction. Hence (X, τ) is somewhat supra connected.

$(2) \Rightarrow (3)$: Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a somewhat supra continuous map, where (Y, σ) is discrete space with at least two points. Then $f^{-1}(y)$ is somewhat supra closed and somewhat supra open for each $y \in Y$. That is (X, τ) is covered by somewhat supra closed and somewhat supra open covering $\{f^{-1}(y) : y \in Y\}$. By assumption, $f^{-1}(y) = \phi$ or $f^{-1}(y) = X$ for each

$y \in Y$. If $f^{-1}(y) = \phi$ for each $y \in Y$, then f fails to be a map. Therefore their exists at least one point say $f^{-1}(y^*) \neq \phi$, $y^* \in Y$ such that $f^{-1}(y^*) = X$. This shows that f is a constant map.

$(3) \Rightarrow (2)$: Let G be both somewhat supra open and somewhat supra closed in (X, τ). Suppose $G \neq \phi$. Let $f:(X,\tau) \longrightarrow (Y,\sigma)$ be a somewhat supra continuous map defined by $f(G) = \{a\}$ and $f(X-G) = \{b\}$ where $a \neq b$ and $a,b \in Y$. By assumption, f is constant so $G = X$.

THEOREM 7.5. Let $f:(X,\tau) \longrightarrow (Y,\sigma)$ be a somewhat supra continuous surjection and (X,τ) is somewhat supra connected. Then (Y,σ) is supra connected.

PROOF. Suppose (Y,σ) is not supra connected. Let $Y = A \cup B$, where A and B are disjoint nonempty supra open subsets in (Y,σ). Since f is somewhat supra continuous, $X = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nonempty somewhat supra open subsets in (X,τ). This disproves the fact that (X,τ) is somewhat supra connected. Hence (Y,σ) is supra connected.

THEOREM 7.6. If $f:(X,\tau) \longrightarrow (Y,\sigma)$ is a somewhat supra irresolute surjection and X is somewhat supra connected, then Y is somewhat supra connected.

PROOF. Suppose that Y is not somewhat supra connected. Let $Y = A \cup B$, where A and B are nonempty somewhat supra open sets in Y. Since f is somewhat supra irresolute and onto, $X = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nonempty somewhat supra open sets in (X,τ). This contradicts the fact that (X,τ) is somewhat supra connected. Hence (Y,σ) is somewhat supra connected.

THEOREM 7.7. Suppose that every somewhat supra closed set in X is supra closed in X and X is supra connected. Then X is somewhat supra connected.

PROOF. Suppose that X is supra connected. Then X cannot be expressed as disjoint union of two nonempty proper supra open subset of X. Suppose X is not somewhat supra connected space. Let A and B be any two somewhat supra open subsets of X such that X = A∪ B, where $A \cap B = \phi$ and $A \subseteq X$, $B \subseteq X$. Since every somewhat supra closed set in X is supra closed in X. So every somewhat supra open set in X is supra open in X. Hence A and B are supra open subsets of X, which contradicts that X is supra connected. Therefore X is somewhat supra connected.

THEOREM 7.8. If the somewhat supra open sets C and D form a separation of X and if Y is somewhat supra connected subspace of X, then Y lies entirely within C or D.

PROOF. By hypothesis C and D are both somewhat supra open sets in X. The sets $C \cap Y$ and $D \cap Y$ are somewhat supra open in Y. These two sets are disjoint and their union is Y. If they were both nonempty, they would constitute a separation of Y. Therefore one of them is empty. Hence Y must lie entirely in C or D.

THEOREM 7.9. Let A be a somewhat supra connected subspace of X. If $A \subseteq B \subseteq SW\ supraCl(A)$, then B is also somewhat supra connected.

PROOF. Let A be somewhat supra connected. Let $A \subseteq B \subseteq SW\ supraCl(A)$. Suppose that $B = C \cup D$ is a separation of B by somewhat supra open sets. Thus by the previous theorem above A must lie entirely in C or D. Suppose that $A \subseteq C$, then $SW\ supraCl(A) \subseteq SW\ supraCl(C)$. Since $SW\ supraCl(C)$ and D are disjoint, B cannot intersect D. This disproves the fact that D is nonempty subset of B. So $D = \phi$ which implies B is somewhat supra connected.

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