

# Connectedness in Soft Minimal Structure

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**Abstract**—In the present paper we introduces the concept of soft connectedness in soft m-structure and studied some of their properties and characterizations.

**Index Terms**—Soft m-structure , Soft m-connectedness and Soft m-connectedness between soft sets.

## I. INTRODUCTION

The concept of soft set is fundamentally important in almost every scientific field. Soft set theory is a new mathematical tool for dealing with uncertainties and is a set associated with parameters and has been applied in several directions. Since in 1999 Molodtsov [19] originated the idea of soft sets. In 2002, Maji et. al [15], gave first practical application of soft sets in decision making problems. Many researchers have contributed toward the algebraic structures of soft set theory ([1], [23]). In 2011 Shabir and Naz [21] initiated the study of soft topological spaces. In the recent past many soft topological concepts such as soft mappings ([12], [25], [9], [10], [13]). Soft regular-open sets[6], soft semi-open sets[17], soft preopen sets [2], soft  $\alpha$ -open sets [3], soft  $\beta$ -open sets [4], soft b-open sets [5], soft connectedness [11], [20], soft semi-connectedness [8], [17], soft preconnectedness [24] etc. play an important part in soft topological spaces. In the present paper we introduces the concept of soft connectedness in soft m-structure and studied some of their properties and characterizations.

## II. PRELIMINARIES

Since we shall require the following known definitions, notations and some properties, we recall them in this section. Let  $U$  is an initial universe set ,  $E$  be a set of parameters ,  $P(U)$  denote the power set of  $U$  and  $A \subseteq E$ .

**Definition 2.1:** [19] A pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F: A \rightarrow P(U)$ . In other words, a soft set over  $U$  is a parameterized family of subsets of the universe  $U$ . For all  $e \in A$ ,  $F(e)$  may be considered as the set of  $e$ -approximate elements of the soft set  $(F, A)$ .

**Definition 2.2:** [16] For two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$ , we say that  $(F, A)$  is a soft subset of  $(G, B)$ , denoted by  $(F, A) \subseteq (G, B)$ , if

- $A \subseteq B$  and
- $F(e) \subseteq G(e)$  for all  $e \in E$ .

**Definition 2.3:** [16] Two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  are said to be soft equal denoted by  $(F, A) = (G, B)$  If  $(F, A) \subseteq (G, B)$  and  $(G, B) \subseteq (F, A)$ .

**Definition 2.4:** [7] The complement of a soft set  $(F, A)$ , denoted by  $(F, A)^c$ , is defined by  $(F, A)^c = (F^c, A)$ , where  $F^c: A \rightarrow P(U)$  is a mapping given by  $F^c(e) = U - F(e)$ , for all  $e \in E$ .

**Definition 2.5:** [16] Let a soft set  $(F, A)$  over  $U$ .

- Null soft set denoted by  $\phi$  if for all  $e \in A$ ,  $F(e) = \phi$ .
- Absolute soft set denoted by  $\tilde{U}$ , if for each  $e \in A$ ,  $F(e) = U$ .

Clearly,  $\tilde{U}^c = \phi$  and  $\phi^c = \tilde{U}$ .

**Definition 2.6:** [7] Union of two sets  $(F, A)$  and  $(G, B)$  over the common universe  $U$  is the soft  $(H, C)$ , where  $C = A \cup B$ , and for all  $e \in C$ ,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ H(e), & \text{if } e \in A \cap B \end{cases}$$

**Definition 2.7:** [7] Intersection of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$ , is the soft set  $(H, C)$  where  $C = A \cap B$  and  $H(e) = F(e) \cap G(e)$  for each  $e \in E$ .

Let  $X$  and  $Y$  be an initial universe sets and  $E$  and  $K$  be the non empty sets of parameters,  $S(X, E)$  denotes the family of all soft sets over  $X$  and  $S(Y, K)$  denotes the family of all soft sets over  $Y$ .

**Definition 2.8:** [12] Let  $S(X, E)$  and  $S(Y, K)$  be families of soft sets. Let  $u: X \rightarrow Y$  and  $p: E \rightarrow K$  be mappings. Then a mapping  $f_{pu}: S(X, E) \rightarrow S(Y, K)$  is defined as:

(i) Let  $(F, A)$  be a soft set in  $S(X, E)$ . The image of  $(F, A)$  under  $f_{pu}$ , written as  $f_{pu}(F, A) = (f_{pu}(F), p(A))$ , is a soft set in  $S(Y, K)$  such that

$$f_{pu}(F)(k) = \begin{cases} \bigcup_{e \in p^{-1}(k) \cap A} u(F(e)), & p^{-1}(k) \cap A \neq \phi \\ \phi, & p^{-1}(k) \cap A = \phi \end{cases}$$

For all  $k \in K$ .

(ii) Let  $(G, B)$  be a soft set in  $S(Y, K)$ . The inverse image of  $(G, B)$  under  $f_{pu}$ , written as

$$f_{pu}^{-1}(G)(e) = \begin{cases} u^{-1}G(p(e)), & p(e) \in B \\ \phi, & \text{otherwise} \end{cases}$$

For all  $e \in E$ .

**Definition 2.9:** [18] Let  $f_{pu}: S(X, E) \rightarrow S(Y, K)$  be a mapping and  $u: X \rightarrow Y$  and  $p: E \rightarrow K$  be mappings. Then  $f_{pu}$  is soft onto, if  $u: X \rightarrow Y$  and  $p: E \rightarrow K$  are onto and  $f_{pu}$  is soft one-one, if  $u: X \rightarrow Y$  and  $p: E \rightarrow K$  are one-one.

**Definition 2.10:** [21] A subfamily  $\tau$  of  $S(X, E)$  is called a soft topology on  $X$  if:

- 1)  $\tilde{\phi}, \tilde{X}$  belong to  $\tau$ .
- 2) The union of any number of soft sets in  $\tau$  belongs to  $\tau$ .
- 3) The intersection of any two soft sets in  $\tau$  belongs to  $\tau$ .

The triplet  $(X, \tau, E)$  is called a soft topological space over  $X$ . The members of  $\tau$  are called soft open sets in  $X$  and their complements called soft closed sets in  $X$ .

**Definition 2.11:** If  $(X, \tau, E)$  is soft topological space and a soft set  $(F, E)$  over  $X$ .

(a) The soft closure of  $(F, E)$  is denoted by  $Cl(F, E)$ , is defined as the intersection of all soft closed super sets of  $(F, E)$  [21].

(b) The soft interior of  $(F, E)$  is denoted by  $Int(F, E)$ , is defined as the soft union of all soft open subsets of  $(F, E)$  [25].

**Definition 2.12:** [25] The soft set  $(F, E) \in S(X, E)$  is called a soft point if there exist  $x \in X$  and  $e \in E$  such that  $F(e) = \{x\}$  and  $F(e') = \phi$  for each  $e' \in E - \{e\}$ , and the soft point  $(F, E)$  is denoted by  $x_e$ .

**Definition 2.13:** [14] Let  $(X, \tau, E)$  be a soft topological space, and  $(A, E), (B, E)$  be two soft sets over  $X$ . The soft sets  $(A, E)$  and  $(B, E)$  are said to soft separated, if  $(A, E) \cap Cl(B, E) = \phi$  and  $Cl(A, E) \cap (B, E) = \phi$ .

**Definition 2.14:** [14] Let  $(X, \tau, E)$  be a soft topological space, and If there exist two non-empty soft separated sets  $(A, E), (B, E)$  such that  $(A, E) \cup (B, E) = \tilde{X}$ , then  $(A, E)$  and  $(B, E)$  are said to be soft disconnection for soft topological space  $(X, \tau, E)$ .  $(X, \tau, E)$  is said to be soft disconnected if  $(X, \tau, E)$  has a soft disconnection. Otherwise,  $(X, \tau, E)$  is said to be soft connected.

**Definition 2.15:** [17] Let  $(X, \tau, E)$  be soft topological space. Two nonempty soft sub sets  $(F, A)$  and  $(F, B)$  of  $S(X, E)$  are called soft semiseparated iff  $scl(F, A) \cap (F, B) = (F, A) \cap scl(F, B) = \phi$ .

**Definition 2.16:** [17] Let  $(X, \tau, E)$  be a soft topological space. If there does not exist a soft semiseparation of  $X$ , then it is said to be soft s-connected.

**Definition 2.17:** [24] Let  $(X, \tau, E)$  be soft topological space. Two nonempty soft sub sets  $(F, A)$  and  $(F, B)$  of  $S(X, E)$  are called soft pre-separated iff  $Pcl(F, A) \cap (F, B) = (F, A) \cap Pcl(F, B) = \phi$ .

**Definition 2.18:** [24] Let  $(X, \tau, E)$  be a soft topological space. If there does not exist a soft pre-separation of  $X$ , then it is said to be soft P-connected.

**Definition 2.19:** A soft set  $(A, E)$  of a soft topological space  $(X, \tau, E)$  is called :

- (a) Soft regular open  $(A, E) = Int(Cl(A, E))$  [6];
- (b) Soft  $\alpha$ -open if  $(A, E) \subset Int(Cl(Int(A, E)))$  [3];
- (c) Soft semiopen if  $(A, E) \subset Cl(Int(A, E))$  [17];
- (d) Soft preopen if  $(A, E) \subset Int(Cl(A, E))$  [2];
- (e) Soft b-open if  $(A, E) \subset Int(Cl(A, E)) \cup Cl(Int(A, E))$  [5].
- (f) Soft  $\beta$ -open if  $(A, E) \subset Cl(Int(Cl(A, E)))$  [4]

The family of all soft regular open (resp. soft  $\alpha$ -open, soft semi open, soft pre open, soft  $\beta$ -open, soft b-open) sets of  $X$  will be denoted by  $SRO(X, E)$  (resp.  $S\alpha O(X, E)$ ,  $SSO(X, E)$ ,  $SPO(X, E)$ ,  $S\beta O(X, E)$ ,  $SbO(X, E)$ ).

**Definition 2.20:** Let  $(A, E)$  be a soft subset of a soft topological space  $(X, \tau, E)$ . Then:

- (a) The intersection of all soft semi open sets containing  $(A, E)$  is called semi closure of  $(A, E)$ . It is denoted by  $scl(A, E)$  [17].
- (b) The intersection of all soft pre open sets containing  $(A, E)$  is called preclosure of  $(A, E)$ . It is denoted by  $pcl(A, E)$  [2].
- (c) The intersection of all soft  $\alpha$  open sets containing  $(A, E)$  is called  $\alpha$ -closure of  $(A, E)$ . It is denoted by  $\alpha cl((A, E))$  [3].
- (d) The intersection of all soft b-open sets containing  $(A, E)$  is called b-closure of  $(A, E)$ . It is denoted by  $bcl(A, E)$  [5].
- (e) The intersection of all soft  $\beta$ -open sets containing  $(A, E)$  is called  $\beta$ -closure of  $(A, E)$ . It is denoted by  $\beta cl(A, E)$  [4].

**Definition 2.21:** A soft mapping  $f_{pu} : (X, \tau, E) \rightarrow (X, \sigma, K)$  is said be :

- (a) Soft continuous if  $f_{pu}^{-1}(U, K) \in \tau$  for every soft set  $(U, K) \in \sigma$  [25].
- (b) Soft  $\alpha$ -continuous if  $f_{pu}^{-1}(U, K) \in S\alpha O(X, E)$  for every soft set  $(U, K) \in \sigma$  [3].
- (c) Soft semi continuous if  $f_{pu}^{-1}(U, K) \in SSO(X, E)$  for every soft set  $(U, K) \in \sigma$  [17].
- (d) Soft pre continuous if  $f_{pu}^{-1}(U, K) \in SPO(X, E)$  for every soft set  $(U, K) \in \sigma$  [2].
- (e) Soft b-continuous if  $f_{pu}^{-1}(U, K) \in SbO(X, E)$  for every soft set  $(U, K) \in \sigma$  [5].
- (f) Soft  $\beta$ -continuous if  $f_{pu}^{-1}(U, K) \in S\beta O(X, E)$  for every soft set  $(U, K) \in \sigma$  [4].

**Definition 2.22:** A soft mapping  $f_{pu} : (X, \tau, E) \rightarrow (X, \sigma, K)$  is said be :

- (a) Soft open if  $f_{pu}(U, E) \in \sigma$  for every soft set  $(U, E) \in \tau$  [26].
- (b) Soft  $\alpha$ -open if  $f_{pu}(U, E) \in S\alpha O(Y, K)$  for every soft set  $(U, E) \in \tau$  [3].
- (c) Soft semi open if  $f_{pu}(U, E) \in SSO(Y, K)$  for every soft set  $(U, E) \in \tau$  [17].
- (d) Soft pre open if  $f_{pu}(U, E) \in SPO(Y, K)$  for every soft set  $(U, E) \in \tau$  [2].
- (e) Soft b-open if  $f_{pu}(U, E) \in SbO(Y, K)$  for every soft set  $(U, E) \in \tau$  [5].
- (f) Soft  $\beta$ -open if  $f_{pu}(U, E) \in S\beta O(Y, K)$  for every soft set  $(U, E) \in \tau$  [4].

**Definition 2.23:** [22] A soft subfamily  $m_{(X, E)}$  of  $S(X, E)$  over  $X$  is called a soft minimal structure (briefly soft m-structure) on  $X$  if  $\phi \in m_{(X, E)}$  and  $\tilde{X} \in m_{(X, E)}$ .

Each member of  $m_{(X, E)}$  is called a soft m-open set and complement of a soft m-open set is called a soft m-closed set.

*Remark 2.24:* [22] Let  $(X, \tau, E)$  be a soft topological space. Then the families  $\tau$ ,  $SO(X, E)$ ,  $SPO(X, E)$ ,  $S\alpha O(X, E)$ ,  $S\beta O(X, E)$ ,  $SbO(X, E)$ ,  $SRO(X, E)$ , are all soft m-structures on  $X$ .

*Definition 2.25:* [22] Let  $X$  be a nonempty set,  $E$  be set of parameters and  $m_{(X, E)}$  be a soft m-structure over  $X$ . The soft  $m_{(X, E)}$ -closure and the soft  $m_{(X, E)}$ -interior of a soft set  $(A, E)$  over  $X$  are defined as follows :

- (1)  $m_{(X, E)}\text{-Cl}(A, E) = \cap \{(F, E) : (A, E) \subset (F, E), (F, E)^c \in m_{(X, E)}\}$   
 (2)  $m_{(X, E)}\text{-Int}(A, E) = \cup \{(F, E) : (F, E) \subset (A, E), (F, E) \in m_{(X, E)}\}$ .

*Remark 2.26:* [22] Let  $(X, \tau, E)$  be a soft topological space and  $(A, E)$  be a soft set over  $X$ . If  $m_{(X, E)} = \tau$  (respectively  $SO(X, E)$ ,  $SPO(X, E)$ ,  $S\alpha O(X, E)$ ,  $S\beta O(X, E)$ ,  $SbO(X, E)$ ,  $SRO(X, E)$ ), then we have:

- (1)  $m_{(X, E)}\text{-Cl}(A, E) = \text{Cl}(A, E)$  (resp.  $\text{SCl}(A, E)$ ,  $\text{PCl}(A, E)$ ,  $\alpha\text{Cl}(A, E)$ ,  $\beta\text{Cl}(A, E)$ ,  $b\text{Cl}(A, E)$ ,  $S_\theta\text{Cl}(A, E)$ ),  
 (2)  $m_{(X, E)}\text{-Int}(A, E) = \text{Int}(A, E)$  (resp.  $\text{SInt}(A, E)$ ,  $\text{PInt}(A, E)$ ,  $\alpha\text{Int}(A, E)$ ,  $\beta\text{Int}(A, E)$ ,  $b\text{Int}(A, E)$ ,  $S_\theta\text{Int}(A, E)$ ).

*Theorem 2.27:* [22] Let  $S(X, E)$  be a family of soft sets and  $m_{(X, E)}$  a soft minimal structure on  $X$ .

For soft sets  $(A, E)$  and  $(B, E)$  of  $X$ , the following holds:

- (a) (i)  $m_{(X, E)}\text{-Int}(A, E)^c = (m_{(X, E)}\text{-Cl}(A, E))^c$  and  
 (ii)  $m_{(X, E)}\text{-Cl}(A, E)^c = (m_{(X, E)}\text{-Int}(A, E))^c$   
 (b) If  $(A, E)^c \in m_{(X, E)}$ , then  $m_{(X, E)}\text{-Cl}(A, E) = (A, E)$  and if  $(A, E) \in m_{(X, E)}$ , then  $m_{(X, E)}\text{-Int}(A, E) = (A, E)$ .  
 (c)  $m_{(X, E)}\text{-Cl}(\phi) = \phi$ ,  $m_{(X, E)}\text{-Cl}(\tilde{X}) = \tilde{X}$ ,  $m_{(X, E)}\text{-Int}(\phi) = \phi$ ,  $m_{(X, E)}\text{-Int}(\tilde{X}) = \tilde{X}$ .  
 (d) If  $(A, E) \subset (B, E)$ , then  $m_{(X, E)}\text{-Cl}(A, E) \subset m_{(X, E)}\text{-Cl}(B, E)$ ,  $m_{(X, E)}\text{-Int}(A, E) \subset m_{(X, E)}\text{-Int}(B, E)$ .  
 (e)  $(A, E) \subset m_{(X, E)}\text{-Cl}(A, E)$  and  $m_{(X, E)}\text{-Int}(A, E) \subset (A, E)$   
 (f)  $m_{(X, E)}\text{-Cl}(m_{(X, E)}\text{-Cl}(A, E)) = m_{(X, E)}\text{-Cl}(A, E)$  and  $m_{(X, E)}\text{-Int}(m_{(X, E)}\text{-Int}(A, E)) = m_{(X, E)}\text{-Int}(A, E)$

### III. CONNECTEDNESS IN SOFT MINIMAL STRUCTURE

*Definition 3.1:* Let  $X$  be a nonempty set,  $E$  be set of parameters and  $m_{(X, E)}$  be a soft m-structure over  $X$  with property **B**. In  $(X, m_{(X, E)})$  two nonempty soft sets  $(A, E)$  and  $(B, E)$  over  $X$  are called soft m-separated iff  $m_{(X, E)}\text{-Cl}(A, E) \cap (B, E) = (A, E) \cap m_{(X, E)}\text{-Cl}(B, E) = \phi$ .

*Remark 3.2:* Let  $(X, \tau, E)$  be a soft topological space over  $X$ . If,  $m_{(X, E)} = \tau$  (respt.  $SSO(X, E)$ ,  $SPO(X, E)$ ,  $SbO(X, E)$ ) and  $m_{(X, E)}\text{-Cl}(A, E) = \text{Cl}(A, E)$  (resp.  $\text{SCl}(A, E)$ ,  $\text{PCl}(A, E)$ ,  $b\text{Cl}(A, E)$ ) we get the definition of soft separated ( resp. soft semiseparated, soft pre-separated, soft b-separated) sets.

*Definition 3.3:* Let  $m_{(X, E)}$  be a soft m-structure over  $X$  with property **B**. Then  $(X, m_{(X, E)})$  is said to be soft m-connected, if there does not exist two nonempty soft m-separated sets  $(A, E)$  and  $(B, E)$  over  $X$ , such that  $(A, E) \cup (B, E) = \tilde{X}$ . Otherwise it is soft m-disconnected. In this case, the pair  $(A, E)$  and  $(B, E)$  is called the soft m-disconnection over  $X$ .

*Remark 3.4:* Let  $(X, \tau, E)$  be a soft topological space over  $X$ . If we replace soft m-separation by soft separated ( resp.

soft semiseparated, soft pre-separated, soft b-separated) sets we get the definition soft connectedness (resp. soft semi connectedness, soft pre connectedness, soft b-connectedness).

*Theorem 3.5:* Let  $(X, m_{(X, E)})$  be a soft m-structure over  $X$  with property **B**. Then the following conditions are equivalent :

- (1)  $(X, m_{(X, E)})$  has a soft m-disconnection.  
 (2) There exist two disjoint soft m-closed sets  $(A, E)$ ,  $(B, E) \in m_{(X, E)}$  such that  $(A, E) \cup (B, E) = \tilde{X}$ .  
 (3) There exist two disjoint soft m-open sets  $(A, E)$ ,  $(B, E) \in m_{(X, E)}$  such that  $(A, E) \cup (B, E) = \tilde{X}$ .  
 (4)  $(X, m_{(X, E)})$  has a proper soft m-open and soft m-closed set over  $X$ .

*Proof:* (1)  $\rightarrow$  (2) : Let  $(X, m_{(X, E)})$  have a soft m-disconnection  $(A, E)$  and  $(B, E)$ , Then  $(A, E) \cap (B, E) = \phi$  and  $m_{(X, E)}\text{-Cl}(A, E) = m_{(X, E)}\text{-Cl}(A, E) \cap ((A, E) \cup (B, E)) = (m_{(X, E)}\text{-Cl}(A, E) \cap (A, E)) \cup (m_{(X, E)}\text{-Cl}(A, E) \cap (B, E)) = (A, E)$ .

Therefore,  $(A, E)$  is soft m-closed set over  $X$ . Similar, we can see that  $(B, E)$  is also a soft m-closed set over  $X$ .

(2)  $\rightarrow$  (3) : Let  $(X, m_{(X, E)})$  has a soft m-disconnection  $(A, E)$  and  $(B, E)$  such that  $(A, E)$  and  $(B, E)$  are soft m-closed. Then  $(A, E)^c$  and  $(B, E)^c$  are soft m-open sets in  $m_{(X, E)}$ . Then it is easy to see  $(A, E)^c \cap (B, E)^c = \phi$  and  $(A, E)^c \cup (B, E)^c = \tilde{X}$ .

(3)  $\rightarrow$  (4) : Let  $(X, m_{(X, E)})$  have a soft m-disconnection  $(A, E)$  and  $(B, E)$  such that  $(A, E)$  and  $(B, E)$  are soft m-open over  $X$ . Then  $(A, E)$  and  $(B, E)$  are also soft closed in  $(X, m_{(X, E)})$ .

(4)  $\rightarrow$  (1) : Let  $(X, m_{(X, E)})$  has a proper soft m-open and soft m-closed set  $(F, E)$  over  $X$ . Put  $(H, E) = (F, E)^c$ . Then  $(H, E)$  and  $(F, E)$  are non-empty soft m-closed set in  $(X, m_{(X, E)})$ .  $(H, E) \cap (F, E) = \phi$  and  $(H, E) \cup (F, E) = \tilde{X}$ . Therefore,  $(H, E)$  and  $(F, E)$  is a soft m-disconnection of  $(X, m_{(X, E)})$ .

*Remark 3.6:* Let  $(X, \tau, E)$  be a soft topological space over  $X$ , if  $m_{(X, E)} = \tau$  (respt.  $SSO(X, E)$ ,  $SPO(X, E)$ ,  $SbO(X, E)$ ) Then the following conditions are equivalent :

- (1)  $(X, \tau, E)$  has a soft disconnection (respt. soft semi disconnection, soft pre disconnection, soft b-disconnection).  
 (2) There exist two disjoint soft closed (respt. soft semi-closed, soft pre-closed, soft b-closed) sets  $(A, E)$ ,  $(B, E)$  such that  $(A, E) \cup (B, E) = \tilde{X}$ .  
 (3) There exist two disjoint soft open (respt. soft semi-open, soft pre-open, soft b-open) sets  $(A, E)$ ,  $(B, E)$  such that  $(A, E) \cup (B, E) = \tilde{X}$ .  
 (4)  $(X, \tau, E)$  has a proper soft open (respt. soft semi-open, soft pre-open, soft b-open) and soft closed (respt. soft semi-closed, soft pre-closed, soft b-closed) set over  $X$ .

*Theorem 3.7:* Let  $(X, m_{(X, E)})$  be a soft m-structure over  $X$  with property **B**. Then the following conditions are equivalent :

- (1)  $(X, m_{(X, E)})$  is a soft m-connected.  
 (2) There exist two disjoint soft m-closed sets  $(A, E)$ ,  $(B, E) \in m_{(X, E)}$  such that  $(A, E) \cup (B, E) = \tilde{X}$ .  
 (3) There exist two disjoint soft m-open sets  $(A, E)$ ,  $(B, E) \in m_{(X, E)}$  such that  $(A, E) \cup (B, E) = \tilde{X}$ .  
 (4)  $(X, m_{(X, E)})$  at most has two soft m-closed and soft m-open sets over  $X$ , that is  $\phi$  and  $\tilde{X}$ .

**Remark 3.8:** Let  $(X, \tau, E)$  be a soft topological space over  $X$ , if  $m_{(X, E)} = \tau$  (respt.  $SSO(X, E), SPO(X, E), SbO(X, E)$ ), Then the following conditions are equivalent :

(1)  $(X, \tau, E)$  is a soft connected (respt. soft semi connected, soft pre connected, soft b-connected).

(2) There exist two disjoint soft closed (respt. soft semi-closed, soft pre-closed, soft b-closed) sets  $(A, E), (B, E)$  such that  $(A, E) \cup (B, E) = \tilde{X}$ .

(3) There exist two disjoint soft open (respt. soft semi-open, soft pre-open, soft b-open) sets  $(A, E), (B, E)$  such that  $(A, E) \cup (B, E) = \tilde{X}$ .

(4)  $(X, \tau, E)$  has a proper soft open (respt. soft semi-open, soft pre-open, soft b-open) and soft closed (respt. soft semi-closed, soft pre-closed, soft b-closed) set over  $X$ .

**Definition 3.9:** Let  $(X, m_{(X, E)})$  be a soft m-structure over  $X$  with property **B**,  $Y \subset X$  in  $(X, m_{(X, E)})$ . The soft space  $(Y, m_{(Y, E)})$  is called a soft m-subspace of  $(X, m_{(X, E)})$  if,  $m_{(Y, E)} = \{(A, E) \cap \tilde{Y} : (A, E) \in m_{(X, E)}\}$ .

**Lemma 3.10:** Let  $(X, m_{(X, E)})$  be a soft m-structure over  $X$  with property **B**,  $(Y, m_{(Y, E)})$  be soft m-subspace of  $(X, m_{(X, E)})$ . If  $(A, E)$  and  $(B, E)$  are soft sets in  $(Y, m_{(Y, E)})$ , then  $(A, E)$  and  $(B, E)$  are a soft m-separation of  $(Y, m_{(Y, E)})$  if and only if  $(A, E)$  and  $(B, E)$  are a soft m-separation of  $(X, m_{(X, E)})$ .

**Proof:** We have,  $m_{(Y, E)}\text{-Cl}(A, E) \cap (B, E) = (m_{(X, E)}\text{-Cl}(A, E) \cap \tilde{Y}) \cap (B, E) = m_{(X, E)}\text{-Cl}(A, E) \cap (B, E)$ .

Similar, we have

$$m_{(Y, E)}\text{-Cl}(B, E) \cap (A, E) = m_{(X, E)}\text{-Cl}(B, E) \cap (A, E).$$

Therefore, the lemma holds.

**Lemma 3.11:** Let  $(X, m_{(X, E)})$  be a soft m-structure over  $X$  with property **B**,  $\tilde{Y} \subset \tilde{X}$ .  $(Y, m_{(Y, E)})$  be soft m-subspace of  $(X, m_{(X, E)})$ .  $(Y, m_{(Y, E)})$  is soft m-connected. If  $(A, E)$  and  $(B, E)$  are a soft m-separation of  $(X, m_{(X, E)})$ , such that  $\tilde{Y} \subset (A, E) \cup (B, E)$ , then  $\tilde{Y} \subset (A, E)$  or  $\tilde{Y} \subset (B, E)$ .

**Proof:** We have,  $\tilde{Y} \subset (A, E) \cup (B, E)$ , we have  $\tilde{Y} = (\tilde{Y} \cap (A, E)) \cup (\tilde{Y} \cap (B, E))$ . By lemma 3.10,  $\tilde{Y} \cap (A, E)$  and  $\tilde{Y} \cap (B, E)$  are a soft m-separation of  $(Y, m_{(Y, E)})$ . Since,  $(Y, m_{(Y, E)})$  is soft m-connected, we have  $\tilde{Y} \cap (A, E) = \phi$  or  $\tilde{Y} \cap (B, E) = \phi$ . Therefore,  $\tilde{Y} \subset (A, E)$  or  $\tilde{Y} \subset (B, E)$ .

**Lemma 3.12:** Let  $\{(X_\alpha, m_{(X_\alpha, E)}) : \alpha \in J\}$  be a soft family non-empty soft m-connected subspaces of soft topological space  $(X, m_{(X, E)})$ . If  $\bigcap_{\alpha \in J} X_\alpha \neq \phi$ , then  $(\bigcup_{\alpha \in J} X_\alpha, m_{(\bigcup_{\alpha \in J} X_\alpha, E)})$  is a soft m-connected subspace of  $(X, m_{(X, E)})$ .

**Proof:** Let  $Y = (\bigcup_{\alpha \in J} X_\alpha)$ . Choose a soft point  $x_e \in \tilde{Y}$ . Let  $(C, E)$  and  $(D, E)$  be a soft m-disconnection of  $(\bigcup_{\alpha \in J} X_\alpha, m_{(\bigcup_{\alpha \in J} X_\alpha, E)})$ . Then,  $x_e \in (C, E)$  and  $x_e \in (D, E)$ , we assume that  $x_e \in (C, E)$ . For each  $\alpha \in J$ , since,  $(X_\alpha, m_{(X_\alpha, E)})$  is soft m-connected, it follows from lemma 3.11 that  $(X_\alpha) \subset (C, E)$  or  $(X_\alpha) \subset (D, E)$ . Therefore, we have  $\tilde{Y} \subset (C, E)$  since  $x_e \in (C, E)$  and then  $(D, E) = \phi$ , which is a contradiction. Thus  $(\bigcup_{\alpha \in J} X_\alpha, m_{(\bigcup_{\alpha \in J} X_\alpha, E)})$  is a soft m-connected subspace of  $(X, m_{(X, E)})$ .

**Theorem 3.13:** Let  $\{(X_\alpha, m_{(X_\alpha, E)}) : \alpha \in J\}$  be a soft family non-empty soft m-connected subspaces of soft topological space  $(X, m_{(X, E)})$ . If  $X_\alpha \cap X_\beta \neq \phi$  for  $\alpha, \beta \in J$ , then

$(\bigcup_{\alpha \in J} X_\alpha, m_{(\bigcup_{\alpha \in J} X_\alpha, E)})$  is a soft m-connected subspace of  $(X, m_{(X, E)})$ .

**Proof:** Let  $\alpha_0 \in J$ . For  $\beta \in J$ , Put  $A_\beta = X_{\alpha_0} \cup X_\beta$ . By lemma 3.12,  $(A_\beta, m_{(X_\beta, E)})$  is soft m-connected. Then,  $\{(A_\beta, m_{(X_\beta, E)}) : \beta \in J\}$  is a family soft m-connected subspace of  $(X, m_{(X, E)})$ , and  $\bigcap_{\beta \in J} A_\beta = X_{\alpha_0} \neq \phi$ . Obvious,  $(\bigcup_{\alpha \in J} X_\alpha, m_{(\bigcup_{\alpha \in J} X_\alpha, E)})$  is a soft m-connected subspace of  $(X, m_{(X, E)})$ .

**Theorem 3.14:** Let  $(X, m_{(X, E)})$  be a soft m-structure over  $X$  with property **B**,  $\tilde{Y} \subset \tilde{X}$ .  $(Y, m_{(Y, E)})$  be soft m-subspace of  $(X, m_{(X, E)})$ . If  $\tilde{Y} \subset \tilde{A} \subset m_{(X, E)}\text{-Cl}(F, E)$ , then  $(A, m_{(A, E)})$  is a soft connected m-subspace of  $(X, m_{(X, E)})$ . In particular,  $m_{(X, E)}\text{-Cl}(F, E)$  is a soft connected m-subspace of  $(X, m_{(X, E)})$ .

**Proof:** Let  $(C, E)$  and  $(D, E)$  be a soft m-disconnection of  $(A, m_{(A, E)})$ . By lemma 3.11, we have  $\tilde{A} \subset (C, E)$  or  $\tilde{A} \subset (D, E)$ . We assume that,  $\tilde{A} \subset (C, E)$ . By lemma 3.10, we have,  $m_{(X, E)}\text{-Cl}(C, E) \cap (D, E) = \phi$ , and hence,  $\tilde{A} \cap (D, E) = \phi$ , which is a contradiction.

**Theorem 3.15:** Let  $f_{pu} : (X, m_{(X, E)}) \rightarrow (Y, m_{(Y, K)})$  be soft continuous mapping, where  $m_{(X, E)}$  and  $m_{(Y, K)}$  are soft minimal structure over  $X$  and  $Y$  respectively, If  $(X, m_{(X, E)})$  is soft m-connected, then the soft image of  $(X, m_{(X, E)})$  is also soft m-connected.

**Proof:** Let  $f_{pu} : (X, m_{(X, E)}) \rightarrow (Y, m_{(Y, K)})$  be soft continuous mapping. Contrarily, Suppose that  $(Y, m_{(Y, K)})$  is soft m-disconnected and pair  $(A, K)$  and  $(B, K)$  is a soft m-disconnection of  $(Y, m_{(Y, K)})$ . Since  $f_{pu} : (X, m_{(X, E)}) \rightarrow (Y, m_{(Y, K)})$  is soft continuous, therefore  $f_{pu}^{-1}(A, K) \in m_{(X, E)}$ ,  $f_{pu}^{-1}(B, K) \in m_{(X, E)}$ . Clearly the pair  $f_{pu}^{-1}(A, K)$  and  $f_{pu}^{-1}(B, K)$  is a soft m-disconnection of  $(X, m_{(X, E)})$ , a contradiction. Hence,  $(Y, m_{(Y, K)})$  is soft m-connected. This completes the proof.

**Remark 3.16:** Let  $(X, \tau, E)$  and  $(Y, \vartheta, K)$  be two soft topological space over  $X$  and  $Y$  respectively, if  $m_{(X, E)} = \tau, m_{(Y, K)} = \vartheta$ .  $f_{pu} : (X, \tau, E) \rightarrow (Y, \vartheta, K)$  is soft continuous mapping. If  $(X, \tau, E)$  is soft connected (respt. soft semi connected, soft pre connected, soft b-connected), then the soft image of  $(X, \tau, E)$  is also soft connected (respt. soft semi connected, soft pre connected, soft b-connected).

**Definition 3.17:** Let  $m_{(X, E)}$  be a soft m-structure over  $X$ , A soft set  $(F, E)$  in  $(X, m_{(X, E)})$  is soft m-connected, if it is soft m-connected as a soft m-subspace.

**Remark 3.18:** Let  $(X, \tau, E)$  be a soft topological space over  $X$ . A soft set  $(F, E)$  in  $(X, \tau, E)$  is soft connected (respt. soft semi-connected, soft pre-connected and soft b-connected), if it is soft connected (respt. soft semi-connected, soft pre-connected and soft b-connected) as a soft subspace.

**Theorem 3.19:** Let  $m_{(X, E)}$  be a soft m-structure over  $X$ , the pair  $(F_1, E)$  and  $(F_2, E)$  of soft sets be a soft m-disconnection in  $(X, m_{(X, E)})$  and  $(F_3, E)$  be a soft m-connected of  $(X, m_{(X, E)})$ . Then  $(F_3, E)$  is contained in  $(F_1, E)$  or  $(F_2, E)$ .

**Proof:** Contrarily suppose that  $(F_3, E)$  is neither contained in  $(F_1, E)$  nor in  $(F_2, E)$ . Then  $(F_3, E) \cap (F_1, E), (F_3, E) \cap (F_2, E)$  are both nonempty soft subsets of  $(F_3, E)$ , such that  $((F_3, E) \cap (F_1, E)) \cap ((F_3, E) \cap (F_2, E)) = \phi$  and  $((F_3, E) \cap (F_1, E)) \cup$

$((F_3, E) \cap (F_2, E)) = (F_3, E)$ . This gives that pair of  $((F_3, E) \cap (F_1, E))$  and  $((F_3, E) \cap (F_2, E))$  is a soft m-disconnection of  $(F_3, E)$ . This contradiction proves the theorem.

**Theorem 3.20:** Let  $m_{(X, E)}$  be a soft m-structure over  $X$ ,  $(G, E)$  be a soft m-connected set in  $(X, m_{(X, E)})$  and  $(F, E)$  be soft set over  $X$  such that  $(G, E) \subset (F, E) \subset m_{(X, E)}\text{-Cl}(G, E)$ . Then  $(F, E)$  is soft m-connected.

**Proof:** It is sufficient to that  $m_{(X, E)}\text{-Cl}(G, E)$  is soft m-connected. On contrary, suppose that  $m_{(X, E)}\text{-Cl}(G, E)$  is soft m-disconnected. Then there exists a soft m-disconnection  $((H, E), (K, E))$  of  $m_{(X, E)}\text{-Cl}(G, E)$ . That is, there are  $((H, E) \cap (G, E)), ((K, E) \cap (G, E))$  soft sets in  $(G, E)$  such that  $((H, E) \cap (G, E)) \cap ((K, E) \cap (G, E)) = ((H, E) \cap (K, E)) \cap (G, E) = \phi$ , and  $((H, E) \cap (G, E)) \cup ((K, E) \cap (G, E)) = ((H, E) \cup (K, E)) \cap (G, E) = (G, E)$ . This gives that pair  $((H, E) \cap (G, E))$  and  $((K, E) \cap (G, E))$  is a soft m-disconnection of  $(G, E)$ , a contradiction. This proves that  $m_{(X, E)}\text{-Cl}(G, E)$  is soft m-connected. Hence the proof.

**Lemma 3.21:** Let  $m_{(X, E)}$  be a soft m-structure over  $X$  with property **B** and  $(A, E)$  and  $(B, E)$  be two soft sets over  $X$ . In  $(X, m_{(X, E)})$  the following statements are equivalent:

- (1)  $\phi, \tilde{X} \in m_{(X, E)}$ .
- (2)  $(X, m_{(X, E)})$  is not the soft union of two disjoint soft sets  $(A, E)$  and  $(B, E) \in m_{(X, E)}$ .
- (3)  $(X, m_{(X, E)})$  is not the soft union of two disjoint soft sets  $(A, E)^c$  and  $(B, E)^c \in m_{(X, E)}$ .
- (4)  $(X, m_{(X, E)})$  is not the soft union of two nonempty soft m-separated sets.

**Remark 3.22:** Let  $(X, \tau, E)$  be soft topological space over  $X$ , we put  $m_{(X, E)} = \tau$  (respt. SSO(X, E), SPO(X, E), SbO(X, E)) and  $(A, E)$  and  $(B, E)$  be two soft sets over  $X$ . In  $(X, \tau, E)$  the following statements are equivalent:

- (1)  $\phi$  and  $\tilde{X}$  are the only soft clopen (respt. soft semi clopen, soft pre clopen, soft b-clopen) sets in  $(X, \tau, E)$ .
- (2)  $(X, \tau, E)$  is not the soft union of two soft disjoint soft open (respt. soft semi open, soft pre open, soft b-open) sets.
- (3)  $(X, \tau, E)$  is not the soft union of two soft disjoint soft closed (respt. soft semi closed, soft pre closed, soft b-closed) sets.
- (4)  $(X, \tau, E)$  is not the soft union of two nonempty soft separated (soft semi separated, soft pre separated, soft b-separated) sets.

**Theorem 3.23:** Let  $m_{(X, E)}$  be a soft m-structure over  $X$  with property **B**. In  $(X, m_{(X, E)})$  the following statements are equivalent:

- (1)  $(X, m_{(X, E)})$  is soft m-connected space.
- (2)  $(X, m_{(X, E)})$  is not the soft union of any two soft m-separated sets.

**Proof:** (1)  $\rightarrow$  (2) : Assume (1), Suppose (2) is false, then let  $(A, E)$  and  $(B, E)$  are two soft m-separated sets such that  $\tilde{X} = (A, E) \cup (B, E)$ . Since  $(X, m_{(X, E)})$  is soft m-connected  $m_{(X, E)}\text{-Cl}(A, E) \cap (B, E) = (A, E) \cap m_{(X, E)}\text{-Cl}(B, E) = \phi$ . Since  $(A, E) \subset m_{(X, E)}\text{-Cl}(A, E)$  and  $(B, E) \subset m_{(X, E)}\text{-Cl}(B, E)$ , then  $(A, E) \cup (B, E) = \phi$ . Now  $m_{(X, E)}\text{-Cl}(A, E) \subset (B, E)^c = (A, E)$ . Hence,  $m_{(X, E)}\text{-Cl}(A, E) = (A, E)$ . Therefore,  $(A, E)^c \in m_{(X, E)}$ . By the same way we show that  $(B, E)^c \in m_{(X, E)}$  which is a contradiction with remark

**Lemma 3.24:** 4.3. This shows that (2) is true. Therefore (1)  $\rightarrow$  (2).

(2)  $\rightarrow$  (1) : Assume that (2) is not true. Let  $(A, E)^c$  and  $(B, E)^c$  are two soft m-disjoint nonempty and  $(A, E)^c$  and  $(B, E)^c \in m_{(X, E)}$  such that  $\tilde{X} = (A, E)^c \cup (B, E)^c$ . Then,  $m_{(X, E)}\text{-Cl}(A, E)^c \cap (B, E) = (A, E) \cap m_{(X, E)}\text{-Cl}(B, E)^c = (A, E)^c \cap (B, E)^c = \phi$ . This contradicts the hypothesis of (2). This show that (1) is true. Therefore, (2)  $\rightarrow$  (1).

**Remark 3.25:** Let  $(X, \tau, E)$  be soft topological space over  $X$ , we put  $m_{(X, E)} = \tau$ . Then, the following statements are equivalent :

- (1)  $(X, \tau, E)$  is soft connected (soft semi connected, soft pre connected, soft b-connected) space.
- (2)  $(X, \tau, E)$  is not the soft union of any two soft separated (soft semi separated, soft pre separated, soft b-separated) sets.

**Remark 3.26:** (1) Let  $m_{(X, E)}$  be a soft m-structure over  $X$  with property **B** and  $(A, E)$  be soft set over  $X$ , If  $\phi \neq (A, E) \subset (X, m_{(X, E)})$  then  $(A, E)$  is a soft m-connected set in  $m_{(X, E)}$  whenever  $(X, m_{(X, E)})$  is a soft m-connected space.

(2) Let  $(X, \tau, E)$  be soft topological space over  $X$ , we put  $m_{(X, E)} = \tau$ . If  $\phi \neq (A, E) \subset (X, \tau, E)$  then  $(A, E)$  is a soft connected (soft semi connected, soft pre connected, soft b-connected) set over  $X$  whenever  $(X, \tau, E)$  is a soft connected (soft semi connected, soft pre connected, soft b-connected) space.

**Theorem 3.27:** Let  $m_{(X, E)}$  be a soft m-structure over  $X$  with property **B**. In  $(X, m_{(X, E)})$ , let soft set  $(A, E)$  be a soft m-connected set. Let  $(B, E)$  and  $(C, E)$  are soft m-separated sets. If  $(A, E) \subset (B, E) \cup (C, E)$ . Then either  $(A, E) \subset (B, E)$  or  $(A, E) \subset (C, E)$ .

**Proof:** Suppose  $(A, E)$  is soft m-connected set and  $(B, E), (C, E)$  are soft m-separated sets such that  $(A, E) \subset (B, E) \cup (C, E)$ . Let  $(A, E)$  notsubset  $(B, E)$  and  $(A, E)$  notsubset  $(C, E)$ . Suppose  $(A_1, E) = (B, E) \cap (A, E) \neq \phi$  and  $(A_2, E) = (C, E) \cap (A, E) \neq \phi$ . Then,  $(A, E) = (A_1, E) \cup (A_2, E)$ . Since,  $(A_1, E) \subset (B, E)$ . Hence,  $m_{(X, E)}\text{-Cl}(A_1, E) \subset m_{(X, E)}\text{-Cl}(B, E)$ . Since,  $m_{(X, E)}\text{-Cl}(B, E) \cap (C, E) = \phi$  then  $m_{(X, E)}\text{-Cl}(A_1, E) \cap (A_2, E) = \phi$ . Since  $(A_2, E) \subset (C, E)$ . Hence,  $m_{(X, E)}\text{-Cl}(A_2, E) \subset m_{(X, E)}\text{-Cl}(C, E)$ . Since,  $m_{(X, E)}\text{-Cl}(C, E) \cap (B, E) = \phi$ . Then  $m_{(X, E)}\text{-Cl}(A_2, E) \cap (A_1, E) = \phi$ . But  $(A, E) = (A_1, E) \cup (A_2, E)$ , therefore,  $(A, E)$  is not soft m-connected space. This is a contradiction. Then either  $(A, E) \subset (B, E)$  or  $(A, E) \subset (C, E)$ .

**Remark 3.28:** Let  $(X, \tau, E)$  be soft topological space over  $X$ , we put  $m_{(X, E)} = \tau$  and let  $(A, E)$  be a soft connected (respt. soft semi connected, soft pre connected, soft b-connected) set. Let  $(B, E)$  and  $(C, E)$  are soft separated (respt. soft semi separated, soft pre separated, soft b-separated) sets. If  $(A, E) \subset (B, E) \cup (C, E)$ . Then either  $(A, E) \subset (B, E)$  or  $(A, E) \subset (C, E)$ .

**Theorem 3.29:** Let  $m_{(X, E)}$  be a soft m-structure over  $X$  with property **B**. In  $(X, m_{(X, E)})$ , let soft set  $(A, E)$  be a soft m-connected set then  $m_{(X, E)}\text{-Cl}(A, E)$  is soft m-connected.

**Proof:** Suppose soft set  $(A, E)$  be a soft m-connected set and  $m_{(X, E)}\text{-Cl}(A, E)$  is not. Then there exist two soft m-separated sets  $(B, E)$  and  $(C, E)$  such that  $m_{(X, E)}\text{-Cl}(A, E) = (B, E) \cup (C, E)$

.But  $(A,E) \subset m_{(X,E)}\text{-Cl}(A,E)$ , then  $(A,E) = (B,E) \cup (C,E)$  and since  $(A,E)$  is soft  $m$ -connected set then by theorem 3.27 either  $(A,E) \subset (B,E)$  or  $(A,E) \subset (C,E)$ .

(i) If  $(A,E) \subset (B,E)$  then  $m_{(X,E)}\text{-Cl}(A,E) \subset m_{(X,E)}\text{-Cl}(B,E)$ . But  $m_{(X,E)}\text{-Cl}(B,E) \cap (C,E) = \phi$ . Hence,  $m_{(X,E)}\text{-Cl}(A,E) \cap (C,E) = \phi$ . Since,  $(C,E) \subset m_{(X,E)}\text{-Cl}(A,E)$ , then  $(C,E) = \phi$  this is a contradiction.

(ii) If  $(A,E) \subset (C,E)$  then the same way we can prove that  $(B,E) = \phi$  which is a contradiction. Therefore,  $m_{(X,E)}\text{-Cl}(A,E)$  is soft  $m$ -connected.

*Remark 3.30:* Let  $(X, \tau, E)$  be soft topological space over  $X$ , we put  $m_{(X,E)} = \tau$  let soft set  $(A,E)$  be a soft connected (respt. soft semi connected, soft pre connected, soft  $b$ -connected) set then  $m_{(X,E)}\text{-Cl}(A,E)$  is soft connected (respt. soft semi connected, soft pre connected, soft  $b$ -connected).

*Theorem 3.31:* Let  $m_{(X,E)}$  be a soft  $m$ -structure over  $X$  with property **B**. In  $(X, m_{(X,E)})$ , let soft set  $(A,E)$  be a soft  $m$ -connected set and  $(A,E) \subset (B,E) \subset m_{(X,E)}\text{-Cl}(A,E)$  then  $(B,E)$  is soft  $m$ -connected.

*Proof:* If  $(B,E)$  is not soft  $m$ -connected, then there exist two soft set  $(C,E)$  and  $(D,E)$  such that  $m_{(X,E)}\text{-Cl}(C,E) \cap (D,E) = (C,E) \cap m_{(X,E)}\text{-Cl}(D,E) = \phi$  and  $(B,E) = (C,E) \cup (D,E)$ . Since,  $(A,E) \subset (B,E)$ , thus either  $(A,E) \subset (C,E)$  or  $(A,E) \subset (D,E)$ . Suppose  $(A,E) \subset (C,E)$  then  $m_{(X,E)}\text{-Cl}(A,E) \subset m_{(X,E)}\text{-Cl}(C,E)$ , thus  $m_{(X,E)}\text{-Cl}(A,E) \subset (D,E) = m_{(X,E)}\text{-Cl}(C,E) \cap (D,E) = \phi$ . But  $(D,E) \subset (B,E) \subset m_{(X,E)}\text{-Cl}(A,E)$ , thus  $m_{(X,E)}\text{-Cl}(A,E) \cap (D,E) = (D,E)$ . Therefore,  $(D,E) = \phi$  which is a contradiction. Thus,  $(B,E)$  is soft  $m$ -connected set.

If  $(A,E) \subset (B,E)$ , then by the same way we can prove that  $(C,E) = \phi$ . This is a contradiction. Thus  $(B,E)$  is soft  $m$ -connected.

*Remark 3.32:* Let  $(X, \tau, E)$  be soft topological space over  $X$ , we put  $m_{(X,E)} = \tau$  let soft set  $(A,E)$  be a soft connected (respt. soft semi connected, soft pre connected, soft  $b$ -connected) set and  $(A,E) \subset (B,E) \subset m_{(X,E)}\text{-Cl}(A,E)$  then  $(B,E)$  is soft connected (respt. soft semi connected, soft pre connected, soft  $b$ -connected).

*Theorem 3.33:* Let  $m_{(X,E)}$  be a soft  $m$ -structure over  $X$  with property **B**,  $(X, m_{(X,E)})$  is soft  $m$ -connected if and only if the only soft sets in  $(X, m_{(X,E)})$  that are both soft open and soft closed over  $X$  are  $\phi$  and  $\tilde{X}$ .

*Proof:* Let  $(X, m_{(X,E)})$  is soft  $m$ -connected. Suppose to the contrary that  $(F,E) \in m_{(X,E)}$  and  $(F,E)^c \in m_{(X,E)}$  over  $X$  different from  $\phi$  and  $\tilde{X}$ . Clearly,  $(F,E)^c \in m_{(X,E)}$  different from  $\phi$  and  $\tilde{X}$ . Now we have  $(F,E)$ ,  $(F,E)^c$  is a soft  $m$ -separation over  $X$ . This is contradiction. Thus the only soft closed and open sets over  $X$  are  $\phi$  and  $\tilde{X}$ . Conversely, let  $(F,E)$ ,  $(G,E)$  be a soft separation over  $X$ .

*Remark 3.34:* Let  $(X, \tau, E)$  be a soft topological space over  $X$  and  $(F,E)$  be soft set over  $X$ .  $(X, \tau, E)$  is soft connected (soft semi connected, pre connected,  $b$ -connected) if and only if there does not exist nonempty soft set  $(E,E)$  over  $X$  which is both soft open (respt. soft semi open, soft pre open, soft  $b$ -open) and soft closed (respt. soft semi closed, soft pre closed, soft  $b$ -closed) set over  $X$ .

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