Characterizations of Fuzzy W – Compactness and Fuzzy W-Closed Spaces in Fuzzy Topological Spaces

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Abstract— The concepts of $\mathrm{W-compactness}$ and $\mathrm{W-closed}$ spaces in the fuzzy setting are defined and investigated. Fuzzy filter bases are used to characterize these concepts.

Keywords— Fuzzy topological space, fuzzy W-open set, quasi-coincident, fuzzy W-compact space, fuzzy W-closed space, fuzzy filter base.

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I. Introduction

The concept of fuzzy set operation was first introduced by Zadeh [15] and subsequently, several authors including Zadeh [15] have discussed various aspects of the theory and applications of fuzzy sets. Fuzzy topological spaces were introduced by Chakrabarty and Ahsanullah (1992) and Chang (1968). Compactness occupies a very important place in fuzzy topology and so do some of its other forms including closed space, countably compactness and Lindelof space. In [7], Talal Al – Hawary introduced the concepts of fuzzy $\mathrm{W-closed}$ sets, fuzzy $\mathrm{W-open}$ sets and Fuzzy $\mathrm{W-generalized}$ closed sets as well as fuzzy $\mathrm{W-g-continuous}$ and fuzzy $\mathrm{W-g-irresolute}$ functions and investigated their some basic properties. The objective of this paper is devoted to introduce and study the concepts of $\mathrm{W-compactness}$ and $\mathrm{W-closed}$ spaces in the fuzzy setting. We use fuzzy filterbases to characterize fuzzy $\mathrm{W-compactness}$ and fuzzy $\mathrm{W-closed}$ spaces. We will also explore several basic properties and characterizations of these concepts.

II. Preliminaries

Let X be a nonempty set and $\mathrm{I}=[0,1]$. A fuzzy set on X is a mapping from X into I. The null fuzzy set 0_X is the mapping from X into I which assumes only the value is 0 and whole fuzzy sets 1_X is a mapping from X into I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha : \alpha \in \Lambda\}$ of fuzzy sets of X is defined to be the mapping $\mathrm{Sup}\{A_\alpha : \alpha \in \Lambda\}$ (resp. $\mathrm{Inf}\{A_\alpha : \alpha \in \Lambda\}$). A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \le B(x)$ for each $x \in X$. A fuzzy point x_β in X is a fuzzy set defined by $x_\beta(y)=\beta$ for $y = x$ and $x_\beta(y)=0$ for $y \ne x$, $\beta \in (0,1]$ and $y \in X$. A fuzzy point x_β is said to be $\mathrm{quasi-coincident}$ with the fuzzy set A denoted by $x_\beta q A$ if and only if $\beta + A(x) > 1$. A fuzzy set A is $\mathrm{quasi-coincident}$ (not $\mathrm{quasi-coincident}$) with a fuzzy set B denoted by AqB $(A\tilde{q}B)$ if and only if there exists a point $x \in X$ such that $A(x)+B(x)>1$ $(A(x)+B(x)\le 1)$. A family T of fuzzy sets of X is called a fuzzy topology on X if X, ϕ belong to T and T is closed with respect to arbitrary union and finite intersection .The members of T are called fuzzy open sets and their complements are fuzzy closed sets. For any fuzzy set A of X, the closure of A (denoted by $\mathrm{Cl}(\mathrm{A})$) is the intersection of all the fuzzy closed supersets of A and the interior of A (denoted by $\mathrm{Int}(\mathrm{A})$) is the union of all fuzzy open subsets of A. Throughout this paper X and Y will mean fuzzy topological spaces. The complement and the support of a fuzzy set U are denoted by U^c and $\mathrm{S}(\mathrm{U})$, respectively.

Definition 2.1. Let A be a fuzzy subset of a fuzzy topological space (X, T). The fuzzy $\mathrm{W-interior}$ of A is the union of all fuzzy open subsets of X whose closures are contained in $Cl(A)$, and is denoted by $\mathrm{W-Int}(\mathrm{A})$. A is called fuzzy $\mathrm{W-open}$ if $\mathrm{A}=\mathrm{W-Int}(\mathrm{A})$. The complement of a fuzzy $\mathrm{W-open}$ subset is called fuzzy $\mathrm{W-closed}$. Alternatively, a fuzzy subset A of X is fuzzy $\mathrm{W-closed}$ if and only if $\mathrm{A}=\mathrm{W-Cl}(\mathrm{A})$, where $\mathrm{W-Cl}(\mathrm{A})=\bigcap_{\alpha\in\Delta}\{A_\alpha : A \le A_\alpha, A_\alpha \text{ is FC-set in X}\}$. Cl early $\mathrm{Int}(A) \le W-\mathrm{Int}(A) \le \mathrm{Cl}(A)$ and $A \le \mathrm{Cl}(A) \le W-\mathrm{Cl}(A)$ and hence every fuzzy $W-\mathrm{closed}$ set is a fuzzy closed set, but the converse needs not be true.

Example 2.2. Suppose that $X=\{a,b,c\}$ and $T=\{0,1,\chi_{\{a\}},\chi_{\{b\}},\chi_{\{a,b\}}\}$. Then the set $\chi_{\{a,b\}}$ is a fuzzy closed set but not a fuzzy $W-\mathrm{closed}$ set since $W-\mathrm{Cl}(\chi_{\{a,b\}})=1$.

The intersection of two fuzzy $W-\mathrm{open}$ subsets need not be fuzzy $W-\mathrm{open}$.

Example 2.3. Let $X=\{a,b,c,d\}$ and $T=\{0,1,\chi_{\{b\}},\chi_{\{c\}},\chi_{\{a,b\}},\chi_{\{b,c\}},\chi_{\{a,b,c\}}\}$. Then $\chi_{\{a,b\}}$ and $\chi_{\{a,c\}}$ are fuzzy $W-\mathrm{open}$ sets, but $\chi_{\{a,b\}} \cap \chi_{\{a,c\}} = \chi_{\{a\}}$ is not a fuzzy $W-\mathrm{open}$ set. The set $\chi_{\{a,b\}}$ is a fuzzy closed set but not a fuzzy $W-\mathrm{closed}$ set since $W-\mathrm{Cl}(\chi_{\{a,b\}})=1$.

In classical topology, the interior of a set is a subset of the set itself. But this is not the case for fuzzy $W-\mathrm{open}$ sets. Next we show that $A \le W-\mathrm{Int}(A)$ and $W-\mathrm{Int}(A) \le A$ need not be true.

Example 2.4. Consider the space in Example 2.3. Then

$$W-\mathrm{Int}(\chi_{\{c\}})=0<\chi_{\{c\}},\chi_{\{a,b\}}<W-\mathrm{Int}(\chi_{\{a,b\}})=1.$$

Next, we state the result as proved in [7] that arbitrary unions of fuzzy $W-\mathrm{open}$ subsets are fuzzy $W-\mathrm{open}$.

Theorem 2.5. If (X,T) is a fuzzy topological space, then arbitrary unions of fuzzy $W-\mathrm{open}$ subsets are fuzzy $W-\mathrm{open}$.

Corollary 2.6. The arbitrary intersection of fuzzy $W-\mathrm{closed}$ subsets are fuzzy $W-\mathrm{closed}$, while finite unions of fuzzy $W-\mathrm{closed}$ subsets need not be fuzzy $W-\mathrm{closed}$.

Corollary 2.7. If A is a fuzzy $W-\mathrm{dense}$ subset of X, $(W-\mathrm{Cl}(A)=1)$, then $W-\mathrm{Int}(A)=1$.

Definition 2.8. A collection ξ of fuzzy subsets of a fuzzy topological space (X,T) is said o form a fuzzy filterbases if and only if for every finite subcollection λ of ξ, $\bigcap_{\alpha\in\lambda} A \ne 0_X$.

Definition 2.9. A collection μ of fuzzy sets in a fuzzy topological space (X,T) is said to be a cover of a fuzzy set U of X if and only if $\left(\bigcup_{\alpha\in\mu} A\right)(x)=1_X$, for every $x \in S(U)$.

Definition 2.10. A fuzzy cover μ of a fuzzy set U in a fuzzy topological space (X,T) is said to have a finite subcover if and only if there exists a finite subcollection η of μ such that $\left(\bigcup_{\alpha\in\eta} A\right)(x) \ge U(x)$, for every $x \in S(U)$.

III. Fuzzy W – Compact Spaces

Definition 3.1. A fuzzy topological space (X,T) is said to be fuzzy $W-\mathrm{compact}$ if and only if for every family μ of fuzzy $W-\mathrm{open}$ sets such that $\bigcup_{A\in\mu} A = 1_X$ there is a finite subfamily $\eta \subseteq \mu$ such that $\bigcup_{A\in\eta} A = 1_X$.

Definition 3.2. A fuzzy set U in a fuzzy topological space (X,T) is said to be fuzzy $W-\mathrm{compact}$ relative to X if and only if for every family μ of fuzzy $W-\mathrm{open}$ sets such that $\bigcup_{A\in\mu} A \ge S(U)$ there is a finite subfamily $\eta \subseteq \mu$ such that $\bigcup_{A\in\eta} A \ge U(x)$ for every $x \in S(U)$.

Theorem 3.3. A fuzzy topological space (X,T) is fuzzy $W-\mathrm{compact}$ if and only if for every collection $\{A_j : j \in J\}$ of fuzzy $W-\mathrm{closed}$ sets of X having the finite intersection property, $\bigcap_{j\in J} A_j \ne 0_X$.

Proof. Let $\{A_j : j \in J\}$ be a collection of fuzzy $W-\mathrm{closed}$ sets with the finite intersection property. Suppose that $\bigcap_{j\in J} A_j = 0_X$. Then $\bigcup_{j\in J} A^c = 1_X$. Since $\{A_j^c : j \in J\}$ is a collection of fuzzy $W-\mathrm{open}$ sets cover of X, then from the $W-\mathrm{compactness}$ of X it follows that there exists a finite subset $F \subseteq J$ such that $\bigcup_{j\in F} A_j^c = 1_X$. Then $\bigcap_{j\in F} A_j = 0_X$ which gives a contradiction and therefore $\bigcap_{j\in J} A_j \ne 0_X$.

Conversely, Let $\{A_j : j \in J\}$ be a collection of fuzzy $W-\mathrm{open}$ sets cover of X. Suppose that for every finite subset $F \subseteq J$, we have $\bigcup_{j\in F} A_j \ne 1_X$. Then

$\bigcup_{j\in F} A_j^c \neq 0_X$. Hence $\{A_j^c : j\in J\}$ satisfies the finite intersection property. Then from the hypothesis, we have $\bigcap_{j\in J} A_j^c \neq 0_X$ which implies $\bigcap_{j\in J} A_j \neq 1_X$ and this contradicts the fact that $\{A_j : j\in J\}$ is a fuzzy $W-\text{open}$ open cover of X. Thus X is fuzzy $W-\text{compact}$.

Now, we give some results of fuzzy $W-\text{compactness}$ in terms of fuzzy filterbases

Theorem 3.4. A fuzzy topological space (X, T) is fuzzy $W-\text{compact}$ if and only if every filterbases ξ in X, $\bigcap_{G\in\xi} W-Cl(G) \neq 0_X$.

Proof. Let μ be a fuzzy $W-\text{open}$ set cover of X and μ has no finite subcover. Then for every finite subcollection λ of μ, there exists $x\in X$ such that $A(x)<1$ for every $A\in\lambda$, Then $A^c(x)>0$, so that $\bigcap_{A\in\lambda} A^c(x)\neq 0_X$. Thus $\{A^c : A\in\mu\}$ forms a filterbases in X. Since μ is fuzzy $W-\text{open}$ set cover of X, then $\left(\bigcup_{A\in\mu} A\right)(x)=1_X$ for every $x\in X$ and hence $\left(\bigcap_{A\in\mu} W-Cl(A^c)\right)(x)=\left(\bigcap_{A\in\mu} A^c\right)(x)=0_X$, which is a contradiction. Then every fuzzy $W-\text{open}$ set cover of X has a finite subcover and hence X is $W-\text{compact}$.

Conversely, suppose there exists a filterbases ξ such that $\left(\bigcap_{G\in\xi} W-Cl(G)\right)(x)=0_X$, so that $\bigcup_{G\in\xi}\left[\left(W-Cl(G)\right)^c\right](x)=1_X$ for every $x\in X$ and hence $\mu=\left\{\left(W-Cl(G)\right)^c : G\in\xi\right\}$ is a fuzzy $W-\text{open}$ set cover of X. Since X is fuzzy $W-\text{compact}$, then μ has a finite subcover. Then there exists a finite subset $\lambda\subseteq\mu$ such that $\left(\bigcup_{G\in\lambda}\left(W-Cl(G)\right)^c\right)(x)=1_X$ and hence $\left(\bigcup_{G\in\lambda} G^c\right)(x)=1_X$, so that $\bigcap_{G\in\lambda} G=0_X$ which is a contradiction since λ is a finite subset of filterbases ξ. Therefore $\bigcap_{G\in\xi} W-Cl(G)\neq 0_X$ for every filterbases ξ.

Theorem 3.5. A fuzzy set U in a fuzzy topological space (X, T) is fuzzy $W-\text{compact}$ relative to X if and only if for every filterbases ξ such that every finite set of members of ξ is $\text{quasi}-\text{coincident}$ with U, and $\left(\bigcap_{G\in\xi} W-Cl(G)\right)\cap U\neq 0_X$.

Proof. Let U not be fuzzy $W-\text{compact}$ relative to X. Then there exists a fuzzy $W-\text{open}$ cover of U such that U has no finite subcover η. Then $\left(\bigcup_{A\in\eta} A\right)(x)<U(x)$ for some $x\in S(U)$, so that $\left(\bigcap_{A\in\eta} A^c\right)(x)>U^c(x)\geq 0$ and hence $\xi=\{A^c : A\in\mu\}$ forms a filterbases and $\left(\bigcap_{A\in\eta} A^c\right)qU$. By hypothesis $\left(\bigcap_{A\in\eta} W-Cl(A^c)\right)\cap U\neq 0_X$ and hence $\left(\bigcap_{A\in\eta} A^c\right)\cap U\neq 0_X$. Then for some $x\in S(U)$, $\left(\bigcap_{A\in\mu} A^c\right)(x)>0_X$, that is $\left(\bigcup_{A\in\mu} A\right)(x)<1_X$, which is a contradiction. Hence U is a fuzzy $W-\text{compact}$ relative to X,

Conversely, suppose that there exists a filterbases ξ such that every finite set of members of ξ is $\text{quasi}-\text{coincident}$ with U and $\left(\bigcap_{G\in\xi} W-Cl(G)\right)\cap U\neq 0_X$. Then for $x\in S(U)$, $\left(\bigcap_{G\in\xi} W-Cl(G)\right)(x)=0_X$ and hence $\left(\bigcup_{G\in\xi}\left(W-Cl(G)\right)^c\right)(x)=1_X$ for every $x\in S(U)$. Thus $\mu=\left\{\left(W-Cl(G)\right)^c : G\in\xi\right\}$ is fuzzy $W-\text{open}$ cover of U. Since U is fuzzy $W-\text{compact}$ relative to X, then there exists a finite subcover, say $\left\{\left(W-Cl(G_1)\right)^c,\dots,\left(W-Cl(G_n)\right)^c\right\}$, such that $\left(\bigcup_{j=1}^{n}\left(W-Cl(G_j)\right)^c\right)(x)\geq U(x)$ for every $x\in S(U)$. Hence $\left(\bigcap_{j=1}^{n}\left(W-Cl(G_j)\right)\right)(x)\leq U^c(x)$ for every $x\in S(U)$, so that $\bigcap_{j=1}^{n}\left(W-Cl(G_j)\right)\tilde{q}\leq U$, which is a contradiction. Therefore for every filterbases ξ such that every finite set of members of ξ is $\mathrm{quasi-coincident}$ with U, $\left(\bigcap_{G\in\xi} \mathrm{W-Cl}(G)\right)\cap \mathrm{U} \neq 0_X$.

The following theorem proves that hereditary property for fuzzy $\mathrm{W-compact}$ spaces.

Theorem 3.6. Every fuzzy $\mathrm{W-closed}$ subset of a fuzzy $\mathrm{W-compact}$ space (X, T) is fuzzy $\mathrm{W-compact}$ relative to X.

Proof. Let ξ be a fuzzy filterbases in X such that $Uq\bigcap\{G : G\in\lambda\}$ holds for every finite subcollection λ of ξ and a fuzzy $\mathrm{W-closed}$ set U. Consider $\xi^* = \{\mathrm{U}\}\cup\xi$. For any finite subcollection λ^* of ξ^*, if $\mathrm{U}\notin\lambda^*$, then $\bigcap\lambda^* \neq 0_X$. If $\mathrm{U}\in\lambda^*$ and since $Uq\bigcap\{G : G\in\lambda^* - \{U\}\}$, then $\bigcap\lambda^*\neq 0_X$. Hence λ^* is a fuzzy filterbases in X. Since X is fuzzy $\mathrm{W-compact}$, then $\bigcap_{G\in\xi^*}\mathrm{W-Cl}(G)\neq 0_X$, so that

$$\left(\bigcap_{G\in\xi}\mathrm{W-Cl}(G)\right)\cap \mathrm{U}$$
$$=\left(\bigcap_{G\in\xi}\mathrm{W-Cl}(G)\right)\cap \mathrm{W-Cl}(\mathrm{U})\neq 0_X.$$

Hence by Theorem 3.5, we have U is fuzzy $\mathrm{W-compact}$ relative to X.

IV. Fuzzy W – Closed Spaces

Definition 4.1. A fuzzy topological space (X, T) is said to be fuzzy $\mathrm{W-closed}$ space if and only if for every family μ of fuzzy $\mathrm{W-open}$ sets such that $\left(\bigcup_{A\in\mu}A\right)(x) = 1_X$ there is finite subfamily $\eta\subseteq\mu$ such that $\left(\bigcup_{A\in\eta}\mathrm{W-Cl}(A)\right)(x) = 1_X$, for every $x\in X$.

Theorem 4.2. A fuzzy topological space (X, T) is fuzzy $\mathrm{W-closed}$ if and only if for every fuzzy $\mathrm{W-open}$ filterbases ξ in X, $\bigcap_{G\in\xi}\mathrm{W-Cl}(G)\neq 0_X$.

Proof. Let μ be a fuzzy $\mathrm{W-open}$ set cover of X and let for every finite subfamily η of μ, $\left(\bigcup_{A\in\eta}\mathrm{W-Cl}(A)\right)(x) < 1_X$, for some $x\in X$. Then $\left(\bigcap_{A\in\eta}(\mathrm{W-Cl}(A))^c\right)(x) > 0_X$ for some $x\in X$. Thus $\xi = \left\{(\mathrm{W-Cl}(A))^c : A\in\mu\right\}$ forms a fuzzy $\mathrm{W-open}$ filterbases in X. Since μ is a fuzzy $\mathrm{W-open}$ set cover of X, then $\bigcap_{A\in\mu}A^c = 0_X$ which implies $\bigcap_{A\in\mu}\mathrm{W-Cl}(\mathrm{W-Cl}(A))^c = 0_X$, which is a contradiction. Then every fuzzy $\mathrm{W-open}$ set μ cover of X has a finite subfamily η such that $\left(\bigcup_{A\in\eta}\mathrm{W-Cl}(A)\right)(x) = 1_X$ for every $x\in X$. Hence X is fuzzy $\mathrm{W-closed}$.

Conversely, suppose there exists a fuzzy $\mathrm{W-open}$ filterbases ξ in X such that $\bigcap_{G\in\xi}\mathrm{W-Cl}(G) = 0_X$, so that $\left(\bigcup_{G\in\xi}(\mathrm{W-Cl}(A))^c\right)(x) = 1_X$ for every $x\in X$ and hence $\mu = \left\{(\mathrm{W-Cl}(G))^c : G\in\xi\right\}$ is a fuzzy $\mathrm{W-open}$ set cover of X. Since X is fuzzy $\mathrm{W-closed}$, then μ has a finite subfamily η such that $\left(\bigcup_{G\in\eta}\mathrm{W-Cl}(\mathrm{W-Cl}(A))^c\right)(x) = 1_X$ for every $x\in X$, and hence $\bigcap_{G\in\eta}\left(\mathrm{W-Cl}(\mathrm{W-Cl}(G))^c\right)^c = 0_X$. Thus $\bigcap_{G\in\eta}G = 0_X$ which is a contradiction, since all the G are members of filterbases.

Definition 4.3. A fuzzy set U in a fuzzy topological space (X, T) is said to be fuzzy $\mathrm{W-closed}$ relative to X if and only if for every family μ of fuzzy $\mathrm{W-open}$ sets such that $\bigcup_{A\in\mu}A = \mathrm{U}$, there is a finite subfamily $\eta\subseteq\mu$ such that $\left(\bigcup_{A\in\eta}\mathrm{W-Cl}(A)\right)(x) = \mathrm{U}(x)$ for every $x\in \mathrm{S}(\mathrm{U})$.

Theorem 4.4. A fuzzy subset U in a fuzzy topological space (X, T) is fuzzy $\mathrm{W-closed}$ relative to X if and only if every fuzzy $\mathrm{W-open}$ filterbases ξ in X, $\left(\bigcap_{G\in\xi}\mathrm{W-Cl}(G)\right)\cap \mathrm{U} = 0_X$, there exists a finite subfamily λ of ξ such that $\left(\bigcap_{G\in\lambda}G\right)\tilde{q}\mathrm{U}$.

Proof. Let U be a fuzzy $\mathrm{W-closed}$ relative to X, suppose ξ is a fuzzy $\mathrm{W-open}$ filterbases in X such that for every finite subfamily λ of ξ, $\left(\bigcap_{G\in\lambda}G\right)q\mathrm{U}$, but

$\left(\bigcap_{G\in\xi} W-Cl(G)\right)\cap U = 0_X$. Then for every $x\in S(U)$, $\left(\bigcap_{G\in\xi} W-Cl(G)\right)(x) = 0_X$ and hence $\left(\bigcup_{G\in\xi}(W-Cl(G))^c\right)(x) = 1_X$ for every $x\in S(U)$. Then $\mu = \left\{(W-Cl(G))^c : G\in\xi\right\}$ is a fuzzy $W-open$ set cover of U and hence there exists a finite subfamily $\lambda\subseteq\xi$ such that $\bigcup_{G\in\lambda} W-Cl(W-Cl(G))^c \geq U$, so that $\bigcap_{G\in\lambda}\left(W-Cl\left((W-Cl(G))^c\right)\right)^c =$ $\bigcap_{G\in\lambda} W-Int(W-Cl(G)) \leq U^c$ and hence $\bigcap_{G\in\lambda} G \leq U^c$. Then $\left(\bigcap_{G\in\lambda} G\right)\tilde{q}U$ which is a contradiction.

Conversely, let U not be a fuzzy $W-closed$ set relative to X, then there exists a fuzzy $W-open$ set μ cover of U such that every finite subfamily $\eta\subseteq\mu$, $\left(\bigcup_{A\in\eta} W-Cl(A)\right)(x) \leq U(x)$ for some $x\in S(U)$ and hence $\left(\bigcap_{A\in\eta}(W-Cl(A))^c\right)(x) > U^c(x) \geq 0$ for some $x\in S(U)$. Thus $\xi = \left\{(W-Cl(A))^c : A\in\mu\right\}$ forms a fuzzy $W-open$ filterbases in X. Let there exists a finite subfamily $\left\{(W-Cl(A))^c : A\in\eta\right\}$ such that $\left(\bigcap_{A\in\eta}(W-Cl(A))^c\right)\tilde{q}U$. Then $U \leq \bigcup_{A\in\eta} W-Cl(A)$. So there exists a finite subfamily $\eta\subseteq\mu$ such that $\bigcup_{A\in\eta} W-Cl(A) \geq U$ which is a contradiction. Then for each finite subfamily $\lambda = \left\{(W-Cl(A))^c : A\in\eta\right\}$ of ξ, we have $\left(\bigcap_{A\in\eta}(W-Cl(A))^c\right)qU$. Hence by the given condition $\left(\bigcap_{A\in\mu} W-Cl(W-Cl(A))^c\right)\cap U \neq 0_X$, so there exists $x\in S(U)$ such that $\left(\bigcap_{A\in\mu} W-Cl(W-Cl(A))^c\right)(x) > 0_X$. Then $\left(\bigcup_{A\in\mu}\left(W-Cl(W-Cl(A))^c\right)^c\right)(x) =$ $\left(\bigcup_{A\in\mu} W-Int(W-Cl(A))\right)(x) < 1_X$, and hence $\left(\bigcup_{A\in\mu} A\right)(x) < 1_X$ which contradicts the fact that μ is a fuzzy $W-open$ set cover of U. Therefore U is fuzzy $W-closed$ relative to X.

Conclusion

In this paper, we have introduced the concepts of fuzzy $W-compactness$ and fuzzy $W-closed$ spaces and have investigated their several properties by making use of fuzzy filter bases. We can extend this concept to introduce the notions of fuzzy W-generalized compactness and fuzzy W-generalized closed spaces and may also investigate several characterizations of these new notions via fuzzy filter bases.

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References

[1] S. S. Benchalli and Jenifer J. Kamel, Fuzzy $b-compact$ and $b-closed$ spaces, International Journal of Computer Applications, Vol. 18, No. 3, 26 – 29 (2011).
[2] M. K. Chakrabarty and T. M. Ahsanullah, Fuzzy topology on fuzzy sets and tolerance topology. Fuzzy Sets and Systems, 45, 103–108. (1992)
[3] C. L. Chang, Fuzzy Topological Spaces J. Math Anal. Appl. 24,182-190 (1968).
[4] D. Coker and A. Haydar Es, On fuzzy $S-closed$ spaces, Doga Tu J. Math., 145-152 (1987).
[5] S. Ganguly and S. Saha, A note on compactness in fuzzy setting, Fuzzy Sets and Systems, 34, 117-124 (1990).
[6] B. Ghosh, Semicontinuous and semiclosed mappings and semi-connectedness in fuzzy setting, Fuzzy Sets and Systems, 35, 345-355 (1990).
[7] Talal Al-Hawary, Fuzzy $W-closed$ sets, Cogent Mathematics, 4(1), 5 Pages (2017).
[8] S. Jafari and R. Latif, "Applications of fuzzy points via fuzzy $\theta-open$ sets and fuzzy $\theta-closure$ operator," The Arabian Journal for Science and Engineering, pp. 1–11, 2008.
[9] A. S. Mashhour, A.A. Allam and K.M. Abd El-Hakeim, On fuzzy semicompact spaces, Bull. Fac. Sci., Assiut Univ., 16(1), 277-285 (1987).
[10] S. Nanda, Strongly compact fuzzy topological spaces, Fuzzy Sets and Systems, 42, 259-262 (1991).
[11] P. Sundram and M. Shekh John, On $W-closed$ sets in topology, Acta Cinica Indica 4, 389-392 (2000).
[12] C. K. Wong, Fuzzy points and local properties of fuzzy topology. Journal of Mathematical Analysis and Applications, 46, 316–328 (1974).
[13] T. H. Yalvac, Semi interior and semi closure of fuzzy sets,. J. math. Anal.Appl. 132 (1988) 356-364.
[14] T.H. Yalvac, Fuzzy sets and functions on fuzzy topological spaces, J. Math. Anal. Appl., 126, 409-423 (1987).
[15] L. A. Zadeh, fuzzy sets, Inform and control 18, 338-353 (1965).