Mathematical model for sounding rockets, using attitude and rotation angles

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Abstract - The paper purpose is to present some aspects regarding the calculus model and technical solutions for multistage sounding rockets used to test spatial equipment and scientific measurements. The calculus methodology consists in numerical simulation of sounding rocket evolution for different start conditions. The rocket model presented will be with six DOF and variable mass. At this item, as novelty of the work we will use simultaneously the rotation angles and the attitude angles for describing the kinematical equations of the movement. The results analyzed will be the flight parameters and the ballistic performances. The conclusions will focus technical possibilities to realize sounding multi-stage rocket recycling military rocket engines.

Keywords— Multi-stage, Mathematic model, Sounding rocket, Simulation, Rotation angles

NOMENCLATURE

α - Attack angle (tangent definition);
β - Sideslip angle (tangent definition);
β_p - Azimuth angle;
λ_p - Geocentric latitude;
ψ - Azimuth angle;
θ - Inclination angle;
ϕ - Bank angle;
ρ - Air density;
$\mathbf{\Omega}$ - Body angular velocity;
$\mathbf{\Omega}_p$ Earth spin:
A, B, C, E - Inertia moments;
$C_x^A; C_y^A; C_z^A$ - Aerodynamic coefficients of force in the mobile frame;
$C_l^A; C_m^A; C_n^A$, - Aerodynamic coefficients of momentum in the mobile frame;
$C_x^T; C_y^T; C_z^T$ - Thrust coefficients in the mobile frame;
$C_l^T; C_m^T; C_n^T$ - Thrust momentum coefficients in the mobile frame.
ξ, η, ζ - Rotation angles
$F_0 = \rho \frac{V^2}{2} S$ - Reference aerodynamic force;
$H_o^A = F_0 l$ - Reference aerodynamic couple;
T_0 - Reference thrust force
$H_o^T = T_0 l$ Reference couple thrust
l - Reference length;
m - Mass;
m_i - Initial mass
m_f - Final mass
p, q, r - Angular velocity components along the axes of mobile frame;
S - Reference area;
$\mathbf{T}$ - Thrust;
I_Σ - Total impulse;
t - Time;
$\mathbf{V}$ - Velocity;
u, v, w - Rocket velocity components in mobile frame;
V_{xp} V_{yp} V_{zp} Velocity components in Earth frame;
$OX_pY_pZ_p$ - Normal Earth-fixed frame;
$Oxyz$ – Body frame (mobile frame);
$x_py_pz_p$ - Mobile coordinates in Earth-fixed frame;
r - The distance between rocket and Earth center:
R_p - Earth radius:

I. INTRODUCTION

It is indisputable that today, the spatial program involves many collateral activities, like preliminary tests for equipment and qualifications. It is well known that those auxiliary activities suppose huge technical and financial effort and this increases the total cost for any space program. Starting from this idea, the paper proposes an economical solution for preliminary tests using small multistage sounding rockets by recycling military rocket engines. The sounding rockets are commonly used to take readings or carry instruments from 50 to 150 km above the surface of the Earth, the altitude generally between weather balloons and satellites. The region above the maximum altitude for balloons is about 40 km and the minimum for satellites is approximately 120 km. A common sounding rocket consists of a solid-fuel rocket motor and a payload. The freefall part of the flight is an elliptic trajectory

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with vertical major axis, allowing the payload to appear to hover near its apogee. The average flight time is less than 40 minutes, usually between 5 and 20 minutes. The rocket consumes its fuel on the first part of the flight, leaving the payload to complete the arc and return to the ground with a parachute. Sounding rockets are advantageous for some research due to their low cost, short lead time (sometimes less than six months) and their ability to conduct research in areas inaccessible to either balloons or satellites. They are also used as test beds for equipment that will be used in more expensive and risky orbital spaceflight missions. The smaller size of a sounding rocket also makes launching from temporary sites possible, allowing for field studies at remote locations, even in the middle of the ocean, if fired from a ship. Finally, they allow us to recycle military subsystems, like rocket engine with solid propellant. Sounding rockets are commonly used for:

- Research in Aeronomy, which requires this tool for *in situ* measurements in the upper atmosphere;
- Ultraviolet and X-Ray astronomy, which require being above the bulk of the Earth's atmosphere;
- Microgravity Research, which benefit from a few minutes of weightlessness on rockets launched to altitudes of a few hundred kilometers.

To approach these problems and in general for evaluating the launching capabilities is necessary to elaborate a complex mathematical model that ensures the rigorous and accuracy evaluation of the flight data and ballistic parameters. The mathematical model presented below, developed with maximum of accuracy, seeks to answer these needs. To solve this we use simultaneously the rotation angles, which presents a number of advantages which will be outlined in section III of the paper and the attitude angles (Euler angles). This model, allows us to evaluate two technical solutions, one of them based on three stages sounding rocket using the engines of short rocket 122 mm (fig. 4), and the second using four boosters, also from short 122 mm rocket around a central body obtained from long rocket 122 mm (fig. 5). These two technical solutions will be evaluated and the flight parameters and ballistic performances will be analyzed

II. The Gravity Acceleration, Complementary Acceleration, Connection Between Earth Frame and Body Frame

In the beginning we will start by analyzing the influence of the secondary parameters like the variation of the gravity acceleration with latitude and altitude and the influence of Earth spine about the sounding rocket trajectory.

The gravity acceleration and complementary acceleration

In order to write the movement equations we will use a geodesic frame [8] connected to the Earth. Due to diurnal spin, beside attraction force, we must consider two supplementary accelerations: carrying acceleration given by:

$$-\mathbf{\Omega}_p \times (\mathbf{\Omega}_p \times \mathbf{r})$$

and complementary acceleration (Coriolis acceleration) given by:

$$-2\mathbf{\Omega}_p \times \mathbf{V} ,$$

where Earth spin has the value:

$$\Omega_p = 7{,}2921 \cdot 10^{-5} \quad s^{-1} .$$

If we designate r - the distance between rocket and Earth center :

$$r = \sqrt{x_p^2 + (R_p + y_p)^2 + z_p^2} \tag{1}$$

where: $x_p ; y_p ; z_p$ are the rocket coordinates in the Earth frame and the Earth radius can be approximated by:

$$R_p \cong a(1 - \alpha \sin^2 \lambda_p) ,$$

then the gravity acceleration components in the Earth frame are:

$$g_{xp} = -g_r \frac{x_p}{r} - g_\omega \frac{\Omega_{xp}}{\Omega_p} ; \; g_{yp} = -g_r \frac{y_p + R_p}{r} - g_\omega \frac{\Omega_{yp}}{\Omega_p} ;$$

$$g_{zp} = -g_r \frac{z_p}{r} - g_\omega \frac{\Omega_{zp}}{\Omega_p} , \tag{2}$$

where the radials and polar components of the gravity acceleration [8], [3] are:

$$g_r = g_{Ar} - \Omega_p^2 r ; \qquad g_\omega = g_{A\omega} + \Omega_p^2 r \sin \lambda_p . \tag{3}$$

and Ω_{xp}; Ω_{yp}; Ω_{zp} - the spin components are given by:

$$\Omega_{xp} = \Omega_p \cos\lambda_p \cos\beta_p ; \quad \Omega_{yp} = \Omega_p \sin\lambda_p ;$$

$$\Omega_{zp} = -\Omega_p \cos\lambda_p \sin\beta_p , \tag{4}$$

where the two angles used are: β_p - azimuth angle and λ_p - geocentric latitude.

On another hand, complementary acceleration is:

$$\mathbf{a}_c = -2\mathbf{\Omega}_p \times \mathbf{V} , \tag{5}$$

with the Earth frame components given by:

$$a_{cxp} = 2(V_{yp}\Omega_{zp} - V_{zp}\Omega_{yp}) ; \; a_{cyp} = 2(V_{zp}\Omega_{xp} - V_{xp}\Omega_{zp}) ;$$

$$a_{czp} = 2(V_{xp}\Omega_{yp} - V_{yp}\Omega_{xp}) , \tag{6}$$

where $V_{xp};V_{yp};V_{zp}$ are the Earth frame velocity components.

The connection between Earth frame and body frame

The Earth frame ($OX_pY_pZ_p$) is a geocentric frame with the origin in the mass center of the Earth with Y_p axis orientated upward (fig. 1). The Earth can be considerate an ellipsoid of revolution [2],[3], [8]. In order to overlapping Earth frame over body frame we are passing through three intermediary frames. The first one is the starting frame $(Ox_s y_s z_s)$, which has y_s axis normal to the tangent plane at the Earth's surface, orientated upward (fig.1). Both frames participant in Earth rotation are not inertial frames. But, if we introduce as corrections the Earth spin influence by transport and complementary acceleration, previous defined, we can consider them as inertial frame.

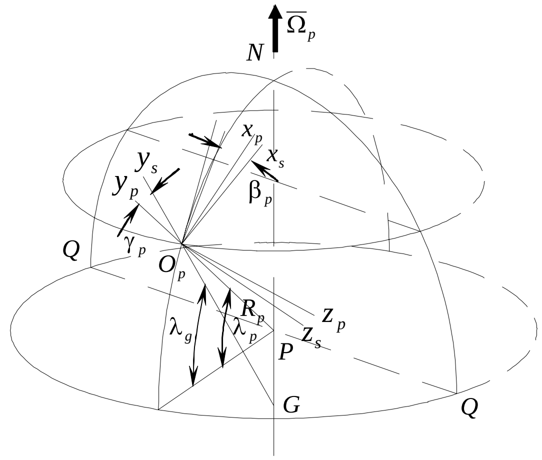

Fig.1 Connection between Earth frame and starting frame

If we denote γ_p the angle between geodesic normal and geocentric normal:

$$\gamma_p = \lambda_g - \lambda_p, \tag{7}$$

the connection between frames is given by:

$$\begin{bmatrix} x_s & y_s & z_s \end{bmatrix}^T = \mathbf{A}_{\gamma p} \begin{bmatrix} x_p & y_p & z_p \end{bmatrix}^T. \tag{8}$$

Overlap of the Earth frame above starting frame can be done by the rotation matrix:

$$\mathbf{A}_{\gamma p} = \begin{bmatrix} \cos\beta_p & 0 & \sin\beta_p \\ 0 & 1 & 0 \\ -\sin\beta_p & 0 & \cos\beta_p \end{bmatrix} \begin{bmatrix} \cos\gamma_p & -\sin\gamma_p & 0 \\ \sin\gamma_p & \cos\gamma_p & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\beta_p & 0 & -\sin\beta_p \\ 0 & 1 & 0 \\ \sin\beta_p & 0 & \cos\beta_p \end{bmatrix} \tag{9}$$

with the elements:

$a_{1,1} = \sin^2\beta_p + \cos^2\beta_p \cos\gamma_p$

$a_{1,2} = -\cos\beta_p \sin\gamma_p$

$a_{1,3} = \sin\beta_p \cos\beta_p (1 - \cos\gamma_p)$

$a_{2,1} = \cos\beta_p \sin\gamma_p$

$a_{2,2} = \cos\gamma_{p\,p}$

$a_{2,3} = -\sin\beta_p \sin\gamma_p$

$a_{3,1} = \sin\beta_p \cos\beta_p (1 - \cos\gamma_p)$

$a_{3,2} = \sin\beta_p \sin\gamma_p$

$a_{3,3} = \cos^2\beta_p + \sin^2\beta_p \cos\gamma_p$

In order to obtain the angle γ_p between geocentric and geodesic normal we must start from the relation [8]:

$$a^2 \operatorname{tg}\lambda_p = b^2 \operatorname{tg}\lambda_g, \tag{10}$$

where a, b are the Earth semi-axis.

From relation: $\alpha = (a-b)/a = 1/298{,}25$ which defines Earth flattering we can obtain the relation:

$$\operatorname{tg}\gamma_p = \frac{\alpha(1-\alpha/2)\sin 2\lambda_p}{1 - 2\alpha(1-\alpha/2)\cos^2\lambda_p}. \tag{11}$$

Obviously, if the start frame origin is on the Equator or on North or South Pole the angle γ_p is null and the matrix $\mathbf{A}_{\gamma p}$ became unitary matrix.

The next intermediary frame is the initial starting frame $(O_0 X_0 Y_0 Z_0)$, which overlap above the starting frame in the launching moment (fig. 2). The starting frame, which is attached to the Earth, is rotating around the polar axe related to the initial starting frame, considerate fix, with an angle equal with rotation angle of the Earth at this time.

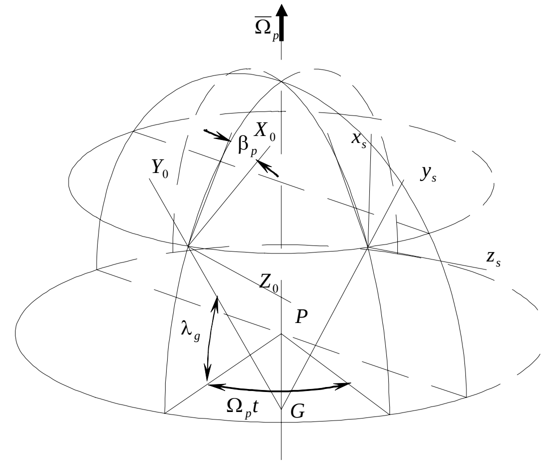

Fig. 2 Connection between starting frame and initial starting frame

The connection between these two frames is given by:

$$\begin{bmatrix} X_0 & Y_0 & Z_0 \end{bmatrix}^T = \mathbf{A}_{\Omega p} \begin{bmatrix} x_s & y_s & z_s \end{bmatrix}^T. \tag{12}$$

where the rotation matrix is:

$$\mathbf{A}_{\Omega p} = \begin{bmatrix} \cos\beta_p & 0 & \sin\beta_p \\ 0 & 1 & 0 \\ -\sin\beta_p & 0 & \cos\beta_p \end{bmatrix} \begin{bmatrix} \cos\lambda_g & -\sin\lambda_g & 0 \\ \sin\lambda_g & \cos\lambda_g & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Omega_p t & -\sin\Omega_p t \\ 0 & \sin\Omega_p t & \cos\Omega_p t \end{bmatrix} \times \times \begin{bmatrix} \cos\lambda_g & \sin\lambda_g & 0 \\ -\sin\lambda_g & \cos\lambda_g & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta_p & 0 & -\sin\beta_p \\ 0 & 1 & 0 \\ \sin\beta_p & 0 & \cos\beta_p \end{bmatrix}, \tag{13}$$

with the elements:

$a_{11} = \cos^2\beta_p \cos^2\lambda_g (1 - \cos\Omega_p t) + \cos\Omega_p t$;

$a_{12} = \cos\beta_p \sin\lambda_g \cos\lambda_g (1 - \cos\Omega_p t) + \sin\beta_p \cos\lambda_g \sin\Omega_p t$;

$a_{13} = -\sin\beta_p \cos\beta_p \cos^2\lambda_g (1 - \cos\Omega_p t) + \sin\lambda_g \sin\Omega_p t$;

$a_{21} = \cos\beta_p \sin\lambda_g \cos\lambda_g (1 - \cos\Omega_p t) - \sin\beta_p \cos\lambda_g \sin\Omega_p t$

$a_{22} = \sin^2\lambda_g (1 - \cos\Omega_p t) + \cos\Omega_p t$;

$a_{23} = -\sin\beta_p \sin\lambda_g \cos\lambda_g (1 - \cos\Omega_p t) - \cos\beta_p \cos\lambda_g \sin\Omega_p t$;

$a_{31} = -\sin\beta_p \cos\beta_p \cos^2\lambda_g (1-\cos\Omega_p t) - \sin\lambda_g \sin\Omega_p t$;

$a_{32} = -\sin\beta_p \sin\lambda_g \cos\lambda_g (1-\cos\Omega_p t) + \cos\beta_p \cos\lambda_g \sin\Omega_p t$;

$a_{33} = \sin^2\beta_p \cos^2\lambda_g (1-\cos\Omega_p t) + \cos\Omega_p t$.

Obviously, if the flight time is short, the matrix $\mathbf{A}_{\Omega p}$ becomes unitary matrix.

Finally, using Euler angle (ψ, θ, ϕ) or Euler modified angle $(\theta^*, \psi^*, \phi^*)$ or Hamilton's quaternion (q_1, q_2, q_3, q_4) or rotation angles ξ, η, ζ [10], we can overlap the initial start frame over the body frame. The rotation matrix $\mathbf{A}_i$, which makes this transformation, will be shown afterwards.

To sum up, in order to pass some elements from Earth frame to the body frame we need three rotations, which can be concentrated in a single matrix:

$$\mathbf{A}_p = \mathbf{A}_i \mathbf{A}_{\Omega p} \mathbf{A}_{\gamma p}, \tag{14}$$

which is the rotation matrix between Earth frame and body frame.

III. General Movement Equations

Kinematical equations

Unlike paper [4], which covers the regular ballistic rockets, where the kinematical equations use Euler angles, in our case, when we have almost vertical initial launching direction, the papers [3], [5], [6], [7], [8] recommend modified Euler angles, which have first rotation in vertical plane. Using kinematical equations written with Euler angles, in addition to benefits related to the significance of physical measurable sizes, the following drawback is involved: the use of trigonometric functions in program algorithms. Although complications related to solve the kinematical equations, the rotation angles can be used for trajectory control, as it will be shown next. So, using partial rotation matrix:

$$\mathbf{A}_{\theta^*} = \begin{bmatrix} \cos\theta^* & \sin\theta^* & 0 \\ -\sin\theta^* & \cos\theta^* & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad \mathbf{A}_{\psi^*} = \begin{bmatrix} \cos\psi^* & 0 & -\sin\psi^* \\ 0 & 1 & 0 \\ \sin\psi^* & 0 & \cos\psi^* \end{bmatrix};$$

$$\mathbf{A}_{\phi^*} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^* & \sin\phi \\ 0 & -\sin\phi^* & \cos\phi^* \end{bmatrix},$$

the complete rotation matrix becomes:

$$\mathbf{A}_i = \mathbf{A}_{\phi^*,\psi^*,\theta^*} = \mathbf{A}_{\phi^*} \mathbf{A}_{\psi^*} \mathbf{A}_{\theta^*}, \tag{15}$$

where, the elements are:

$a_{1,1} = \cos\psi^* \cos\theta^*$

$a_{1,2} = \cos\psi^* \sin\theta^*$.

$a_{1,3} = -\sin\psi^*$

$a_{2,1} = -\cos\phi^* \sin\theta^* + \sin\phi^* \sin\psi^* \cos\theta^*$

$a_{2,2} = \cos\phi^* \cos\theta^* + \sin\phi^* \sin\psi^* \sin\theta^*$

$a_{2,3} = \sin\phi^* \cos\psi^*$

$a_{3,1} = \sin\phi^* \sin\theta^* + \cos\phi^* \sin\psi^* \cos\theta^*$

$a_{3,2} = -\sin\phi^* \cos\theta^* + \cos\phi^* \sin\psi^* \sin\theta^*$

$a_{3,3} = \cos\phi^* \cos\psi^*$

In order to obtain the connection between the derivatives of Euler angles and components of rotation velocity in the body frame, starting with relation:

$$\mathbf{\Omega} = \dot{\boldsymbol{\theta}}^* + \dot{\boldsymbol{\psi}}^* + \dot{\boldsymbol{\phi}}^*, \tag{16}$$

we can write:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \mathbf{A}_{\phi^*,\psi^*,\theta^*} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}^* \end{bmatrix} + \mathbf{A}_{\phi^*,\psi^*} \begin{bmatrix} 0 \\ \dot{\psi}^* \\ 0 \end{bmatrix} + \mathbf{A}_{\phi^*} \begin{bmatrix} \dot{\phi}^* \\ 0 \\ 0 \end{bmatrix} = \mathbf{A}_{\phi^*,\psi^*} \begin{bmatrix} 0 \\ \dot{\theta}^* \\ \dot{\psi}^* \end{bmatrix} + \begin{bmatrix} \dot{\phi}^* \\ 0 \\ 0 \end{bmatrix}.$$

Because the rotation matrix is:

$$\mathbf{A}_{\phi^*,\psi^*} \equiv \mathbf{A}_{\phi^*,\psi^*,0},$$

we obtain:

$$\begin{bmatrix} p & q & r \end{bmatrix}^T = \mathbf{U}_A^* \begin{bmatrix} \dot{\phi}^* & \dot{\psi}^* & \dot{\theta}^* \end{bmatrix}^T.$$

where:

$$\mathbf{U}_A^* = \begin{bmatrix} 1 & 0 & -\sin\psi^* \\ 0 & \cos\phi^* & \sin\phi^* \cos\psi^* \\ 0 & -\sin\phi^* & \cos\phi^* \cos\psi^* \end{bmatrix} \tag{17}$$

Denoting $\mathbf{W}_A^*$ the connection matrix:

$$\mathbf{W}_A^* = U_A^{*-1} = \begin{bmatrix} 1 & \sin\phi^* \tan\psi^* & \cos\phi^* \tan\psi^* \\ 0 & \cos\phi^* & -\sin\phi^* \\ 0 & \sin\phi^* / \cos\psi^* & \cos\phi^* / \cos\psi^* \end{bmatrix}. \tag{18}$$

we can finally write:

$$\begin{bmatrix} \dot{\phi}^* & \dot{\psi}^* & \dot{\theta}^* \end{bmatrix}^T = \mathbf{W}_A^* \begin{bmatrix} p & q & r \end{bmatrix}^T, \tag{19}$$

Besides the kinematical relations with the attitude angles described above in the model we use the rotation angles that we will introduce next.

In paper [10] a group of three angles, called the rotation angles, were first introduced. The sizes were used to describe the aircraft movement.

Angles of rotation have the advantage that they can be measured easily on board of the aircraft or rocket. They retain the advantages of quaternion, removing singularity from kinematical equations written with the attitude angles (19). Also, allow the polynomial expression of the kinematical equations, an important advantage in building high-speed algorithms and easily implemented on hardware support. Angles of rotation retain the advantage of angles Euler type, that of being quantities directly measurable with a concrete physical meaning.

It is well known that a sequence of rotations of a rigid body with a fixed point can be replaced by a single rotation σ around an axis through the fixed point.

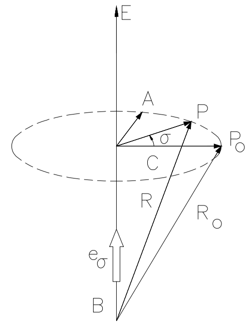

Fig. 3 Single rotation from fixed frame to mobile frame

In order to build kinematical equations we will use two frames:

$OX_0Y_0Z_0$ - The fixed frame with unitary vectors: $\mathbf{I},\mathbf{J},\mathbf{K}$;

Oxyz – The mobile frame, linked by body with unitary vectors $\mathbf{i},\mathbf{j},\mathbf{k}$;

We suppose that the body has angular velocity $\mathbf{\Omega}$ with the components (*p, q, r*) in mobile frame *Oxyz:*

$$\mathbf{\Omega} = \mathbf{i}p + \mathbf{j}q + \mathbf{k}r\ . \qquad (20)$$

Axis E is the axis around which a single rotation σ is necessary to overlap frame $OX_0Y_0Z_0$ over frame *Oxyz* (fig 3). Unitary vector for axis E is $\mathbf{e}_\sigma$:

$$\mathbf{e}_\sigma = \mathbf{I}l + \mathbf{J}m + \mathbf{K}n\ , \qquad (21)$$

Taken into account the notations from figure 3, we can write:

$$\mathbf{A} = \mathbf{e}_\sigma \times \mathbf{R}_0; \quad \mathbf{B} = \mathbf{e}_\sigma \cdot (\mathbf{e}_\sigma \cdot \mathbf{R}_0); \quad \mathbf{C} = \mathbf{R}_0 - \mathbf{B}\ . \qquad (22)$$

In this case, the relation between the position vectors of the point P and the point P_0 became successively:

$$\mathbf{R} = \mathbf{B} + \mathbf{A}\sin\sigma + \mathbf{C}\cos\sigma;$$

$$\mathbf{R} = \mathbf{e}_\sigma \cdot (\mathbf{e}_\sigma \cdot \mathbf{R}_0) + (\mathbf{e}_\sigma \times \mathbf{R}_0)\sin\sigma + [\mathbf{R}_0 - \mathbf{e}_\sigma \cdot (\mathbf{e}_\sigma \cdot \mathbf{R}_0)]\cos\sigma;$$

$$\mathbf{R} = \mathbf{R}_0\cos\sigma + \mathbf{e}_\sigma \cdot (\mathbf{e}_\sigma \cdot \mathbf{R}_0)(1-\cos\sigma) + (\mathbf{e}_\sigma \times \mathbf{R}_0)\sin\sigma. \quad (23)$$

If the point P_0 is located initially on the axis X_0, the point P will be finally on-axis x.

Because the vectors $\mathbf{R}$ and $\mathbf{R}_0$ are equal in module, we can substitute in relation (23):

$$\mathbf{R} \to \mathbf{i}; \quad \mathbf{R}_0 \to \mathbf{I}.$$

Similarly, if the point P is located on the axis y or z , we can substitute:

$$\mathbf{R} \to \mathbf{j}; \quad \mathbf{R}_0 \to \mathbf{J}; \quad \mathbf{R} \to \mathbf{k}; \quad \mathbf{R}_0 \to \mathbf{K}.$$

Finally we obtain the system:

$$\begin{aligned}
\mathbf{i} &= \mathbf{I}\cos\sigma + (\mathbf{I}l + \mathbf{J}m + \mathbf{K}n)l(1-\cos\sigma) + (\mathbf{I}l + \mathbf{J}m + \mathbf{K}n)\times\mathbf{I}\sin\sigma;\\
\mathbf{j} &= \mathbf{J}\cos\sigma + (\mathbf{I}l + \mathbf{J}m + \mathbf{K}n)m(1-\cos\sigma) + (\mathbf{I}l + \mathbf{J}m + \mathbf{K}n)\times\mathbf{J}\sin\sigma;\\
\mathbf{k} &= \mathbf{K}\cos\sigma + (\mathbf{I}l + \mathbf{J}m + \mathbf{K}n)n(1-\cos\sigma) + (\mathbf{I}l + \mathbf{J}m + \mathbf{K}n)\times\mathbf{K}\sin\sigma.
\end{aligned} \qquad (24)$$

If we note $c = \cos\sigma$; $s = \sin\sigma$, we obtain the relation:

$$[\mathbf{i} \quad \mathbf{j} \quad \mathbf{k}]^T = \mathbf{A}_i[\mathbf{I} \quad \mathbf{J} \quad \mathbf{K}]^T,$$

where, $\mathbf{A}_i$ is the direct rotation matrix:

$$\mathbf{A}_i = \begin{bmatrix} c + l^2(1-c) & lm(1-c) + ns & ln(1-c) - ms \\ lm(1-c) - ns & c + m^2(1-c) & mn(1-c) + ls \\ ln(1-c) + ms & mn(1-c) - ls & c + n^2(1-c) \end{bmatrix} \qquad (25)$$

which coincides with that defined by the relation (15) using attitude angles.

Thus, the overall rotation angle can be expressed by the superposition of three simultaneous rotations along the mobile frame axes:

$$\xi = \sigma l; \ \eta = \sigma m; \quad \zeta = \sigma n \qquad (26)$$

The sizes are called the rotation angles:

ξ - Rotation angle around x axis;

η - Rotation angle around y axis;

ζ - Rotation angle around z axis.

The angles check the relation:

$$\sigma^2 = \xi^2 + \eta^2 + \zeta^2\ . \qquad (27)$$

Using rotation angles, from (25) the rotation matrix becomes:

$$\mathbf{A}_i = \begin{bmatrix} a\xi^2 + c & a\eta\xi + b\zeta & a\zeta\xi - b\eta \\ a\xi\eta - b\zeta & a\eta^2 + c & a\zeta\eta + b\xi \\ a\xi\zeta + b\eta & a\eta\zeta - b\xi & a\zeta^2 + c \end{bmatrix} \qquad (28)$$

where:

$$a = \frac{1-c}{\sigma^2}; \quad b = \frac{s}{\sigma}; \ c = \cos\sigma; \ s = \sin\sigma \qquad (29)$$

Using the inverse of this matrix

$$\mathbf{B}_p = \mathbf{A}_p^{-1} = \mathbf{A}_p^T \qquad (30)$$

we can obtain the components of acceleration in the Earth frame from components of acceleration in the body frame, used in dynamical equations.

Like the kinematical equations we can write the relation:

$$[\dot{x}_p \quad \dot{y}_p \quad \dot{z}_p]^T = [V_{xp} \quad V_{yp} \quad V_{zp}]^T, \qquad (31)$$

where $[V_{xp} \quad V_{yp} \quad V_{zp}]^T$ are components of the velocity in the Earth frame.

Because the rotation matrix (15) and (28) is the same regardless of the variables used, we obtain the following relationships between different variables (Euler angles, rotation angles)

The attitude angles from the rotation angles are given by::

$$\tan\phi^* = \frac{a_{2,3}}{a_{3,3}} = \frac{a\zeta\eta + b\xi}{a\zeta^2 + c}\ ; \tan\theta^* = \frac{a_{1,2}}{a_{1,1}} = -\frac{a\eta\xi + b\zeta}{a\xi^2 + c}$$

$$\sin\psi^* = -a_{1,3} = -a\zeta\xi + b\eta\ ; \qquad (32)$$

Also, we can obtain the rotation angles from attitude angles using the relations:

$$\xi = \frac{a_{2,3} - a_{3,2}}{2b}\ ; \ \eta = \frac{a_{3,1} - a_{1,3}}{2b} \quad \zeta = \frac{a_{1,2} - a_{2,1}}{2b} \qquad (33)$$

where:

$$b=\frac{\sin\sigma}{\sigma}\ ;\ \sigma=\arccos c\quad c=\left(a_{1,1}+a_{2,2}+a_{3,3}-1\right)/2\,. \quad (34)$$

Next, we will try to obtain the connection between the derivatives of rotation angles and components of rotation velocity in the body frame.

Thus, as rotation around axis E is an equivalent transformation in terms of the two systems, it follows that the vector $\mathbf{e}_\sigma$ projections are identical:

$$\mathbf{e}_\sigma=\mathbf{I}l+\mathbf{J}m+\mathbf{K}n=\mathbf{i}l+\mathbf{j}m+\mathbf{k}n\ . \quad (35)$$

If this relationship is derived with respect to time we obtaine:

$$\mathbf{I}\dot{l}+\mathbf{J}\dot{m}+\mathbf{K}\dot{n}=\mathbf{i}\dot{l}+\mathbf{j}\dot{m}+\mathbf{k}\dot{n}+\mathbf{\Omega}\times\mathbf{e}_\sigma\,, \quad (36)$$

where:

$$\mathbf{\Omega}\times\mathbf{e}_\sigma=\mathbf{i}(qn-rm)+\mathbf{j}(rl-pn)+\mathbf{k}(pm-ql), \quad (37)$$

thus:

$$\mathbf{I}\dot{l}+\mathbf{J}\dot{m}+\mathbf{K}\dot{n}=\mathbf{i}(\dot{l}+qn-rm)+\mathbf{j}(\dot{m}+rl-pn)+\mathbf{k}(\dot{n}+pm-ql). \quad (38)$$

If we multiply successively by $\mathbf{i},\mathbf{j},\mathbf{k}$ results:

$$\begin{bmatrix}\mathbf{I}\cdot\mathbf{i} & \mathbf{J}\cdot\mathbf{i} & \mathbf{K}\cdot\mathbf{i}\\ \mathbf{I}\cdot\mathbf{j} & \mathbf{J}\cdot\mathbf{j} & \mathbf{K}\cdot\mathbf{j}\\ \mathbf{I}\cdot\mathbf{k} & \mathbf{J}\cdot\mathbf{k} & \mathbf{K}\cdot\mathbf{k}\end{bmatrix}\begin{bmatrix}\dot{l}\\ \dot{m}\\ \dot{n}\end{bmatrix}=\begin{bmatrix}\dot{l}\\ \dot{m}\\ \dot{n}\end{bmatrix}+\begin{bmatrix}0 & n & -m\\ -n & 0 & l\\ m & -l & 0\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix},$$

or otherwise:

$$\left[\mathbf{I}-\mathbf{A}_i\right]\begin{bmatrix}\dot{l}\\ \dot{m}\\ \dot{n}\end{bmatrix}=\begin{bmatrix}0 & -n & m\\ n & 0 & -l\\ -m & l & 0\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix}. \quad (39)$$

Introducing the matrix $\mathbf{A}_i$ given by (25), the left member of the relationship becomes:

$$\begin{bmatrix}1-c & -ns & ms\\ ns & 1-c & -ls\\ -ms & ls & 1-c\end{bmatrix}\begin{bmatrix}\dot{l}\\ \dot{m}\\ \dot{n}\end{bmatrix}-(1-c)\begin{bmatrix}l^2 & lm & nl\\ lm & m^2 & mn\\ nl & mn & n^2\end{bmatrix}\begin{bmatrix}\dot{l}\\ \dot{m}\\ \dot{n}\end{bmatrix}\equiv$$

$$\equiv\frac{2t}{1+t^2}\begin{bmatrix}t & -n & m\\ n & t & -l\\ -m & l & t\end{bmatrix}\begin{bmatrix}\dot{l}\\ \dot{m}\\ \dot{n}\end{bmatrix}-(1-c)(l\dot{l}+m\dot{m}+n\dot{n})\begin{bmatrix}l\\ m\\ n\end{bmatrix}, \quad (40)$$

where we noted $t=\mathrm{tg}(\sigma/2)$

Since the projections of unitar vector $\mathbf{e}_\sigma$ satisfying the relationship:

$$l^2+m^2+n^2=1, \quad (41)$$

results from differentiation:

$$l\dot{l}+m\dot{m}+n\dot{n}=0\,, \quad (42)$$

making the last term of the previous development to be null.

On the other hand reverse matrix of the first term of relation (40) is

$$\begin{bmatrix}t & -n & m\\ n & t & -l\\ -m & l & t\end{bmatrix}^{-1}=\frac{1}{t(1+t^2)}\begin{bmatrix}l^2+t^2 & lm+nt & nl-mt\\ lm-nt & m^2+t^2 & mn+lt\\ nl+mt & mn-lt & n^2+t^2\end{bmatrix}. \quad (43)$$

Multiplying by inverse matrix thus defined, relation (39) becomes:

$$2t\begin{bmatrix}\dot{l}\\ \dot{m}\\ \dot{n}\end{bmatrix}=\begin{bmatrix}1-l^2 & -lm-nt & -nl+mt\\ -lm+nt & 1-m^2 & -mn-lt\\ -nl-mt & -mn+lt & 1-n^2\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix}, \quad (44)$$

which leads to algebraic relations:

$$\begin{aligned}2\dot{l}&=rm-qn+(p-l\dot{\sigma})/t;\\ 2\dot{m}&=pn-rl+(q-m\dot{\sigma})/t;\\ 2\dot{n}&=ql-pm+(r-n\dot{\sigma})/t,\end{aligned} \quad (45)$$

where:

$$\dot{\sigma}=\mathbf{\Omega}\cdot\mathbf{e}_\sigma=pl+qm+rn\ . \quad (46)$$

By derivation of the definition relations (26) we obtaine:

$$\begin{aligned}\dot{\xi}&=(1-h)l\dot{\sigma}+h(p-ntq+mtr)\,;\\ \dot{\eta}&=(1-h)m\dot{\sigma}+h(ntp+q-ltr)\,;\\ \dot{\zeta}&=(1-h)n\dot{\sigma}+h(-mtp+ltq+r)\,,\end{aligned} \quad (47)$$

where derivatives of the angles of rotation can be put in the form:

$$\begin{bmatrix}\dot{\xi} & \dot{\eta} & \dot{\zeta}\end{bmatrix}^T=\mathbf{W}_R\begin{bmatrix}p & q & r\end{bmatrix}^T \quad (48)$$

in which, with notations:

$$h=\frac{\sigma}{2t};\quad f=\frac{1-h}{\sigma^2}, \quad (49)$$

connection matrix $\mathbf{W}_R$ is given by:

$$\mathbf{W}_R=f\begin{bmatrix}\xi^2 & \eta\xi & \zeta\xi\\ \xi\eta & \eta^2 & \zeta\eta\\ \xi\zeta & \eta\zeta & \zeta^2\end{bmatrix}+\frac{1}{2}\begin{bmatrix}0 & -\zeta & \eta\\ \zeta & 0 & -\xi\\ -\eta & \xi & 0\end{bmatrix}+h\mathbf{I}\,, \quad (50)$$

or, in compact form:

$$\mathbf{W}_R=\begin{bmatrix}f\xi^2+h & f\eta\xi-\zeta/2 & f\zeta\xi+\eta/2\\ f\xi\eta+\zeta/2 & f\eta^2+h & f\zeta\eta-\xi/2\\ f\xi\zeta-\eta/2 & f\eta\zeta+\xi/2 & f\zeta^2+h\end{bmatrix} \quad (51)$$

Relation (31) together with the relation (48) represents kinematical equations written using rotation angles. The relation (48) is equivalent with relation (18) which is written using attitude angles.

Dynamical equations

Developing cross products from paper [3], with the supplementary notations from [3], [4] we can obtain matrix representations of the dynamical equations:

- Force equations in the Earth frame

$$\begin{bmatrix}\dot{V}_{xp}\\ \dot{V}_{yp}\\ \dot{V}_{zp}\end{bmatrix}=\frac{1}{m}\mathbf{B}_p\left\{F_0\begin{bmatrix}C_x^A\\ C_y^A\\ C_z^A\end{bmatrix}+T_0\begin{bmatrix}C_x^T\\ C_y^T\\ C_z^T\end{bmatrix}\right\}+\begin{bmatrix}g_{xp}\\ g_{yp}\\ g_{zp}\end{bmatrix}+\begin{bmatrix}a_{cxp}\\ a_{cyp}\\ a_{czp}\end{bmatrix}; \quad (52)$$

- Moment equations in the body frame

$$\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix}=\mathbf{J}^{-1}\left\{H_e^A\begin{bmatrix}C_l^A\\ C_m^A\\ C_n^A\end{bmatrix}+H_e^T\begin{bmatrix}C_l^T\\ C_m^T\\ C_n^T\end{bmatrix}\right\}+\mathbf{J}^{-1}\begin{bmatrix}(B-C)qr\\ (C-A)rp\\ (A-B)pq\end{bmatrix}, \quad (53)$$

where we denoted:

$$\mathbf{J}^{-1} = \begin{bmatrix} 1/A & 0 & 0 \\ 0 & 1/B & 0 \\ 0 & 0 & 1/C \end{bmatrix} . \quad (54)$$

The inertial moment inverse matrix, where the inertial moments are given by:

$$A = \int (y^2 + z^2)\,\mathrm{d}\,m;\ B = \int (z^2 + x^2)\,\mathrm{d}\,m;\ C = \int (x^2 + y^2)\,\mathrm{d}\,m . \quad (55)$$

and the matrix $\mathbf{B}_p = \mathbf{A}_p^T$ is given by relation (30).

For the aerodynamic coefficients we used the method indicated in [4] and [9], based on polynomial series expanding:

$$\begin{aligned}
C_x^A &= a_1 + a_2(\alpha^2 + \beta^2) + a_3(\alpha^4 + \beta^4) + a_4\alpha^2\beta^2 + a_{13}(\hat{z}_p - \hat{z}_{pc});\\
C_y^A &= -b_1\alpha - b_2\alpha^3 - b_3\beta^2\alpha - b_4\hat{r};\\
C_z^A &= b_1\beta + b_2\beta^3 + b_3\alpha^2\beta + b_4\hat{q};\\
C_l^A &= c_2(\alpha^2 + \beta^2)\alpha\beta + c_3\hat{p} + c_5(\alpha\hat{q} - \beta\hat{r});\\
C_m^A &= d_1\beta + d_2\beta^3 + d_3\beta\alpha^2 + d_4\hat{q};\\
C_n^A &= d_1\alpha + d_2\alpha^3 + d_3\alpha\beta^2 + d_4\hat{r},
\end{aligned} \quad (55)$$

where, by definition [9]

$$\alpha = -\arctan(v/u)\,, \beta = \arctan(w/u)\,. \quad (56)$$

The following notations are used for the non dimensional angular velocities:

$$\hat{p} = p\,l/V\ ;\ \hat{q} = q\,l/V\ ;\ \hat{r} = r\,l/V\ ; \quad (57)$$

Through the thrust coefficients we insert the commands:

$$\begin{aligned}
&C_x^T = C_{xT\delta}(\delta_l, \delta_m, \delta_n)\ ;\ C_y^T = C_{yT\delta}\delta_n\ ;\ C_z^T = C_{yT\delta}\delta_m\\
&C_l^T = C_{lT\delta}\delta_l\ ;\ C_m^T = C_{mT\delta}\delta_m\ ;\ C_n^T = C_{nT\delta}\delta_n
\end{aligned} \quad (58)$$

IV. GUIDANCE COMMAND

Although the sounding missiles do not require a guided fly, we suppose a guided system which maintain the angle of inclination close to 90 degrees (vertical trajectory) and cancel lateral deviations. Also it maintains the roll angle at a prescribed value (45 deg). Although the core of calculation used angles of rotation, applying relations (32) we get as secondary size the attitude angles, which we can use to build the guidance command.

Resuming [3], the guidance commands for the ballistic guided rocket are the simple form:

$$\mathbf{u} = \mathbf{K}_1\mathbf{A}_p\begin{bmatrix}0 & 0 & u_z\end{bmatrix}^T + \mathbf{U}_A^*\begin{bmatrix}u_{\phi^*} & u_{\psi^*} & u_{\theta^*}\end{bmatrix}^T , \quad (59)$$

where the main control signals are:

$$\begin{aligned}
&u_z = k_u^h h_z;\quad u_{\Phi^*} = k_u^{\phi}\tilde{\phi}^* + k_u^{\dot{\phi}}\dot{\tilde{\phi}}^*\\
&u_{\Theta^*} = k_u^{\theta}\tilde{\theta}^* + k_u^{\dot{\theta}}\dot{\tilde{\theta}}^* + k_u^{I_\theta} I_{\theta^*};\quad u_{\psi^*} = k_u^{\psi}\tilde{\psi}^* + k_u^{\dot{\psi}}\dot{\tilde{\psi}}^*
\end{aligned} , \quad (60)$$

The matrix $\mathbf{A}_P$ and $\mathbf{U}_A^*$, were the previously presented and the signification of $\mathbf{K}_1$ is:

$$\mathbf{K}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \quad (61)$$

The parameters relative $\tilde{\theta}^*; \tilde{\psi}^*; \tilde{\phi}^*; h_z$ are given by:

$\tilde{\theta}^* = \theta_d^* - \theta^*;\ \tilde{\psi}^* = \psi_d^* - \psi^*;\ \tilde{\phi}^* = \phi_d^* - \phi^*;\ h_z = z_{pd} - z_p$

where $\theta_d^*;\ \psi_d^*\ \phi_d^*\ ;\ z_{pd}$ are input reference values.

The integrals term are defined hereby:

$$\begin{bmatrix}\dot{I}_{\phi^*} & \dot{I}_{\theta^*} & \dot{I}_{\psi^*}\end{bmatrix} = \begin{bmatrix}\tilde{\phi}^* & \tilde{\theta}^* & \tilde{\psi}^*\end{bmatrix}. \quad (62)$$

The connection between derivatives of Euler angles and the body angular velocity components are given by (19) where the matrix $\mathbf{W}_A^*$ is given by (18), and his inverse by (17). On another hand, the matrix $\mathbf{A}_p$ used in relation (59) can be approximated by matrix $\mathbf{A}_i$ which is given directly by relation (15). In this case, the relation (59) can be written in the scalar form:

$$\begin{aligned}
u_l &= u_{\phi^*} - u_{\theta^*}\sin\psi^*;\\
u_m &= u_{\psi^*}\cos\phi^* + u_{\theta^*}\sin\xi^*\cos\psi^* - u_z\cos\phi^*\cos\psi^*;\\
u_n &= u_{\Theta^*}\cos\phi^*\cos\psi^* - u_{\psi^*}\sin\phi^* + u_z\sin\phi^*\cos\psi^*;
\end{aligned} \quad (63)$$

The guidance commands are applied to the actuators which are approximated in the paper [3] by the matrix form:

$$[\dot{\delta}_l \quad \dot{\delta}_m \quad \dot{\delta}_n]^T = \mathbf{D}_\delta[\delta_l \quad \delta_m \quad \delta_n]^T + \mathbf{D}_u\mathbf{u}\,, \quad (64)$$

and the scalar form:

$$\dot{\delta}_l = -\frac{\delta_l}{\tau_{\delta l}} + \frac{k_{\delta l}^u u_l}{\tau_{\delta l}};\quad \dot{\delta}_m = -\frac{\delta_m}{\tau_{\delta m}} + \frac{k_{\delta m}^u u_m}{\tau_{\delta m}};\quad \dot{\delta}_n = -\frac{\delta_n}{\tau_{\delta n}} + \frac{k_{\delta n}^u u_n}{\tau_{\delta n}}. \quad (65)$$

where $\tau_{\delta l}; \tau_{\delta m}; \tau_{\delta n}$ are time constants and $k_{\delta l}^u; k_{\delta m}^u; k_{\delta n}^u$ gain constants.

In the next item of the presentation, using the simulation results, we will show that the benefit in performance of the guidance system does not justify the technical effort to use such system for sounding missile.

V. INPUT DATA, CALCULUS ALGORITHM AND RESULTS

Input data for the model

Figure 4 shows the first model, called „VLS T3”, and main characteristics are included in Table 1.

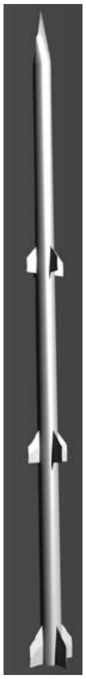

TABLE I. VLS T3 CHARACTERISTICS

VLS T3	m_i [kg]	m_f [kg]	Length [m]
I Stage	96	85	4.2
II Stage	69	58	3.0
III Stage	42	31	1.8
Final stage	16	16	0.6

Three stage launcher using the engines of short rocket 122 mm. Total impulse of the short engine: $I_\Sigma = 2.5 kNs$

Fig. 4 VLS T3 configuration

Fig. 5 shows the second model, called „VLS B4”, and main characteristics are included in Table II.

Table II. VLS B4 Characteristics

VLS B4	m_i [kg]	m_f [kg]	Length [m]
I Stage (with boosters)	164	120	2,4
II Stage	57	31	2,4
Final stage	16	16	0,6

Central body from long rocket 122 mm and four boosters, from short rocket around. Total impulse of the long engine: $I_\Sigma = 6.2 kNs$

Fig. 5 VLS B4 configuration

Calculus algorithm

The calculus algorithm consists in multi-step method Adams' predictor-corrector with variable step integration method: [1] [17]. Absolute numerical error was 1.e-12, and relative error was 1.e-10.

Test calculus

For the test calculus the following initial conditions were used:
Geographic orientation:

- Azimuth angle $\beta_p = 90°$ (towards the East);
- Geocentric latitude $\lambda_p = 45°$ (Romania latitude);
- Altitude: $y_0 = 1[m]$

Initial velocity $V_0 = 40[m/s]$;

Initial inclination angle $\theta_0 = 85°$

Results

First item consists in choosing between two possible integration models: first performs a separate integration for each stage, and the second performs continuum integration along entire trajectory. For the first model the final moving conditions for the previous work stage became initial conditions for the next work stage. First approach provides us a better model for the mass variation in the moment of stage separation, when the rocket mass has a decreasing jump. Contrary, the second model gives us wrong simulation of the mass jump as we can see in fig 6.

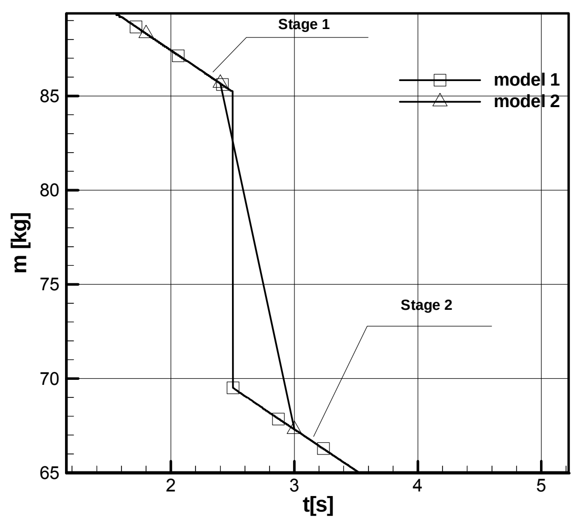

Fig. 6 Mass diagram – different integration model VLST3

Second item consists in the advantage and disadvantage of using a guidance system for fly control. From the figure 7 we can see a comparison between guided and unguided trajectory. We can observe that the difference in altitude is insignificant related to technical effort due for trajectory control. Accordingly, in the next development we will use unguided trajectory.

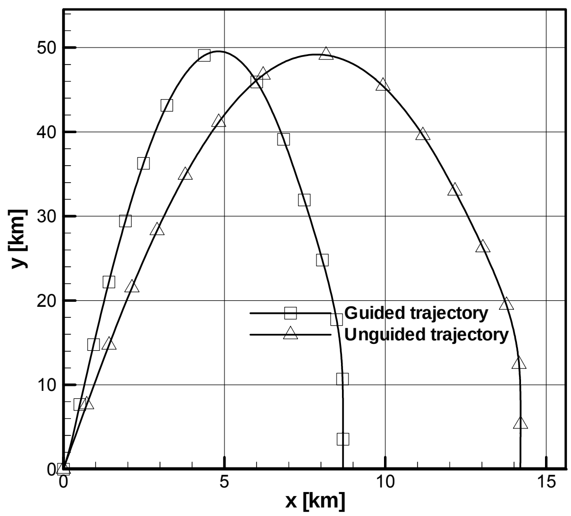

Fig. 7 Comparison between guided and unguided trajectory VLS B4

In the figures 8-11 are comparatively shown the flight parameters and the ballistic performances of the two models.

Fig. 8 shows the mass variation in time along the trajectory.

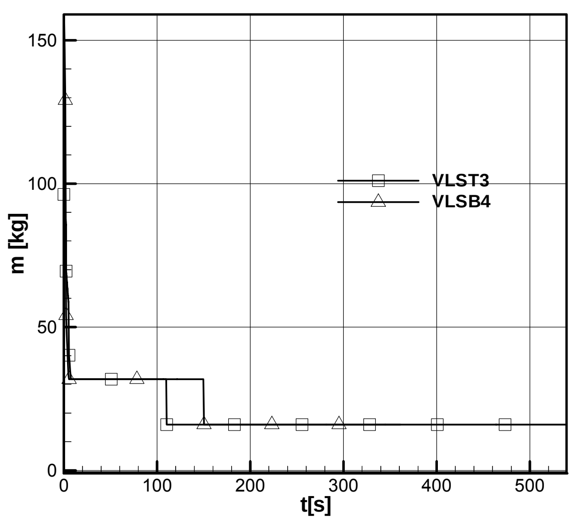

Fig. 8 Mass comparison between VLST3 and VLSB4

We can see a quickly mass decrease, followed by constant period, after burnout. In fig 9 is presented the velocity diagram. It can be observed the difference between the models, the velocity of the second being greater.

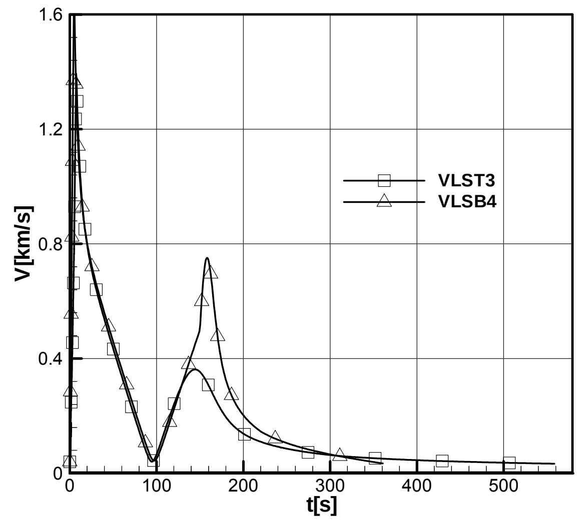

Fig. 9 Velocity comparison between VLST3 and VLSB4

Consequently, due to the velocity difference the trajectory of the second is higher (fig. 10). Finally, in fig. 11 we have shown the inclination angle in time. It can be observed that at trajectory apex we have increase instability especially for the second model which attend higher trajectory.

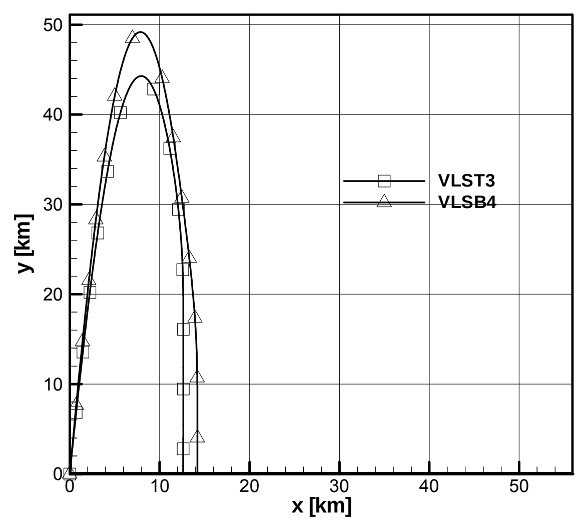

Fig. 10 Ballistic trajectory

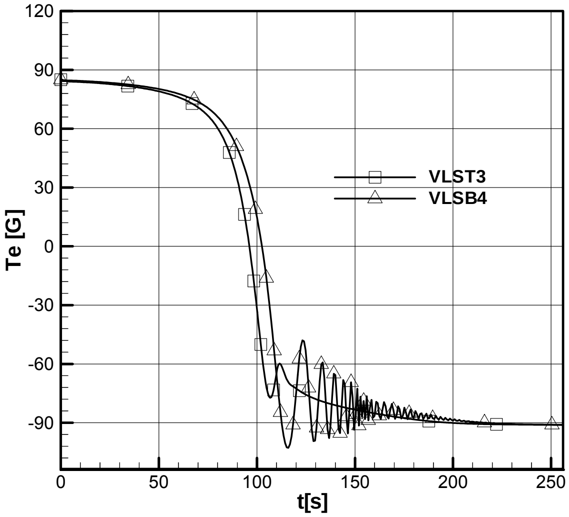

Fig. 11 Inclination angle diagram

VI. Conclusions

The paper presents synthesis aspects of the simulation model, developed for the calculation of an operational rocket - which will launch equipment that will be tested with the aim of integrating them into complex spatial systems. The application is made for two variants of the sounding rockets, which will be tested in the national projects. We considered more possible solutions and we presented and analyzed the flight parameters for two sounding rockets that can be used. Since the major objective of the sounding rockets consists of testing solutions for assembling and detachment of multi-stage rockets and to launch at large angles, to eliminate the additional risk, it will be used the rocket motors in current production connected in parallel or in tandem, solutions what where presented during the work. In conclusion, the main sub-assemblies of the sounding rockets, will be provided from current production, the engine being developed in Romania and being in significant amounts in deposits. Assembly and testing of the ground will be made at the plant where usually such systems are manufactured, and finally the sounding rockets will be tested by firing in an area at the Black Sea.

References

[1] BAKHVALOV, N. Methodes Numeriques – Analyse, algebre, equations differentielles ordinaires Ed. Mir Moscou , 1976.

[2] BOIFFIER, J.L. The Dynamics of Flight – The Equations, John Wiley & Sons , Chichester, New York, Weinheim, Brisbane, Singapore, Toronto , ISBN 0-471-94237-5, 1998.

[3] CHELARU T.V. Dinamica Zborului–Racheta dirijată, Ediția a-II-a revizuită şi adaugită ((*Dynamic flight – guided missile – 2-nd edition)*. ,Ed.Printech, Bucureşti, ISBN 973-718-013-5 , mai 2004.

[4] CHELARU T.V. Dinamica Zborului–Racheta nedirijată (Dynamic flight – unguided missile) ,Ed. Printech, Bucureşti, ISBN 973-718-428-9, 155 pag. ,martie 2006.

[5] CHELARU, T.V., COMAN, A, Some theoretical aspects regarding the mathematical model for launching a small satellite on low orbit - International Workshop on SMALL SATELLITES, NEW MISSIONS AND NEW TECHNOLOGIES, Istanbul, Turkey, 05-07 June 2008.

[6] CHELARU T.V., BARBU C.,CHELARU A., *Mathematical model and technical solution for small multistage suborbital launchers,* Proceedings of the 4th International Conference on Recent Advances in Space Technologies-RAST2009, ISBN:978-1-4244-3626-2, Istambul, Turkey, 11-13 Jun 2009.

[7] Kuzovkov, N.T., Sistemî stabilizații letatelnîh appararov ,balisticeschih i zenitnîh raket, Ed. Vîsşaia Şkola , Moskva, 1976.

[8] Lebedev, A.A., Gerasiota, N.F., Balistika raket, Ed. Maşinostroenie, Moskva, 1970.

[9] Nielsen, J.N., Missile Aerodynamics, McGraw-Hill Book Company, Inc., New-York, Toronto, London, 1960.

[10] Niță, M.M., Chelaru,T.V., PÂRVU, P., Some considerations on modeling flight vehicles movement, Revue Roumaine des Sciens Techniques Mecanique Appliquee, Tome 36 Nr 5-6 , septembre-decembre - 1991, Bucureşti

[11] Jerzy Stefan Respondek- Observability of 4th Order Dynamical Systems *Proceedings of the 8th WSEAS International Conference on SIMULATION, MODELLING and OPTIMIZATION (SMO '08)* Santander, Cantabria, Spain, September 23-25, 2008

[12] Anton V. Doroshin - Synthesis of Attitude Motion of Variable Mass Coaxial Bodies, *WSEAS TRANSACTIONS on SYSTEMS AND CONTROL* Volume 3, 2008 ISSN: 1991-8763

[13] Piotr Kulczycki- Nonparametric Estimation for Control Engineering, *Proceedings of the 4th WSEAS/IASME International Conference DYNAMICAL SYSTEMS and CONTROL (CONTROL'08)* Corfu, Greece, October 26-28, 2008

[14] Zheng Jiewang1,Wei Li and Shijun Guo - Lateral Control for an Aircraft of Folding Wing *Proceedings of the 8th WSEAS International Conference on SIMULATION, MODELLING and OPTIMIZATION (SMO '08)* Santander, Cantabria, Spain, September 23-25, 2008

[15] Tain-Sou Tsay - High Accurate Positioning Technique for AUV *WSEAS TRANSACTIONS on SYSTEMS AND CONTROL* Volume 4, 2009 ISSN: 1991-8763

[16] ISO 1151 -1:1988; -2:1985-3:1989

[17] SLATEC Common Mathematical Library, Version 4.1, July 1993

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