# Scaled Image Edge Detection Based on The Total Variation Functional

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***Abstract*—We present total variation anisotropic edge detection. We derive the total variation functional from measure theory, distribution theory and vector gradient method. The Euler-Lagrange equation of the total variation functional gives a steady state equation. The steady state equation acts as an anisotropic filter on an image. The total variation filtered images are compared to Laplacian filtered images. A subsequent application of the zero crossing algorithm works quite well for the traditional Marr-Hildreth method but gives poorer results for the total variation filtered images. It was found that thresholding methods work better and saves computational time. Also, our results show that total variation edge detection overcomes some drawbacks associated with the Marr-Hildreth method.**

***Index Terms*—Anisotropic, Euler-Lagrange, LoG, Total Variation**

## I. Introduction

EDGE detection is a fundamental operation in image processing. Most methods for edge detection can be grouped into two basic categories:

1. Search-based methods which compute the edge strength using first order derivatives. An example of this is the Canny edge detection method.
2. Zero-crossing-based methods which search for zero-crossings in a second order derivative expression computed from the image. This could be the zero crossings of the Laplacian or a nonlinear differential expression. A popular method is the Marr-Hildreth method.

Historically, the Marr-Hildreth edge detection method has been the most popular edge detection method based on second order partial differential equations. It uses the Lapalacian to filter an image. The zero-crossings algorithm is then applied on the image to detect the edges. If the edge threshold is zero, the edges form a closed loop. This is one of the drawbacks of the method. The Laplacian used in the Marr-Hildreth model is a linear partial differential equation and it has isotropic filtering properties. We approach the problem differently. We begin from the total variation functional for a two dimensional image. We minimize the functional by the Euler-Lagrange method. This results in a nonlinear steady state second order partial differential equation. Because the operator is nonlinear, applying it to an image results in an anisotrpic filtering of the image. The filtered image when compared to images filtered using the Marr-Hildreth operator shows stronger image edges. It will be observed that the TV-filtered image is devoid of double edges as is the case with the Laplacian operator.

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## II. Total Variation

A function $f$ defined on an interval $[a, b]$ is said to be of bounded variation if there is a constant $C > 0$ such that

$$\sum_{k=1}^{n} |f(x_k) - f(x_{k-1})| \leq C$$

for every partition

$$a = x_0 < x_1 < ... < x_n = b$$

of $[a, b]$ by points of subdivision $x_0, x_1, ..., x_n$. If $f$ is of bounded variation, then by the total variation of $f$ is meant the quantity

$$V_a^b(f) = \sup \sum_{k=1}^{n} |f(x_k) - f(x_{k-1})|$$

where the least upper bound is taken over all partitions of the interval $[a, b]$. The above definitions work quite well for Riemann integrable functions. However, there are functions that are not Riemann integrable. For this reason, we turn to a more complete understanding of the total variation in the light of Lebesgue integration.

Let $\lambda$ be a charge on $X$ and let $P, N$ be a Hahn decomposition for $\lambda$. The positive and negative variations of $\lambda$ are the finite measures$\lambda^+, \lambda^-$ defined for $E$ in $X$ by

$$\lambda^+(E) = \lambda(E \cap P)$$

$$\lambda^-(E) = -\lambda(E \cap N)$$

The total variation of $\lambda$ is the measure $|\lambda|$ defined for $E$ in $X$ by

$$|\lambda| = \lambda^+(E) + \lambda^-(E)$$

If $f$ is Lebesgue-integrable and $f$ belongs to $L(X, \mathbf{X}, \mu)$ with respect to a measure $\mu$ on $X$, and if $\lambda$ is defined for $E$ in $X$ by

$$\lambda(E) = \int_E f d\mu,$$

then $\lambda$ is a charge and

$$\lambda^+(E) = \int_E f^+ d\mu$$

$$\lambda^-(E) = \int_E f^- d\mu$$

so that

$$|\lambda|(E) = \int_E |f| d\mu = \int_E f^+ d\mu + \int_E f^- d\mu \quad (1)$$

Here, $f$ is any Lebesgue-integrable function. The sum of the positive and negative variations of the function $f$ gives its total variation.

A function $\mu \in L^1(\Omega)$ whose partial derivatives in the sense of distributions are measures with finite total variation in $\Omega$ is called a function of bounded variation. This class of functions is usually denoted by $BV(\Omega)$. Thus, $u \in BV(\Omega)$ if and only if there are signed measures $\mu_1, \mu_2, ..., \mu_n$ defined in $\Omega$ such that for $i = 1, 2, ..., n$,

$$|Du|(\Omega) < \infty$$

and

$$\int u D_i \varphi dx = -\int \varphi d\mu_i \quad (2)$$

for all $\varphi \in C_0^\infty(\Omega)$. The gradient of $\mu$ will therefore be a vector valued measure with finite total variation:

$$\begin{aligned} &\|Du\| = \\ &\quad \sup\{\int_\Omega u \mathrm{div} v dx : v = (v_1, ..., v_n) \in C_0^\infty(\Omega; R^n), \\ &\qquad |v(x)| \leq 1 \forall x \in \Omega\} < \infty \end{aligned} \quad (3)$$

The divergence of a vector field is denoted by $\mathrm{div} v$ and is defined by

$$\mathrm{div} v = \sum_{i=1}^n D_i v_i = \sum_{i=1}^n \frac{\partial v_i}{\partial x_i}$$

If $u \in BV(\Omega)$, the total variation $\|Du\|$ may be regarded as a measure, for if $f$ is a non-negative real-valued continuous function with compact support in $\Omega$, we may define

$$\begin{aligned} &\|Du\|(f) = \\ &\quad \sup\{\int_\Omega u \mathrm{div} v dx : v = (v_1, ..., v_n) \in C_0^\infty(\Omega; R^n), \\ &\qquad |v(x)| \leq f(x) \forall x \in \Omega\} \end{aligned} \quad (4)$$

[11] shows that $||Du||$ is additive, continuous under monotone convergence and a non-negative Radon measure on $\Omega$. The space of absolutely continuous $u$ with $u' \in L^1(R^1)$ is contained in $BV(R^1)$. In the same manner in $R^n$, a Sobolev function is also $BV$. That is, $W^{1,1}(\Omega) \subset BV(\Omega)$. If $u \in W^{1,1}(\Omega)$ then,

$$\int_\Omega u \mathrm{div} v dx = -\int_\Omega \sum_{i=1}^n D_i u v dx \quad (5)$$

and the gradient of $u$ has finite total variation with

$$||Du||(\Omega) = \int_\Omega |Du| dx. \quad (6)$$

(6) is related to (1) in the sense that (6) measures both the positive and negative variations implicitly rather than explicitly as in (1). If $u \in C^1(\Omega)$, then [3] showed that

$$\int_\Omega |Du| dx = \int_\Omega |\nabla u(x)| dx \quad (7)$$

Let $\Omega \in R^2$ be an image surface. Then from (7), we have

$$\int_\Omega |\nabla u(x)| dx = \int\int \sqrt{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2} dx dy \quad (8)$$

which is the total variation of an image $u$ with two independent variables $x, y$. Furthermore, we derive the total variation of an image using the vector gradient method. Images are two dimensional spatial functions. To find the total variation of an $n$-dimensional mathematical object, we consider the directional derivative of a scalar function $f(\vec{x}) = f(x_1, x_2, ..., x_n)$ along a unit vector $\vec{u} = (u_1, ..., u_n)$. The directional derivative is defined to be the limit

$$\nabla_{\vec{u}} f(\vec{x}) = \lim_{h \to 0^+} \frac{f(\vec{x} + h\vec{u}) - f(\vec{x})}{h} \quad (9)$$

We assume the function $f$ to be differentiable at $\vec{x}$. This means that the directional derivatives exist along any unit vector $\vec{u}$, and one has

$$\nabla_{\vec{u}} f(\vec{x}) = \nabla f(\vec{x}) \cdot \vec{u} \quad (10)$$

For an image we represent the directional derivatives by **i,j** so that

$$\begin{cases} \nabla_x f(\vec{x}) = \nabla f(\vec{x}) \cdot \mathbf{i} & = \frac{\partial f}{\partial x} \\ \nabla_y f(\vec{x}) = \nabla f(\vec{x}) \cdot \mathbf{j} & = \frac{\partial f}{\partial y} \end{cases} \quad (11)$$

The components $\nabla_x f(\vec{x})$ and $\nabla_y f(\vec{x})$ are orthogonal so that

$$\nabla f(\vec{x}) = \mathbf{i}\frac{\partial f}{\partial x} + \mathbf{j}\frac{\partial f}{\partial y} \quad (12)$$

and the inner product is

$$\langle \nabla f(\vec{x}), \nabla f(\vec{x}) \rangle = |\nabla f| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} \quad (13)$$

Therefore, total variation in two dimensions can be written as

$$\int\int |\nabla f| \, dx dy \quad (14)$$

which is the total variation of $f$ over the entire image surface. An important property of the total variation of a function $f$ is that it relies on the derivative of $f$ to measure the total change in $f$.

## III. The Total Variation of an Image

Equation (8) is a norm and the vectorial components of $\nabla f$ are $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. So, the norm

$$\int\int |\nabla f| dx dy = \int\int \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} dx dy \quad (15)$$

In the discrete form,

$$\frac{\partial f}{\partial x} = f(x+1, y) - f(x, y)$$

$$\frac{\partial f}{\partial y} = f(x, y+1) - f(x, y)$$

So,

$$|\nabla f| = \sqrt{[f(x+1, y) - f(x, y)]^2 + [f(x, y+1) - f(x, y)]^2} \quad (16)$$

Therefore, the discrete total variation is written as:

$$\sum_x \sum_y |\nabla f| = \sum_x \sum_y \sqrt{[f(x+1, y) - f(x, y)]^2 + [f(x, y+1) - f(x, y)]^2} \quad (17)$$

## IV. The Euler Equation of the Total Variation Functional

We restate the total variation functional (following the general convention) as

$$J(u) = \int |\nabla u| dx dy \quad (18)$$

where $u$ is our image function and it depends spatially on two independent variables $x$ and $y$. The total variation functional has one dependent variable (the image function $u$) and two indepedent variables $x$ and $y$. First, we state the form of the Euler-Lagrange equation for the case of one dependent variable and several independent variables. Let

$$I(f) = \int_\Omega F(x_1, x_2, ..., x_n, f, f_{x_1}, f_{x_2}, ..., f_{x_n}) dx \quad (19)$$

where $f_{x_i} = \frac{\partial f}{\partial x_i}$. (19) is extremized only if $f$ satisfies the partial differentiation equation:

$$\frac{\partial F}{\partial f} - \sum_{i=1}^{n} \frac{\partial}{\partial x_i} \frac{\partial F}{\partial f_{x_i}} = 0 \quad (20)$$

We observe that the total variation functional can be written as

$$J(x, y, u, |\nabla u|) = \int |\nabla u| dx dy \quad (21)$$

The right hand side of the equation tells us that the functional doses not depend on $u$ but it depends on its gradient because

$$|\nabla u| = \sqrt{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2} = f \quad (22)$$

and there is no term in $u$ on the right hand side. So the Euler equation for the total variation is:

$$\frac{\partial f}{\partial u} - \sum_{i=1}^{2} \frac{\partial}{\partial x_i} \frac{\partial f}{\partial x_i} = 0 \quad (23)$$

but $\frac{\partial f}{\partial u} = 0$ since the functional $J$ does not depend on $u$. So we have

$$\sum_{i=1}^{2} \frac{\partial}{\partial x_i} \frac{\partial f}{\partial x_i} = \frac{\partial}{\partial x_1} \frac{\partial f}{\partial x_1} + \frac{\partial}{\partial x_2} \frac{\partial f}{\partial x_2}. \quad (24)$$

In an image, $x_1 = x$ and $x_2 = y$ so (24) becomes

$$\sum_{i=1}^{2} \frac{\partial}{\partial x_i} \frac{\partial f}{\partial x_i} = \frac{\partial}{\partial x} \frac{\partial f}{\partial x} + \frac{\partial}{\partial y} \frac{\partial f}{\partial y}. \quad (25)$$

This is equivalent to differentiating $|\nabla u|$ with respect to $x$ and $y$. We made $|\nabla u| = f$ for simplicity. So differentiating with respect to $x$, we get

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right]^{\frac{1}{2}} = \frac{1}{\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right]^{\frac{1}{2}}} \cdot \frac{\partial u}{\partial x} = \frac{1}{|\nabla u|} \cdot \frac{\partial u}{\partial x} \quad (26)$$

Also,

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right]^{\frac{1}{2}} = \frac{1}{\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right]^{\frac{1}{2}}} \cdot \frac{\partial u}{\partial y} = \frac{1}{|\nabla u|} \cdot \frac{\partial u}{\partial y} \quad (27)$$

So (25) becomes

$$\frac{\partial}{\partial x} \cdot \frac{\frac{\partial u}{\partial x}}{|\nabla u|} + \frac{\partial}{\partial y} \cdot \frac{\frac{\partial u}{\partial y}}{|\nabla u|} = \frac{1}{|\nabla u|}\left(\frac{\partial}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial}{\partial y}\frac{\partial u}{\partial y}\right) \quad (28)$$

We can write (28) in vector form as:

$$\frac{1}{|\nabla u|}\left(\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y}\right) \cdot \left(\mathbf{i}\frac{\partial u}{\partial x} + \mathbf{j}\frac{\partial u}{\partial y}\right) = \nabla \cdot \frac{\nabla u}{|\nabla u|}. \quad (29)$$

This can be rewritten as

$$\nabla \cdot \frac{\nabla u}{|\nabla u|} = \frac{1}{|\nabla u|}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = \frac{\nabla^2 u}{|\nabla u|} = \frac{\Delta u}{|\nabla u|}. \quad (30)$$

From the foregoing analysis, the Euler equation of the total variation functional is

$$\min J(u) = J_{\min}(u) = \nabla \cdot \frac{\nabla u}{|\nabla u|} \quad (31)$$

or,

$$J_{\min}(u) = \frac{\nabla^2 u}{|\nabla u|}. \quad (32)$$

## V. Partial Differential Equations Image Filters

Prior to edge detection, a partial second order partial differential equation (usually a Laplacian) is used to filter an image. The result of this filtering is then tested for zero crossings. This is based on the Marr-Hildreth(1980) model. The use of the Laplacian operator

$$L = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} \tag{33}$$

to operate on an image $u$ produces a Laplacian filtered image. This is a basic step in image segmentation using partial differential equations. An example of such segmentation is edge detection. The approach basically consists of defining a discrete formulation of the second order derivative and then constructing a filter mask based on the formulation. The mask is then convolved with the image to produce the filtered image. The Laplacian filter is isotropic. On the other hand the discretized form of the Laplacian can be applied directly to the image. The results are the same. Isotropic filters are rotation invariant in the sense that rotating the image and then applying the filter gives the same result as applying the filter first and then rotating the image. The simplest isotropic derivative operator is the Laplacian [16], which for a function(image) of two variables, is defined as

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \tag{34}$$

Because derivatives of any order are linear operators, the Laplacian is also a linear operator. To implement the Laplacian on a digital image, we discretize it as follows:

$$\begin{cases} \frac{\partial^2}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y) \\ \frac{\partial^2}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y) \end{cases} \tag{35}$$

So, adding the two equations,

$$\begin{aligned} \nabla^2 f(x,y) = \\ & f(x+1,y) + f(x-1,y) + f(x,y+1) \\ & + f(x,y-1) - 4f(x,y) \end{aligned} \tag{36}$$

which filters the image. This equation can also be implemented using the filter mask:

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | -4 | 1 |
| 0 | 1 | 0 |

TABLE I: Mask 1

The mask gives an isotropic result for rotations. Figure 1 shows the effect of this mask on an image. A variant of Mask 1 which works better is Mask 2 shown in Table 2. It is also isotropic.

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | -8 | 1 |
| 1 | 1 | 1 |

TABLE II: Mask 2

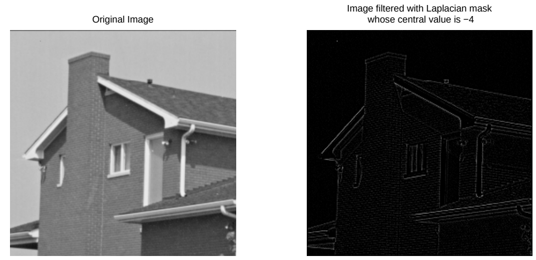


Fig. 1: Image Filtered with Mask 1

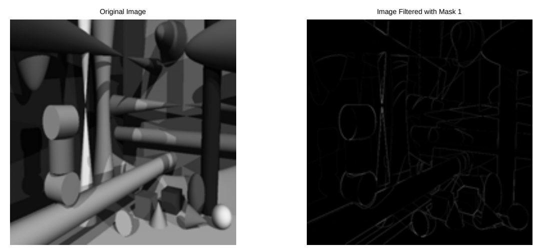


Fig. 2: Image Filtered with Mask 1

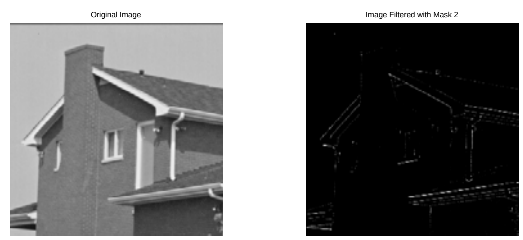


Fig. 3: Image Filtered with Mask 2

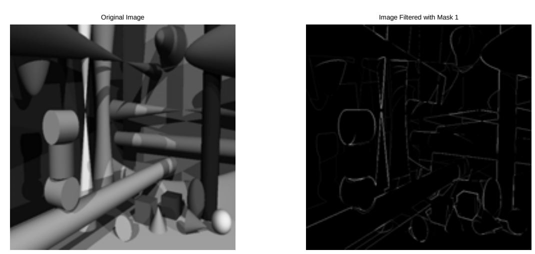


Fig. 4: Image Filtered with Mask 2

## VI. The Total Variation Filter

The total variation of a function(image) is usually used in the form of its Euler equation.

$$J_{\min}(u) = \frac{\nabla^2 u}{|\nabla u|}. \tag{37}$$

But we can see that the $J_{\min}(u)$ is related to the Laplacian. The numerator of $J_{\min}(u)$ is the Laplacian. The difference is that the denominator is $|\nabla u|$. Now, $|\nabla u|$ is a function. It takes different values at different pixels in the image. Both the Laplacian and $J_{\min}(u)$ represent diffusion. The Laplacian is isotropic but the total variation filter is anisotropic. Below we present images filtered with the total variation filter.

Fig. 5: Original Image(House)

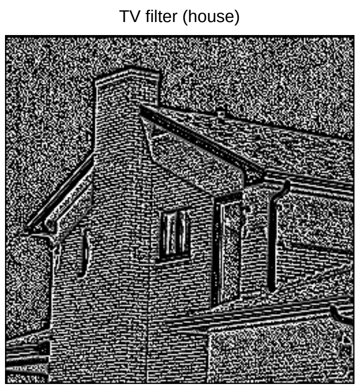


Fig. 6: TV Filtered Image(House)

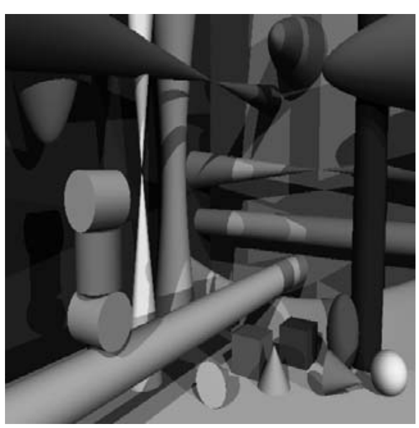

Fig. 7: Original Image(Shapes)

The anisotropic property of the TV filter gives it some advantages over the traditional Laplacian filter. We present these advantages in the next section.

## VII. Advantages of The Total Variation Filter

We state some reasons that give the total variation filter an advantage over the traditional Laplacian filter:

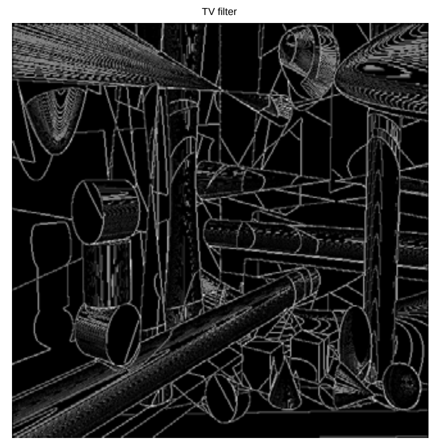


Fig. 8: TV Filtered Image of Shapes

1. The total variation filter is better at edge-sensitive filtering than the Laplacian. A comparison of the TV-filtered images (Figures 5 to 8) to the other images (Figures 1 to 4) shows that the edges in the TV-filtered images are conspicuously more pronounced and detailed.
2. The total variation filter is quite texture-sensitive. This is very obvious in Figure 8. The detailed changes in the texture of the images are clearly captured in the filtered image.
3. A drawback of the Laplacian is that during the filteration process, a double edge is formed. The total variation filter overcomes this disadvantage (see Figures 13 and 14). There is only one edge formed and it is appreciably strong. Unlike the Laplacian filter, the TV filter does not show false edges. The images being compared below show this clearly. To illustrate this point, we show an image with sharp edges (Figure 9). The image was filtered with Mask 1, Mask 2 and the total variation filter. Mask 1 and Mask 2 are Laplacian filters and so the corresponding images are scaled to show the presence of double edges. This is not so in the case of the total variation filter. There is only one edge.
4. A major challenge in edge detection is disconnection of curves. This is very evident when the Laplacian is used to detect edges in images. The TV filter gives stronger edges and so mitigates the problem caused by weak Laplacian edges.

## VIII. Detecting Edges

We have shown the effect of image filtering with the Laplacian and total variation filters. However, in practice, this is insufficient to properly detect edges because the Laplacian operator is very sensitive to noise and small oscillations and so we may not obtain the real edges in an image. To remedy this problem [6] used a 2D Gaussian function

$$G(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}} \tag{38}$$

to smooth the image $u(x, y)$. This is accomplished by a convolution of both functions i.e. $G(x, y) \star u(x, y)$. $\sigma$ is

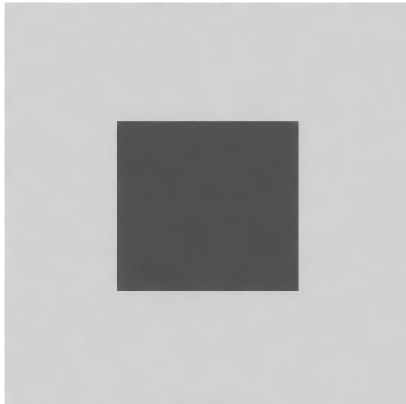

Fig. 9: Original Image (Noise Square)

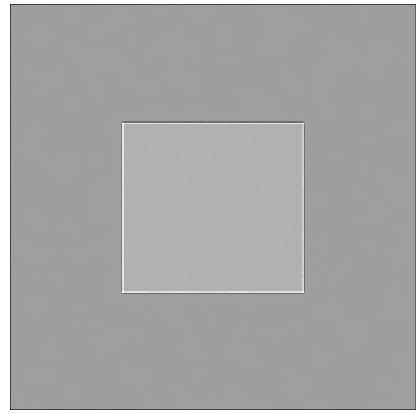

Fig. 11: Image filtered with Mask 2: It is Scaled to Show Double Edges

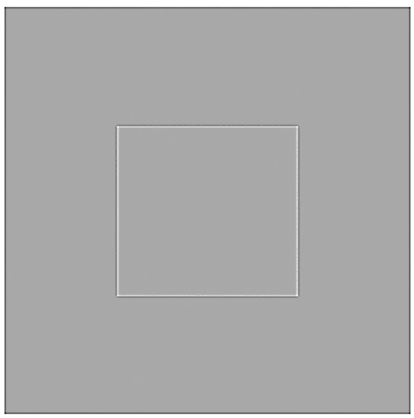

Fig. 10: Image filtered with Mask 1: It is Scaled to Show Double Edges

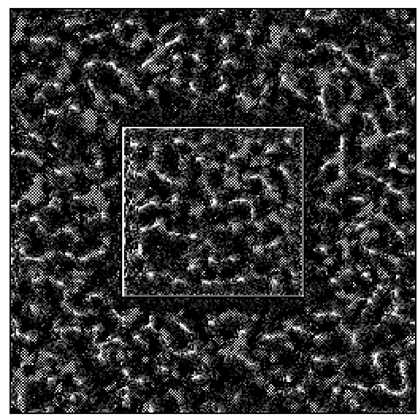

Fig. 12: Image filtered with TV Filter: No double Edges

the standard deviation of $G(x, y)$ and it acts as a scaling factor, blurring out noise and structures with scales below $\sigma$. Therefore, we get no absolute definition of edges. We only talk about edges at a certain scale. The convolution

$$D(x, y) = G(x, y) \star u(x, y) \tag{39}$$

yields a smoothed image at a scale $\sigma$ to which the Laplacian operator is applied i.e. $\nabla^2 (G(x, y) \star u(x, y))$. This operation is commutative and gives the same result as

$$\left(\nabla^2 G(x, y)\right) \star u(x, y) = \nabla^2 D(x, y) \tag{40}$$

The expression $\nabla^2 G$ is called the Laplacian of a Gaussian (LoG) and is expressed as

$$\nabla^2 G(x, y) = \left[\frac{x^2 + y^2 - 2\sigma^2}{\sigma^4}\right] e^{-\frac{x^2+y^2}{2\sigma^2}} \tag{41}$$

The scaling factor $\sigma$ enables us to tune the edge detection process. Likewise, for the total variation approach, we first smooth the image by convolving $u(x, y)$ and the Gaussian function $G(x, y)$ just as in the case of the Laplacian approach. We then apply the total variation operator to the convolved image $D(x, y)$ i.e. $\frac{\nabla^2 D(x,y)}{|\nabla D|}$. This operation is not commutative as in the case of the LoG. Below we show results of edge detection based on the LoG images and total variation filtered images. Edge detection for both kinds of images do not follow exactly the same procedure. The traditional method of detecting edges in LoG images is by zero crossings. Figure 8 shows an example of this. However, this method is not effective for the total variation filtered images. The zero crossings do not give strong edges as expected. Therefore, for application purposes, it is best to avoid the use of zero crossings in total variation filtered images. Generally, the edges in total variation filtered images are stronger than edges in LoG images. the pixels in

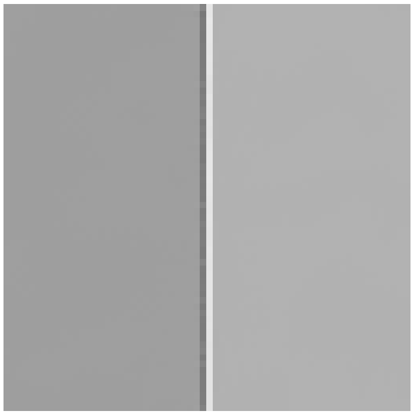

Fig. 13: Magnified Region of Interest Showing Double Edges

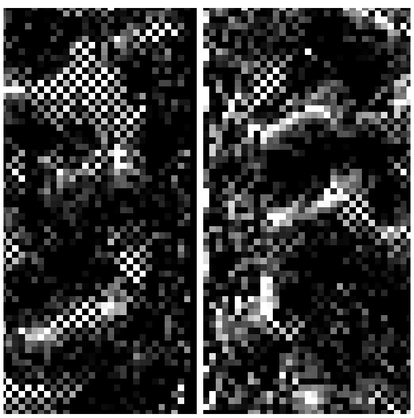

Fig. 14: Magnified Region of Interest Showing Single Edge in TV-Filtered Image

the LoG images were magnified by a factor of 30 to give the result in Figure 8. But even after a magnification of the pixel strength, the edges in the LoG image still do not possess the same strength and detail as the edges in the total variation filtered image. We present further examples of image edge detection at various scales $\sigma = 3, 4, 5$.

## IX. Conclusion

We have compared the total variation edge detection method to the Marr-Hildreth method and found that:

1. The TV method gives stronger edges and detail than the Marr-Hildreth method.
2. The TV filter produces only a single strong edge rather than the double edges in the Laplacian filtered image.
3. The zero crossing method used in detecting edges in the LoG images is not effective in TV filtered images. Simple thresholding is more effective.

Fig. 15: Gaussian Smoothed Image with Standard Deviation = 2

Fig. 16: Zero Crossing of LoG Image with Pixel Strength Magnification of 30

4. Thresholding LoG images at 0 after the zero-crosings algorithm is applied produces closed-loop edges. The total variation method does not exhibit this drawback.

## References

[1] Tony F. Chan and Jianchong Shen, *Image Processing and Analysis: Variational, PDE, Wavelet and Stochastic Methods*, SIAM, 2005.

[2] Rafael Gonzalez and Richard E. Woods, *Digital Image Processing*, Pearson Prentice Hall, 2008.

[3] Gilles Aubert and Pierre Kornprobst, *Mathematical Problems in Image Processing: Partial Differential Equations and the Calculus of Variations, Second Edition*, Springer, 2006.

Fig. 17: Edge Detection in Total Variation Filtered Image with Thresholding at 0

Fig. 19: Laplacian Edge Detection with Pixel Strength Magnification of 30 and $\sigma = 3$

Fig. 18: Gaussian Smoothed Image with $\sigma = 3$

Fig. 20: Total Variation Edge Detection at $\sigma = 3$

[4] April Khademi, Farhang Sahba and Anastasios Venetsanopoulos, Multiscale Edge Detection and Classification for Automatic Diagnosis of Mammographic Lesions, *8th International Conference on Signal Processing, Computational Geometry and Artificial Vision, Rhodes, Greece,* August 20-22, 2008.
[5] Al Bovik, *Handbook of Image and Video Processing,* Academic Press, 2000.
[6] D. Marr and E. Hildreth, Theory of Edge Detection, *Proc. R. Soc. Lond. B 207, 187-217,* 1980.
[7] Mark Nixon and Alberto Aguado, *Feature Extraction and Image Processing, Second Edition,* Academic Press, 2008.
[8] Lawrence O'Gorman, Michael J. Sammon and Michael Seul, *Practical Algorithms for Image Analysis, Second Edition,* Cambridge University Press, 2008.
[9] Azzam Sleit et al., An Edge Detection Algorithm for Online Image Analysis, *Recent Advances in Applied Mathematics,* WSEAS Press, 2010.
[10] J.R. Parker, *Algorithms for Image Processing and Computer Vision,* Wiley Computer Publishing, 1997.
[11] William P. Ziemer, *Weakly Differentiable Functions,* Springer Verlag, 1989.
[12] Fritz John, *Partial Differential Equations, Fourth Edition,* Springer, 1982.
[13] Zhou Liang and Zheng Jianguo, A New Approach for Color Image Edge Detection Using Improved PCNN, *Proceedings of The 7th WSEAS International Conference on Simulation, Modelling and Optimization, Beijing, China,* September 15-17, 2007.
[14] Michael E. Taylor, *Partial Differential Equations III: Nonlinear Equations,* Springer, 1996.
[15] J.D. Logan, *An Introduction to Nonlinear Partial Differential Equations, Second Edition,* John Wiley and Sons, Inc., 2008.
[16] Rosenfeld A. and Kak A.C., *Digital Picture Processing,* Academic Press, 1976.
[17] Ehsan Nadernajad, Sara Sharifzadeh and Hamid Hassanpour, Edge Detection Techniques: Evaluation and Comparisons *Ap-*

Fig. 21: Gaussian Smoothed Image with $\sigma = 4$



Fig. 22: Laplacian Edge Detection with Pixel Strength Magnification of 30 and $\sigma = 4$

*plied Mathematical Sciences,*Vol. 2, 2008, no. 31, 1507-1520.

[18] Robert G. Bartle, *The Elements of Integration and Lebesgue Measure,* John Wiley and Sons, Inc., 1995.

[19] Liming Zhang, A Direction Based Novel Edge Detector, *Proceedings of the 7th WSEAS International Conference on Automation Information, Cavtat, Croatia (pp 116-121),* June 13-15, 2006.

[20] A.N. Kolmogorov and S.V. Fomin, *An Introductory Real Analysis,* Dover Publications, Inc., 1970.

[21] Walter Rudin, *Real and Complex Analysis, Third Edition,* WCB/McGraw Hill, 1987.

[22] Marek Capinski and Ekkerhard Kopp, *Measure, Integral and Probability, Second Edition,* Springer-Verlag, 2004.

[23] Serge Lang, *Real and Functional Analysis, Third Edition,* Springer-Verlag, 1993.

Fig. 23: Total Variation Edge Detection at $\sigma = 4$

Fig. 24: Gaussian Smoothed Image with $\sigma = 5$

Fig. 25: Laplacian Edge Detection with Pixel Strength Magnification of 30 and $\sigma = 5$

Fig. 26: Total Variation Edge Detection at $\sigma = 5$