Monocriteria and multicriteria based placement of reactive power sources in distribution systems

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***Abstract*—**This paper presents results of research on formulating and solving the problem of capacitor placement in distribution systems within the framework of monocriteria and multicriteia models. The application of the multicriteria approach is aimed, first of all, at overcoming the difficulties of simultaneous observation of contradictory constraints for the upper and lower permissible voltage limits at different buses of distribution systems as well as other important conditions in operating capacitors. The solution of the capacitor placement problem in the monocriteria statement is based on applying the generalized algorithms of discrete optimization. The solution of the problem in the multicriteria statement is based on combining the Bellman-Zadeh approach to decision making in a fuzzy environment with the application of the generalized algorithms of discrete optimization. The paper results are illustrated by computational experiments with a real distribution system.

***Keywords*—**Discrete optimization, Distribution systems, Muticriteria decision making, Reactive power compensation.

I. Introduction

CAPACITORS are widely used in distribution systems for reactive power compensation to achieve power and energy loss reduction and to improve the system voltage profile. Traditionally, the problem solution is directed at the determination of the locations, sizes, and types of capacitors to minimize the objective function of an economical character, while the constraints on voltage magnitudes at different load levels are satisfied.

The methods for solving the reactive power compensation problem are analyzed and classified in [1]. The following groups of methods are classified: analytical methods, numerical methods, heuristic methods, and artificial intelligence based methods (associated with applying genetic algorithms, expert systems, simulated annealing, artificial neural networks, and fuzzy set theory). All works considered in [1] are directed at solving the problem within the framework of monocriteria models.

The majority of more recent works related to reactive power compensation in distribution systems (for example, [2]-[6]) is also directed at solving the problem within the monocriteria framework.

The present paper is devoted to formulating and solving the capacitor placement problem within the framework of monocriteria as well multicriteria models. The results of the work are based on applying the generalized algorithms of discrete optimization (with the use of the Bellman-Zadeh approach to decision making in a fuzzy environment in the case of the multicriteria statement). The rationality of the utilization of the multicriteria approach is associated with the following considerations.

The necessity to simultaneously observe constraints on the upper and lower permissible voltage limits at different buses of distribution systems creates essential difficulties (it is not uncommon that these constraints generate situations when the corresponding feasible regions are empty). These difficulties can be overcome by minimizing the objective function of an economical character as well as the objective function which reflects a volume of poor energy (energy consumed with voltage magnitudes outside of the permissible limits) consumption. The flexibility of including the objective function reflecting energy quality is confirmed by the results of [7]. These results are based on application of fuzzy logic. However, they do not permit one to take directly into account the discrete nature of the capacitor placement problem.

Thus, if the problem is associated with the determination of the locations and sizes of fixed capacitors, it can be approximated by a bicriteria model. At the same time, if we talk about the determination of the locations, sizes, and types (fixed or switched) of capacitors, the number of the objective

Manuscript received May X, 2011: Revised version received XXXX XX, 2011. This research was supported by the National Council for Scientific and Technological Development of Brazil (CNPq) under Grant 307474/2008-9, Minas Gerais State Research Foundation (FAPEMIG) under grant BPD-00235-10, and Minas Gerais State Energy Company (CEMIG).

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functions is to be more. In particular, one of the most important questions in operating switched capacitors is the observation of a desirable (permissible) number of their commutations per time unit [8]. However, although the paper results are of a general character, considering its limited size, the problem formulation is related to the determination of locations and sizes of capacitors.

II. Problem Statement

Considering the discrete nature of the capacitor placement problem, the generalized algorithms of discrete optimization are used for its solution. These algorithms, firstly, have been presented in [9]. The results of their development are reflected, for instance, in [10]-[12]. The algorithms are associated with the method of normalized functions [13] and combine formal and informal procedures. In particular, the algorithms are based on applying the ideas of greedy heuristics [14], [15] which provide the best heuristic among possible heuristics with *a priori* estimates and offer a basis for effective approximated approaches. The algorithms allow one to obtain quasi-optimal solutions after a small number of steps, overcoming the NP-completeness. They do not require analytical specification of objective functions and constraints. This ensures the flexibility and the possibility to correctly reflect diverse types of initial data using the so-called discrete sequences

$$x_{s_i}, \rho_{s_i}, \tau_{s_i}, ..., \quad s_i = 1,..., v_i \tag{1}$$

where $\rho_{s_i}, \tau_{s_i}, ...$ are technical, economical, and other characteristics required for forming objective functions, constraints, and their increments, which correspond to the sth discrete (integer, Boolean) value of the variable x_i. Considering this, it is rational to formulate the problem of capacitor placement as follows.

If capacitors can be installed on the middle and low voltage levels, then information on capacitors can be presented as the increasing discrete sequences

$$Q_{(m)}, \ K_{(m)}, \ \mathrm{tg}\delta_{(m)}, \quad m = 1,..., p \tag{2}$$

for capacitors of the middle voltage level and

$$Q_{(l)}, \ K_{(l)}, \ \mathrm{tg}\delta_{(l)}, \quad l = 1,..., q \tag{3}$$

for capacitors of the low voltage level, respectively.

In (2) and (3), $Q_{(m)}$ and $Q_{(l)}$ are standard sizes of the mth and lth capacitors for the middle and low voltage levels, respectively; $K_{(m)}$ and $K_{(l)}$ are their total costs; $\mathrm{tg}\delta_{(m)}$ and $\mathrm{tg}\delta_{(l)}$ are their specific losses. It is natural that different discrete sequences can be applied to different buses. However, without loss of generality, we consider the same discrete sequences for the middle and low voltage buses to simplify the problem statement.

From the discrete sequences (2) and (3), it is necessary to choose $Q_{(m)i^M}, m = 1,..., p, i^M \in I^M \equiv \{1,..., n^M\}$ (I^M is a set of the middle voltage buses where capacitors can be installed) and $Q_{(l)i^L}, l = 1,..., q, i^L \in I^L \equiv \{1,\dots, n^L\}$ (I^L is a set of the low voltage buses where capacitors can be installed) to minimize the following objective function of the net present value (NPV) of the project's lifetime:

$$\begin{aligned}
&Z(Q_{(m)1},..., Q_{(m)i^M},..., Q_{(m)n^M}, Q_{(l)1},..., Q_{(l)i^L},..., Q_{(l)n^L}) = \\
&\sum_{i^M=1}^{n^M} K_{(m)i^M} + \sum_{i^L=1}^{n^L} K_{(l)i^L} + \\
&\sum_{\omega=1}^{\Omega} \frac{1}{(1+r)^{\omega}} \left\{ \left[\left(\frac{p_{o\&m} + p_d}{100} \right) \left(\sum_{i^M=1}^{n^M} K_{(m)i^M} + \sum_{i^L=1}^{n^L} K_{(l)i^L} \right) + \right.\right. \\
&c_e \sum_{t=1}^{T} \left(\sum_{i^M=1}^{n^M} \mathrm{tg}\,\delta_{(m)i^M} Q_{(m)i^M} + \sum_{i^L=1}^{n^L} \mathrm{tg}\,\delta_{(l)i^L} Q_{(l)i^L} \right) + \\
&c_p \sum_{t\in p} \left(\sum_{i^M=1}^{n^M} \mathrm{tg}\,\delta_{(m)i^M} Q_{(m)i^M} + \sum_{i^L=1}^{n^L} \mathrm{tg}\,\delta_{(l)i^L} Q_{(l)i^L} \right) + \\
&c_{op} \sum_{t\in op} \left(\sum_{i^M=1}^{n^M} \mathrm{tg}\,\delta_{(m)i^M} Q_{(m)i^M} + \sum_{i^L=1}^{n^L} \mathrm{tg}\,\delta_{(l)i^L} Q_{(l)i^L} \right) + \\
&\frac{c_e}{10^3 V^2} \sum_{f=1}^{g} R_f \sum_{t=1}^{T} \left(\sum_{\substack{i=1\\ f\leftarrow i}}^{n} Q_i^t - \sum_{\substack{i^M=1\\ f\leftarrow i^M}}^{n^M} Q_{(m)i^M} - \sum_{\substack{i^L=1\\ f\leftarrow i^L}}^{n^L} Q_{(l)i^L} \right)^2 + \\
&\frac{c_p}{10^3 V^2} \sum_{f=1}^{g} R_f \sum_{t\in p} \left(\sum_{\substack{i=1\\ f\leftarrow i}}^{n} Q_i^t - \sum_{\substack{i^M=1\\ f\leftarrow i^M}}^{n^M} Q_{(m)i^M} - \sum_{\substack{i^L=1\\ f\leftarrow i^L}}^{n^L} Q_{(l)i^L} \right)^2 + \\
&\frac{c_{op}}{10^3 V^2} \sum_{f=1}^{g} R_f \sum_{t\in op} \left(\sum_{\substack{i=1\\ f\leftarrow i}}^{n} Q_i^t - \sum_{\substack{i^M=1\\ f\leftarrow i^M}}^{n^M} Q_{(m)i^M} - \sum_{\substack{i^L=1\\ f\leftarrow i^L}}^{n^L} Q_{(l)i^L} \right)^2
\end{aligned} \tag{4}$$

where $t = 1,...,T$ is a current index of load curve steps; $t \in p$ means that a step belongs to power system peak load time; $t \in op$ means that a step is out of power system peak load time; $f \leftarrow i$ means that load of bus i flows through branch f; $f \leftarrow i^M$ means that reactive power of a capacitor i^M flows through branch f; $f \leftarrow i^L$ means that reactive power of a capacitor i^L flows through branch f; V is a network nominal voltage; $R_f, f = 1,..., g$ is a resistance of branch f; $\omega = 1,...,\Omega$ is a current year; r is a discount rate; $p_{o\&m}$ and p_d are relative expenses associated with capacitor operation and maintenance and depreciation, respectively; c_e is energy cost; c_p and c_{op} are transportation tariffs for power system peak load time and out of power system peak load time.

The minimization of (4) must be executed while the constraints on voltage magnitudes at different load levels are satisfied. These constraints can be related to the most remote consumers and to the nearest consumers of low voltage networks. Considering this, the constraints for the inferior permissible voltage levels can be presented as

$$V_j^{re,t} + \frac{1}{10V^2}\left(\sum_{i^M=1}^{n^M} Q_{(m)i^M} \sum_{\substack{f=1 \\ f\in W_j}}^{g} X_f + \sum_{i^L=1}^{n^L} Q_{(l)i^L} \sum_{\substack{f=1 \\ f\in W_j}}^{g} X_f \right) \ge V^-,$$
$$j \in J^-,\ t = 1,...,T \tag{5}$$

where J^- is a set of the most remote consumers of low voltage networks with voltage levels $V_j^{re,t} < V^-$ (V^- is the permissible inferior voltage level); X_f is a reactance of branch f; $f \in W_j$ means that branch f belongs to the way of supplying a low voltage network $j = 1,...,J$.

The constraints for the nearest consumers of low voltage networks can be presented as follows:

$$V_j^{ne,t} + \frac{1}{10V^2}\left(\sum_{i^M=1}^{n^M} Q_{(m)i^M} \sum_{\substack{f=1 \\ f\in W_j}}^{g} X_f + \sum_{i^L=1}^{n^L} Q_{(l)i^L} \sum_{\substack{f=1 \\ f\in W_j}}^{g} X_f \right) \le V^+,$$
$$j = 1,...,J,\ t = 1,...,T \tag{6}$$

where $V_j^{ne,t}$ is a voltage level of the nearest consumer of a low voltage network j; V^+ is the superior permissible voltage level. It is straightforward to construct the constraints related to voltage magnitudes of middle voltage buses similar to (5) and (6).

The constraints (5) can be presented in the following form:

$$\sum_{i^M=1}^{n^M} a_{ji^M} Q_{(m)i^M} + \sum_{i^L=1}^{n^L} a_{ji^L} Q_{(l)i^L} \ge b_j^t > 0,$$
$$j \in J^-,\ i^M \in I^M,\ i^L \in I^L,\ t = 1,...,T \tag{7}$$

where

$$a_{ji^M} = \sum_{\substack{f=1 \\ f\in W_j}}^{g} X_f,\ j \in J^-,\ i^M \in I^M;$$

$$a_{ji^L} = \sum_{\substack{f=1 \\ f\in W_j}}^{g} X_f,\ j \in J^-, i^L \in I^L;$$

$$b_j^t = 10V^2(V^- - V_j^{re,t}),\ j \in J^-,\ t = 1,...,T.$$

At the same time, the constraints (6) can be presented as

$$\sum_{i^M=1}^{n^M} a_{ji^M} Q_{(m)i^M} + \sum_{i^L=1}^{n^L} a_{ji^L} Q_{(l)i^L} \le d_j^t > 0,$$
$$j = 1,...,J,\ t = 1,...,T \tag{8}$$

where

$$a_{ji^M} = \sum_{\substack{f=1 \\ f\in W_j}}^{g} X_f,\ j = 1,...,J,\ i^M \in I^M;$$

$$a_{ji^L} = \sum_{\substack{f=1 \\ f\in W_j}}^{g} X_f,\ j = 1,...,J,\ i^L \in I^L;$$

$$d_j^t = 10V^2(V^+ - V_j^{ne,t}),\ j = 1,...,J,\ t = 1,...,T.$$

III. Monocriteria Based Placement of Capacitors

Let us consider the Boolean problem of maximization of

$$F(x) = \sum_{i=1}^{n} c_i x_i \tag{9}$$

while satisfying the constraints

$$\sum_{i=1}^{n} a_{ji} x_i \le b_j, \quad j = 1,...,m \tag{10}$$

where $c_i > 0$, $i = 1,...,n$, $a_{ji} > 0$, $j = 1,...,m$, $i = 1,...,n$, $b_j > 0$, $j = 1,...,m$.

The idea of one of the most popular methods, related to the class of heuristic methods, may be illustrated by considering the problem (9) and (10) for $j = 1$ (the 0-1 knapsack problem). It is possible to assume that x_i, $i = 1,...,n$ are arranged as follows:

$$\frac{c_1}{a_1} \ge \frac{c_2}{a_2} \ge ... \ge \frac{c_n}{a_n}. \tag{11}$$

It permits one to try to maximize (9) on the basis of the largest $\frac{c_i}{a_i}$, taking $x_1 = 1$, then $x_2 = 1$, and so on until (10) is observed. Similar methods are called greedy methods. In spite of their "naivety", in many cases they represent the best heuristic among other heuristics with *a priori* estimates. However, a range of problems is not restricted by the case of $m = 1$. Considering this, the results of [9]-[11] permit one to construct algorithms for the general case ($m > 1$) to solve problems (linear as well as nonlinear), which can include not only Boolean, but integer and discrete variables as well.

When analyzing the model (9) and (10) for $m=1$, maximization is reached by expending only one resource type. If $m>1$, the optimization process is stopped when a remaining amount of only one of resources is not sufficient for next incrementing any of x_i, $i=1,...,n$. It is possible to speak about "equivalence" of different types of resources from the standpoint of termination of the process of maximizing (9). Thus, it is expedient to have a single measure for different resources. This consideration leads to the idea of normalization [13]. For example, the constraints (10) are reduced to a single arbitrary resource b as

$$a_{ji}^{(\theta)} = a_{ji} \frac{b}{b_{ji}^{(\theta-1)}}, \; j=1,...,m, \; i=1,...,n \tag{12}$$

where θ is the optimization step number.

Using (12), it is possible to convert the constraints (10) to equal conditions. For instance, before the first optimization step we have

$$\sum_{i=1}^{n} a_{ji}^{(0)} x_i \le b, \; j=1,...,m. \tag{13}$$

The algorithms of [9]-[11], based on the idea of normalization as well as on the use of elements of greedy heuristics, permit one to solve two types of problems:

1) maximization of an objective function interpreted as concave while satisfying constraints interpreted as convex;
2) minimization of an objective function interpreted as convex while satisfying constraints interpreted as concave.

The modification of the algorithm of analyzing the minimization problem is used below to solve the problem of determining locations of installation and sizes of fixed capacitors. However, it is applied at the second stage of the solution process. The first stage is associated with minimizing (4) observing only the constraints (8). The second stage is associated, if necessary, with the continuation of the optimization process (the initial state for the second stage is a result obtained at the first stage) to satisfy the constraints (7).

A. Algorithm of the first stage

1. The increments of the constraints (8) $\Delta g_{ji}^{(\theta)}$, $j=1,...,J$, $i \in I^{(\theta)} = I^{M(\theta)} \cup I^{L(\theta)}$ are calculated as

$$\Delta g_{ji}^{(\theta)} = a_{ji^M} (Q_{(m+1)i^M}^{(\theta)} - Q_{(m)i^M}^{(\theta)}),$$
$$j=1,\dots,J, \; i^M \in I^{M(\theta)} \tag{14}$$

or

$$\Delta g_{ji}^{(\theta)} = a_{ji^L} (Q_{(l+1)i^L} - Q_{(l)i^L}), \quad j=1,...,J, i^L \in I^{L(\theta)}. \tag{15}$$

In (14) and (15), $I^{M(\theta)}$ and $I^{L(\theta)}$ are sets of variables for the middle voltage level and the low voltage level, respectively, for the θth optimization process step. For the first step (θ=1), $Q_{(m)i^M}^{(1)} = Q_{(1)i^M}, i^M \in I^{M(1)}$ and $Q_{(l)i^L}^{(1)} = Q_{(1)i^L}, i^L \in I^{L(1)}$.

2. Refine the set $i \in I^{(\theta)}$ of variables on which optimization is possible at θth step:

$$I^{(\theta)} = \{i \big| \Delta g_{ji}^{(\theta)} \le d_j^{t(\theta-1)}\}, \; j=1,...,J, i \in I^{(\theta)}, \; t=1,...,T \tag{16}$$

In (16), for the first step (θ=1), we have $d_j^{t(\theta-1)} = d_j^{t(0)} = d_j^t$.

3. Check for nonemptiness of the set $I^{(\theta)}$. If $I^{(\theta)} \ne \varnothing$, then go to operation 4; otherwise go to operation 11.
4. The components of the increment vector of the objective function $\{\Delta Z_i^{(\theta)}\}$, $i \in I^{(\theta)} = I^{M(\theta)} \cup I^{L(\theta)}$ are calculated as

$$\begin{aligned}
&\Delta Z_i^{(\theta)} = \\
&Z(Q_{(m)1},..., Q_{(m+1)i^M},..., Q_{mn^M}, Q_{(l)1},..., Q_{(l)i^L},..., Q_{(l)n^L}) \\
&- Z(Q_{(m)1},..., Q_{(m)i^M},..., Q_{(m)n^M}, Q_{(l)1},..., Q_{(l)i^L},..., Q_{(l)n^L}), \\
&i^M \in I^{M(\theta)}
\end{aligned} \tag{17}$$

or

$$\begin{aligned}
&\Delta Z_i^{(\theta)} = \\
&Z(Q_{(m)1},..., Q_{(m)i^M},..., Q_{mn^M}, Q_{(l)1},..., Q_{(l+1)i^L},..., Q_{(l)n^L}) \\
&- Z(Q_{(m)1},..., Q_{(m)i^M},..., Q_{(m)n^M}, Q_{(l)1},..., Q_{(l)i^L},..., Q_{(l)n^L}), \\
&i^L \in I^{L(\theta)}.
\end{aligned} \tag{18}$$

5. Refine the set $I^{(\theta)}$ of variables on which optimization is possible at the θth step:

$$I^{(\theta)} = \{i \mid \Delta Z_i^{(\theta)} < 0, i \in I^{(\theta)}\}. \tag{19}$$

6. Check for nonemptiness of the set $I^{(\theta)}$. If $I^{(\theta)} \ne \varnothing$, then go to operation 7; otherwise go to operation 11.
7. The index $i = s_\theta$ of the most promising variable to be incremented is determined from

$$\Delta Z_{s_\theta}^{(\theta)} = \max \left| \Delta Z_i^{(\theta)} \right|, \quad i \in I^{(\theta)}. \tag{20}$$

8. Recalculate the current values of the quantities:

$$Q^{(\theta)}_{(m)i^M} = \begin{cases} Q_{(m)i^M}, & \text{if } i \neq s_\theta, i \in I^{M(\theta)}, \\ Q_{(m+1)i^M}, & \text{if } i = s_\theta, \end{cases} \quad (21)$$

or

$$Q^{(\theta)}_{(m)i^M} = \begin{cases} Q_{(l)i^L}, & \text{if } i \neq s_\theta, i \in I^{L(\theta)}, \\ Q_{(l+1)i^L}, & \text{if } i = s_\theta, \end{cases} \quad (22)$$

$$d_j^{t(\theta)} = d_j^{t(\theta-1)} - \Delta g^{(\theta)}_{js_\theta}, \quad j = 1,...,J, t = 1,...,T. \quad (23)$$

9. Refine the sets $I^{M(\theta)}$ and $I^{L(\theta)}$:

$$I^{M(\theta)} = \{i \mid m_i < p, i \in I^{M(\theta)}\} \quad (24)$$

and

$$I^{L(\theta)} = \{i \mid l_i < q, i \in I^{L(\theta)}\} \quad (25)$$

10. Make a check for nonemptiness of the set $I^{(\theta)}$. If $I^{(\theta)} = I^{M(\theta)} \cup I^{L(\theta)} \neq \emptyset$, then go to operation 1 taking θ=θ+1; otherwise go to operation 11.
11. The calculations are completed because the solution is obtained.

When applying the algorithm of the second stage, it is assumed that the constraints (7) are already normalized and have the following form:

$$\left(\sum_{i^M=1}^{n^M} a_{ji^M} Q_{(m)i^M} + \sum_{i^L=1}^{n^L} a_{ji^L} Q_{(l)i^L} \right) \frac{b}{b_j^t} \geq b, \ j \in J^-, \qquad i^M \in I^M,$$
$$i^L \in I^L, \ t = 1,...,T. \quad (26)$$

The algorithm of solving the problem of minimization of (4) while satisfying the constraints (7) and (8) can be written as follows:

B. Algorithm of the second stage

1. The increments of the constraints (8) $\Delta g^{(\theta)}_{ji}$, $j = 1,...,J$, $i \in I^{(\theta)} = I^{M(\theta)} \cup I^{L(\theta)}$ are calculated on the basis of (14) and (15).
2. Refine the set $i \in I^{(\theta)}$ of variables on which optimization is possible at θth step in accordance with (16).
3. Check for nonemptiness of the set $I^{(\theta)}$. If $I^{(\theta)} \neq \emptyset$, then go to operation 4; otherwise go to operation 13.
4. The components of the constraint increment vector $\{\Delta G_i^{(\theta)}\}$ are evaluated as

$$\Delta G_i^{(\theta)} = \sum_j \Delta g^{(\theta)}_{ji}, \ j \in J^{-(\theta)}, i \in I^{(\theta)} = I^{M(\theta)} \cup I^{L(\theta)}. \quad (27)$$

In (27),

$$\Delta g^{(\theta)}_{ji} = a_{ji^M} (Q^{(\theta)}_{(m+1)i^M} - Q^{(\theta)}_{(m)i^M}) \frac{b_j^{(\theta-1)}}{b}, \ j \in J^-,$$
$$i^M \in I^{M(\theta)}, \ t = 1,...,T \quad (28)$$

or

$$\Delta g^{(\theta)}_{ji} = a_{ji^M} (Q^{(\theta)}_{(l+1)i^L} - Q^{(\theta)}_{(l)i^L}) \frac{b_j^{(\theta-1)}}{b}, \ j \in J^-,$$
$$i^L \in I^{L(\theta)}, \ t = 1,...,T. \quad (29)$$

5. The components of the increment vector of the objective function $\{\Delta Z_i^{(\theta)}\}$, $i \in I^{(\theta)} = I^{M(\theta)} \cup I^{L(\theta)}$ are calculated with the use of (17) and (18).
6. The components of the vector $\{V_i^{(\theta)}\}$ are calculated as

$$V_i^{(\theta)} = \frac{\Delta Z_i^{(\theta)}}{\Delta G_i^{(\theta)}}, \quad i \in I^{(\theta)}. \quad (30)$$

7. The index $i = s_\theta$ of the most promising variable to be incremented is determined from

$$\Delta Z^{(\theta)}_{s_\theta} = \max_i V_i^{(\theta)}, \quad i \in I^{(\theta)}. \quad (31)$$

8. Recalculate the current values of the quantities $Q^{(\theta)}_{(m)i^M}$, $i^M \in I^{M(\theta)}$ and $Q^{(\theta)}_{(l)i^L}$, $i^L \in I^{L(\theta)}$ using (21) and (22), respectively, and

$$d_j^{t(\theta)} = d_j^{t(\theta-1)} - \Delta g^{(\theta)}_{js_\theta} \frac{b}{b_j^{t(\theta-1)}}, \quad j = 1,...,J, t = 1,...,T, \quad (32)$$

$$b_j^{t(\theta)} = b_j^{t(\theta)} - \Delta g_{js_\theta} \frac{b}{b_j^{t(\theta-1)}}, \quad j \in J^{-(\theta)}, t = 1,...,T. \quad (33)$$

9. Refine the set $J^{-(s)}$:

$$J^{-(s)} = \{\mid b_j^{t(\theta-1)} > 0, j \in J^{-(\theta)}\}. \quad (34)$$

10. Check for nonemptiness of the set $J^{-(\theta)}$. If $J^{-(s)} \neq \emptyset$, then go to operation 11; otherwise go to operation 14.
11. Refine the sets $I^{M(\theta)}$ and $I^{L(\theta)}$ in accordance with (24) and (25), respectively.
12. Check for nonemptiness of the set $I^{(\theta)}$. If $I^{(\theta)} = I^{M(\theta)} \cup I^{L(\theta)} \neq \emptyset$, then go to operation 1, taking

$\theta=\theta+1$; otherwise go to operation 13.

13. The calculations are completed because the problem has no solution.
14. The calculations are completed because the problem solution is obtained.

IV. MULTICRITERIA BASED PLACEMENT OF CAPACITORS

As it was indicated above, the simultaneous observation of (5) and (6) meets difficulties. In particular, any violation of the constraint (6) for any bus stops the optimization process. Considering this, it is rational to change the monocriteria problem (2)-(6) by the problem (2)-(4) and the following additional objective function

$$W(Q_{m1}^M,...,Q_{mi}^M,...,Q_{mn}^M,Q_{l1}^L,...,Q_{li}^L,...,Q_{\ln}^L)= W^+ + W^- = \sum_{j=1}^{J}\sum_{t=1}^{T} W_j^{+t} + \sum_{j=1}^{J}\sum_{t=1}^{T} W_j^{-t} \tag{35}$$

reflecting a volume of poor energy consumption.

In (35), W^+ is an overall volume of energy consumption with the voltage levels superior V^+; W^- is an overall volume of energy consumption with the voltage levels inferior V^-; W_j^{+t} is a volume of poor energy consumption with the voltage levels superior V^+ by the *j*th low voltage network for *t*th load curve step; W_j^{-t} is a volume of poor energy consumption with the voltage levels inferior V^- by the *j*th low voltage network for *t*th load curve step.

If parameters of a low voltage network as well as its loads are available, the evaluation of W_j^{+t} or W_j^{-t} creates no difficulties. If they are not available, it is possible to apply so-called low voltage network models.

A. Evaluation of Poor Energy Consumption

Suppose that active $I_{a,j}^t$ and reactive $I_{r,j}^t$ loads of the *j*th low voltage network have an uniform distribution along the length L of its model. This model can be defined as a function of a maximum voltage drop $\Delta V_{j,\max}$ corresponding to a maximal load $I_{j,\max}$. The estimation of $\Delta V_{j,\max}$ is associated with difficulties. Taking this into account, it is possible to utilize a value defined by project norms of the utility.

A voltage drop ΔV_j^t can be defined as

$$\Delta V_j^t = \Delta V_{j,\max} - \frac{I_j^t}{I_{j,\max}^t} = \Delta V_{n,\max}\frac{S_j^t}{S_{j,\max}^t}. \tag{36}$$

A current flow through an elementary section of the model corresponding to a distance X from the nearest consumer can be defined by

$$I_{j,e}^t = i_{j,0}^t(L_j^t - X_j^t) = \frac{I_j^t}{L_j^t}(L_j^t - X_j^t). \tag{37}$$

The corresponding voltage drop then can be calculated as

$$\Delta V_{j,e}^t = i_{j,0}^t(L_j^t - X_j^t)Z_{j,0} = \frac{I_j^t}{L_j^t}(L_j^t - X_j^t)Z_{j,0} \tag{38}$$

where $Z_{j,0}$ is a specific impedance.

A voltage drop from the nearest consumer to the point X_j^t includes two components. The first one is associated with an uniform load distribution along X_j^t. The second one is associated with a concentrated load, which is equal to a total load obtained along of $L_j^t - X_j^t$. Thus, considering (37) and (38), it is possible to write the following expression:

$$\Delta V_{X_j^t} = 0.5\frac{I_j^t}{L_j^t}X_j^t Z_{j,0}X_j^t + \frac{I_j^t}{L_j^t}(L_j^t - X_j^t)Z_{j,0}X_j^t \tag{39}$$

which can be reduced to

$$0.5 I_j^t Z_{j,0}\left(\frac{X_j^t}{L_j^t}\right)^2 - I_j^t Z_{j,0}L_j^t\left(\frac{X_j^t}{L_j^t}\right) + \Delta V_{X_j^t} = 0. \tag{40}$$

The ratio $\Psi_j''^t = \frac{X_j^t}{L_j^t}$ reflects part of consumers of a low voltage network with a voltage drop less than or equal to $\Delta V_{X_j^t}$. Considering this, the expression (40) can be presented as

$$(\Psi_j''^t)^2 - 2\Psi_j''^t + \frac{\Delta V_{X_j^t}}{\Delta V_j^t} = 0. \tag{41}$$

The solution of (41) is

$$\Psi' = 1 - \sqrt{1 - \frac{\Delta V_{X_j^t}}{\Delta V_j^t}}. \tag{42}$$

At the same time, part of consumers placed between the point X_j^t and the end of the line is

$$\Psi'' = \sqrt{1 - \frac{\Delta V_{X_j^t}}{\Delta V_j^t}}. \tag{43}$$

Considering $V_j^{re,t} = V_j^{ne,t} - \Delta V_j^t$ and applying (42) and (43), it is possible to estimate the energy $W_j^{+,t}$ as

$$W_j^{+t} = \left[1 - \sqrt{1 - \frac{\left(V_j^{ne,t} - V^+\right)}{\Delta V_j^t}} \right] W_j^t \tag{44}$$

and the energy W_j^{-t} as

$$W_j^{+t} = \left[\sqrt{1 - \frac{\left(V_j^{ne,t} - V_j^t - V^-\right)}{\Delta V}} \right] W_j^t . \tag{45}$$

B. The Bellman-Zadeh Approach to Decision Making in a Fuzzy Environment as Applied to Multicriteria Based Placement of Capacitors

When analyzing multicriteria models, a vector of objective functions $F(X) = \{F_1(X),...,F_q(X)\}$ is considered, and the problem consists in simultaneous optimizing all objective functions, i.e.,

$$F_p(X) \to \underset{X \in L}{\text{extr}}, \quad p = 1,...,q \tag{46}$$

where L is a feasible region in R^n.

The first step in analyzing (46) is associated with determining a set of Pareto solutions $\Omega \subseteq L$ [16]. This step is useful. However, it does not permit one to obtain unique solutions. It is necessary to choose a particular Pareto solution on the basis of information provided by a decision maker (DM).

Three approaches to using this information are classified in [17]: *a priori*, *a posteriori*, and adaptive. The most preferable approach is the adaptive one. In this approach, the procedure of successive improving the solution quality is performed as a transition from $X_\alpha^0 \in \Omega \subset L$ to $X_{\alpha+1}^0 \in \Omega \subset L$, considering the information I_α provided by a DM.

When analyzing multicriteria problems, it is necessary to solve questions related to normalizing criteria, selecting principles of optimality, and considering priorities of criteria. Their solution and, therefore, the development of multicriteria methods are carried out in several directions [18]. Without discussion of them, it is necessary to point out that an important question in multicriteria decision making is the solution quality. It is considered high if levels of satisfying criteria are equal or close to each other (harmonious solutions), when all objective functions have the same importance [19], [21]. It is not difficult to extend this concept for the case when the importance levels of objective functions are different. From this point of view, it should be recorded the validity and advisability of the direction related to the principle of guaranteed result, which can be realized [17], [19] on the basis of applying the Bellman-Zadeh approach to decision making in a fuzzy environment [20].

The Bellman-Zadeh approach permits one to realize a computationally effective and rigorous (from the standpoint of obtaining solutions $X^0 \in \Omega \subseteq L$) method of analyzing multicriteria models. Its use also allows one to preserve a natural measure of uncertainty in decision making and to take into account indices, criteria, and constraints of qualitative character.

When using the approach, each objective function $F_p(X)$ is replaced by a fuzzy objective function or a fuzzy set:

$$A_p = \{X, \mu_{A_p}(X)\}, \quad X \in L, \quad p = 1,...,q \tag{47}$$

where $\mu_{A_p}(X)$ is a membership function of A_p [17].

A fuzzy solution D with the given fuzzy sets (47) is turned out as a result of the intersection $D = \bigcap_{p=1}^{q} A_p$ with a membership function

$$\mu_D(X) = \min_{1 \le p \le q} \mu_{A_p}(X), \quad X \in L . \tag{48}$$

The use of the intersection (48) permits one to obtain the solution proving the maximum degree

$$\max \mu_D(X) = \max_{X \in L} \min_{1 \le p \le q} \mu_{A_p}(X) \tag{49}$$

of belonging to the fuzzy solution D. Therefore, the problem (18) is reduced to

$$X^0 = \arg \max_{X \in L} \min_{1 \le p \le q} \mu_{A_p}(X) . \tag{50}$$

To obtain the solution (50), it is necessary to build the membership functions $\mu_{A_p}(X)$, $p = 1,...,q$ reflecting a degree of achieving "own" optima by $F_p(X)$, $X \in L$, $p = 1,...,q$. This condition is satisfied by the use of membership functions

$$\mu_{A_p}(X) = \left[\frac{F_p(X) \quad - \min_{X \in L} F_p(X)}{\max_{X \in L} F_p(X) - \min_{X \in L} F_p(X)} \right]^{\lambda_p} \tag{51}$$

for maximized objective functions or by the use of membership functions

$$\mu_{A_p}(X) = \left[\frac{\max F_p(X) \ - \ F_p(X)}{\max_{X \in L} F_p(X) - \min_{X \in L} F_p(X)} \right]^{\lambda_p} \tag{52}$$

for minimized objective functions. In (51) and (52), λ_p, $p = 1,...,q$ are corresponding importance factors.

The construction of (51) and (52) demands to solve the following problems:

$$F_p(X) \to \min_{X \in L}, \quad (53)$$

$$F_p(X) \to \max_{X \in L} \quad (54)$$

providing the solutions $X_p^0 = \arg\min_{X \in L} F_p(X)$ and $X_p^{00} = \arg\max_{X \in L} F_p(X)$. In this manner, the solution of (18) demands analysis of $2q+1$ monocriteria problems (53), (54), and (49), respectively.

Since the solution X^0 is to belong to $\Omega \subseteq L$, it is necessary to build

$$\overline{\mu}_D(X) = \min\left\{\min_{1 \le p \le q} \mu_{A_p}(X), \mu_\pi(X)\right\} \quad (55)$$

where $\mu_\pi(X) = 1$ if $X \in \Omega$ or $\mu_\pi(X) = 0$ if $X \notin \Omega$.

The procedures for solving the problem (21), discussed in [17], provide a line in obtaining $X^0 \in \Omega \subseteq L$ in accordance with (27). Thus, it can be said about equivalence of $\overline{\mu}_D(X)$ and $\mu_D(X)$. It permits one to give up the necessity of implementing a cumbersome procedure for building the set $\Omega \subseteq L$.

The existence of s additional conditions (indices, criteria, and/or constraints) of qualitative character, defined by linguistic variables [17], reduces (50) to

$$X^0 = \arg\max_{X \in L} \min_{1 \le p \le q+s} \mu_{A_p}(X) \quad (56)$$

where $\mu_{A_p}(X)$, $X \in L$, $p = q+1,...,s$ are membership functions of fuzzy values of linguistic variables which reflect these additional conditions.

Taking the above into account, the solution of the multicriteria based capacitor placement problem is reduced to modifying the algorithms of discrete optimization discussed above to solve the *maxmin* problem (49).

V. Illustrative Example

The presented results have served for elaborating a custom-developed Electric Power Distribution Analysis (EPODIAN) software. This software is implemented in Java/C++ to provide flexible power flow model to optimization algorithms while supplying rich visualization and analysis capabilities to the user. A modified backward-forward sweep algorithm [22] is implemented along with the techniques of parallel processing for obtaining high performance results on large scale models (for instance, networks with over 10000 busses), allowing application of the paper results to the real-world networks on conventional desktop computers.

Let us consider the results obtained from EPODIAN for the problem of placing fixed capacitors in a distribution network 13.8/0.22 kV of one of substations 138/13.8 kV of the Minas Gerais State Energy Company (CEMIG). This network includes 3 feeders feeding 19 primary consumers and 2092 distribution transformers with 19756 secondary consumers. The total length of networks is 1036 km.

The following solution alternatives are presented in Table 1: *I* – initial state; *A* – monocriteria solution which minimizes the objective function (4) observing the constraints for the superior voltage levels (6); *B* – monocriteria solution which minimizes the objective function (4); *C* – monocriteria solution which minimizes the objective function (35); M_1 – multicriteria solution which provides a compromise between solutions *A* and *B*; M_2 – multicriteria solution which provides a compromise between solutions *B* and *C*.

Table 1. Solution results

Alternative	Objective function Z , R$	Objective function W, MWh
I	2,048,495	63.40
A	1,928,496	60.83
B	1,844,457	66.44
C	5,873,414	29.43
M1	1,849,665	60.59
M2	1,973,829	50.14

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