PDEM Estimator for Digital Elevation Model: Utilities

José Francisco Zelasco, Patrick Julien, Gaspar Porta, Kevin Ennis, Judith Donayo

***Abstract*—** The measurement of the precision of a DSM model in relation to another model of the same physical surface is done primarily by estimating the expectation of the squares of differences between pairs of points (called homologous points) which correspond to the same feature of the physical surface. But frequently there aren't homologous points. In these cases, the procedure that is generally used has been to square the vertical distances between the models at selected points in a preferred (relative to a 'natural' horizontal plane) direction. This procedure only addresses the vertical component of the error, thus giving a biased estimate when the surface is not horizontal. In this paper we describe the Perpendicular Distance Evaluation Method (PDEM). The PDEM allows the estimation of planimetric errors in the x,y plane, not obtainable by other methods, and estimates for the vertical component which are superior to those obtained from the vertical distances method because they are not affected by the bias introduced by slanted surfaces. The planimetric estimates improve if the surface is relatively irregular. The PDEM provides estimates for the three dimensional error components when applied to a DSM (general case). This is the case of Dem obtained by IFSAR technology that has also a problem of correlation. When planimetric isometry is acceptable, a considerable simplification of the method is possible. For this last case we present the utilities allowing better understand the method and apply it appropriately.
The PDEM is a useful tool for evaluating digital three dimensional surface model precision.

***Keywords*—** Digital Surface Model (DSM); Digital Elevation Model (DEM) geometric accuracy assessment, Digital three dimensional surface model precision, Perpendicular Distance Evaluation Method (PDEM).

I. Introduction and Description of Challenges

A Digital Surface Model (DSM) is a numerical surface model which is formed by a set of points, the coordinates of which are the result of measurements of points of the surface of a real object. In the case of a terrain surface, this model is called a Digital Elevation Model (DEM). The points might be arranged over a regular square grid or over a triangular grid. As a result the altitude is known at vertices of squares or triangles and the altitude within any square or triangle is obtained by an often linear interpolation which results in a representation of the object's surface formed by squares or triangles. In this paper we use the triangular grid. The set of all these triangles is referred to as $\{T_i\}$.

Here we are concerned with the measurement of the precision of a DSM model, which we call the evaluated DSM (e-DSM). This is done by comparing this e-DSM to a numerical surface model of a certain surface which we consider to be exact, and which we refer to as the reference DSM (r-DSM). We assume that the errors in the r-DSM are negligible. Indeed, there is no way to compare the e-DSM with the real surface, except through the use of an r-DSM obtained by any mean. Therefore, we obtain the precision of the e-DSM with reference to the r-DSM, and not with reference to the real surface.

The evaluation of the precision of the e-DSM model in relation to the r-DSM consists mainly in estimating the standard deviation of the discrepancies between them. The expectation of the squares of differences between both surfaces gives the variance—under the assumption of a null mean. For the rest of this paper, we assume the hypothesis that the measurement errors are independent random variables with components e_x, e_y, e_z in the three main orthogonal directions, with a three dimensional normal (Laplace-Gauss) distribution, as is the usual case. This is the setting for a DEM obtained by optical techniques. Otherwise, it is a standard statistical procedure to obtain the main components by suitable rotations of the coordinate system.

To carry out this comparison, two methods stand out in the literature. We describe them briefly in the following two paragraphs.

The measurement of vertical distances between the models has been the procedure of choice. These distances are taken from the points in one model to the interpolated surface of the other model. It is natural to choose the points M_i in the e-DSM, and intersect vertical lines through these points with the r-DSM calling these intersections W_i. The distances between the M_i and the W_i give the measured vertical distances. The

J. F. Zelasco, Lab of Estereología y Mecánica Inteligente, Dto de Mecánica Facultad de Ingeniería Buenos Aires University. Phone and fax 54 11 49 42 99 90; e-mail: jfzelasco@fi.uba.ar.

P. Julien., Laboratoire Matis, Institut Géographique National, France. patrik.julian@ign.fr

G. Porta Assistant Professor, Mathematics and Statistics Department, Washburn University. e.mail: gaspar.porta@washburn.edu .

K. Ennis, Universidad del Centro de la provincia de Buenos Aires. kevinennis@fibertel.com.ar

J. Donayo, Zelasco, Lab of Estereología y Mecánica Inteligente, Dto de Mecánica Facultad de Ingeniería Buenos Aires University. eMail; juddonay@gmail.com

limitation of this standard method is that the expectancy of the vertical square distance between a point M_i in the e-DSM and points W_i on the r-DSM has a systematic bias as an estimator of the vertical component of the variance because the surfaces of the r-DSM are not always horizontal.

Alternatively, measurements of discrepancies are done by comparison with *benchmark* points of the r-DSM. We call the collection of these *benchmark* points $\{P_i\}$ and the corresponding points in the e-DSM $\{M_i\}$. A drawback of this method is that the measurements between homologous points—pairs of these *benchmark* points with their corresponding points in the e-DSM—might be subjected to special conditions. These special conditions may include better measurements because of some outstanding feature which might imply a precision different to that of the rest of the e-DSM. Also, given a point M_i of the e-DSM, it is often impossible to obtain the homologous point of the r-DSM because the latter is not easy to recognize since the grids seldom coincide. To summarize, the points in the e-DSM have no identifiable homologous points in the r-DSM model, or those points that do have identifiable homologous points are either too few or do not form a representative sample of the surface (in particular, they might come from a privileged part of the surface) to evaluate precision.

In this paper we describe a method for measurement of vertical standard deviations which uses the perpendicular distances according to the normal vectors of the planes of the triangles in the set $\{T_i\}$. This method is called Perpendicular Distance Estimation Method (PDEM). The PDEM does not introduce a systematic error in the evaluation of the vertical error, like the vertical distance model does.

II. Background and Previous Treatments of the Problem

The evaluation of a DEM error is an important topic [1] [2] [3] [4] [5] [6]. In the literature several attempts have been made for the evaluation of a DSM error with respect to a more precise reference DSM.

Somewhat analogous problems were studied for horizontal errors in maps [7] [8] [9]. These authors found that the discrepancy model-reference was the most important factor to determine corresponding pairs. They use Hausdorff distances to evaluate the errors in maps which does not allow for decomposition of errors in the two main directions (x ans y).

Several solutions have been proposed for the *punctual control method* (recognition algorithms, filtering methods, adjustment of histograms to theoretical laws) without obtaining practical results [10]. We also mention some work concerning the DEM´s quality [1] [2] [3] [4] [5] [6] [10] [11], but no solution to the simultaneous evaluation of vertical and horizontal errors is proposed.

A critical problem for error estimation is to establish the corresponding (homologous) point in the r-DSM for each selected point of the e-DSM. In some cases, even though there are pairs of homologous points available these are either special ones, or there are not enough of them to establish a good random sample [12].

The DEM Quality Assessment chapter in [13], states that horizontal accuracy, although recognized as a part of DEM quality, is generally considered difficult to evaluate in the absence of an image coincident with the e-DEM (check points or benchmarks), or of clearly discernable surface features.

We note that most of the existing methods for quality control of an e-DSM are confined to computing distances between check points, or *benchmarks*, or vertical distances from the points of one of the models to the surface of the other (with the inconvenient bias we have mentioned above). This explains the lack, or at least scarcity, of work devoted to the horizontal accuracy of a DEM.

Regarding the solution proposed here, see [14] [15] and [16]: in the first one, the method was intuitively described and a study was undertaken, using simulated r-DEM, according to the type of surface and the number of sample points. The subsequent papers are mostly users' tutorials of the method.

In the present paper we give a better description of the PDEM. The PDEM allows us to get statistical information about the horizontal and vertical standard deviations without forming full three dimensional error vectors. Indeed, we show that only the component perpendicular to the r-DSM surface is required.

III. Description of the PDEM

The Perpendicular Distance Estimation Method (PDEM), as opposed to the *vertical distance* methods, yields vertical standard deviation results and allows us to obtain the horizontal variance under the condition of enough surface roughness [21].

The r-DSM is a parameterized **three dimensional** surface – that is to say, a surface whose points $V_i = [x_i, y_i, z_i]^T, \quad i \in \{1,2,\ldots,n\}$, are determined by the **two dimensional** position parameters x, y – and the e-DSM has points which are also functions of x, y, and will be denoted $M_i = [x_i, y_i, z_i]^T = M(x, y)$. The error is the vector function which denotes the discrepancy between both surfaces, and is defined for each point M_i and its homologous point P_i in the r-DEM. The points M_i define the e-DSM surface, but their homologous points P_i are in general not necessarily vertices of the r-DSM surface triangles. If they were, their identification would be easy, and our problem would be trivial. However, we want to deal with the more common case in which the homologous points are not readily identifiable. The error vector is assumed to be the result of three stochastically independent components, e_x, e_y, e_z one in each of the basic axis of the x, y, z coordinate system. Notice that this error vector is not constrained to be vertical, nor necessarily orthogonal to any of the surfaces.

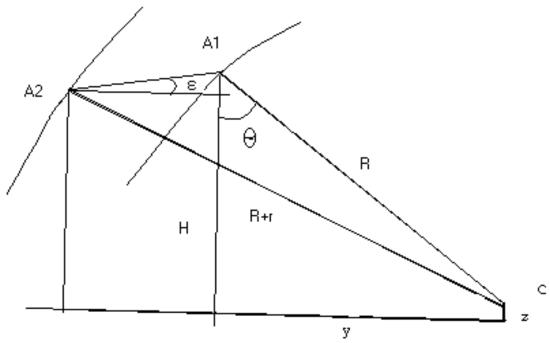

Fig. 1: IFSAR Geometry

$$\theta(x_i, y_i) = M_i(x_i, y_i) - P_i = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} \in R^3 \qquad (1)$$

If the r-DSM has no systematic error, the expectation is

$$E(e) = \left(E(e_x), E(e_y), E(e_z)\right)^T = 0 \qquad (2)$$

For each $P_i = [x_i, y_i, z_i]^T$, the error vector $\theta(x_i, y_i) = M_i(x_i, y_i) - P_i \quad i \in \{1,2\ldots,n\}$ cannot usually be determined because of the difficulties in establishing the homologous point P_i, as we mentioned above. However, even if the homologous point P_i cannot be identified, the triangle $T_i \in$ r-DSM containing it can usually be identified (The difficulties are addressed in section 4.4).

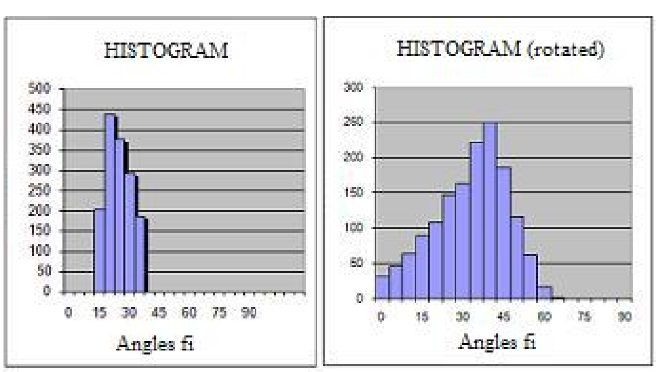

Fig. 2: Previous Method PDEM Histogram.

Fundamental property: an important property is that the projection of the error vector $\theta(x_i, y_i) = M_i(x_i, y_i) - P_i$ on a unitary vector N_i orthogonal to the surface $T_i \in$ r-DSM remains invariant if the point P_i is replaced by any other point $Q \in T_i$.

For the projection of the error vector on Ni the scalar product is

$$[M_i(x_i, y_i)] \cdot N_i = [Q - P_i] \cdot N_i = M_i - M_i' \qquad (3)$$

where M_i is the normal projection of M_i relative to the surface of the triangle T_i (the point of the triangle determined by the line normal to that triangle and which passes through M_i).

For any point Q belonging to the surface of the triangle T_i, we define a vector, which we will also call Q, the projection of the difference $M_i - Q$ on N coincides with the projection, on N, of $\theta(x_i, y_i) = M_i(x_i, y_i) - P_i$. Both projections are equal to $M_i - M_i'$. The fundamental property resulting from relation (3) is the reason for the choice of the name PDEM (Perpendicular Distance Estimation Method).

This relation implies that the length of the projection of the error vector may be computed knowing only the triangle $T_i \in$ r-DSM that contains P_i, even without knowing the exact position of P_i.

The mathematical expectation of the length of the projection of the error vector is

$$E\{M_i - M_i'\} = E\{(M_i - P_i') \cdot \cos\phi\} =$$
$$= E\{(M_i - P_i')\} \cdot \cos\phi = 0 \cdot \cos\phi = 0$$

where ϕ is the angle between the error vector and the unit vector N_i orthogonal to the surface T_i and where $E\{(M_i - P_i')\} = 0$ because it is the expectation of the error, which in (2) we have assumed to be zero.

Consequently, the variance of $M_i - M_i'$ is

$$Var\{M_i - M_i'\} = E\left\{|M_i - M_i'|^2\right\} = \sigma_{N_i}^2 =$$
$$= \sigma_x^2 \cdot \cos^2\alpha_i + \sigma_y^2 \cdot \cos^2\beta_i + \sigma_z^2 \cdot \cos^2\gamma_i \qquad (4)$$

where $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are known, because they are the direction cosines of the normal unit vector, obtained from the data of the surface triangle $T_i \in$ r-DSM, with

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \qquad (5)$$

An estimator for this $\sigma_{N_i}^2$ is therefore the $M_i - M_i'$ If n observations were available, the estimator would be

$$\frac{\sum_{j=1}^{n} |M_i - M_i'|^2}{n}$$

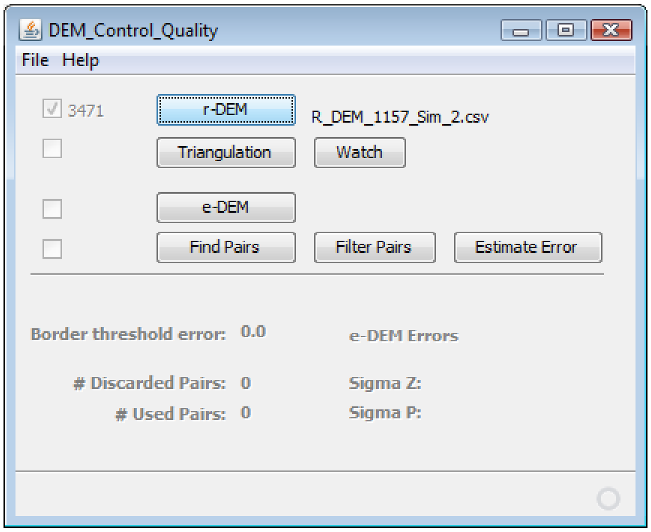

Fig. 3: Data entry screen.

(the mean of the modulus of the vectors $M_i - M_i'$ is zero and we divide by n instead of by $n-1$ because it is assumed that the r-DEM does not have any errors, and it is not the result of a mean therefore it gives one more degree of freedom). We will usually have only one observation to estimate $\sigma_{N_i}^2$ for each triangle T_i, because there will be only one normal unit vector to each triangle $T_i \in$r-DSM. But we have n triangles for which the homologous points are not readily identifiable.

For different triangles we have different normal vectors, and different values of $\sigma_{N_i}^2$. Moreover, we have one observation for each, and one estimate for each. This gives us one relation (4) for each. In these expressions, the coefficients $\cos^2\alpha_i$, $\cos^2\beta_i$, $\cos^2\gamma_i$ are known, and σ_x^2, σ_y^2, σ_z^2 are unknown, and they are what we need to estimate. Our n expressions (4) give us an observation matrix, or design matrix

$$M = \begin{bmatrix} \cos^2\alpha_1 & \cos^2\beta_1 & \cos^2\gamma_1 \\ \cos^2\alpha_2 & \cos^2\beta_2 & \cos^2\gamma_2 \\ \dots & \dots & \dots \\ \cos^2\alpha_i & \cos^2\beta_i & \cos^2\gamma_i \\ \dots & \dots & \dots \\ \cos^2\alpha_n & \cos^2\beta_n & \cos^2\gamma_n \end{bmatrix} \tag{6}$$

The n expressions (4) allow us to establish n estimates for $\sigma_{N_i}^2$, which form a vector

$$L = \begin{bmatrix} \sigma_{N_1}^2 \\ \sigma_{N_2}^2 \\ \dots \\ \sigma_{N_i}^2 \\ \dots \\ \sigma_{N_n}^2 \end{bmatrix} \tag{7}$$

We may now estimate σ_x^2, σ_y^2, σ_z^2 as if they were the parameters of an ordinary linear regression of the output variable $\sigma_{N_i}^2$ as a linear function of the three variables $\cos^2\alpha_i$, $\cos^2\beta_i$, $\cos^2\gamma_i$, given by (4). We have at our disposal n points, and the estimates may be obtained by the usual least squares regression method. We seek the values of σ_x^2, σ_y^2, σ_z^2, which we may write as a vector

$$\sigma^2 = \begin{bmatrix} \sigma_x^2 \\ \sigma_y^2 \\ \sigma_z^2 \end{bmatrix} \tag{8}$$

and we seek to minimize the sum of squares of differences

$$\sum_{i=1}^{n}\left[\sigma_{N_i}^2 - \left(\sigma_x^2\cdot\cos^2\alpha_i + \sigma_y^2\cdot\cos^2\beta_i + \sigma_z^2\cdot\cos^2\gamma_i\right)\right]^2 =$$

$$= \sum_{i=1}^{n}\varepsilon_i^2 = \left(L - M\cdot\sigma^2\right)^T\cdot\left(L - M\cdot\sigma^2\right)$$

IV. Discussion

In this section we discuss a variety of limitations, simplifications, computational particularities, extensions, and potential adaptations to our method.

A. *Horizontal estimation*

The estimate of σ^2 will improve if the values of the observations are spread adequately, which means that our design matrix M should have sufficiently different values of the observations $\cos^2\alpha_i$, $\cos^2\beta_i$, $\cos^2\gamma_i$. In other words, if the values of the angles α_i, β_i, γ_i are very similar, the estimate of σ^2 will have a larger variance. This implies that a relatively uneven surface will improve the precision of the estimate of the DEM error in the x, y components. The estimates of the horizontal components of the errors will be more unreliable in a relatively flat surface where all angles are small.

B. *Error Correlation of the DEM Obtained by IFSAR*

The DEM's obtained by means of the interferometry radar: IFSAR (Interferometry Synthesis Aperture Radar) needs a Reformulation.

The IFSAR geometry carries a vertical correlation between y –axes (range axis) and the z –axis (Fig. 1).

It is get a diagonal covariance-matrix from a rotation round x –axis. The rotation joins the z'–axis with the direction of the radar. This hypothesis is justified by the formulas presented in articles [10] and [11] [20]. After the rotation, in the new referential, the x –axis parallel to the trajectory of the satellite don't change of position, and the y' and z' correspond to the normal plane of the trajectory and this way the random variables in the three directions are not correlated.

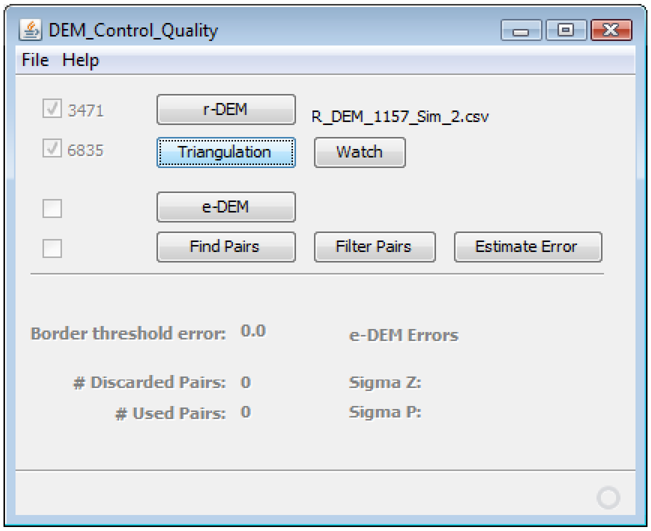

Fig. 4: Data entry screen. 1

B is the base (distance between antennas $A1$ y $A2$), H is the height of the antennae $A1$ and R is the distance from $A1$ to the bull-eye C. The value of z (height of the bull-eye) is given by: $z = H - R\cos\theta$ where H and R are known and $\cos\theta$ is incognito.

Been: $\cos\theta = \cos\varepsilon \cdot \cos(\theta-\varepsilon) - \sin\varepsilon \sin(\theta-\varepsilon)$ y, $\cos(\theta-\varepsilon) = \left(1-\sin^2(\theta-\varepsilon)\right)^{1/2}$

In the triangle of vertices $A1$ $A2$ C formed by the two antennas and the bull-eye, $R+r$ is the distance between $A2$ and the bull-eye C, and a the angle in $A1$.

So: $(R+r)^2 = B2 + R2 + 2BR\cos\alpha$

Where R, B, r (r is functional to the phase-difference and the wave length) are known and $\alpha = 90 + \theta - \varepsilon$. It means: $\cos\alpha = \sin(\theta-\varepsilon)$, the height z of the bull-eye is:

$$z = H - R\cdot\cos\theta \tag{4}$$

$$y = R\cdot\sin\theta \tag{5}$$

After rotation, see the histogram Fig. 2, slopes are increased which favors the accuracy of the assessment of the error in the y direction.

C. *Isotropic horizontal error*

In most practical problems (with the mentioned exception of the IFSAR case among others) we may consider the measurement process to have the same planimetric errors in the horizontal directions ${\sigma_x}^2 = {\sigma_y}^2$. This isotropy allows for some simplifications.

The error vector is no longer assumed to be the result of three stochastically independent components, e_x, e_y, e_z, one in each of the basic axis of the x, y, z, coordinate system, but of only two: one planimetric component, $e_x = e_y = e_p$, and a vertical component e_z. The pairs of measurements in the "horizontal" components are considered as different realizations of the same random variable. Relation (1) takes the form

$$\theta(x_i, y_i) = M_i(x_i, y_i) - P_i = \begin{bmatrix} e_p \\ e_z \end{bmatrix} \in R^2 \tag{1'11}$$

We assume that the e-DSM has no systematic error. But, relation (2) is replaced by

$$E(e) = \left(E(e_p), E(e_z)\right)^T = 0$$

and our equation (4) is replaced by

$$\begin{aligned} &Var\{M_i - M_i'\} = E\left\{\left|M_i - M'\right|^2\right\} = \sigma_{N_i}^2 = \\ &= \sigma_p^2 \cdot \left(\cos^2\alpha_i + \cos^2\beta_i\right) + \sigma_z^2 \cdot \cos^2\gamma_i = \\ &= \sigma_p^2 \cdot \left(1 - \cos^2\gamma_i\right) + \sigma_z^2 \cdot \cos^2\gamma_i = \\ &= \sigma_p^2 \cdot \left(\sin^2\gamma_i\right) + \sigma_z^2 \cdot \cos^2\gamma_i \end{aligned} \tag{4'13}$$

(where we used relation (5)). Our estimates of e_p, e_z are now obtained from the linear regression problem of fitting (4´) where

$$L = \begin{bmatrix} \sigma_{N_1}^2 \\ \sigma_{N_2}^2 \\ \vdots \\ \sigma_{N_i}^2 \\ \vdots \\ \sigma_{N_n}^2 \end{bmatrix} = M \cdot \sigma'^2 + \varepsilon =$$

$$= \begin{bmatrix} 1-\cos^2\gamma_1 & \cos^2\gamma_1 \\ 1-\cos^2\gamma_2 & \cos^2\gamma_2 \\ \cdots & \cdots \\ 1-\cos^2\gamma_i & \cos^2\gamma_i \\ \cdots & \cdots \\ 1-\cos^2\gamma_n & \cos^2\gamma_n \end{bmatrix} \begin{bmatrix} \sigma_p^2 \\ \sigma_z^2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_i \\ \vdots \\ \varepsilon_n \end{bmatrix} \tag{9'14}$$

Notice that only the γ_i are measured. Only the inclination of the surface is of interest, regardless of the direction of inclination. In the three dimensional problem knowledge of the direction of the surface inclination was necessary to distinguish the errors in each of the x, y directions.

D. *How to avoid ambiguous cases*

If the line normal to the triangle which passes through M_i intersects that triangle near to on one of its sides, a doubt arises about whether that particular triangle contains the homologous point. The homologous point might instead be in a neighboring triangle. When this happens, it is safe to avoid using that point in the estimation of the error by eliminating the square of the length of the modulus of that particular vector

$M_i - M_i'$ when appropriate. In practice, we compute the distance between M_i' and the corresponding triangle edge, and discard the estimate if that distance is significantly small. Equivalently, one needs to verify that the error ellipse has a high enough probability (based on the probability distribution assumed for errors) of not containing any triangle edges.

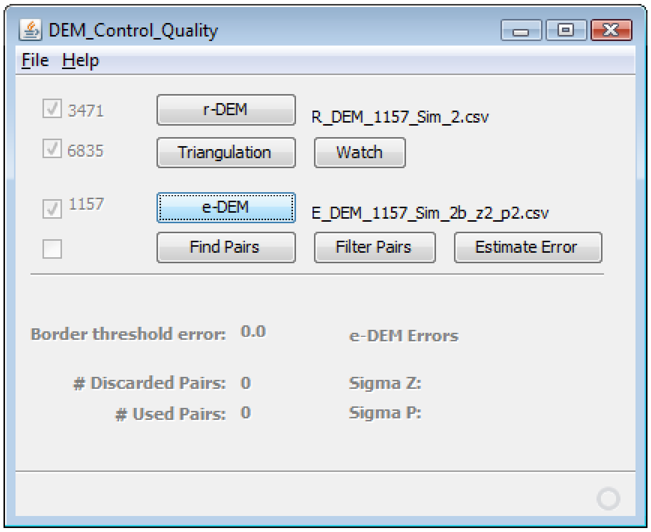

Fig. 5: Data entry screen.

V. Some Examples of Applications and Simulations

Knowledge of the discrepancy between a numerical surface model and a reference surface is of interest in many applications. Examples of these include face recognition using three dimensional models obtained by stereoscopic techniques, evaluation of the deformation of pathological organs or cells regarding healthy surfaces, or in search algorithms for certain multidimensional structures, etc. In other words, our focus is on situations in which tests for discrepancies require error estimates. The most frequently analyzed application seems to be in cartography, for the DEMs stored in a Geographic Information System (GIS). In particular, the DEMs of a GIS frequently overlap, which allows us to evaluate the precision of one measurement in terms of another. The second measurement would be our new reference model whose fidelity would be considered unquestionable. The motivation for this study is rooted in this very application pertaining to a particular, very large GIS and the need to compare the DEMs in it.

An idea that is currently being considered is to use the PDEM in the evaluation of the existing topographical framework in a country in which a telecommunication investor wants to build a tower network. Before embarking on the expensive process of starting from scratch, it makes sense to evaluate whether the error of the existing framework is within the tolerance of the project requirements. A representative, small portion of the large area to be covered is measured very carefully with a much more precise (and thus expensive) method over a small area and the resulting DEM is used as an r-DEM. Then the overlapping part is compared to this measurement as the e-DEM. Depending on how well it measures up, the expensive measurement of the whole area can be avoided.

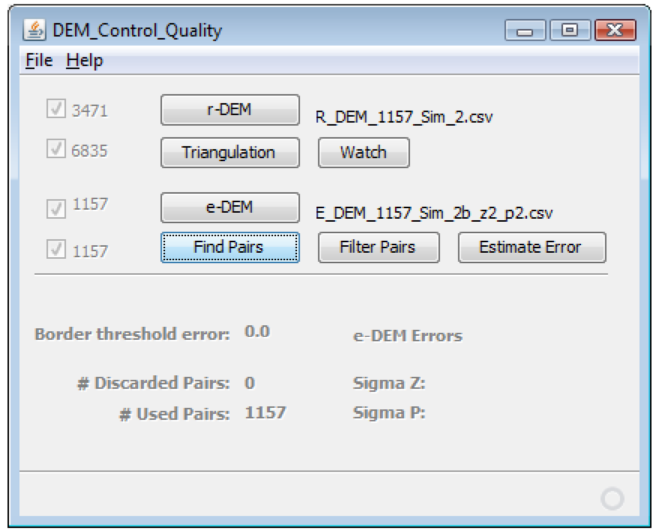

Fig. 6: Data entry screen.

Another application is to use the PDEM to verify the integrity of a DEM from a GIS. Comparisons of the overlapping parts of different measurements can be used as e-DEM and r-DEM pairs. Then the consistency of the measurement of error can be measured. Since there is less bias, and less systematic error in the PDEM, a more accurate reading of the uniformity of errors in overlapping parts can be detected. For a study showing the concern with this kind of situation see [17].

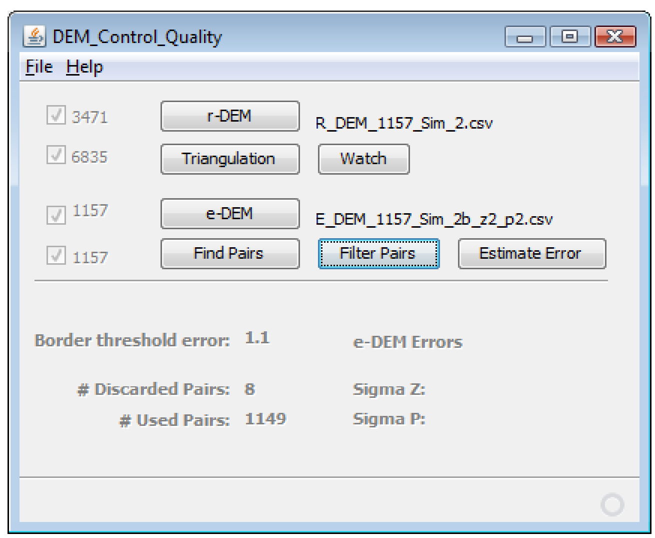

Fig. 7: Data entry screen.

If we think of the measurements in the vertical direction as the values of a random variable, the surface inclination causes us to record values that are sometimes smaller and sometimes larger than the actual error would give. The sum of the squares

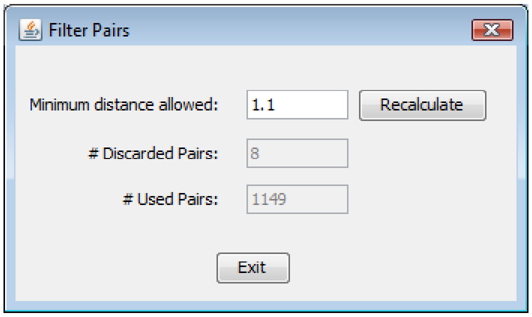

Fig. 8: Data entry screen.

of these longer and shorter distances will be systematically larger than the sum of squares of the vertical components' actual errors. Although it is possible to fabricate situations where this does not happen, generically it will—this conclusion is a corollary to the Cauchy-Schwartz theorem [18]. In consequence, the estimate of the vertical component of the variance based on those squares of vertical distances is a biased estimate, unless the surfaces of all triangles were horizontal—that is, a uniformly flat plane—which is not a very interesting case.

The Perpendicular Distance Evaluation Method (PDEM) described in the present paper provides estimates for vertical and horizontal components of errors, and avoids the bias in the error estimates that is present when vertical distances are used. The exaggerated estimates of the error in measurement indicated by the vertical distance strategy can be a cause for concluding that the practical error is more significant than would be appropriate. For comparisons between the vertical distance method and the PDEM as well as simulations to evaluate the quality of the PDEM see [19].

VI. An Example of an Implementation Using a Java Application

We have developed a Java application to measure the error of an e-DEM in relation to an r-DEM. In what follows we describe the use of this application to gage the error of a measurement with introduced errors of a real topographic surface.

1) First we select the file having the list of points that make up the r-DEM. In this example this list consists of 2471 three dimensional points formatted as a .csv file. The file is uploaded as the r-DEM. (Fig. 3)
2) We make a triangulation from the r-DEM in the XY plane; the application allows viewing the triangulation although we do not show it here. In this case we have obtained 1157 triangles. (Fig. 4)
3) Next, we upload the e-DEM. In this example this is another .csv file containing it. (Fig. 5)
4) The application identifies the triangles T_i of the r-DEM corresponding to each point P_i of the e-DEM. (Fig. 6)
5) The application allows entering a threshold corresponding to the minimal distance (in the XY plane) between the projection of the point and the edges of the triangle. The application discards ambiguous triangles based on this threshold. When pertinent, we refer to this process as addressing a boundary problem. (Fig.7 y 8)
6) Finally, the application calculates the vertical and horizontal errors. (Fig. 9)

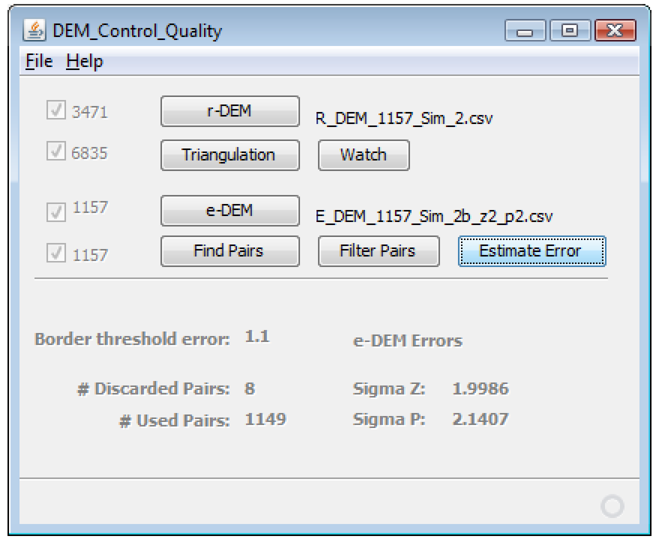

Fig. 9: Data entry screen.

Eventually, a histogram can be established to monitor the level of surface irregularity. (Fig. 10)

In this example we have made the e-DEM selecting 1157 triangles from the r-DEM. We have added a Gaussian noise to the three coordinates of the triangles' centre of mass with the same standard deviation (equal to 2) in each dimension.

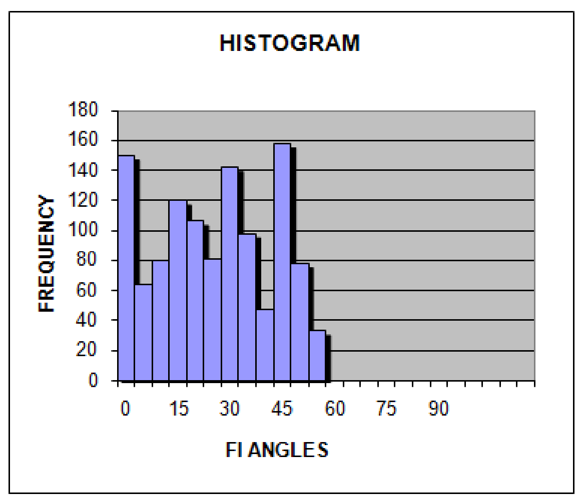

Fig. 10: Histogram

Consequently, the error results presented by the PDEM can be compared with the actual error value. The PDEM results are 1.9986 in the vertical direction and 2.1407 in the horizontal plane. The relative differences with the expected values are less than 1% and around 7% respectively.

VII. Conclusion

The PDEM (Perpendicular Distance Estimation Method) provides estimates for the three dimensional error components when applied to a DEM. This includes the planimetric errors in the x,y plane, not obtainable by other methods, and estimates for the vertical component which are superior to those obtained from the vertical distances method because they are not affected by the bias introduced by slanted surfaces. The planimetric estimates improve if the surface is relatively irregular. When planimetric isotropy is acceptable, a considerable simplification of the method is possible. Thus, the PDEM provides a useful tool for evaluating the error of Digital Surface Models (DSM).

Acknowledgment

We thank Mr. Hugo RYCKEBOER for his comments and suggestions.

References

[1] S. C. Guptill, J. L. Morrison. "Elements of spatial data quality". Guptill, S. C., Morrison, J. L., eds. Elsevier, Oxford, 1995.

[2] F. Harvey. "Quality Needs More Than Standards. Data Quality in Geographic Information: From Error to Uncertainty". Goodchild, M., Jeansoulin, R., eds. Hermès, Paris, 1997.

[3] R. Laurini, F. Milleret-Raffort. "Les bases de données en géomatique". Hermès, Paris 340 pages, 1993.

[4] K. P. Udagepola, L. Xiang, Y. Xiaozong, A. W. Wijeratne. "Review of data Consistency and Integrity Constraints in Spatial Databases". Proceedings of the 5th WSEAS Int. Conf. on Artificial Intelligence, Knowledge Engineering and Data Bases, Madrid, Spain, February 15-17, 2006 (pp348-353).

[5] N. H. Arshad, F. A. Hanifah. "Issues and Challenges in NSDI Implementation". Selected Topics in System Science and Simulation in Engineering. (ICOSSSE 2010) ISBN: 978-960-474-230-1. ISSN: 1792-507X.

[6] T. Ubeda, S. Servigne. "Geometric and Topological Consistency of Spatial Data". Proceedings of the 1st International Conference on Geocomputation, Leeds, United Kingdom, Vol 1: 830-842, 1996.

[7] I. Abbas. "Base de données vectorielles et erreur cartographique: problèmes posés par le contrôle ponctuel; une méthode alternative fondée sur la distance de Hausdorff: le contrôle linéaire": doctoral dissertation, Université Paris 7, France, 120 pages, 1994.

[8] P. Grussenmeyer, P. Hottier, I. Abbas. "Le contrôle topographique d'une carte ou d'une base de données constituées par voie photogrammétrique". Journal XYZ, Association Française de Topographie 59 : 39-45, 1994.

[9] P. Hottier. "Qualité géométrique de la planimétrie. Contrôle ponctuel et contrôle linéaire Dossier: La notion de précision dans le GIS". Journal Géomètre 6: 34-42, 1996.

[10] R. Dunn, A. R. Harrison, J. C. White. "Positional accuracy and measurement error in digital databases of land use: an empirical study". International Journal of Geographic Information Systems 4 (4): 385-398, 1990.

[11] M. Lester, N. Chrisman. "Not all slivers are skinny: a comparison of two methods for detecting positional error in categorical maps". GIS/LIS 2: 648-656, 1991.

[12] A. Hirano, R. Welch, H. Lang. "Mapping from ASTER stereo image data: DEM validation and accuracy assessment". ISPRS Journal of Photogrammetry and Remote Sensing 54: 356-370, 2003.

[13] D. F. Maune. "Digital Elevation Model Technologies and Applications: The DEM Users Manual". The American Society for Photogrammetry and Remote Sensing, Bethesda, 2001.

[14] J. F. Zelasco. "Contrôle de qualité des modèles numériques des bases de données géographique". Journal Association Française de Topographie XYZ 90: 50-55, 2002.

[15] J. F. Zelasco, G. Porta, J. L. Fernández Ausinaga. "Geometric Quality in Geographic Information". In: Doorn JH, Rivero LC, Ferraggine VE (eds) Encyclopedia of Database Technologies and Applications, 1st edn. Idea Group Inc., Hershey, 266-270, 2005.

[16] J. F. Zelasco, J. Donayo, K. Ennis, J. L. Fernández Ausinaga. "Geometric Quality in Geographic Information: IFSAR DEM Control". In: Ferraggine VE, Doorn JH, Rivero LC (eds) Handbook of research on Innovations in Database Technologies and Applications: Current and Future Trends, 1st edn. IG I Global, Hershey, 396-402, 2008.

[17] S. J. Buckley, J. P. Mills, H. L. Mitchel. "Improving the accuracy of photogrametric absolute orientation using surface matching". In: Geo-Imagery Bridging Continents, XXth ISPRS Congress, 12-23 July 2004 Istanbul, Turkey, Commission 3.

[18] J. M. Steele. "The Cauchy-Schwarz Master Class". Cambridge University Press, 306 pages, 2004.

[19] J. F. Zelasco, J. L. Fernandez Ausinaga, P. Julien, G. Porta, H. Ryckeboer. "Geometric Quality Control of a DEM obtained by Optical Remote Sensing Techniques: Evaluation of Effectiveness". Submited to Computer & Geosciences, 2010.

[20] J. F. Zelasco, J. Donayo, J. L. Fernandez Ausinaga, G. Porta, B. Sulpis, B. "PDEM Assessment for Precision Estimation of IFSAR DEM'S". WSEAS, International Conference on Mathematical Models for Engineering Science (MMES '10). Puerto De La Cruz, Tenerife, November 30-December 2, 2010

[21] J. F. Zelasco, P. Julien, G. Porta, K. Ennis, J. Donayo. "A Vertical and Horizontal Error Estimator for Digital Surface Models". Proceedings of the 6th WSEAS European Computing Conference (ECC '12). Prague, Czech Republic. September 24-26, 2012

José F. Zelasco is graduated as surveyor from the Buenos Aires University. Then he was working several years in Africa (Nigeria and Algeria). Afterwards he graduated in informatics from the Pierre et Marie Curie University (France) and obtained his Diplôme D'Études Approfondies (French master) in geodetic sciences, from the École Nationale de Sciences Géographiques. Finally he got his PhD in applied mathematics from the Montpellier II University (Académie de Sciences et Techniques du Languedoc).

He has been member, co director and director of several research projects. He is professor at the Buenos Aires University (Computer Sciences Department and Mechanical Department. He is director of the LEMI laboratory (University of Buenos Aires).

He is author of about 60 papers published in several journals and conference proceedings, and invited book chapters.