# Transportation Problem and Related Tasks with Application in Agriculture

Miloš Šeda, Jan Roupec, and Jindřiška Šedová

***Abstract*****—**In this paper, we deal with well-known distribution problems and discuss their restrictions, extensions and modifications including a possible application in agriculture. We show that the transportation problem can be transformed to an allocation, location and set covering problem using special constraints, but because of NP-hardness of the last problem it needs quite different methods of its solving. Another modification of the transportation problem, the crop problem, has an application in agriculture, but we must deal with uncertain data. We propose a genetic algorithm and fuzzy logic approach for solving these problems

***Keywords*****—**crop problem, set covering problem, transportation, fuzzy number.

## I. Introduction

THE Hitchcock *transportation problem* with $m$ sources (supply points, factories) and $n$ destinations (demand points, clients) can be formulated using linear programming as follows [1], [2]:
Minimise

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{1}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, \dots, m \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, \dots, n \tag{3}$$

$$x_{ij} \geq 0, \quad i = 1, \dots, m, \ j = 1, \dots, n, \tag{4}$$

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where
$z$ is the total transportation cost,
$c_{ij}$ is the unit shipping cost from source $i$ to destination $j$,
$x_{ij}$ is the number of units shipped from source $i$ to destination $j$,
$a_i$ is the supply of the source $i$,
$b_j$ is the demand of destination $j$,
and only a single commodity is transported.

Since there is only one commodity, a destination can receive its demand from more than one source. Therefore, the objective is to determine how much should be shipped from each source to each destination so as to minimise the total transportation cost.

If total commodity supply equals to total demand, the problem is said to be a *balanced transportation problem*:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{5}$$

If (5) is not satisfied, then it becomes an *unbalanced transportation problem*.

$$\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j \tag{6}$$

If total supply exceeds total demand, see (6), we can balance the problem by adding a dummy destination to absorb the excess supply. Shipments to this destination are assigned a cost of zero.

$$\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j \tag{7}$$

If a transportation problem has a total supply that is strictly less than total demand, see (7), the problem has no feasible solution, because one or more demands cannot be satisfied. In such situations a penalty cost is often associated with unmet demand and the total penalty cost is desired to be minimal.

There are several methods for solving the balanced transportation problem as follows:

- The Northwest Corner Method
- The Least Cost Method

- Vogel's Approximation Method

We can also solve the transportation problem using specialised software tools, e.g. GAMS, LINDO, LINGO, or MS Excel Solver.

If it is possible to both ship into and out of the same point of the transport network, then we speak about a *transshipment* (or *transhipment*) *problem*. For the transshipment problem, you can ship from one supply point to another or from one demand point to another.

The Hitchcock formulation of the transportation problem may also be extended considering fixed charges associated with supply points (e. g. warehouses), means of transport, their capacity, cost of transport by vehicles to 1 km, which enables to determine the number of trips due to volume, transport in two levels: primary source – warehouses – destinations, admitting the possibility of direct transport from the primary source to destinations, etc.

Instead, we turn our attention to problems that seem to have nothing with transportation, but their formulation can be obtained from the basic model of the transportation problem.

## II. Allocation Problem

Consider the distribution problem where all supplier capacities (resources, warehouse) $a_i$ , $i$ = 1,2, … , $m$ are sufficient to cover the requirements of all consumers (customers) $b_j$ , $j$ = 1,2, … , $n$, but the demands of every consumer must be covered by only one supplier.

If $c_{ij}$, $i$= 1,2, ..., $m$, $j$ = 1,2, ..., $n$ is the cost of transportation from the $i$-th supplier to the $j$-th consumer, $x_{ij}=1$ or $x_{ij}=0$, depending on whether it will be transported from the $i$-th supplier to the $j$-th consumer or not, the mathematical model of this task can be expressed as follows:

Minimise

$$z=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij} \tag{8}$$

subject to

$$\sum_{i=1}^{m}x_{ij}=1, \quad j=1,\dots,n \tag{9}$$

$$x_{ij}\in\{0,1\}, \quad i=1,\dots,m,\ j=1,\dots,n \tag{10}$$

If the capacities of suppliers are limited, then the previous model needs to be supplemented by further constraint. This is expressed by formula (13) in the following model.

Minimise

$$z=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij} \tag{11}$$

subject to

$$\sum_{i=1}^{m}x_{ij}=1, \quad j=1,\dots,n \tag{12}$$

$$\sum_{j=1}^{n}b_j x_{ij}\le a_i, \quad i=1,\dots,m \tag{13}$$

$$x_{ij}\in\{0,1\}, \quad i=1,\dots,m,\ j=1,\dots,n \tag{14}$$

## III. Location Problem

Consider $m$ sites, which can be used to operate the stores, and they will supply $n$ consumers with demands $b_j$ , $j$ = 1,2, … , $n$.

Let store operation cost in the $i$-th location costs be $f_i$, $i$ = 1,2, … , $m$ for the required period, and $c_{ij}$ , $i$ = 1,2, … , $m$, $j$ = 1,2, … , $n$ represent the total cost for the assignment of the $j$-th consumer to the $i$-th location.

The task is to decide which locations operate a store and find the assignment of consumers to operated stores so that the value of the total cost of operating the system was minimal. It is assumed as in the allocation problem that the requirements of each consumer must be covered by only one store.

Note:

Locational problem is a generalization of the allocation problem. Moreover, there are bivalent variable $y_i$, where $y_i$ = 1 indicates that a store in the $i$-th location will be operated and $y_i$ = 0, that there will not be operated.

Bivalent variables $x_{ij}$, where $x_{ij}=1$ or $x_{ij}=0$, similarly to the allocation task indicate that the $j$-th consumer will or will not be assigned to a store in the $i$-th location.

Note:

Decisions that correspond to bivalent variables $y_i$ and $x_{ij}$ have a close relation.

The set of constraints in the case of a decision not to operate a store in a location must ensure that it could not be assigned to any consumer, on the other hand, if we assign a consume to a location, there must be a store operated in it.

These conditions can be formulated as follows:

Minimise

$$z=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij} \tag{15}$$

subject to

$$\sum_{i=1}^{m} x_{ij} = 1, \quad j = 1, \dots, n \tag{16}$$

$$x_{ij} \le y_i, \quad i = 1, \dots, m, \; j = 1, \dots, n \tag{17}$$

$$x_{ij} \in \{0, 1\}, \quad i = 1, \dots, m, \; j = 1, \dots, n \tag{18}$$

$$y_i \in \{0, 1\}, \quad i = 1, \dots, m \tag{19}$$

From (18) we get

$$x_{i1} \le y_i, \quad i = 1, \dots, m$$
$$x_{i2} \le y_i, \quad i = 1, \dots, m$$
$$\dots$$
$$x_{in} \le y_i, \quad i = 1, \dots, m$$

thus

$$x_{i1} + x_{i2} + \dots + x_{in} \le y_i + y_i + \dots + y_i, \quad i = 1, \dots, m,$$

and hence we get that formula (18) may be alternatively expressed by (18')

$$\sum_{j=1}^{n} x_{ij} \le n y_{i,} \quad i = 1, \dots, m \tag{18'}$$

If the capacities of suppliers $a_i$, $i = 1,2, \dots, m$ are limited, then the previous model must be modified as follows:

Minimise

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{19}$$

subject to

$$\sum_{i=1}^{m} x_{ij} = 1, \quad j = 1, \dots, n \tag{20}$$

$$x_{ij} \le y_i, \quad i = 1, \dots, m, \; j = 1, \dots, n \tag{21}$$

or

$$\sum_{j=1}^{n} x_{ij} \le n y_{i,} \quad i = 1, \dots, m \tag{21'}$$

$$\sum_{j=1}^{n} b_j x_{ij} \le a_i, \quad i = 1, \dots, m \tag{22}$$

$$x_{ij} \in \{0, 1\}, \quad i = 1, \dots, m, \; j = 1, \dots, n \tag{23}$$

$$y_i \in \{0, 1\}, \quad i = 1, \dots, m \tag{24}$$

## IV. Dmax Set Covering Problem

Assume that the transport network contains $m$ vertices, that can be used as operating service centers, and $n$ vertices to be served, and for each pair of vertices $v_i$ (considered as service centers) and $v_j$ (serviced vertex) their distance $d_{ij}$ is given and $D_{\max}$ is the maximum distance which will be accept for operation between the service centers and serviced vertices.

The aim is to determine which vertices must be used as service centers so that each vertex was covered by at least one of the centers and the total number of operating centers was minimal.

Note:

1. A necessary condition for solvability of the task is that all of the serviced vertices were reachable from at least one place where an operating service center is considered.
2. Serviced vertex $v_j$ is reachable from vertex $v_i$, which is considered as an operating service center if $d_{ij} \le D_{\max}$. If this inequality is not satisfied, vertex $v_j$ is unreachable from $v_i$.

If variables $a_{ij}$, where $a_{ij}$=1 or $a_{ij}$=0, express whether operated vertex $v_j$ is reachable from vertex $v_i$, which is considered as operating service center, respectively. is not reachable, then the set covering problem can be described by the following mathematical model:

Minimise

$$z = \sum_{i=1}^{m} y_i \tag{25}$$

subject to

$$\sum_{i=1}^{m} a_{ij} y_i \ge 1, \qquad j = 1, \dots, n \tag{26}$$

$$y_i \in \{0,1\}, \qquad i = 1, \dots, m \tag{27}$$

The objective function represents the number of operating centers, constraint (26) means that each serviced vertex is assigned at least one operating service center.

Example:

serviced vertices (customers locations)

$$\begin{array}{c c}
\text{service centers} & \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 5 & 41 & 50 & 26 & 38 & 60 & 44 & 59 \\ 49 & 82 & 13 & 67 & 68 & 20 & 32 & 31 \\ 45 & 17 & 61 & 45 & 67 & 48 & 53 & 127 \\ 37 & 170 & 195 & 32 & 77 & 88 & 90 & 30 \\ 58 & 42 & 25 & 101 & 133 & 32 & 21 & 78 \end{pmatrix}
\end{array}$$

Consider the previous *distance matrix* which expresses service centers and serviced vertices (= customer locations) and $D_{\max}$=40.

From $D_{\max}$=40 we get the *reachability matrix* of serviced vertices from service centers.

$$\begin{array}{c}
\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{pmatrix}
1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 0
\end{pmatrix}
\end{array}$$

Since only service center 3 is reachable to the second serviced vertex (serviced vertex 2 is covered by the 3rd service center) and only service center 1 is reachable to service center 5, these service centers must not be omitted. These two centers cover serviced vertices 1, 4, 5 and 2

It remains to find the service centers which cover the remaining uncovered vertices 3, 6, 7 and 8. This can be achieved either by selecting the service centers 2 and 5, or 4 and 5.

Thus the example has two solutions, where four vertices are sufficient to cover serviced vertices (either 1, 3, 2, 5 or 1, 3, 4, 5). $\boxtimes$

Note:

In the general case, however, the selection of service centers for $k$ uncovered vertices has $2^k$ possibilities and thus the complexity of tasks increases exponentially with the number of uncovered vertices.

For large $k$ we must solve it using a heuristic method [3], [4], [5], [7], [8], e.g., by a genetic algorithm.

## V. Genetic Algorithm for Set Covering Problem

The skeleton for GA can be described as follows [5]:

```
generate an initial population ;
evaluate fitness of individuals in the population ;
repeat
    select parents from the population;
    recombine (mate) parents to produce children ;
    evaluate fitness of the children ;
    replace some or all of the population by the children
until a satisfactory solution has been found ;
```

Since the principles of GAs are well-known, we will only deal with GA parameter settings for the problems to be studied. Now we describe the general settings [3], [6].

Individuals in the population (*chromosomes*) are represented as binary strings of length $n$, where a value of 0 or 1 at the $i$-th bit (*gene*) implies that $x_i$ = 0 or 1 in the solution respectively.

The *population size* $N$ is usually set between $n$ and $2n$. Many empirical results have shown that population sizes in the range [50, 200] work quite well for most problems.

*Initial population* is obtained by generating random strings of 0s and 1s in the following way: First, all bits in all strings are set to 0, and then, for each of the strings, randomly selected bits are set to 1 until the solutions (represented by strings) are feasible.

The *fitness function* corresponds to the objective function to be maximised or minimised.

There are three most commonly used methods of *selection* of two parent solution for *reproduction*: proportionate selection, ranking selection, and tournament selection. The tournament selection is perhaps the simplest and most efficient among these three methods. We use the *binary tournament selection* method where two individuals are chosen randomly from the population. The more fit individual is then allocated a reproductive trial. In order to produce a child, two binary tournaments are held, each of which produces one parent.

The *recombination* is provided by the *uniform crossover* operator, which has a better recombination potential than do other crossover operators as the classical *one-point* and *two-point* crossover operators. The uniform crossover operator works by generating a random crossover mask $B$ (using Bernoulli distribution) which can be represented as a binary string $B = b_1 b_2 b_3 \cdots b_{n-1} b_n$ where $n$ is the length of the chromosome. Let $P_1$ and $P_2$ be the parent strings $P_1[1]$, **...** ,$P_1[n]$ and $P_2[1]$, **...** ,$P_2[n]$ respectively. Then the child solution is created by letting: $C[i] = P_1[i]$ if $b_i = 0$ and $C[i] = P_2[i]$ if $b_i = 1$. *Mutation* is applied to each child after crossover. It works by *inverting* $M$ randomly chosen bits in a string where $M$ is experimentally determined. We use a mutation rate of $5/n$ as a lower bound on the optimal mutation rate. It is equivalent to mutating five randomly chosen bits per string.

When $v$ child solutions have been generated, the children will replace $v$ members of the existing population to keep the population size constant, and the reproductive cycle will restart. As the replacement of the whole parent population does not guarantee that the best member of a population will survive into the next generation, it is better to use *steady-state* or *incremental replacement* which generates and replaces only a few members (typically 1 or 2) of the population during each generation. The *least-fit* member, or a randomly selected member with *below-average fitness*, are usually chosen for replacement.

*Termination of a GA* is usually controlled by specifying a maximum number of generations $t_{max}$ or relative improvement of the best objective function value over generations. Since the optimal solution values for most problems are not known, we choose $t_{max} \leq 5000$.

The chromosome is represented by an $n$-bit binary string $S$ where $n$ is the number of columns in the SCP. A value of 1 for the $j$-th bit implies that column $j$ is in the solution and 0 otherwise.

Since the SCP is a minimisation problem, the lower the fitness value, the more fit the solution is. The fitness of a chromosome for the unicost SCP is calculated by (8).

$$f(S) = \sum_{j=1}^{n} S[j] \tag{28}$$

As to the crossover operation, we can use the traditional two-point crossover, where middle parts of the parent chromosomes are changed, see Fig.1, where P1 and P2 are parents and O1 and O2 are offsprings.

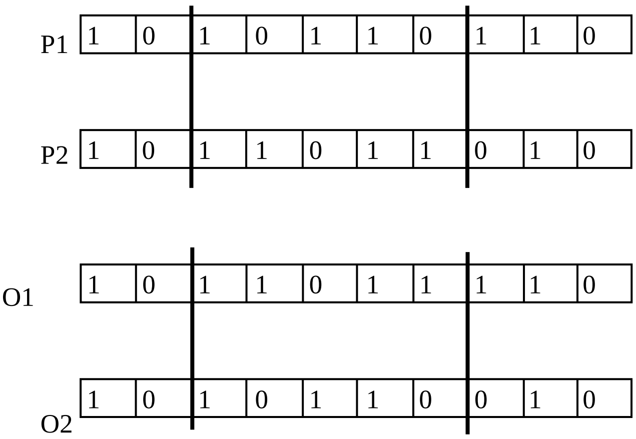


Fig. 1. Two-point crossover

For mutation we considered three operators:

- *exchange mutation* (it exchanges two randomly selected positions in a permutation),
- *shift mutation* (it removes a value at one position and puts it at another position), see Fig. 2, and
- *mutation* inspired by well-known *Lin*-2-*Opt change operator* usually used for solving the travelling salesman problem [9]. Here first two elements are added to the permutation (into positions 0 and $|n|+1$) and then the same values are assigned to them to simulate a cyclic tour. Two 'edges' (pairs of neighbour elements) are randomly chosen (($p_1$, $p_2$) and ($q_1$, $q_2$) say), the inner elements $p_2$, $q_1$ are swapped and the elements between $p_2$ and $q_1$ are reversed.

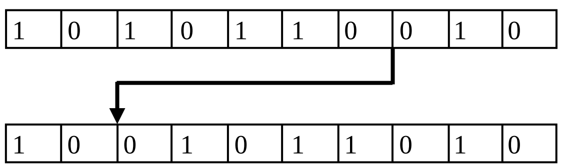


Fig. 2. Shift mutation

The binary representation causes problems with generating infeasible chromosomes, e.g. in initial population, in crossover and/or mutation operations. To avoid infeasible solutions a *repair operator* is applied.

Let

$I = \{1, \dots, m\}$ = the set of all rows;
$J = \{1, \dots, n\}$ = the set of all columns;
$\alpha_i = \{j \in J \mid a_{ij} = 1\}$ = the set of columns that cover row $i$, $i \in I$;
$\beta_j = \{i \in I \mid a_{ij} = 1\}$ = the set of rows covered by column $j$, $j \in J$;
$S$ = the set of columns in a solution;
$U$ = the set of uncovered rows;
$w_i$ = the number of columns that cover row $i$, $i \in I$ in $S$.

The repair operator for the unicost SCP has the following form:

initialise $w_i := |S \cap \alpha_i|, \forall i \in I$;
initialise $U := \{ i \mid w_i = 0, \forall i \in I \}$;
**for each** row $i$ in $U$ (in increasing order of $i$) **do**
  **begin** find the first column $j$ (in increasing order of $j$)
    in $\alpha_i$ that minimises $1/|U \cap \beta_j|$;
    $S := S + j$;
    $w_i := w_i + 1, \ \forall i \in \beta_j$;
    $U := U - \beta_j$
  **end** ;
**for each** column $j$ in $S$ (in decreasing order of $j$) **do**
  **if** $w_i \geq 2, \forall i \in \beta_j$
   **then begin** $S := S - j$;
      $w_i := w_i - 1, \ \forall i \in \beta_j$
    **end** ;
{ $S$ is now a feasible solution to the SCP and contains no redundant columns }

Initialising steps identify the uncovered rows. **For** statements are “greedy” heuristics in the sense that in the 1st **for**, columns with low cost-ratios are being considered first and in the 2nd **for**, columns with high costs are dropped first whenever possible.

## VI. Crop Problem

Let us denote:

$p_1, \dots, p_m$ = grounds

$r_1, \dots, r_m$ = area of grounds

$k_1, \dots, k_n$ = crops

$c_{ij}$, $i = 1,2, \dots, m$, $j = 1,2, \dots, n$

= profit from 1 ha of ground sown by crop $k_j$

$x_{ij}$ = number of hectares of ground $p_i$ sown by crop $k_j$

TABLE I. Crop Problem

| crops / grounds | $k_1$ | $k_2$ | | $k_n$ | area [ha] |
|---|---|---|---|---|---|
| $p_1$ | $c_{11}$ | $c_{12}$ | … | $c_{1n}$ | $r_1$ |
| $p_2$ | $c_{21}$ | $c_{22}$ | … | $c_{2n}$ | $r_2$ |
| … | | | | | … |
| $p_m$ | $c_{m1}$ | $c_{m2}$ | … | $c_{mn}$ | $r_m$ |

In the crop problem [10], [11] is to find the optimum sowing of areas by crops for given yields of crops (in quintals per hectare) and contractual purchase prices so as to maximise the total profit [12].

From Table II we get the following system of equations:

$$\begin{aligned} x_{11} + x_{12} + \dots + x_{1n} &\le r_1 \\ x_{21} + x_{22} + \dots + x_{2n} &\le r_2 \\ &\dots \\ x_{m1} + x_{m2} + \dots + x_{mn} &\le r_m \end{aligned}$$

$$x_{ij} \ge 0, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n$$

$$z = c_{11}\, x_{11} + c_{12}\, x_{12} + \dots + c_{1n}\, x_{1n} + c_{21}\, x_{21} + c_{22}\, x_{22} + \dots + c_{2n}\, x_{2n} + \dots + c_{m1}\, x_{m1} + c_{m2}\, x_{m2} + \dots + c_{mn}\, x_{mn} \to \max$$

It can be expressed as follows:
Maximise

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{29}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \le r_i, \quad i = 1, \dots, m \tag{30}$$

$$x_{ij} \ge 0, \quad i = 1, \dots, m, \; j = 1, \dots, n \tag{31}$$

If we require for each crop that were sown at a certain minimum area, then the task has become an example of maximisation version of the *generalized distribution problem*. It is included in Table III and the corresponding model follows:

TABLE II. CROP PROBLEM WITH MINIMUM REQUIREMENTS

| crops / grounds | $k_1$ | $k_2$ | | $k_n$ | area [ha] |
|---|---|---|---|---|---|
| $p_1$ | $c_{11}$ | $c_{12}$ | … | $c_{1n}$ | $r_1$ |
| $p_2$ | $c_{21}$ | $c_{22}$ | … | $c_{2n}$ | $r_2$ |
| … | | | | | … |
| $p_m$ | $c_{m1}$ | $c_{m2}$ | … | $c_{mn}$ | $r_m$ |
| minimum requirements for crop sowing area | $d_1$ | $d_2$ | | $d_n$ | |

Maximise

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{32}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \le r_i, \quad i = 1, \dots, m \tag{33}$$

$$\sum_{i=1}^{m} x_{ij} \ge d_j, \quad j = 1, \dots, n \tag{34}$$

$$x_{ij} \ge 0, \quad i = 1, \dots, m, \; j = 1, \dots, n \tag{35}$$

## VII. CROP PROBLEM WITH UNCERTAIN YIELDS

Of course, yields of crops are only estimated and in real conditions cannot be considered as deterministic.

This situation can be solved with techniques inspired by PERT.

Denote

$a$ = estimate of the crop yields under the most favourable conditions

$b$ = estimate of the crop yields under the least favourable conditions

$m$ = most likely value for the crop yields

PERT requires the assumption that estimated parameter follows a beta distribution. Then its mean value may be approximated by the following equation:

$$c = \frac{a + 4m + b}{6} \tag{36}$$

Since the beta distribution is not guaranteed, we propose a fuzzy approach.

Let us assume now that crop yields are given by fuzzy numbers [13], [14].

A *fuzzy number A* is a fuzzy set represented by 4-tuple $(a_1, a_2, a_3, a_4)$ and a piecewise continuous membership function with the following properties:

- $a_1 \le a_2 \le a_3 \le a_4$
- $\mu_{A(x)} = 0$ for $x \le a_1$, $x \ge a_4$
- $\mu_{A(x)} = 1$ for $a_2 \le x \le a_3$
- $\mu_A$ is increasing on $[a_1, a_2]$ and decreasing on $[a_3, a_4]$.

The fuzzy set defined by the membership function is an example of fuzzy number. In this paragraph we consider trapezoidal fuzzy numbers, see (37) and Fig. 3.

$$\Pi(x,a,b,c,d)=\begin{cases}0 & \text{for } x<a_1\\ \dfrac{x-a_1}{a_2-a_1} & \text{for } a_1\le x\le a_2\\ 1 & \text{for } a_2<x\le a_3\\ \dfrac{a_3-x}{a_4-a_3} & \text{for } a_3<x\le a_4\\ 0 & \text{for } x>a_4\end{cases} \tag{37}$$

The addition of fuzzy numbers can be derived using the extension principle and it is determined as follows:

$$X^F \oplus Y^F = (x_1, x_2, x_3, x_4) \oplus (y_1, y_2, y_3, y_4) = = (x_1+y_1, x_2+y_2, x_3+y_3, x_4+y_4) \tag{38}$$

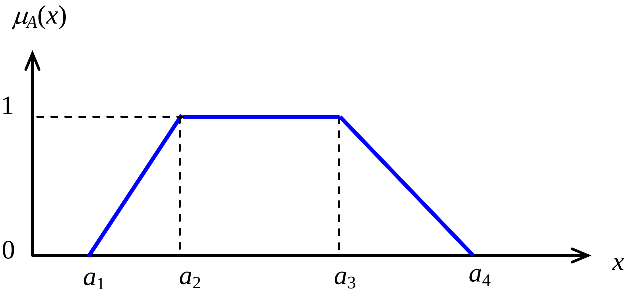


Fig. 3. Trapezoidal fuzzy number

When the maximum operation would be derived in the same way, then its results may not be trapezoidal fuzzy numbers. Therefore we approximate this operation as follows.

$$\max(X^F,Y^F)=(\max(x_1, y_1), \max(x_2, y_2), \max(x_3, y_3), \max(x_4, y_4))$$

To find a solution of the crop problem which maximises the total profit, we must compare fuzzy numbers in some way, which is a difficult problem. An *ordering relation* ≤ can be defined e.g. as follows:

$$X^F \le Y^F \Leftrightarrow (x_1\le y_1)\wedge(x_2\le y_2)\wedge(x_3\le y_3)\wedge(x_4\le y_4) \tag{39}$$

However, this relation is not a complete ordering relation, as fuzzy numbers $X^F$, $Y^F$ satisfying

$$(\exists\, i,j \in\{1,2,3,4\}): (x_i<y_i) \wedge (x_j >y_j) \tag{40}$$

are not comparable by ≤.

It is evident that, for non-comparable fuzzy numbers $X^F$, $Y^F$, this fuzzy max operation results in a fuzzy number different from both of them. For example, for $X^F$ =(4,9,12,16) and $Y^F$ =(6,8,13,15), we get from (23) a fuzzy max (6,9,13,16) which differs from $X^F$ and $Y^F$.

This problem can be solved by assigning a scalar value to each resulting fuzzy number and comparing these scalars.

We use the fuzzy ranking method described in [15], modified for the case of trapezoidal fuzzy numbers. This method uses inverse functions $g_A^L:[0,1]\to[a_1,a_2]$ and $g_A^R:[0,1]\to[a_3,a_4]$ derived from functions $f_A^L:[a_1,a_2]\to[0,1]$ and $f_A^R:[a_3,a_4]\to[0,1]$, respectively.

From $y=\dfrac{x-a_1}{a_2-a_1}$ (increasing part) and $y=\dfrac{x-a_4}{a_3-a_4}$ (decreasing part) we can derive that

$$g_A^L = a_1+(a_2-a_1)y, \quad g_A^R = a_4+(a_3-a_4)y \tag{41}$$

The ranking function is defined as the distance between the *centroid point* ($x_0$, $y_0$) and the origin

$$R(A)=\sqrt{(x_0)^2+(y_0)^2} \tag{42}$$

where

$$x_0=\frac{\int\limits_{\mathrm{Supp}\,A} x\mu_A(x)dx}{\int\limits_{\mathrm{Supp}\,A}\mu_A(x)dx},\quad y_0=\frac{\int_0^1 y g_A^L dy+\int_0^1 y g_A^R dy}{\int_0^1 g_A^L dy+\int_0^1 g_A^R dy} \tag{43}$$

and Supp $A$ is the support of $A$.

Fuzzy numbers $A$, $B$ are then ranked by their ranking function values $R(A)$ and $R(B)$.

Let us now consider the described approach on specific dates. The Czech Farm Weekly magazine states that barley yield ranged from 3.5 to 6 tons per hectare, rape yields are between 3.5 and 5 t/ha and wheat yields between 5.5 and 7.5 t/ha. Commodity prices are constantly changing. Price of barley is roughly from 3300 to 3500 CZK/t.,(3.20) wheat from 3700 to 4000 CZK /t and rape price is between 8000 and 8500 CZK/t.

The interval boundaries of products of yields and prices correspond to the values $a_1$, $a_4$ in the fuzzy number by Fig. 2, or $b$ and $a$ in (26). The parameters $a_2$, $a_3$ and $m$ must be estimated. Let us assume that corresponding values evaluated by (36) or (42) are as follows in Table IV.

Using these data we can compute by (32)-(35) the value of decision variables $x_{ij}$ maximising the total income. This model was implemented in GAMS (*General Algebraic Modelling System*) [16]. This package was developed by A. Meeraus and A. Brooke at the World Bank especially for the tasks of linear, nonlinear and mixed integer programming.

In the calculation we assumed decision variables restricted to integers. Computational results are summarised in Table V. The total income is 7 512 247 CZK

TABLE III. DATA FOR 3 CROPS AND 10 GROUNDS

| crops / grounds | barley | rape | wheat | area [ha] |
|---|---|---|---|---|
| $p_1$ | 15625 | 34436 | 25291 | 22 |
| $p_2$ | 13355 | 37556 | 22756 | 15 |
| $p_3$ | 12881 | 31263 | 28350 | 38 |
| $p_4$ | 15522 | 39245 | 26423 | 10 |
| $p_5$ | 13895 | 36328 | 25689 | 42 |
| $p_6$ | 18254 | 40634 | 27105 | 16 |
| $p_7$ | 17425 | 38231 | 26725 | 23 |
| $p_8$ | 16233 | 33252 | 24490 | 17 |
| $p_9$ | 15880 | 34567 | 25562 | 31 |
| $p_{10}$ | 16540 | 37843 | 26243 | 25 |
| minimum requirements for crop sowing area | 30 | 90 | 80 | |

TABLE IV. COMPUTATIONAL RESULTS

| crops / sown areas | barley | rape | wheat | area [ha] |
|---|---|---|---|---|
| 22 | 13 | 0 | 9 | 22 |
| 15 | 0 | 15 | 0 | 15 |
| 38 | 0 | 0 | 38 | 38 |
| 10 | 0 | 10 | 0 | 10 |
| 42 | 0 | 40 | 2 | 42 |
| 16 | 0 | 16 | 0 | 16 |
| 23 | 0 | 23 | 0 | 23 |
| 17 | 17 | 0 | 0 | 17 |
| 31 | 0 | 0 | 31 | 31 |
| 25 | 0 | 25 | 0 | 25 |
| total sown areas | 30 | 129 | 80 | |

## VIII. CONCLUSIONS

In this paper we studied the well-known transportation problem and presented several modifications which are important in various application areas.

We showed that the allocation, location and set covering problem can be derived from the linear transportation problem, but the last problem cannot be solved for large instances using linear programming methods and heuristics must be used. We presented a genetic algorithm (GA) approach and GA parameter settings. Since traditional operators generate infeasible solutions, a repair operator was proposed.

Finally, we investigated the crop problem, important in agriculture engineering, and generalised it for case of uncertain crop yields. Besides traditional interval and PERT approach, we propose a fuzzy algebra based on fuzzy numbers. Using the Cheng ranking function based on the distance between the centroid point and the origin was the problem defuzzified and we could use the deterministic mixed integer model.

In the future, in spite of no free lunch theorem, we foresee further tests with other stochastic heuristics.

## REFERENCES

[1] M. B. Stroh, A Practical Guide to Transportation and Logistics. Dumont: Logistics Network, 2006.

[2] M. Šeda, J. Roupec, and J. Šedová, “Distribution Problems, Their Modifications and Applications,” Int. Conf. on Applied Mathematics and Computational Methods. Venice, pp. 284-289, 2013.

[3] I. Zelinka, Z. Oplatková, M. Šeda, P. Ošmera and F. Včelař, Evolutionary Computational Techniques. Principles and Applications. Prague: BEN, 2009.

[4] D. H. Wolpert, W. G. McReady, “No Free Lunch Theorems for Optimization.” IEEE Transactions on Evolutionary Computations, vol. 1, no. 1, pp. 67-82, 1997.

[5] Z. Michalewicz.: Genetic Algorithms + Data Structures = Evolution Programs. Springer-Verlag, Berlin, 1996.

[6] M. Šeda, “Heuristic Set-Covering-Based Postprocessing for Improving the Quine-McCluskey Method.” International Journal of Computational Intelligence, vol. 4, no. 2, pp. 139-143, 2007.

[7] D. E. Goldberg, The Design of Innovation (Genetic Algorithms and Evolutionary Computation). Dordrecht: Kluwer Academic Publishers, 2002.

[8] Z. Michalewicz and D.B. Fogel, How to Solve It: Modern Heuristics. Berlin:,Springer-Verlag, 2002.

[9] G. Gutin, and A. P. Punnen (eds.). The Traveling Salesman Problem and Its Variations. Dordrecht: Kluwer Academic Publishers, 2002.

[10] N. H. Mitchell, Mathematical Applications in Agriculture, 2nd ed. London: Cengage Learning, 2011.

[11] J Thornley, and J. France, Mathematical Models in Agriculture, 2nd ed., Wallingford, Oxfordshire: CABI, 2006.

[12] A. Bechar, and G. Vitner, “Work Planning in Packing Houses of Flowers Mixed Farms to Increase the yield,” Agricultural Engineering International: CIGR Journal, vol. 13, no. 3, 2011, 10 pp., manuscript no. 1901.

[13] L. A. Zadeh, “Fuzzy sets.” Information and Control, vol. 8, pp. 338–353, 1965.

[14] V. Novák, Fuzzy Sets and their Applications. Bristol: Adam Hilger, 1989.

[15] C.-H Cheng, “A New Approach for Ranking Fuzzy Numbers by Distance Method,” Fuzzy Sets and Systems, vol. 95, pp. 307-317, 1998.

[16] A. Brooke, D. Kendrick and A. Meeraus,. GAMS, release 2.25. A User’s Guide. Danvers, Massachussetts: Boyd & Fraser Publishing Company, 1992.