Properties of the cost matrix and the *p*-median problem

Marta Janackova, Alzbeta Szendreyova

***Abstract*—**This contribution deals with the *p*-median problem and with the properties of the distribution network.We study the dependence of the computational time of exact method for *p*-median problem on characteristics of underlying transportation network. The research is motivated by previous experiences in experimental work with approximate method for the *p*-median problem. Networks obtained from the internet were used in mentioned experiment. The networkswere notgiven by their graphical structure, but by the cost matrix. During these experiments it was found that the distribution of the distance values may considerable influence the effectiveness of the solving process, which is based on the branch and bound method. We want to verify the similar hypothesis for the specific algorithm developed for the *p*-median problem. We also solve a problem of the network deformation. We check up changing of the computational time of the *p*-median problem after rounding the values of the cost matrix. We found out what differences are between the solutions before and after deformation of the network. We are interested in the impact of the deformation on the computational time, on the availability of the service, on the objective values and on the changing of the service centers locations as well.

***Keywords*—**cost matrix**,** distance matrix, frequency, network, *p*-median problem.

I. Introduction

Many services (health, education, public administration, etc.) are provided through a so-called public service systems. The ground of the public service system design is, inter alia, a decision on the service centers locations [2], [6], [12], [21]. Customers go to the service centers for the same services and theirs requirements are subsequently satisfied from these locations.Municipal offices, courts, hospitals and schools belong to these services.

Some services are provided directly at customers.For example, emergency medical services, fire brigades, removal of the crashes or evacuation of the population belong to these services [7], [11], [19]. A lot of constrains influence the decision about location of the service center to the certain point.Standard demand is the economicpoint of view. It is necessary to minimize the total costs of the accessibility of the service. These include the costs for building and operating the centers and the variable costs as well. The variable costs are usually given by the kilometric availability or by the time availability.It also depends on how often the customer service is used by a customer.

At present, a simple location problem is solvable for thousands of nodes.In real life, there often occur problems, which need toobserveseveral constraints during their solution. One of them is, for example,the capacity limitation (production capacity, permeability of the distribution channel).

When the storages of the dangerous material are located, the greatest distances from the populated area are required. On the contrary, a providing of the emergency services and other rescue centers need minimization of the distance between the costumer and the located center.This requirement may be even stronger.We can require the availability which is limited by the given time or by the given distance [10].

A specific number of the centers must be located to achieve a certain level of the comfort. Sometimes it is necessary to locate no more than a required number of the centers (absence of the finance), some other time it is necessary to locate at least required number of the centers. Each of these constraints changes the uncapacitated location problem to a specific problem [16], [17], [18].The problems are sometimes solved by commercial IP solver [26], [27]. In other cases, special methods are searched for solving the particular problem [20], [22], [23], [24]. The computational time of solving the location problem together with the additional constraints significantly grows.

In all mentioned distribution problems,the placement of the centers is often influenced by some geographical conditions. The facility location problems are often solved on large networks and it causes that large number of variables must be used. The exact method based on the principle of branch and bound method, can handle the network containing in order of thousands of nodes. The bigger the number of nodes is the higher the computational time is. This forces us to find other approximate methods for solving the location tasks in shorter time [5]. When one of these methods (radial method) was tested to solve *p*-median problem [8], [9], the analysis of the results showed some anomalies - the gap of theobjective values between the optimal solution and the solution obtained by this method grew into 50% and the time consumption grew

This work was supported by the research grants VEGA 1/0296/12 “Public Service Systems with Fair Access to Service” and APVV-0760-11 “Designing of Fair Service Systems on Transportation Networks”.

We would also like to thank to „Centre of excellence for systems and services of intelligent transport“ (ITMS 26220120050) for built up the infrastructure, which was used.

Marta Janackova Author is with the Department of Applied Mathematics, University of Zilina, Univerzitna 1, 010 26 Zilina, SlovakRepublik, (e-mail: Marta.Janackova@fstroj.uniza.sk).

AlzbetaSzendeyova Author is with the Department of Transportation Networks, University of Zilina, Univerzitna 1, 010 26 Zilina, Slovak Republic, (e-mail: Alzbeta.Szendreyova@fri.uniza.sk).

into some hours. On the contrary, no anomalies were found, when solving problems on the real road network of Slovak Republic. During experiments, it was found that distribution of distance values may considerable influence the effectiveness of the solving process. The following hypothesis was formulated: The anomalies mentioned above relate to the frequency of the lengths in distance matrix.

In this contribution, we try to answer two questions:

a) Whether the atypical distribution of lengths in the distance matrix influences the exact location problem solving method based on the principle of branch and bound.

b) How the deformation of the real network influences the computational time Procedure for Paper Submission

II. *P*-MEDIAN PROBLEM

Recently we looked at the deployment of emergency medical service centers. This problem belongs to the tasks with a limited number of center locations so-called the *p*-median problem. Therefore we try to locate no more than *p* centers so as to get the best value of the objective function. The criterion of "best" can be understood as the best average time of service availability, minimum total time for service availability of the most disadvantaged customer, etc.

For the purpose of this paper, we solve the *p*-median problem using networks, which are specified by the distance matrix. The distance matrices mutually differ in distribution of lengths. As the criterion for optimization we understand the sum of the distances between customers and their nearest located service centers.

We can use a commercial IP-solver for solving tasks of smaller size (in order of hundreds of nodes). This solver, known as Xpress, is based on branch and bounds principle [4]. On the same principle it was developed a specific method *BBDual* in our workplace [13]. Its advantage is that it can solvetasks of larger size (in order of thousands of nodes) in real time. The method *BBDual* solves the uncapacitated location problem and it was modified by Lagrangean multiplier to the ability to solve the *p*-median problem tasks. It means that we solve the tasks of the *p*-median problem by the iterative way.

The *p*-median problem can be formulated as follows:

Let I denote the set of the possible center locations and let J denote the set of the customers. The customers are situated at the dwelling places of a network and the number of inhabitants at $j \in J$ is denoted by b_j. We assume that each inhabitant performs the same number of visits at his/her service center. Let d_{ij} denote the distance between the center location $i \in I$ and the customer's location $j \in J$. The segment between i and j is evaluated by the total traveled distance $c_{ij}=b_j d_{ij}$ for each possible center location $i \in I$ and for each dwelling place $j \in J$. Our task is to locate limited number p of service centers to some nodes from the set I to minimize the sum of traveled kilometers. The decision on locating or not locating a service center i at a place $i \in I$ is modeled by a variable y_i, which takes the value of 1, if the center is located at place i and it takes the value of 0 otherwise. The decision on allocation of the customer from node j to the center at the place i is modeled by a variable z_{ij}. It takes the value of 1 if the customer j will be served from the center i and takes the value of 0 otherwise.The model has the following form:

$$Minimize \ \sum_{i \in I} \sum_{j \in J} c_{ij} z_{ij} \quad (1)$$

$$Subjectto \quad \sum_{i \in I} z_{ij} = 1 \quad forj \in J \quad (2)$$

$$z_{ij} \le y_i fori \in I, j \in J \quad (3)$$

$$\sum_{i \in I} y_i = p \quad (4)$$

$$z_{ij} \in \{0,1\} fori \in I, j \in J \quad (5)$$

$$y_i \in \{0,1\} \quad for\ i \in I \quad (6)$$

The coefficients in the model have the following meanings:

I … the set of possible facility locations

J …the set of customers (dwelling places)

c_{ij} … value of the edge between nodes i and j

p … required number of facilities (centers).

There was also designed another way, based on the covering principle, to solve large tasks of the *p*-median problem. A simplified description of the model [14], [15] follows:

The approximate approach is based on a relaxation of the assignment of a service centre to a customer. In the approximate approach, the distance between a customer and the nearest facility is approximated unless the facility must be assigned. The range $<0, max\{d_{ij}: i \in I, j \in J\}>$ of all possible distances is partitioned the into $r+1$ zones. The zones are separated by finite ascending sequence of dividing points D^1, D^2, …, D^r, where $0 = D_0 < D_1$ and $D_r < D_m = max\{d_{ij}: i \in I, j \in J\}$. A zone k corresponds with the interval $(D_k, D_{k+1}>$, the first zone corresponds with the interval $(D_1, D_2>$ and so on, till the *r-th* zone, which corresponds with interval $(D_r, D_m>$. A width of the *k-th* interval is denoted by e_k for $k= 0, \dots, r$.

In addition to the zero-one variable $y_i \in \{0,1\}$, which takes the value of 1 if a facility should be located at location i, and which takes the value of 0 otherwise. An auxiliary zero-one variable x_{jk} for $k=0, \dots, r$ is introduced. The variable takes the value of 1 if the distance of the customer $j \in J$ from the nearest located center is greater than D_k and this variable takes the value of 0 otherwise. Then the expression $e_0 x_{j0} + e_1 x_{j1} + e_2 x_{j2} + e_3 x_{j3} + \dots + e_r x_{jr}$ is an upper approximation of d_{ij}. If the distance d_{ij} belongs to the interval $(D_k, D_{k+1}>$, it is estimated by upper bound D_{k+1} with a possible deviation e_k.

Similarly to the covering model, we introduce zero-one constant a_{ij}^k for each triple $<i, j, k> \in I \times J \times \{1, \dots, r\}$. The constant a_{ij}^k is equal to 1 if and only if the distance between the customer j and the possible location I is less or equal to D_k, otherwise a_{ij}^k is equal to 0. Then a covering-type model can be formulated as follows:

$$Minimize \sum_{j \in J} \sum_{k=0}^{r} e_k x_{jk} \quad (7)$$

$$Subject\ to \ \ x_{jk} + \sum_{i \in I} a_{ij}^k y_i \ge 1 \quad forj \in J, k = 0, \dots, r \quad (8)$$

$$\sum_{i \in I} y_i = p \quad (9)$$

$$x_{jk} \ge 0 \quad forj \in J, k = 0, \dots, r \quad (10)$$

$$y_i \in \{0,1\} \quad for\ i \in I \quad (11)$$

In this model, the objective function (7) gives the upper bound of the sum of the original distances. The constraints (8) ensure that the variables x_{jk} are allowed to take the value of 0, if there is at least one center located in radius D^k from the customer j. The constraint (9) limits the number of located facilitiesby p.

III. Properties of the cost matrix

Values of d_{ij} have a different character for networks with dense and equable settlement and different for mountain areas, where a larger number of nodes are located away from the main settlement. In the first case, the centers can be located so that each customer is "near" to one of them, in the second case, the presence of large values of d_{ij} can be found in the distance matrix. Similarity or differences in the character of the distance matrix can be described by the frequency of occurrence of each value d_{ij} in the matrix. Because this frequency depends on the cardinality of the sets I and J and the distribution of the frequency depends on the scale of values of d_{ij}, it is possible to compare the task results of the networks, which have the same ranges of the distance matrices and approximately the same maximum values of d_{ij}. For different sizes of networks we can standardize the matrix values. Differences in the number of assignments (i.e. dimension of the matrix) can be solved by registering the relative frequencies of occurrences of d_{ij}instead of absolute registering. As regards to the nature of the p-median problem we assumed that the assignments with the highest valuation will not be used in the optimization process. Therefore the relative frequencies with the values less than half, respectively, two-thirds of the maximum value of d_{ij}, are interesting for the purpose of research for this paper. For illustration three graphs are presented in Fig. 1-3. The first one shows the matrix with fast growth of the frequency value, the second one with slow growth of the frequency value.

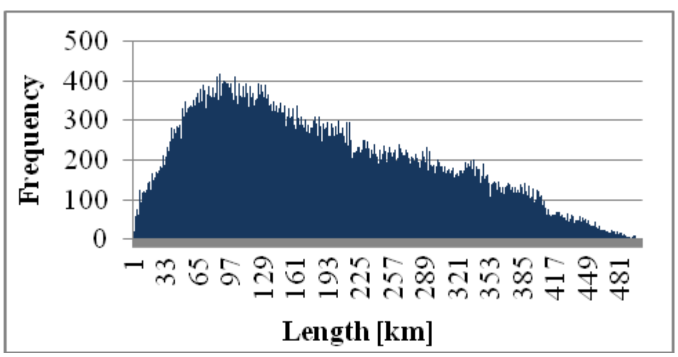

Fig.1 Frequencies of values in the distance matrix - 100x1000 (Slovak Republic)

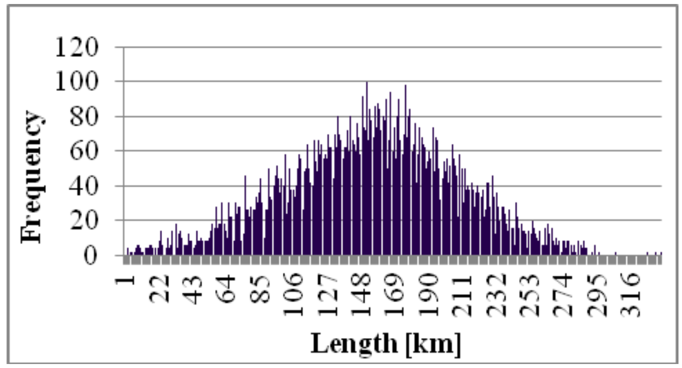

Fig.2 Frequencies of values in the distance matrix - 100x100 (Beasley's benchmark)

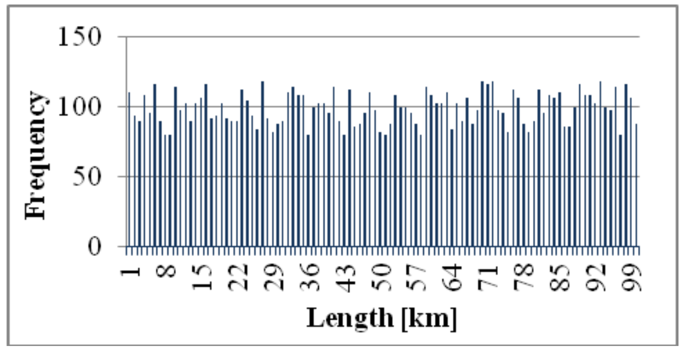

Fig.3 Frequencies of values in the generated distance matrix - 100x100

IV. Frequencies of the values in cost matrix

To check the exact method sensitivity, which depends on the character of the frequency values in the distance matrix, we calculated the p-median problem using the multiple sets of networks. These networks varied in size and character of the task. One part of networks was picked from the road network of the Slovak Republic. First, it was the entire network of the Slovak Republic, and then several selections of its subset, e.g. networks of 100x1000 matrix type and networks of the Slovakia regions as well as a selection of them with 100x100 matrices. In the smallest networks, the set of the possible center locations consisted only of the district towns of the region and the set of customers was formed from all the municipalities of the region. All these networks have real character, although some of them can be considered to be someway deformed in view of the criterion used for the selection.

Another set of networks we obtained from internet. They were not based on map, but they were created only as the distance matrices among the nodes (Beasley's networks). There were just the networks, which showed significant differences in the objective values and computational times, when exact location-allocation approach and approximate covering approach were compared. We chose some Beasley's matrices of sizes 100x100 and 100x1000 and we compared their solutions of associated problems with results obtained for real networks. For our experiments, we generated also some networks with similar sizes. These networks were specified only by distance matrices.

We calculated the absolute and relative frequencies of occurrence of each value in the distance matrix.

We obtained the following results: An optimal solution was found for each studied task and no calculation lasted unreasonably long. The characteristic of the frequency values in the distance matrix were different in generated networks in comparison with Beasley's networks and the networks which were derived from the real road network of the Slovak Republic. The occurrence of the frequencies of values less than one-third of the maximum frequency value for real network ranged from 20% to about 40%, for Beasley's network it was from 33% to 63% and for the generated networks ranged from 0% to about 5%. When the occurrences of frequencies were less than a half of the maximum frequency value, the differences were more significant. In real networks, the frequencies of those values varied between 37% and 58% of all occurrences, with Beasley's networks it ranged from 71% to 82% and for the generated networks ranged from 0% to 18%.

The results of the measured values for some of the networks with a matrix of size 100x100 are shown in the table I.

Columns A and C in the table I denote the frequencies of the value less than or equal to 1/3 (A) and 1/2 (C) of the maximal frequency and columns B and D represent the relative frequencies of the value less than or equal to 1/3 (B) and 1/2 (D) of the maximal frequency [%].

Table I Occurrences of the frequencies

Task size 100x100	freq≤ (max freq) /3		freq≤ (max freq)/2	
	A	B	C	D
TT-real net	57	40.71%	82	58.57%
ZA-real net	42	27.27%	58	37.66%
BB-real net	73	42.20%	88	50.87%
B1-Beasley	200	59.70%	241	71.94%
B2-Beasley	198	63.46%	227	72.76%
Gen1	0	0.00%	0	0.00%
Gen2	0	0.00%	0	0.00%
Gen3	0	0.00%	0	0.00%
Gen4	0	0.00%	13	9.42%
Gen5	0	0.00%	11	7.97%
Gen6	0	0.00%	14	10.29%
Gen7	5	3.33%	28	18.67%
Gen8	6	3.95%	25	16.45%
Gen9	8	5.26%	27	17.76%

We solved the p-median problem for all the networks. We defined the required number of p so that it always represented the same percentage of the number of possible center locations in the current network. We followed up the value of objective function, and we measured the computation time.

We used a java implementation of the branch and bound method with Erlenkotter'sapproach [5] for the experiments. This implementation was named *BBDual* and it was developed at the Department of Transportation Network, University of Zilina, Slovakia. One component of this implementation (*pBBDual*) solves the p-median problem so that it solves uncapacitated facility location problem with limited number of the locations (maximum p). *BBDual*always gives optimal solution for the uncapacitated location problem. It may not always be so, when the p-median problem is solved.

The algorithm *BBDual* works iteratively. It solves the uncapacitated location problem. If the number of locations is higher than p, the penalty would be added. The Lagrangean multiplier is introduced for the constraint$\sum_{i \in I} y_i \leq p$. The number of the iteration depends on several factors. The adjustment of the fixed values, the way of ending the iterative cycle, the repeated occurrences of values in cost matrix, these all influence the computational time. We assume that the frequent occurrence of the same values in the matrix significantly affects the branching diagram in branch and bound method. We verified the calculations using the XPRESS solver (professional solver for linear and integer programming).

The table II shows the results of task solutions for p equal to one-third and to a half of the cardinality of the possible center locations set. The computation time is presented in seconds.

Table II Computation times

Task size 100x100	p≤ (number of candidates)/3 = 33			p≤ (number of candidates)/2 = 50		
	Exact number of p	Comp. time *pMBBDual* [s]	Comp. time *XPRESS* [s]	Exact number of p	Comp. time *pMBBDual* [s]	Comp. time *XPRESS* [s]
TT-real net	33	0.22	0.73	50	0.11	0.72
ZA-real net	31	0.10	0.62	48	0.08	0.62
BB-real net	33	0.07	0.73	45	0.08	0.87
B1-Beasley	33	0.08	0.64	50	0.07	0.64
B2-Beasley	33	0.08	0.06	50	0.09	0.62
Gen1	31	89.74	0.78	50	90.12	0.71
Gen2	32	52.20	0.78	41	51.72	0.71
Gen3	32	76.98	1.5	49	77.03	0.73
Gen4	28	153.46	0.77	40	153.46	0.71
Gen5	30	68.73	0.74	46	68.90	0.69
Gen6	31	287.58	0.76	50	285.51	0.71
Gen7	33	197.82	0.77	45	197.83	0.71
Gen8	32	154.13	1.1	42	153.77	0.70
Gen9	29	186.63	1.5	47	186.50	0.72

The computation times were mostly greater when it was solved using the program *pBBDual* in comparison with the computational times necessity for the professional program XPRESS. The calculation of the tasks with real and Beasley's networks lasted up to one second for *pBBDual* and for XPRESS as well. The calculation times of the tasks with generated networks varied from 90 seconds to 300 seconds when program *pBBDual* was used for solving, and it lasted up to 1 second in case of program XPRESS. But it is important that the time consumption of all the tested tasks lasted only tens of seconds.

Solution of the *p*-median problem using exact methods based on the principle of a branch and bound method in our examples did not confirm time consumption differences depending on the distribution of frequency values in the distance matrix. We assumed that the rapidity of solution of the *pBBDual* method was positively influenced by the special structure of coefficients c_{ij}, which acquired different values despite of the fact that a lot of them were repeated. This causes the variety of the objective function values in the branches and hence, the faster progress of the optimization process.

V. An influence of the deformation of the cost matrix on *p*-median

Based on the result of the table II we state, that there are no differences in computational time, when the algorithm Xpress solved the tasks with networks of different types. Algorithm *pBBDual*"worked" well on real networks and worse on artificially obtained ones. The question arises, what causes the difference in time consumption, when algorithm *pBBDual* is used for solving the tasks of the *p*-median problem.

It can be the difference in occurrence of the value frequencies between real cost matrix and generated cost matrix.The difficulty can follow from the fact, that the networks which are given by generated matrices do not have the properties of the real networks (e.g. they do not meettheEuclidean metric).

We propose the following experiment to verify the hypothesis that the deformation of the real network does not significantly affect the properties that influence the computation of the *p*-median problem. We round up the distances among the nodes of real network so that theywill be divisible by 5. We obtain another set of task when we round up the distances to the values that are divisible by 10. The aim of this deformation is to obtain matrices with greater occurrence of the same values. These values will occur in the distance matrix with greater frequency. We will solve the *p*-median problem by algorithm *pBBDual*for both the original and the deformed data. The request for parameter *p*will be the same in both cases.

We are interested in how the computational time of the *p*-median problem will change after the deformation of the distance matrix.We will monitor the computational time, changes in the set of the service center locations and the objective value. We will compute the objective value as the total availability of service calculated from the real distances among the customers and the locations of the service centers. We will also use the real distances to calculate the objective function of the tasks with deformed data.They will differ because of the different sets of the located centers.

We will evaluate the difference between the availability of the service on the real and on the deformed networks.To evaluate the differences among locations of the centers before and after the deformation of the network we will use the Hamming distance. The Hamming distance between two vectors of equal length is the number of coefficients in which they differ. The length of the vector in our task is equal to the number of the candidates for the locations. The vector on the *i-th* position takes the value of 1, when a service center is located in the *i-th*node; otherwise it takes the value of 0.

We performed the testing on road networks, which correspond to the regions of Slovakia and to the whole Slovakia. The networks of the regions contain from 87 to 664 nodes. Both the set of the candidates and the set of the customers consist of all nodes of the particular region. The road network of Slovakia contains 2916 nodes and the set of the candidates in this case is formed from 79 district capitals.

We created another group of calculations so that we selected some nodes from the networks mentioned above and we obtained matrices of sizes 100x100. We solved the *p*-median problem for two value of *p* for each network. The value of *p* corresponds to one third of the candidates in one case and to one half in the second one.

The table III shows the comparison of the computation time of the real networks and of the deformed ones, which are specified by the matrices of sizes 100x100. In all cases, *p* takes the value of 33 except the Bratislava region (BA), where *p* is equal to 29 because the total number of nodes of this region is only 87.

Table III Time consumption of the algorithm BBDual in seconds [s]

Net	***p***	**without demands**			**with demands**		
		real	**def=5**	**def=10**	**real**	**def=5**	**def=10**
BA	29	0.040	0.023	0.054	0.009	0.005	0.004
BB	33	0.051	0.022	0.046	0.009	0.007	0.007
KE	33	0.040	0.043	0.023	0.007	0.006	0.005
NR	33	0.060	0.038	0.027	0.007	0.006	0.007
PO	33	0.044	0.026	0.031	0.012	0.006	0.006
TN	33	0.052	0.031	0.146	0.012	0.011	0.006
TT	33	0.154	0.039	0.111	0.011	0.007	0.005
ZA	33	0.077	0.037	0.018	0.006	0.005	0.005

The demands of the customer also influence the solution of the task. The term "customer" can represent a node of the network – dwelling place (town, village). In another case, we consider the customer aseach resident of the village. We use both approaches in our research. When the cost matrix contains of the distances among the nodes of network, we talk about the distance matrix ($c_{ij}=d_{ij}$). We solve the problem with demands, when each customer is an inhabitant. In this case we talk about the cost matrix. We calculate the elements c_{ij} of the cost matrix as product of the shortest distance between the nodes $i\in I$and $j\in J$ and the number of inhabitants b_j in the node $j\in J(c_{ij}=b_j.d_{ij})$.The real number of inhabitants was divided by 100 for each node $j\in J$. We always apply the deformation of the network (the rounding of the values) to the distances d_{ij}.

The table IV shows the objective values for the same set of networks as it was used in the table III (matrices of sizes 100x100, p=33, BA with matrix 87x87 and *p*=29).

Table IV Objective values

Net	*p*	without demands			with demands		
		real	def=5	def=10	real	def=5	def=10
BA	29	227	274	313	4504	4581	4968
BB	33	472	485	593	9051	9148	9220
KE	33	365	402	425	9498	10007	10600
NR	33	420	505	568	9543	9734	10220
PO	33	393	427	574	9103	9147	9165
TN	33	320	355	451	6671	7610	7561
TT	33	360	378	523	7334	7602	7862
ZA	33	422	428	441	10363	10541	10901

There are results for *p*=50 in the table V. The set of the networks was the same as in the table III. Bratislava region (BA) has only 87 nodes, therefore the *p*-median problem was solved on the matrix of the size 87x87 and *p*=43.

Table V Time consumption of the algorithm BBDual in seconds [s]

Net	*p*	without demands			with demands		
		real	def=5	def=10	real	def=5	def=10
BA	43	0.050	0.031	0.064	0.019	0.015	0.015
BB	50	0.064	0.026	0.041	0.008	0.007	0.003
KE	50	0.041	0.042	0.034	0.006	0.007	0.005
NR	50	0.058	0.035	0.020	0.010	0.007	0.008
PO	50	0.044	0.025	0.022	0.007	0.007	0.009
TN	50	0.052	0.030	0.086	0.011	0.008	0.006
TT	50	0.053	0.034	0.074	0.010	0.008	0.008
ZA	50	0.074	0.033	0.022	0.007	0.003	0.006

The objective values for the same set of networks are displayed in the table VI.

Table VI Objective values

Net	*p*	without demands			with demands		
		real	def=5	def=10	real	def=5	def=10
BA	43	149	196	313	2045	2226	2688
BB	50	315	302	383	4476	4520	4912
KE	50	235	295	386	4960	5125	5422
NR	50	293	284	353	5120	5187	5877
PO	50	241	275	339	4574	4752	5185
TN	50	215	284	358	3479	3570	3884
TT	50	242	336	375	4017	4076	5059
ZA	50	246	264	303	5500	5696	6489

Fig. 4 shows the differences of the objective function in percent for both deformations (the distances are rounded up to the values that are divisible by 5 and by 100) and for networks with the demands of the customers.Parameter *p* takes the value of one half of the candidates for the location.

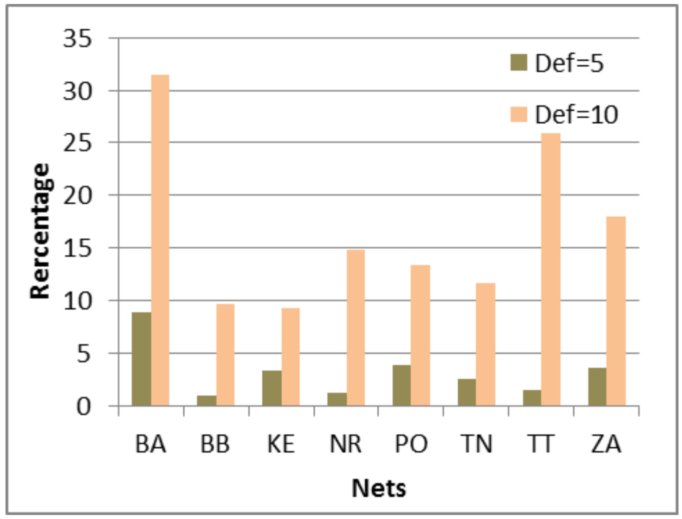

Fig.4 Objective values differences in [%]

We evaluated the Hamming distances from the vectors of the locations between problems solved on real network and problems solved on networks after the deformation. The relative frequenciesof the deviationsfor the same networks as it was presented in table VI (with demands) and Fig. 4are shown on Fig. 5.

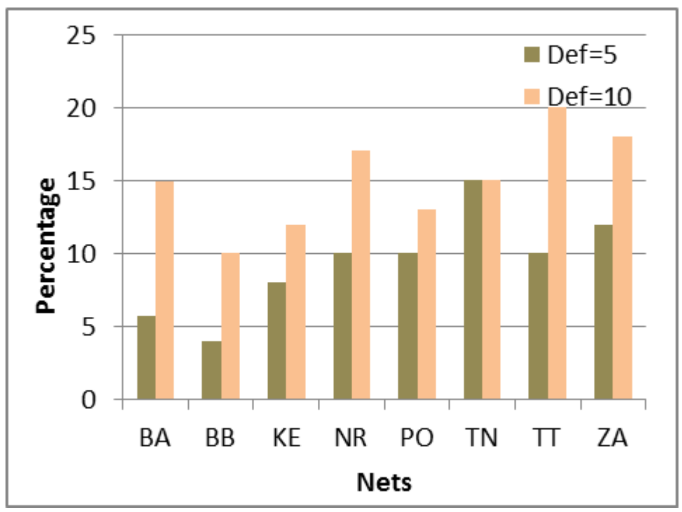

Fig.5 Hemming distances

The next set of the network that was used in our experiments contains the road networks of the regions of Slovakia and the road network of the whole Slovak republic. The numbers of the nodes of these networks are higher. The numbers of the candidates are also higher except the network of whole Slovakia and they differ from each other. Therefore, we always chose the parameter *p*in the same ratio to the number of the candidates for the location. This is why the values of *p* are not the same in the next tasks.

The comparison of the computational time is shown in the table VII. The networks of the regions are given by the real matrices and the deformed matrices (the distances are rounded up to the values that are divisible by 5).Parameter p is equal to the one third of the candidates for the location.

Table VII Computation time of the algorithm BBDual in seconds [s]

Net	m number of candidates	$p=m/3$	without demands		with demands		
			real	def=5	real	def=5	def=10
BA	87	29	0.04	0.02	0.01	0.01	0.00
BB	515	171	48.50	165.49	1.40	0.72	0.50
KE	460	153	812.58		0.96	0.71	0.28
NR	350	116	76.94		0.76	0.79	0.29
PO	640	221	1075.09		7.76	3.94	1.29
TN	276	92	0.92	0.95	0.16	0.11	0.05
TT	249	83	5.61	37.60	0.13	0.06	0.08
ZA	315	105	3.43	7.89	0.29	0.11	0.09
SR	79	26	7.53	4.15	0.88	0.74	0.64

The computation times of the p-median problem differ from each other in this set of tasks for the particular regions. The computation of some tasks on real (not deformed) networks without demands of the customers took more than 10 minutes. When the networks were deformed and no demands of the customers were added, the computation in some cases did not end even after 12 hours and the computation was interrupted (blank cells in the table VII).

The computational times have no significant differences, when the distance matrices of the particular problems were multiplied by the demands of the customers ($c_{ij}=b_j.d_{ij}$ for $i\in I$ and$j\in J$).Therefore, we will introduce only the results of the problems, where cost matrices ($c_{ij}=b_j.d_{ij}$ for $i\in I$ and$j\in J$) were used in our experiment instead of distance matrices ($c_{ij}=d_{ij}$ for $i\in I$ and$j\in J$).

The table VIII shows the objective values for the networks of the regions of Slovakia and of whole Slovakia, where cost matrices are used and where parameter p is equal to the one third of the candidates for the locations.

Table VIII Objective values

Net	m number of candidates	$p=m/3$	Onjective		
			real	def=5	def=10
BA	87	29	4504	4581	4968
BB	515	171	4849	5188	5615
KE	460	153	5675	6110	7491
NR	350	116	6768	7320	8244
PO	640	221	5555	6002	7864
TN	276	92	4113	4648	4859
TT	249	83	5581	5845	6734
ZA	315	105	5674	6222	6969
SR	79	26	877874	877874	885356

Because of individual regions are of unequal size, we calculated the relative differences between the values after deformation of the matrix compared to the values of the real matrix. Fig. 6 shows the differences of the objective values in percent for both deformations (the distances are rounded up to the values that are divisible by 5 and by 100).

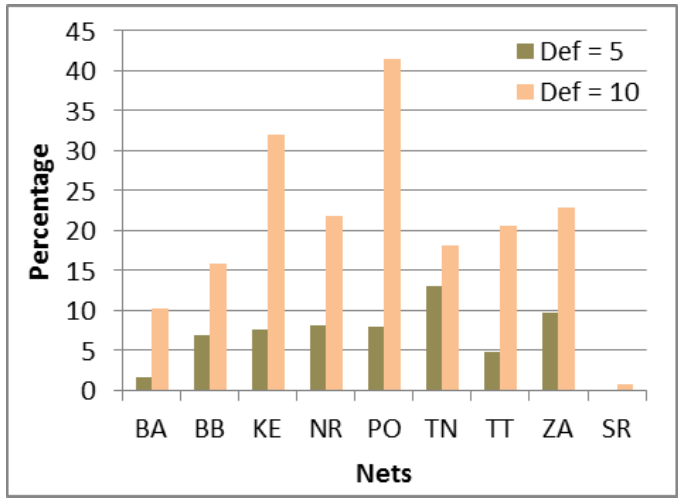

Fig.6 Objective values differences in [%]

We evaluated the Hamming distances from the vectors of the locations between problems solved on real network and problems solved on networks after the deformation. The relative frequenciesof the deviationsfor the same networks as it was presented in table VII and Fig. 6are shown on Fig. 7.

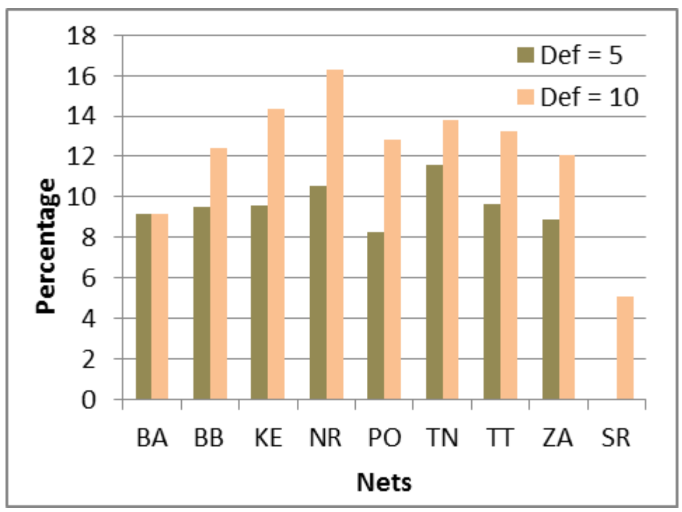

Fig.7 Hamming distances

VI. Conclusion

We study two problems in this paper. At first, it is the influence of the frequency of the values in a distance matrix on the computation time of the solving the p-median problem by the specific algorithm.

Although they were some deflections of the frequency valuesin the networks with real matrices, we noticed no differences of the computation times. The computation times several times grew when the algorithm *pBBDual* solved the p-median problem on the generated matrices. It was confirmed that the algorithm is not suitable for solving the tasks on artificial networks for its specific properties.

The second problem that we study in this paperis the influence of the real network deformation on both the computation time and final solution of the task solved by specific algorithm.

The results of the task solved on deformed networks are not uniform. The computation time of the tasks with the sets of candidates of numbers from 87 to 100 were not changed after the deformation. Some differences were only in time between the tasks with demands and tasks without demands of the customers (computation times grew up about 10 times). The computation times took some milliseconds that are why the differences are negligible.

There were differences in the set of tasks with the matrices of higher sizes. The computation times grew up to 2-15 minutes for three tested regions whenwe tested the networks with real distance matrices without the demands of the customers. In these three cases, the deformations were strongly manifested. We knew no results even after 12 hours, so we stopped the solving process. On the contrary, when we solved the tasks of higher sizes and with demands of the customers, no problems with the solving process occurred and no significant differences arose.

The real distance matrices contained approximately 500 different values. The number of different values was reduced to less than 100 after the first deformation (the distances were rounded up to the values that are divisible by 5) and itwas reduced to less than 50 after the second one (the distances were rounded up to the values that are divisible by 10). The tested matrices contained again a high number of different values after multiplying the distance matrices by the demands of the customers and the deformations of the networks were negated. Nevertheless, differences in the objective values arose in all tested tasks. The higherthe differences of the computation time were the higher the differences of the objective valueswere.Differences of the objective functions grew with the growth of the deformation.

The results of our experiment show that even a small deformation of the real network has an impact on the changes of the resultant service centers locations. The differences go up with the enlarging of the number of the candidates.

REFERENCES

[1] B Bertsimas, D., Farias, V.F., Trichakis N. 2011. The price of fairness, Operations Research, 2011, Vol. 59, No. 1, pp. 17-31.

[2] Current, J., Daskin, M., Schilling, D., Discrete network location models, in Facility location - Applications and theory (Z. Drezner ed.), Berlin, Springer, 2002, pp. 81-118.

[3] Doerner, K., Gutjahr, W., Hartl, R., Karral, Reimann, M., Heuristic solution of an extended double-coverage location problem for Austria, Central European Journal of Operations Research, No. 13, 2005, pp. 325-340.

[4] Erlenkotter, D. (1978): A Dual-BasedProcedureforUncapacitatedFacilityLocation,*OperationsResearch*, Vol. 26, No. 6, pp 992-1009

[5] García, S., Labbé, M., and Marín, A., Solving large p-median problems with a radius formulation, INFORMS Journal on Computing 23 (4), 2011, 546-556.

[6] Ingolfsson, A., Budge, S., Erkut, E., Optimal ambulance location with random delays and travel times, Health Care management science, Vol. 11, No. 3, 2008, pp. 262-274.

[7] Janáček, J., The Medical Emergency Service System Design. In: . Mikulski, J. Advances in Transport Systems Telematics, Katowice, 2006, pp443-449.

[8] Janáček, J.,Approximative covering models for emergency system design. In: Proceedings of the Conference “Quantitative Methods in Economics” , TataranskaLomnica, 2008

[9] Janáček, J., Approximate Covering Models of Location Problems, in Lecture Notes in Management Science - Proceedings of the 1st International Conference ICAOR ´08, Vol. 1, Sept. 2008, Yerevan, Armenia, pp.53-61.

[10] Janáček, J., Modeling of utility in public service system design problem, in Proceedings of the workshop “Optimization of region covering by public service systems”, (in Czech) Pardubice, April 26, 2013, pp. 50-59

[11] Janáček, J., Emergency Public Service System Design Using IP-Solver, in Proceedings of the 1st WSEAS International Conference on Optimization Techniques in Engineering (OTENG’13), Antalya 2013, pp. 97-101

[12] Janacek, J., Buzna, L., Facility location in distribution systems, EDIS-Zilina University Publisher, 2007

[13] Janacek, J., Buzna, L., An acceleration of Erlenkotter-Körkel'salgoriths for uncapaci-tated facility location problem, *Annals of Operations Research*, Vol. 164, pp 97-109

[14] Janáček, J., Kvet, M., Approximate solving of large p-median problems, in ORP3-Operational research peripatetic post-graduate programme, Cádiz, Spain, September 13-17, 2011.

[15] Janacek, J., Kvet, M., Sequential zone adjustment for approximate solving of large *p*-median problems, *Operations Research Proceedings 2011, (OR 2011),* Zurich, Switzerland, Springer-Verlag, 2011, pp. 269-274

[16] Janáčková, M., Szendreyová, A:Animpactofsomeparameters on correctnessofthelocationproblemsolution. In.: Transport Problems. InternationalScientificJournal, WydawnictwoPolitechnikiSlaskiej, Gliwice 2008, p 39-45, ISSN 1896-0596.

[17] Janáčková, M., Szendreyová, A:Territorydecompositionparametersofdistributiontasks. Karviná – ČR, sept. 2012, s.373-379, ISBN 978-80-7248-779-0.

[18] Janáčková, M., Szendreyová, A., An Influence of the Lengths Distribution in the Distance Matrix on the Efficiency of the Exact p-Median Algorithm, in Proceedings of the 1st WSEAS International Conference on Optimization Techniques in Engineering (OTENG’13), Antalya 2013, pp. 97-101

[19] Jánošíková, L., Emergency Medical Service Planning, Communications – Scientific Letters of the University of Žilina, Vol. 9, No. 2, 2007, pp. 64-68.

[20] Jiann-Horng, L., Li-Ren, H., Chaotic bee swarn optimization algorithm for path planning of mobile robots, in Proceedings of the 10th WSEAS international conference on evolutionary computing, 2009, pp. 84-89

[21] Marianov, V., Serra, D., Location problems in the public sector, in Facility location - Applications and theory (Z. Drezner ed.), Berlin, Springer, 2002, pp 119-150.

[22] Ozdamar, L., A dual sequence simulated annealing algorithm for constrained optimization, in Proceedings of the 10th WSEAS International Conference on applied mathematics, 2006, pp. 557-564

[23] Tuba, M., Subotic, M., Stanarevic, T., Modified cuckoo search algorithm for unconstrained optimization problems. in Proceedings of the European Computing Conference (ECC ’11), Paris, France April 28-30, 2011, Published by WSEAS Press, pp 263-268

[24] Tuba, M., Bacanin, N., Stanarevic, T., Guided Artificial Bee Colony Algorithm, in Proceedings of the European Computing Conference (ECC ’11), Paris, France April 28-30, 2011, Published by WSEAS Press, pp 398-404

[25] Um, J. At AL: Scaling laws between ppulation and facility densities. In: Proceedings of the National Academy of Sciences of the USA, Vol. 106,No 34, p. 14236-14240, 2009

[26] Xpress-MP Manual “Getting Started”. Dash Associates, Blisworth, 2005, p. 105

[27] Xpress-Mosel “User guide”. Dash Associates, Blisworth, 2005, p. 99