

Multidimensional Systems Optimization Developed from Perfect Torus Groups

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Abstract— This paper concerns the innovative techniques for improving the quality indices of two- or multidimensional systems with optimizing arranged of structural elements in spatially or temporally distributed systems (e.g. vector data coding of signals) with respect to system capabilities, transmission speed, and data redundancy. Novel design based on remarkable properties of proposed combinatorial structures, namely the concept of Ideal Vector Rings (IVR)s, which can be used for finding optimal solutions for wide classes of technological problems is proposed. These design techniques make it possible to configure multidimensional systems with fewer components than at present, while maintaining or improving on information reliability, resolving ability, and the other significant operating characteristics of the system. In particular, these results are useful for synthesis of non-uniformly spaced thinned antenna arrays with low level of side lobes. This work relates to development of new directions in fundamental and applied research in systems engineering based on idea of the Ideal Vector Rings. Identification of proposed structures with standard combinatorial configurations such as cyclic difference sets and cyclic groups is given.

Keywords— Ideal Vector Ring, Glory to Ukraine Star, perfect torus group, non-redundant vector monolithic code.

I. INTRODUCTION

General problem of distributed system theory relates to finding the optimal placement of structural elements (or components) in spatially or temporally distributed systems, for example positioning of elements in an antenna array in order to avoid interference of signal components of the same spatial frequency. The classical theory of the combinatorial configuration arise in many problems of pure mathematics, in algebraic number theory and topology, including the appropriate algebraic constructions based on cyclic groups in extensions of Galois fields, cyclic difference sets and projective geometry [1]. Combinatorial theory has many applications not only in pure mathematics but also in systems engineering, experimental design optimization and computer science. Combinatorial optimization is one of the most acceptable in systems theory, and other engineering areas.

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II. OPTIMUM ORDERED COMBINATORIAL SEQUENCES

A. Optimum Chain Ordered Sequences

The "ordered chain" approach to the study of elements and events is known to be of widespread applicability, and has been extremely effective when applied to the problem of finding the optimum ordered arrangement of structural elements in a distributed technological systems.

Let us calculate all S_n sums of the terms in the numerical n-stage chain sequence of distinct positive integers $C_n = \{k_1, k_2, \dots, k_n\}$, where we require all terms in each sum to be consecutive elements of the sequence. Clearly the maximum such sum is the sum $S_c = k_1 + k_2 + \dots + k_n$ of all n elements; and the maximum number of *distinct* sums is

$$S_n = 1 + 2 + \dots + n = n(n+1)/2 \quad (1)$$

If we regard the chain sequence C_n as being *cyclic*, so that k_n is followed by k_1 , we call this a *ring sequence*. A sum of consecutive terms in the ring sequence can have any of the n terms as its starting point, and can be of any length (number of terms) from 1 to $n-1$. In addition, there is the sum S_n of all n terms, which is the same independent of the starting point. Hence the maximum number of distinct sums S_{max} of consecutive terms of the ring sequence is given by

$$S_{max} = n(n-1) + 1 \quad (2)$$

Comparing the equations (1) and (2), we see that the number of sums S_{max} for consecutive terms in the ring topology is nearly double the number of sums S_n in the daisy-chain topology, for the same sequence C_n of n terms.

B. Ideal Numerical Rings

An n-stage ring sequence $C_n = \{k_1, k_2, \dots, k_n\}$ of natural numbers for which the set of all $S = S_{max}$ circular sums consists of the numbers from 1 to $S_{max} = n(n-1) + 1$ (each number occurring exactly once) is called an "Ideal Numerical Ring" [2]. Here is an example of an Ideal Numerical Ring with $n = 4$ and $S_{max} = n(n-1) + 1 = 4(4-1) + 1 = 13$, namely $\{1, 2, 6, 4\}$ (Fig.1).

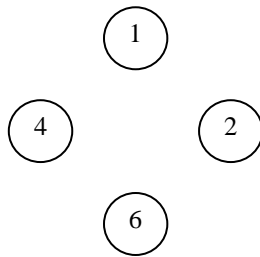


Fig.1. Ideal Numerical Ring {1,2,6,4} with $n=4$.

To see this, we observe:

$$\begin{array}{lll}
 1 = 1 & 5 = 4+1 & 9 = 1+2+6 \\
 2 = 2 & 6 = 6 & 10 = 6+4 \\
 3 = 1+2 & 7 = 4+1+2 & 11 = 6+4+1 \\
 4 = 4 & 8 = 2+6 & 12 = 2+6+4 \quad 13 = 1+2+6+4
 \end{array}$$

Note that if we allow summing over more than one complete revolution around the ring, we can obtain all positive integers as such sums. Thus:

$$14 = 1+2+6+4+1, \quad 15 = 1+2+6+4+1+2, \text{ etc.}$$

A more general type of Ideal Numerical Ring generates the S_{max} circular sums which give each integer value from 1 to N , for some integer N , exactly R times, as well as the value $S_{max} = N+1$ (the sum of all n terms) exactly once.

An example with $n = 4$ and $R=2$, so that $N = 6$, is the ring sequence {1,1,2,3} (Fig.2).

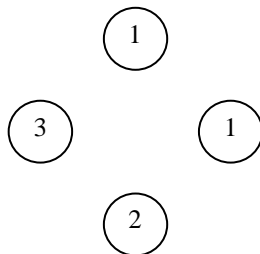


Fig.2. Ideal Numerical Ring {1,1,2,3} with $n=4$ and $R=2$.

For this ring sequence {1,1,2,3} the sums of consecutive terms are:

$$\begin{array}{ll}
 1, \text{ and } 1 & 4 = 3+1, \text{ and } 4=1+1+2 \\
 2, \text{ and } 2 = 1+1 & 5 = 2+3, \text{ and } 5 = 3+1+1 \\
 3, \text{ and } 3 = 1+2 & 6 = 1+2+3, \text{ and } 6 = 2+3+1 \\
 & 7 = 1+1+2+3
 \end{array}$$

We see that each circular sum from 1 to 6 occurs exactly twice ($R=2$). So, this Ideal Numerical Ring has the parameters $n=4, R=2$.

Next, we consider two-dimensional combinatorial constructions with remarkable properties evolved from simpler Ideal Numerical Rings.

C. Two-dimensional Vector Rings

Let us calculate all S sums of the terms in the n -stage ring sequence of non-negative integer 2-stage sub-sequences (2D vectors) of the sequence $C_{n2} = \{(k_{11}, k_{12}), (k_{21}, k_{22}), \dots, (k_{11}, k_{12}), \dots, (k_{n1}, k_{n2})\}$ as being *cyclic*, so that (k_{n1}, k_{n2}) is followed by (k_{11}, k_{12}) , where we require all terms in each *modular 2D vector sum* to be consecutive elements of the *cyclic sequence*, and a modulo sum m_1 of k_{12} and a modulo sum m_2 of k_{22} are taken, respectively. A *modular 2D vector sum* of consecutive terms in this sequence can have any of the n terms as its starting point, and can be of any length (number of terms) from 1 to $n-1$. Hence the maximum number of such sums is given by

$$S = n(n-1) \tag{3}$$

If we require all *modular vector sum* of consecutive terms give us each vector value exactly R times, then

$$S_R = \frac{n(n-1)}{R} \tag{4}$$

Let $n = m_1, n - 1 = m_2$, then a space coordinate grid $m_1 \times m_2$ forms a frame of two modular (close-loop) axes modulo m_1 and modulo m_2 , respectively, over a surface of torus as an orthogonal two modulo cyclic axes of the system being the product of two ($t=2$) circles. We call this two-dimensional Ideal Vector Ring (2D IVR), shortly "Vector Ring".

Example: Let $n=3, m_1=2, m_2=3, R=1$, and complete set of the IVRs takes four variants as follows:

- (a) $\{(0,1),(0,2),(1,2)\};$ (b) $\{(0,1),(0,2),(1,1)\};$ (5)
- (c) $\{(0,1),(0,2),(1,0)\};$ (d) $\{(1,0),(1,1),(1,2)\}.$

To see this, we observe that *ring* sequence $\{(0,1), (0,2), (1,2)\}$ gives the next circular vector sums to be consecutive terms in this sequence:

$$\left. \begin{array}{l}
 (0,1) + (0,2) = (0,0) \\
 (0,2) + (1,0) = (1,2) \\
 (1,2) + (0,1) = (1,0)
 \end{array} \right\} (\text{mod } 2, \text{ mod } 3)$$

So long as the terms $(0,1), (0,2), (1,2)$ of the three-stage ($n=3$) ring sequence themselves are two-dimensional vector sums also, the set of the modular vector sums ($m_1=2, m_2=3$) forms a set of nodal points of annular reference grid over 2×3 exactly once ($R=1$):

$$\begin{array}{lll}
 (0,0) & (0,1) & (0,2) \\
 (1,0) & (1,1) & (1,2)
 \end{array}$$

Schematic model of two-dimensional Vector Ring in torus system of reference is given below (Fig.3) as the simplest and well useful for analytic study of two-dimensional Vector Rings.

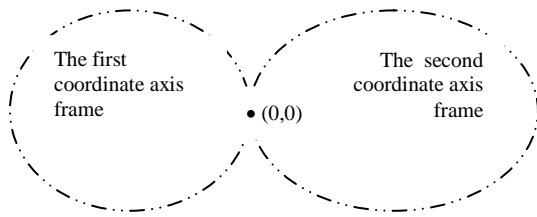


Fig.3. Schematic model of two-dimensional Vector Ring in torus system of coordinates with ground coordinate (0,0).

Easy check to see, that the rest of ring sequences have the principal property of forming reference grid 2x3 over a torus using only three (n=3) two-stage (t=2) terms of these circular sequences.

III .MULTIDIMENSIONAL VECTOR CYCLIC GROUPS

A. Principal Consideration

To discuss concept of Vector Cyclic Groups (VCG)s let us regard structural model of t-dimensional vector ring as ring n-sequence $C_{nt} = \{K_1, K_2, \dots, K_i, \dots, K_n\}$ of t-stage sub-sequences (terms) $K_i = (k_{i1}, k_{i2}, \dots, k_{in})$ each of them to be completed with nonnegative integers (Fig.4).

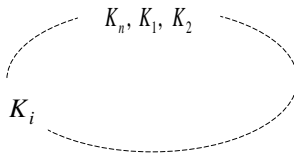


Fig.4. Schematic model of t-dimensional n-stage ring sequence.

Here is an example of 3D Vector Ring with $n= 6, m_1=2, m_2=3, m_3=5,$ and $R=1$ which contains circular 6-stage sequence of 3-stage (t=3) sub-sequences $\{K_1, \dots, K_6\}$: $K_1 \Rightarrow (k_{11}, k_{21}, k_{31}) = (0,2,3); K_2 \Rightarrow (k_{12}, k_{22}, k_{32}) = (1,1,2); K_3 \Rightarrow (k_{13}, k_{23}, k_{33}) = (0,2,2); K_4 \Rightarrow (k_{14}, k_{24}, k_{34}) = (1,0,3); K_5 \Rightarrow (k_{15}, k_{25}, k_{35}) = (1,1,1); K_6 \Rightarrow (k_{16}, k_{26}, k_{36}) = (0,1,0)$. The set of all circular sums over the 6-stage sequence, taking 3-tuple (t=3) modulo (2,3,5) gives the next result:

$$\begin{aligned} (0,0,1) &= ((0,2,2) + (1,0,3) + (1,1,1)), \\ (0,0,2) &= ((1,1,2) + (0,2,2) + (1,0,3)), \\ (0,0,3) &= ((0,2,3) + (0,1,0)), \\ (0,0,4) &= ((0,2,2) + (1,0,3) + (1,1,1) + (0,1,0) + (0,2,3)), \\ (0,1,1) &= ((0,2,2) + (1,0,3) + (1,1,1) + (0,1,0)), \\ (0,1,2) &= ((1,0,3) + (1,1,1) + (0,1,0) + (0,2,3)), \\ (0,1,3) &= ((1,1,1) + (0,1,0) + (0,2,3) + (1,1,2) + (0,2,2)), \\ (0,1,4) &= ((0,1,3) + (1,1,1)), \\ (0,2,0) &= ((0,2,3) + (1,1,2) + (0,2,2) + (1,0,3)), \end{aligned}$$

$$(0,2,1) = ((1,1,1) + (0,1,0) + (0,2,3) + (1,1,2)),$$

$$(1,2,4) = ((0,2,3) + (1,1,2) + (1,1,1) + (1,0,3) + (0,1,0)).$$

Easy to see this verify of the next conditions:

$$\prod_1^t m_i = \frac{n(n-1)}{R}, \text{ or } \prod_1^t m_i = \frac{n(n-1)}{R} + 1$$

$$(m_1, m_2, \dots, m_t) = 1 \tag{6}$$

Here $n, R,$ and m_1, m_2, \dots, m_t are numerical parameters of a t-dimensional Ideal Ring [3].

Remarkable properties of 3D Vector Rings can be used for improving the quality indices of optic or acoustic systems with non-uniform structure (e.g. overlapping masks utilizing the entire ultra-acoustic aperture) with respect to resolving ability due to avoid the interference of signal components of the same spatial frequency [4-6].

B. Characteristics of the Ideal Rings

The data on the complete sets of t-dimensional IRs for $n \leq 7, R=1, t=1,2,3$ are given in Table 1.

TABLE I. CHARACTERISTICS OF IDEAL RINGS

n	Cardinal number of Ideal Rings			Grids $m_1 \times m_2$	Grids $m_1 \times m_2 \times m_3$
	ID	2D	3D		
2	1	1	-	1x2	-
3	1	4	-	2x3	-
4	2	24	-	3x4	-
5	1	272	-	4x5, 3x7	-
6	5	256	128	5x6, 3x10	2x3x5
7	0	360	180	6x7, 3x14	2x3x7

One can see that the numbers of tabled two- and three-dimensional IRs are increasing with rising number of terms. Besides, dimensionality and two-dimensional IRs outnumber in part analogous to cyclic difference sets heavily.

C. Vector Ring Sequences as Cyclic Groups

Next, we consider a set of Vector Rings with $n=3, m_1=2, m_2=3, R=1$ as a cyclic multiplicative group of a finite field. With this aim let us multiply the VR $\{(0,1),(0,2),(1,2)\}$ (5,a) trough by 2D (t=2) coefficient (1,2) taking both (mod 2), and (mod 3) as follows: $(0,1) \cdot (1,2) \Rightarrow (0,2), (0,2) \cdot (1,2) \Rightarrow (0,1), (1,2) \cdot (1,2) \Rightarrow (1,1)$. As a result of this transformation we got circular sequence $\{(0,2),(0,1),(1,1)\}$ that differs from the previous (5,a) but it is the same as the reverse sequence (5,b), while multiplying circular sequences (5,b) or (5,c) through by (1,2) no transform them to others variants of the sequences, only

reflection and cyclic shifting we can see here. Hence, the complete family of four VRs with $n=3$, $m_1=2$, $m_2=3$, and $R=1$ contains both two isomorphic, and two non-isomorphic variants of the sequences, each of them makes it possible to cover the set of nodal points over torus grid 2×3 exactly once ($R=1$) using only three ($n=3$) basic vectors for configure optimum specify coordinates with respect to torus surface frame of reference. A new type of cyclic groups is among the most perfect Vector Rings which properties hold for the same set of the VRs in varieties permutations of terms in the set, e.g. set of two-dimensional ($t=2$) Vector Rings $\{(1,0), (1,1), (1,2), (1,3), (1,4)\}$ and $\{(1,0), (1,2), (1,4), (1,1), (1,3)\}$. We call this family of Vector Rings the “Glory to Ukraine Star” family. We have found numerous families of the Glory to Ukraine Stars vector cyclic group.

D. Definitions of Vector Cyclic Group families

a) *Vector Cyclic Group* is a multiplicative group of integer vectors lengths m_1, m_2, \dots, m_t that form a group under t -dimensional multiplication.

b) *Perfect Torus Cyclic Group* is a Vector Cyclic Group of n -stage Vector Rings such that each of them covers the set of nodal points of reference grid $m_1 \times \dots \times m_t$ over torus exactly R -times, $m_1 \cdot m_2 \dots \cdot m_t = n(n-1)/R$ or $m_1 \cdot m_2 \dots \cdot m_t = n(n-1)/R+1$, $m_1 \cdot m_2 \dots \cdot m_t = n(n-1)/R$.

c) *Glory to Ukraine Cyclic Group* is Perfect Torus Cyclic Group such that its vector elements are the same for all family of the group.

Here is the simplest and well useful for analytic study and applications of the underlying properties of Torus Groups for development of new mathematical, physical and technological results.

IV. OPTIMUM MONOLITHIC VECTOR CODES

A. Useful Properties of Optimum Vector Codes

The remarkable properties of Vector Ring Sequences are that all ring sums of vectors in the sequence exhaust the set of vectors of a finite modular vector space by R ways exactly, which allows on binary encoding of two- and multidimensional vectors as sequences of the same signals or characters in ring code combination length. This makes it possible to use *a priori* maximal number of combinatorial varieties of ring sums for coded design of signals (6). As an example it is chosen the “Glory to Ukraine Star” $\{(1,0), (1,1), (1,2), (1,3), (1,4)\}$ with $n=5$, $m_1=4$, $m_2=5$, $R=1$. Here digit weight of the first position is vector value (1,0), the next – (1,1), ..., and (1,4). Result of the 2D vector code design is given as a set ring sums modulo $m_1=4$, and $m_2=5$ respectively for consecutive vector values of the Star (Table 2).

TABLE 2.

2D vector data code based on “Glory to Ukraine Star” group $\{(1,0), (1,1), (1,2), (1,3), (1,4)\}$ with $n=5$, $m_1=4$, $m_2=5$, $R=1$

Vector Code	Digit weights				
	(1,0)	(1,1)	(1,2)	(1,3)	(1,4)
(0,0)	0	1	1	1	1
(0,1)	1	1	1	1	0
(0,2)	1	1	1	0	1
(0,3)	1	1	0	1	1
(0,4)	1	0	1	1	1
(1,0)	1	0	0	0	0
(1,1)	0	1	0	0	0
(1,2)	0	0	1	0	0
(1,3)	0	0	0	1	0
(1,4)	0	0	0	0	1
(2,0)	0	0	1	1	0
(2,1)	1	1	0	0	0
(2,2)	0	0	0	1	1
(2,3)	0	1	1	0	0
(2,4)	1	0	0	0	1
(3,0)	1	1	0	0	1
(3,1)	0	1	1	1	0
(3,2)	1	0	0	1	1
(3,3)	1	1	1	0	0
(3,4)	0	0	1	1	1

We can see that 2D vector sequence $\{(1,0), (1,1), (1,2), (1,3), (1,4)\}$ forms complete set of ring code combinations on 2D ignorable array 4×5 , and each of its occurs exactly once ($R=1$). So, we have non-redundant code, that forms massive arranged (solid parts of bits) both symbols “1” and of course “0” for each code combinations as being circular. This property makes such vector ring codes useful in applications for high performance vector computer systems with improving of noise immunity, and advanced coded design of signals [3,5-7,9].

B. Definitions of the Ring Monolithic Vector Codes

a) *Ring Monolithic Code* is a set of ring sequence code combinations which the same characters arranged all together into the code combinations.

b) *Numerical Optimum Ring Code* is weighted binary Ring Monolithic Code which ring n - sequence of positive integer digit weights forms a set of binary n -digital code combinations of a finite interval $[1,S]$, the sums of connected digit weights taken modulo $S=n(n-1)/R$ enumerate the set of integers $[1,S]$ exactly R -times.

c) *Two-dimensional Optimum Ring Code* is weighed binary Ring Monolithic Code which set of connected 2-stage modular sums taken modulo m_1 and m_2 , respectively, allows an enumeration of nodal points of reference grid $m_1 \times m_2$ over torus exactly R -times with respect to torus surface frame of axes, $m_1 \cdot m_2 = n(n-1)/R$.

d) *Multidimensional Optimum Ring Code* is weighed binary Ring Monolithic Code which set of connected t -stage modular sums taken modulo m_1, m_2, \dots, m_t , respectively, allows an enumeration of nodal points of reference grid $m_1 \times \dots \times m_t$ over hypertorus exactly R -times, $m_1 \cdot m_2 \dots \cdot m_t = n(n-1)/R$.

e) *Glory to Ukraine Code* is t -dimensional Optimum Ring Monolithic Code that forms under Perfect Torus Cyclic Group such that its vector elements are the same for all family of the group.

V. CONCLUSION

Concept of the systems optimizations provides, essentially, a new model of technical systems. Moreover, the optimization has been embedded in the underlying combinatorial models. The favorable qualities of the Ideal Vector Rings provide breakthrough opportunities to apply them to numerous branches of science and advanced technology, with direct applications to optical, and acoustic engineering [4-6], radio-engineering and computer sciences [7-9], vector data coding and telecommunications [3, 8-10], information technology and control [10-11]. Structural perfection and harmony has been embedded not only in the abstract models but in the real world also [12].

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Current research interests: The scientific basis of multidimensional optimum distributed systems theory, including the appropriate algebraic structures based on cyclic groups in extensions of Galois fields, and the generalization of these methods and results to optimization of a larger class of technological systems; development of fundamental and applied research in systems engineering for improving such quality indices as reliability, precision, speed, resolving ability, and functionality, using innovative methodologies based on combinatorial techniques; better understanding of the fundamental role of summery and asymmetry relationships in the worldwide harmony laws. Previous research interests: Design and engineering of an improved devices and process engineering for industrial automation of power stations.

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