Computational Method for Reliability Analysis of Complex Systems Based on the One Class Markov Models

Igor Kabashkin

Abstract**—**Markov analysis is a powerful modelling and analysis technique with strong applications in time-based reliability and availability analysis. The reliability behavior of a system is represented using a state-transition diagram, which consists of a set of discrete states that the system can be in, and defines the speed at which transitions between those states take place. Markov models consist of comprehensive representations of possible chains of events, i.e. transitions within systems which, in the case of reliability and availability analysis, correspond to sequences of failures and repair. The paper describes specific computational approach to reliability analysis of complex systems, which behavior is described by the Markov chain finite-state transition diagram which contains two no crossing sets of arbitrary configuration states, transitions between which is possible only through an one intermediate state. The method of calculation of stationary probabilities of states of the original system includes it decomposition into two separate subsystems and calculation of stationary probabilities of the original model from the known values of stationary probabilities of subsystems using the proposed transitional equations.

Keywords**—**Markov chain model, reliability analysis, state transition diagram.

I. Introduction

Reliability of complex systems often is described by the homogeneous Markov process [1]. In general case the precise equations for the definition of the reliability indexes of such systems may be quite complex.

Therefore, for the practice it is interesting to use applications which simplify case studies. Different methods designed to simplify the description of different models on the base of discrete-time Markov chain: failure biasing [2], [3]; selective failure biasing [4]; distance-based selected failure biasing [5] and others.

This work presents the simple method of the computing of the state's stationary probabilities for the systems with two nonoverlapping subsets of states π_1 and π_2 when the transitions between them are possible only via a certain intermediate P_n state.

This work was supported in part by the project of the Latvian state research program "The Next Generation of Information and Communication Technologies (Next IT)" (2014-2017).

Igor Kabashkin is with the Transport and Telecommunication Institute, 1 Lomonosova iela, Riga, LV-1019, Latvia (phone: +371-29215392; e-mail: kiv@tsi.lv).

For example, to this class of systems it is possible to put technical objects with the computer-aided diagnostics where the regime of the operation and control do no coincide in time as well as the wide class of the systems with the preventive maintenance when the equipment is switched off. In the mentioned systems state P_n presents the initial state of normal operation, multitude π_1 describes the process of systems operation in the working mode, and, the multitude π_2 characterizes the behavior of the system in the mode of diagnostics or during the technical maintenance.

Another example of Markov models with two nonoverlapping subsets of states is complex systems with redundancy. In this class of models the multitude π_1 describes the failure process of main unit of redundant system; state P_n presents the transition state of work from main to standby unit; the multitude π_2 characterizes the behavior of the system in the standby mode.

II. Formulation of the problem

The behavior of the complex system is described by the Markov Chain set of states transition diagram (Fig.1):

$$\pi = \pi_1 \cup \{P_n\} \cup \pi_2$$

where $\pi_1 = \{P_i : i = 1, \dots, n-1\}$, $\pi_2 = \{P_j : j = n+1, \dots, m\}$.

The set of states π includes the aggregate of nonoverlapping subsets π_1 and π_2, each of them is ensemble of states are connected among themselves and with P_n by the derivative way, and the direct connections between two subsets π_1 and π_2 are absent.

Let us make a decomposition of the initial Markov Chain set of states into the two independent subsets $\{H_i : i = 1, \dots, n\}$ and $\{R_j : j = n, \dots, m\}$. The graph of the first states transition diagram $\{H_i\}$ has identical appearance to the part $\pi_1 \cup \{P_n\}$ of the initial Markov Chain set of states, and the graph of the second states transition diagram $\{R_j\}$ has identical appearance to the part $\{P_n\} \cup \pi_2$ of the initial Markov Chain set of states.

Let us solve the problem of determination of the stationary probabilities p_k of states $P_k, k = 1, \dots, m$ of the initial system according to the known probabilities h_i and r_j of the states H_i and R_j of the sub-systems obtained as a result of the decomposition of initial system.

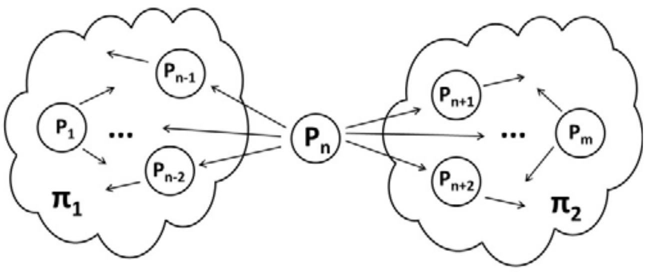

Fig.1. Model of the studied Markov Chain

III. Solution for the System with the Transitive Transition Graph of Markov Process

In the mathematical field of graph theory, a vertex-transitive graph is a graph G such that, given any two vertices v_1 and v_2 of G, there is some automorphism $f: V(G) \rightarrow V(G)$ such that $f(v_1) = v_2$. In other words, a graph is vertex-transitive if its automorphism group acts transitively upon its vertices [6].

For the systems described by the Markov Chain set of states with the transitive transition graph the stationary probabilities of the initial system states are defined by the expressions [7]

$$p_k = \theta_k p_1 \; (k = 1, \dots, m) \,, \tag{1}$$

where

$$\theta_k = \frac{\lambda_{12}\lambda_{23}\cdots\lambda_{k-1,k}}{\mu_{21}\mu_{32}\cdots\mu_{k,k-1}} \tag{2}$$

$\lambda_{k-1,k}$ and $\mu_{k,k-1}$ - the intensity of the direct and reverse transitions from the state P_{k-1} into the state P_k accordingly.

For the stationary probabilities h_k and r_k of the states H_k and R_k of the sub-systems the following equations are valid:

$$h_k = \theta_k h_1 \;\; (k = 1, \dots, n) \tag{3}$$
$$r_k = \theta_k r_1 \;\; (k = n, \dots, m) \tag{4}$$

where θ_k is defined by the equation (2). Comparing (1) with (2) and (3) we obtain

$$\frac{p_k}{p_n} = \begin{cases} \frac{h_k}{h_n} \,, k = 1, \dots, n \\ \frac{r_k}{r_n} \,, k = n, \dots, m \end{cases} \tag{4}$$

Using the received expressions it is possible to write

$$\sum_{k=1}^{m} p_k = p_n \left(\frac{1 - h_n}{h_n} + \frac{1}{r_n} \right) = 1$$

from which

$$p_n = \frac{r_n h_n}{r_n + h_n - r_n h_n} \tag{5}$$

Unknown probabilities p_k can be defined by known probabilities r_k and h_k putting p_n from equation (5) into expressions (4).

IV. Solution for the System with the Arbitrary Transition Graph of Homogeneous Markov Process

Let us consider the transition rate matrix A which is an array of numbers describing the rate a continuous time Markov chain moves between states [8].

After the elimination from matrix A the line and column with number n we will get matrix A_n which has a block type structure

$$A_n = \begin{vmatrix} A' & 0 \\ 0 & A" \end{vmatrix}$$

where A' and $A"$ - infinitesimal matrixes of the transitions of subsystems derived as a result of decomposition with the eliminated elements of the *n*-th line and *n*-th column.

If we denote as $\bar{a}_n$ the *n*-th column of matrix A without the element of the *n*-th line, and as $\bar{p}_n$, $\bar{r}_n$, $\bar{h}_n$ - vectors of the stationary probabilities of the corresponding Markov chains without *n*-th state, it is possible to write the following matrix equations [8]:

$$A_n \bar{p}_n = \bar{a}_n p_n \;\; , \; A' \bar{h}_n = \overline{a'}_n h_n \; , \; A" \bar{r}_n = \overline{a"}_n r_n$$

From these relations we have

$$\bar{p}_n = A_n^{-1} \bar{a}_n p_n, \; \bar{h}_n = (A')^{-1} \overline{a'}_n h_n, \; \bar{r}_n = (A")^{-1} \overline{a"}_n r_n \tag{6}$$

In accordance as

$$A_n^{-1} = \begin{vmatrix} (A')^{-1} & 0 \\ 0 & (A")^{-1} \end{vmatrix}$$

from equations (6) we can obtain the equation (4). At the same time the value of p_n determined by the expression (5).

V. An Illustrative Example

The proposed approach is illustrated with the following example. Given a backup system with repair permitted for either component with a single repair crew and no failures while in standby. A system has two modes of reliability: a degraded mode and failed mode.

A system described by the states: P_1 - system is in fully operational state; P_2 – main system operates in degraded mode; P_3 – failed mode of main system, backup system begins to work, it is in fully operational state; P_4 – backup system operates in degraded mode; P_5 - failed mode of backup system.

The behavior of the examined system is described by the Markov Chain state transition diagram (Fig. 2), where λ_1 is rate of transition to degraded mode, λ_2 – rate of transition from degraded mode to failed mode, λ_3 – rate of transition from fully operational state to failed mode, μ – repair rate.

Let's define the stationary probabilities p_k of states P_k, $k = 1, \dots, 5$.

Steps of solution:

1. The transition graph of the initial system (Fig.2) divided into two independent graphs (Fig.3, Fig.4).

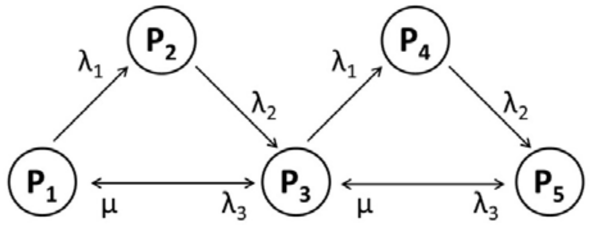

Fig. 2. State transition diagram of the examined system.

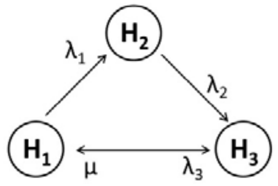

Fig. 3. The first graph obtained after transformation

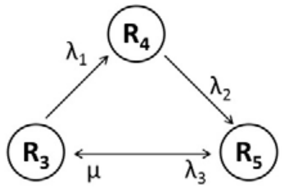

Fig. 2. The second graph obtained after transformation

2. The state probabilities for each graph of the subsystems (Fig.3, Fig.4) are possible to take from [9]:

$$h_1 = r_3 = \gamma \ , h_2 = r_4 = \alpha\gamma \ , h_3 = r_5 = \beta\gamma \ ,$$

where $\alpha = \frac{\lambda_1}{\lambda_2}, \beta = \frac{\lambda_1+\lambda_3}{\mu}, \gamma = (1+\alpha+\beta)^{-1}$.

3. In accordance with (5) we can obtain the equation for p_3:

$$p_3 = \frac{r_3 h_3}{r_3 + h_3 - r_3 h_3}$$

.

4. In accordance with (4) we can obtain the equations for other states $p_i, i = 1,2,4,5$:

$$p_1 = h_1 p_3 / h_3 = \gamma\varphi, p_2 = h_2 p_3 / h_3 = \alpha\gamma\varphi,$$
$$p_4 = h_4 p_3 / h_3 = \alpha\beta\gamma\varphi, p_5 = h_5 p_3 / h_3 = \beta^2\gamma\varphi,$$

where $\varphi = (1+\beta-\beta\gamma)^{-1}$.

VI. Conclusion

The paper describes specific computational approach to reliability analysis of complex systems, which behavior is described by the Markov chain finite-state transition diagram which contains two no crossing sets of arbitrary configuration states, transitions between which is possible only through an one intermediate state.

The method of calculation of stationary probabilities of states of the original system includes it decomposition into two separate subsystems and calculation of stationary probabilities of the original model from the known values of stationary probabilities of subsystems using the proposed transitional equations.

The proposed method is particularly comfortable in case if earlier the system has been already investigated, but with more strong limitations, for example, without factors of maintenance, redundancy or others. In this case in the framework of the submitted method there is a possibility to use the obtained earlier results and, therefore, the simplification of the conducted calculations.

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Igor Kabashkin was born in Riga (Latvia). He is currently Professor in the Department of Electronics and Telecommunication at Transport and Telecommunication Institute, Riga, Latvia. He received Diploma of Engineer (1977) in Avionics from Riga Civil Aviation Engineering Institute, Doctor Degree (1981) in Aviation from Moscow Aviation University, Doctor Habilitatus Degree (1993) in Aviation from Riga Aviation University.

He was lecture and Professor at Riga Aviation University from 1981 to 1999, Vice-Rector for Research and Development Affairs at Riga Aviation University from 1994 to 1999, Professor and President at Transport and Telecommunication Institute from 1999 to 2014. He received the Award from Cabinet of Ministers of the Republic of Latvia in 2014 for the significant contribution to the research and educational work in the transport, logistics and information technology, Award and Memorial Medal from Latvian Academy of Sciences in 2002 for outstanding research in computer science and informatics, Award and Memorial Medal from Latvian Council of Sciences in 2001 for outstanding research in air navigation, communication and radar technologies, Award from the American Center for Advancement of Teaching and Learning in 1998 for Innovative Excellence in Teaching, Learning and Technology, Honorary Title of Latvian Honored Inventor from Latvian Parliament in 1989. His research interests are analysis and modelling of complex systems, intelligent transport systems, logistics, reliability engineering, decision support systems, avionics, and air traffic control systems.