Distributed Backstepping Control With Actuator Delay For Active Suspension System

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***Abstract*—The actuator time delay problem for a linear active suspension system using the theory of backstepping control design is examined in this study. Time-delay may arise in active suspension systems because of transport phenomenons, information processing, sensors or some mechanical reasons. Designing the controller without taking into account the actuator time delay may degrade the performance of the controller or even destabilize the closed loop control system. It is aimed to improve the ride comfort of passengers without degrading road holding. Therefore, a backstepping controller was designed which takes into account the actuator time delay by combining a first order hyperbolic partial differential equation(PDE) with the linear suspension system. The numerical results confirm the success of the controller.**

***Index Terms*—Active suspension system, actuator time delay, distributed backstepping control.**

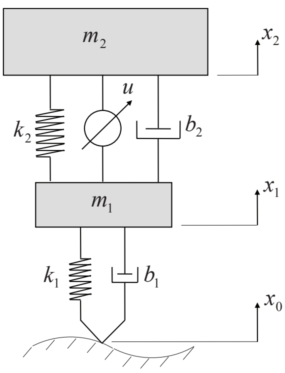

Figure 1. Quarter car with active suspension

I. Introduction

Vehicle suspensions are generally classified as passive, semi-active and active systems. Passive ones are composed of spring and damper elements whereas semi-active ones include variable damping elements such as electrorheological [1] and magnetorheological [2] dampers. In active suspension systems hydraulic, pneumatic actuators or linear electric motors can be placed generally parallel to the suspension elements. Active suspensions provide promising performance for suppression of vehicle body vibrations compared with passive and semi-active ones. Therefore, this research area has remained attractive for many years and various control strategies such as PID [3], fuzzy logic [4], [5], [6], H_∞ [7], sliding mode [8], fuzzy sliding mode [9],[10], backstepping control [11] have been proposed. Most of the previously mentioned studies neglect the time delays during the controller design. However in practice it is not possible to calculate and apply the needed control action to the system without any time delay. Therefore in reality effect of time delay should be taken into account. If not, the performance of the controlled system may degrade or even cause instability of the system. Various approaches have been used in literature for the control of active suspensions with actuator delay. In [12] constrained optimization was used to calculate state feedback gains along with a scheme for stability chart strategy for quarter active suspension system. H_∞ control design have also been proposed for vehicle active suspension system with actuator delay in [13] and [14].

In [15], to obtain a controller for linear time invariant (LTI) systems with desired performance, a boundary backstepping controller is designed by combining a first order hyperbolic partial differential equation (PDE) with LTI system. Mostly, boundary control is used for distributed systems by using backstepping design [16]. In [17] it is shown that this methodology can also be used for the delay systems by solving a coupled LTI-PDE system. With the same methodology of the designing backstepping controller, if a target stable system is chosen for the partial differential system, one can define a controller for the investigated delay system [17] by using the the transformation between the original and the target systems. At the end a controller can be derived as smith predictor. As the main contribution of this study, we have used that distributed backstepping approach presented in [15] for the vibration suppression of a quarter car active suspension system where actuator time delay exists.

II. Vehicle Model

Quarter car active suspension system model, presented in Figure 1, is used in this study. Mathematical model of the system is given by

$$m_1\ddot{x}_1 = -(k_1+k_2)x_1 + k_2x_2 - (b_1+b_2)\dot{x}_1 + b_2\dot{x}_2 + k_1x_0 + b_1\dot{x}_0 - u(t-D), \quad (1)$$

$$m_2\ddot{x}_2 = k_2x_1 - k_2x_2 + b_2\dot{x}_1 - b_2\dot{x}_2 + u(t-D). \quad (2)$$

The model has two degrees of freedom which are body bounce x_2 and displacement of the wheel x_1 that are both in vertical directions. Here, x_0 is the road surface input representing the road surface unevenness. The m_1 and m_2 represent the mass of the wheel-axle assembly and the vehicle main body, respectively; k_1 and b_1 are the stiffness and damping constants of the tire; similarly k_2 and b_2 stand for the stiffness and damping constant of the suspension spring and damper, respectively; $u(t-D)$ is the control signal with time delay D. Numerical values of the vehicle parameters are given in Table I.

Table I
VEHICLE PARAMETERS

$m_1 = 45\ kg$	$m_2 = 320\ kg$	$k_1 = 211180\ N/m$
$k_2 = 27000\ N/m$	$b_1 = 20\ Ns/m$	$b_2 = 935\ Ns/m$

III. CONTROLLER DESIGN

The road input applied to the vehicle suspension is shown in Figure 2. The vehicle travels over that road profile with a constant velocity of 20 m/s. In order to show the effects of the actuator delay on the system performance, the time responses of the vehicle body are presented in Figure 3. State Feedback Control(SFC) was chosen as the first controller since it is one of the basic control methods that stability of the controlled system can be investigated easily. There are two cases for the controlled suspensions namely, the case with (w) time delay and the case without (w/o) time delay. The case without actuator delay may be thought as the desired force u is produced by the actuator immediately, that is without any time delay. On the other hand in reality it is not possible due to actuator dynamics. Therefore, the performance of this controller with time delay, $D = 35ms$ is also presented here. It is seen that the displacements grow up rapidly for the SFC case with actuator delay, that is time delay destabilized the suspension system. Equations of motion of the quarter car suspension model are presented below in vector matrix form.

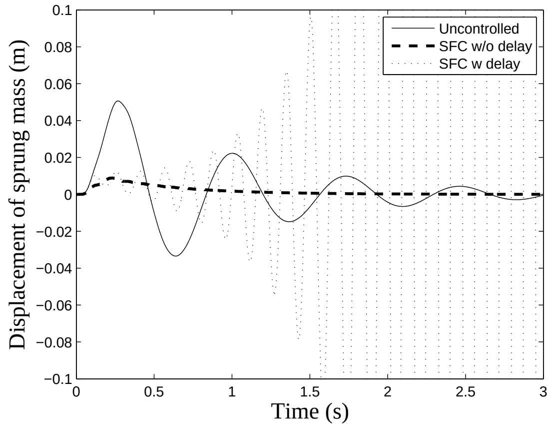

Figure 3. The open and closed-loop response of the quarter-car model with state feedback controller. Displacement of sprung mass with state feedback controller without delay (dashed line); state feedback controller with delay (dotted line); Passive system (solid line).

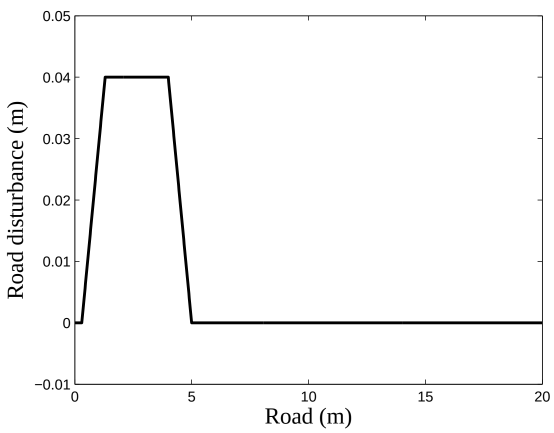

Figure 2. The road disturbance acted on the suspension system.

$$\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}t} = \mathbf{A}\,\mathbf{X} + \mathbf{B}\,\mathbf{U}(t-D) + \mathbf{W}\,\mathbf{X_0}. \quad (3)$$

where $\mathbf{X} = [x_1\, x_2\, x_3\, x_4]$ is the state vector which includes the displacement x_1 and the velocity x_3 of the wheel and similarly displacement x_2 and velocity x_4 of vehicle body. $\mathbf{X_0} = [x_0\ \frac{dx_0}{dt}]^T$ is the road excitation vector, $\mathbf{U}(t-D)$ is the actuator control signal with time delay, D, and related matrices of the state equation of the vehicle model are given by

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_1+k_2)}{m_1} & \frac{k_2}{m_1} & \frac{-(b_1+b_2)}{m_1} & \frac{b_2}{m_1} \\ \frac{k_2}{m_2} & \frac{-k_2}{m_2} & \frac{b_2}{m_2} & \frac{-b_2}{m_2} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & -\frac{1}{m_1} & \frac{1}{m_2} \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} 0 & 0 & \frac{k_1}{m_1} & 0 \\ 0 & 0 & \frac{b_1}{m_1} & 0 \end{bmatrix}^T$$

$(\mathbf{A},\mathbf{B})$ is a controllable pair. The control rule for SFC that do not take into account the actuator time delay is

$$\mathbf{U} = [\mathbf{K}]\mathbf{X} \quad (4)$$

where $[\mathbf{K}]$ is chosen to make $\mathbf{A}+\mathbf{BK}$ stable, namely $[\mathbf{K}] = [-5000 - 5000 - 300 - 15500]$ The controlled case is compared with the passive suspension (uncontrolled) case, and for the active cases the actuator time delay was chosen to be $D = 35ms$, which is within the ranges of typical values in literature, namely it is between $25-50ms$ [18]. By using [15], we modelled the actuator delay by using a first-order hyperbolic partial differential equation

$$u_t(x,t) = u_x(x,t), \tag{5}$$
$$u(D,t) = U(t), \tag{6}$$

which has a solution as $u(x,t) = U(t+x-D)$. Therefore, we get the delayed input as $u(0,t) = U(t-D)$, [15]. By using the following backstepping transformation

$$w(x,t) = u(x,t) - \int_0^x q(x,y)u(y,t)dy - \gamma(x)^T \mathbf{X} \tag{7}$$

which gets system 5-6 into the following system

$$\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}t} = (\mathbf{A}+\mathbf{B}\,\mathbf{K})\mathbf{X} + \mathbf{B}\,w(0,t), \tag{8}$$
$$w_t(x,t) = w_x(x,t), \tag{9}$$
$$w(0,t) = 0. \tag{10}$$

Here $[K]$ is the state feedback control gain vector which stabilizes the system without time delay. To derive necessary functions with straightforward calculations we get the following functions as

$$\gamma(x)^T = \mathbf{K}\, e^{\mathbf{A}x}, \tag{11}$$
$$q(x,y) = \mathbf{K}\, e^{\mathbf{A}\,(x-y)}\, \mathbf{B}. \tag{12}$$

One can see the detailed solution of the function $\gamma(x)^T$ and $q(x,y)$ in [15]. Therefore controller for the linear quarter-car suspension system is given by

$$U(D) = \int_0^D \mathbf{K}\, e^{\mathbf{A}\,(D-y)}\, \mathbf{B}\, u(y,t)\, dy + \mathbf{K}\, e^{\mathbf{A}\,D}\, \mathbf{X} \tag{13}$$

By using the system 5-6 with a transformation the control law for distributed backstepping controller(DBC) can be derived as

$$U(t) = \mathbf{K}\left[e^{\mathbf{A}\,D}\,\mathbf{X} + \int_{t-D}^{t} e^{\mathbf{A}\,(t-\tau)}\,\mathbf{B}\,U(\tau)\,d\tau\right]. \tag{14}$$

For the stability analysis, [15] can be helpful. During numerical implementation because of the second term of the control law 14, some problems described in [**?**] such as numerical instabilities can occur. To solve this problem, we use ordinary differential equation

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \mathbf{A}\,z(t) + \mathbf{B}\,u(t) - e^{\mathbf{A}\,\mathbf{D}}\,\mathbf{B}\,u(t-D), \tag{15}$$

which has the solution as

$$z(t) = \int_{t-D}^{t} e^{\mathbf{A}\,(t-\tau)}\,\mathbf{B}\,U(\tau)\,d\tau. \tag{16}$$

Since our open-loop system is stable, 16 can be used to calculate the second term in control law 14.

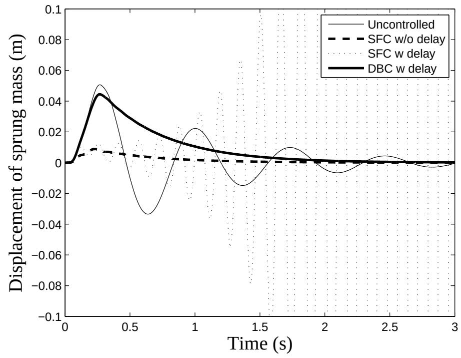

Figure 4. The open and closed-loop response of the quarter-car model. Displacement of sprung mass with; State feedback controller without delay (dashed line); State feedback controller with delay (dotted line); Distributed backstepping controller with delay (thick solid line); Passive system (thin solid line).

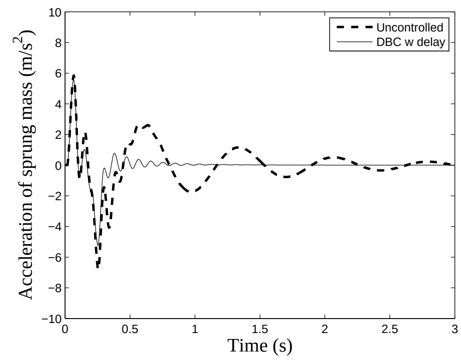

Figure 5. Acceleration of the quarter-car model without control (solid line); with backstepping controller(dashed line).

IV. Numerical Results

The time responses for the vehicle body displacement are presented in Figure 4 for the passive suspension, active suspension without delay using SFC and active suspension with delay using designed DBC. When there is no delay the SFC suppresses vehicle vibrations effectively as seen from the figure. When the actuator delay, $D = 35ms$, is in effect the SFC vehicle body displacements grow up that is system is destabilized. On the other hand it is seen from the same figure that the designed DBC stabilizes the system while satisfactorily suppressing the vehicle body displacements. Since acceleration of the vehicle body is also an important measure of the ride comfort, it is also presented in Figure 5. If compared with the passive case it is seen that designed controller suppresses the acceleration of the vehicle body which means that the ride comfort is improved. The delayed control signal applied to the quarter-car suspension is shown in Figure 6. Suspension travel response of the investigated vehicle active suspension system is presented in Figure 8. It is seen that there is not any control effort at the beginning due to the time delay. Figure 7 presents the time history of

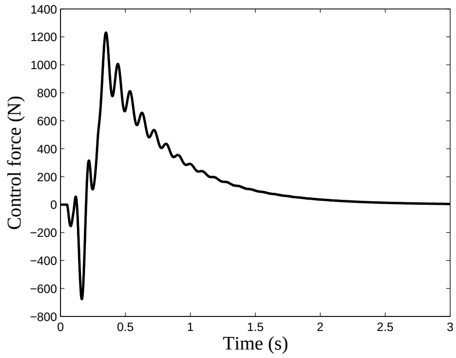

Figure 6. Distributed backstepping control force applied to the quarter-car active suspension system.

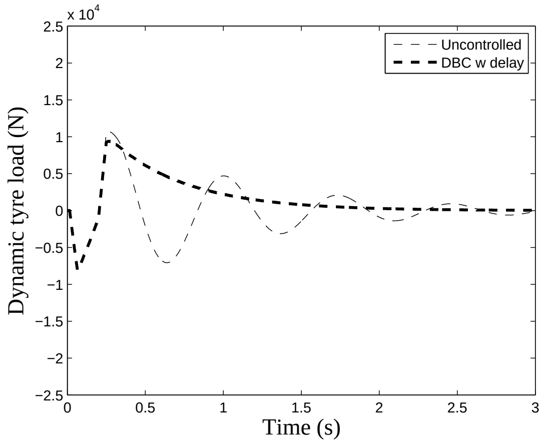

Figure 7. Comparison of dynamic tire load of the quarter-car model without control and with backstepping controller.

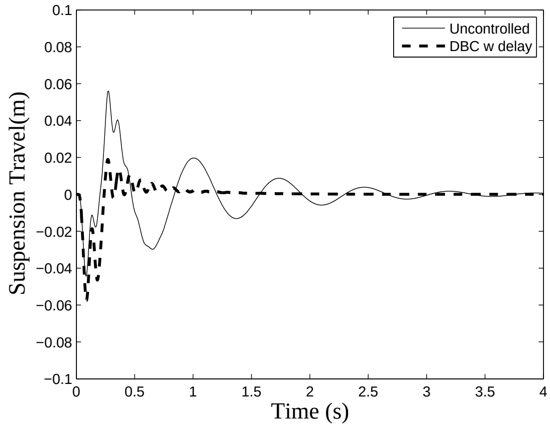

Figure 8. Suspension travel responses of the vehicle system

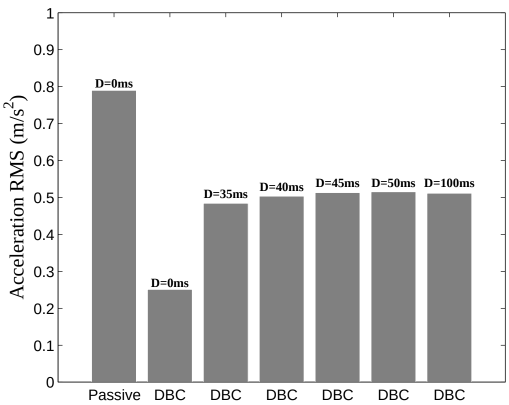

Figure 9. Acceleration RMS by using different delays

the dynamic tire load for the vehicle. It is seen that dynamic tire loads are not increased during ride comfort improvement. Moreover, the dynamic tire load was also reduced to a some degree indicating that road holding was also improved. It is seen from this figure that the magnitudes of the suspension travel response for the DBC case do not exceed the suspension travel response magnitudes of uncontrolled suspension system. As a measure of the ride comfort, the root mean square(RMS) values of the acceleration of the vehicle body are presented in Figure 9. It was seen that designed DBC reduced the RMS values if compared with the passive suspension which means that ride comfort was improved. From this figure it is also concluded that designed controller continues to suppress vehicle vibrations though actuator time delay takes different values.

V. Conclusion

General aim was to improve the ride comfort without reducing road holding. Actuator time delays should be taken into consideration during controller design if not they may give rise to instability of the closed loop system. A distributed backstepping controller was designed that has taken into account the actuator time delay by means of first-order hyperbolic partial differential equation as the primary aim of this study. Then this controller was applied to a quarter vehicle active suspension system with actuator time delay. The time responses have demonstrated that this controller improved ride comfort without reducing road holding of the vehicle along with guaranteed stability of the system.

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