Assessing the results of exposure to computers on problem solving skills

Michael Gr. Voskoglou

***Abstract*—** Problem solving affects our daily lives in direct or indirect ways for ages. Nowadays, in our knowledge society the attitude to think critically has become a necessary condition for solving non-routine problems. However, Critical Thinking is not always a sufficient condition too for problem solving, especially when tackling complicated technological problems, where computers are frequently used as a supporting tool. In this case the need for Computational Thinking is another prerequisite for problem solving. The present paper utilizes the traditional calculation of the mean values and of the Grade Point Average (GPA) index for assessing the effect of the student exposure to computers when taught mathematics on their mathematical problem solving skills. A classroom experiment is performed in which the outcomes of the two assessment methods are compared to each other and interesting conclusions are drawn.

***Keywords*—** Problem Solving (PS), Critical Thinking (CrT), Computational Thinking (CT), Mathematical Modelling (MM), Grade Point Average (GPA) Index.

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I. Introduction

THE Problem Solving (PS) process affects our daily lives in direct or indirect ways for ages. Volumes of research have been written about PS and attempts have been made by many educationists and psychologists to make it accessible to all in various ways [1].

The failure of introduction of the "new mathematics" in school education during the 60's and 70's [2] turned the attention of researchers and educators to PS processes and in particular to the process of *Mathematical Modelling* (MM) *and Applications* dealing with a special type of problems generated by corresponding real situations [3].

The present paper utilizes principles of Fuzzy Logic (FL) for developing an assessment method of student MM skills. The rest of the paper is organized as follows: In Section II we describe the way in which computational thinking synthesizes the existing knowledge with critical thinking and applies them for modelling and solving complicated problems, while in Section III we give a brief account of the MM process as a tool of teaching and learning mathematics. In Section IV we present a classroom experiment connecting the student abilities for solving MM problems with the use of computers. Two traditional methods are used for assessing students MM skills, the calculation of the mean value and of the Grade Point Average (GPA) index. In Section V we compare the outcomes obtained from the application of the above two assessment methods (mean value, GPA) on the data of our experiment, while our last Section VI is devoted to our conclusion and to some hints for future research.

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II. Critical and Computational Thinking in Problem Solving

In our modern society, with the explosion of the information technology and moving from an industrial society to a knowledge society the attitude to think critically became a necessary condition for solving non-routine problems. *Critical thinking* (CrT) is a higher mode of thinking for which, due to its complexity, there is no definition universally accepted. However, most of CrT skills are agreed upon by many authors as involving analysis and synthesis, making judgements, abstraction, uncertainty, application of multiple criteria, decision making, reaching to warranted conclusions and generalisations, reflection, self-regulation, etc. (e.g. see [4]).

Nevertheless, CrT is not always a sufficient condition too for PS, especially when tackling complicated technological problems, where computers are frequently used as a supporting tool. In this case the need for *Computational Thinking* (CT) is another prerequisite for PS. Computation is an increasingly essential tool for doing scientific research. It is expected that future generations of scientists and engineers will need to engage and understand computing in order to work effectively with information systems, technologies and methodologies. CT, named so for its extensive use of computer science techniques [5], is a type of analytical thinking that employs mathematical and engineering thinking to understand and solve complex problems within the constraints of the real world.

The relationship between CT and CrT, the two basic modes of thinking for PS, has not been clearly established yet. In [6] we have attempted to shed some light into this relationship. The conclusions of our study can be summarized with the help of Figure 1, where a 3 - dimensional model for the PS process is presented. According to this model the existing knowledge serves as the connecting tool between CrT and CT, while the problem's solution appears to be the "product" of a simultaneous application of the above three components (knowledge, CrT and CT) to the PS process. This approach is based on the hypothesis that, when the already existing knowledge is adequate, the necessary for the problem's solution new knowledge is obtained through CrT, while CT is applied to design and to obtain the solution. The type of each problem dictates the order of the application of the above three components, which (order) can have in certain, relatively simple, cases the linear form of Figure 2.

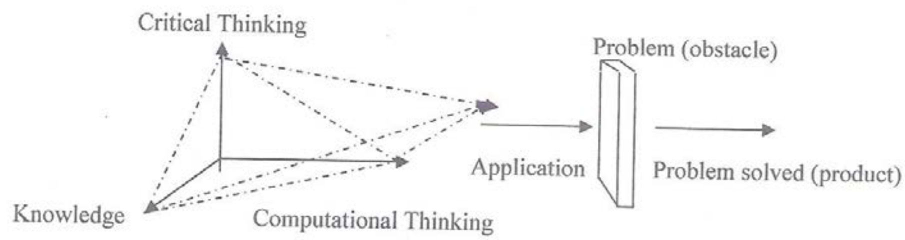

Fig. 1: The 3- dimensional model for the PS process

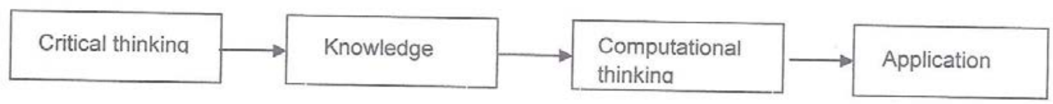

Fig. 2: The linear model for the PS process

The above model (Figures 1 and 2) can be used in formulating the PS process of the complex problems of our everyday life and especially of the complicated technological problems.

According to Liu and Wang [7] CT is a hybrid of other modes of thinking including abstract, logical, modelling and constructive thinking. It synthesizes the existing knowledge with CrT and applies them for modelling and solving complicated problems, for building engineering systems, for interpreting data, etc.

III. THE PROCESS OF MATHEMATICAL MODELLING

We recall that a *model* is simplified representation of a real situation including only the real system's entities and features related to the corresponding situation. It becomes clear that the study of a system's behaviour through the model saves time and reduces the relevant cost.

There are several types of models in use according to the form of the corresponding problem ([8], section 1.3.1). The representation of a system's operation through the use of a *mathematical model* is achieved by a set of mathematical expressions (equalities, inequalities,, etc) and functions properly related to each other. The solutions provided by a mathematical model are more general and accurate than those provided by the other types of models. However, in cases where a system's operation is too complicated to be described in mathematical terms (e.g. biological systems), or the corresponding mathematical relations are too difficult to deal with in providing the problem's solution, a *simulation model* can be used, which is usually constructed with the help of computers.

Until the middle of 1970's MM was mainly a tool in hands of scientists and engineers for solving real world problems related to their disciplines (physics, industry, constructions, economics, etc). One of the first who described the process of MM in such a way that it could be used for teaching and l4earning mathematics was Pollak [9]. He represented the interaction between mathematics and the real world with the scheme shown in Figure 3, which is known as the *circle of MM*.

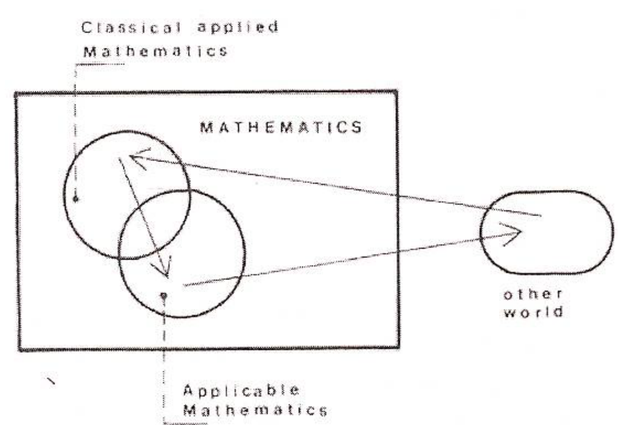

Fig. 3: The Pollak's circle of MM

From the history of mathematics it is well known that many of its topics, which were initially developed on a purely theoretical basis, have found later, or could be found in future, practical applications in real situations; e.g. the Riemann's Geometry in Einstein's General Relativity Theory, Knot Theory in the study of the mechanisms of DNA, etc. Such kind of mathematics is termed in Pollak's scheme (Figure 3) as *Applicable Mathematics* in contrast to the *Classical Applied Mathematics*, which refers to mathematics developed through the scientists' efforts to construct mathematical models representing real situations. But the most important feature of Pollak's scheme (Figure 3) is the direction of the arrows, representing a looping between the "universe" of mathematics and the other (real) world, including all the other sciences and the human activities of our day to day life: Starting from a problem of the real world we transfer to the other part of the scheme, where we use or develop suitable mathematics for its solution. Then we return to the other world interpreting and testing on the corresponding real situation the mathematical results obtained. If these results are not providing a satisfactory solution to the real problem, then we repeat the same circle again one or more times.

From the time that Pollak presented this scheme in ICME-3 (Karlsruhe, 1976) until nowadays much effort has been placed to analyze in detail the process of MM. A brief but comprehensive account of the different models used for the description of the MM process can be found in Haines & Crouch [10] including the present author's model [11, 12]**,** where the MM circle is treated as a Markov chain process dependent upon the transition between the successive discrete steps of the MM process.

MM appears today as a dynamic tool for teaching and learning mathematics, because it connects mathematics with our everyday life giving the possibility to students to understand its usefulness in practice and therefore increasing their interest about mathematics [13]. But we must be careful! The process of MM could not be considered as a general and therefore applicable in all cases method for teaching mathematics. In fact, such a consideration could lead to far-fetched situations, where more emphasis is given to the search of the proper applications rather, than to the consolidation of the new mathematical knowledge [14].

IV. THE CLASSROOM EXPERIMENT

As we have seen in Section II, modelling thinking is a principal mode of CT. On the other hand, exploratory investigations have demonstrated how exposure to computers enhances the way students approach real world problems; [15] – [18], etc. In an attempt to explore the effect of the exposure to computers on student MM skills we performed the following classroom experiment with subjects students of the School of Technological Applications (prospective engineers) of the Graduate Technological Educational Institute (TEI.) of Western Greece attending the course "Higher Mathematics I" of their first term of studies. This course involves Differential and Integral Calculus in one variable, Elementary Differential Equations and Linear Algebra.

The students, who had no previous experience with computers apart from the basics learned in secondary education, were divided in two equivalent groups according to their grades obtained in the national maths exam for entrance in higher education.

For the control group the lectures were performed in the classical way on the board, followed by a number of exercises and examples connecting mathematics with real world applications and problems. The students participated in solving these problems. The difference with the experimental group was that about the 1/3 of the lectures and exercises were performed in a computer laboratory. There the instructor presented the corresponding mathematical topics with the help of computers, while the students themselves, divided in small groups and making use of known mathematical software packages solved the problems and real world applications with the help of computers. The teaching schedule of both courses involved six hours per week including, for the experimental group, the time spent in the laboratory.

Note that, the use of the computers was consuming much more time than the classical way of teaching on the board did. Consequently, the students of the control group had the opportunity to participate to the solution of more exercises and problems. Thus, the control group's overall performance during the assessment process was normally expected to be significantly better.

At the end of the term the students of both groups participated to a common (the instructor was the same person) written exam for the assessment of their progress. The exam involved a number of theoretical questions and exercises covering all the mathematical topics taught and three simplified real world problems (see Appendix) requiring MM techniques for their solution. The instructor marked the students' papers in a scale from 0 to 100, separately for the questions and exercises and separately for the problems. The student performance was characterized as follows: A (85-100) = Excellent, B (75-84) = Very Good, C (60-74) = Good, D (50-59) = Satisfactory and F (0-49) = Unsatisfactory.

The performance of the control group was found to be better, as it was expected, concerning the student answers to the theoretical questions and exercises. Here we shall present and evaluate in detail the results of the two groups concerning the solution of MM problems. The scores achieved by the students of the two groups for this task were the following:

Experimental group (G_1): 100(2 times), 99(1), 98(2), 95(3), 94(2), 92 (3), 90(2), 89(1), 88(3), 85(1), 82(2), 80(4), 78(3), 76(2), 75(4), 72(3), 70(1), 68(2), 60(1), 58(2), 57(1), 56(2), 55(2), 54(1), 50(2), 45(3), 42(2), 40(2), 35(1).

Control group (G_2) *:* 100(2), 99(1), 98(1), 97(1), 95(2), 92(4), 91(1), 90(2), 88(1), 85(5), 82(2), 80(6), 78(9), 75(13), 70(3), 64(4), 60(8), 58(2), 56(3), 55(3), 50(7), 45(2), 40(3).

The above data are depicted in Table 1:

Table 1: Characterization of the student performance

Characterizations	G_1	G_2
A	20	20
B	15	30
C	7	15
D	10	15
F	8	5
Total	60	85

The evaluation of the above data was performed in two traditional ways:

a) Calculation of the mean values: A straightforward calculation gives that the means of the student scores are approximately 73.28 and 71.91 for the experimental and the control group respectively. This shows that the *mean performance* of both groups was good (C), with the performance of the experimental group being better.

b) Application of the GPA index: We recall that the *Grade Point Average* (*GPA*) index is a weighted mean (frequently used in the USA and some other Western Countries), where more importance is given to the higher scores, by assigning greater coefficients (weights) to them. In other words, the GPA index focuses on the *quality performance* rather, than on the mean performance of the student groups.

In order to calculate the GPA index from the data of our experiment let us denote by n_A, n_B, n_C, n_D and n_E the numbers of students whose performance was characterized by A, B, C, D and E respectively and by n the total number of students of each group. .Then the GPA index is calculated by the formula GPA $=\frac{n_D+2n_C+3n_B+4n_A}{n}$ [19]. Since the GPA's maximal value corresponds to the case $n_A = n$, $n_B = n_C = n_D = n_F = 0$, while its minimal value corresponds to the case $n_F = n$, $n_A = n_B = n_C = n_D = 0$, we obviously have that $0 \leq \text{GPA} \leq 4$.

In our case, replacing the data of Table 1 to the above formula and making the corresponding calculations one finds that the GPA index is 2.48 for the experimental and 2.52 for the control group. These values indicate a more than satisfactory quality performance of both groups, since they are greater than the half of the GPA's maximal value (4:2=2). Nevertheless, the control group demonstrated a slightly better performance.

V. Comparison of the Experimental Results

The application of the above two methods for assessing students MM skills resulted to different conclusions. However, this is not embarrassing, since, in contrast to the calculation of the mean values of the student individual scores, which focuses to the mean performance of a student group, the GPA 9ndex focuses on its quality performance by assigning greater coefficients to the higher scores.

Concerning the effect of the use of computers on student MM skills, according to the mean values of their scores and in contrast to what was normally expected, enhanced the mean performance of the experimental with respect to the control group. On the contrary the values found for the GPA index demonstrated a slight superiority of the control group (in both cases a clear superiority of the control group was expected). Therefore, the exposure to computers enhanced significantly the performance of the mediocre students (lower scores) of the experimental group, but it had a much smaller effect on the performance of the good students (higher scores). A possible explanation about this could be that the figures' animation, the quick transformations of the numerical and algebraic representations, the easy and accurate construction of the graphs, etc, which are comfortably achieved using the computers, increased the mediocre students' imagination and helped them in using effectively their intuition for designing and constructing the solutions of the corresponding problems. On the contrary, all the above had a much smaller effect on the performance of the good students, who had already developed high MM skills.

VI. Discussion and Final Conclusion

In the present paper a classroom experiment connecting student MM skills with their exposure to computers was presented. The fact that, in contrast to what it was normally expected, the experimental group demonstrated a significantly better mean performance and only a slightly worse quality performance than the control group, gives a strong .indication for the positive effect of the exposure to computers for enhancing student MM skills. Nevertheless, since the differences appeared between the two groups were relatively small, more experimental research is needed to allow a stronger conclusion.

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APPENDIX

List of MM problems used in our experiment

Problem 1: Among all the cylindrical towers having a total surface of 180π m^2, which one has the maximal volume?

Problem 2: Let us correspond to each letter the number showing its order into the alphabet (A=1, B=2, C=3 etc). Let us correspond also to each word consisting of 4 letters a 2X2 matrix in the obvious way; e.g. the matrix $\begin{bmatrix} 19 & 15 \\ 13 & 5 \end{bmatrix}$ corresponds to the word SOME. Using the matrix E=$\begin{bmatrix} 8 & 5 \\ 11 & 7 \end{bmatrix}$ as an encoding matrix how you could send the message LATE in the form of a camouflaged matrix to a receiver knowing the above process and how he (she) could decode your message?

Problem 3: The population of a country is increased proportionally. If the population is doubled in 50 years, in how many years it will be tripled?

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