Two Term Control Strategy For Position Control Of Twin Rotor System

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Abstract**—** In this paper, a two term control law is developed for position control of Twin rotor multiple-input-multiple-output (MIMO) system (TRMS). The proposed controller is an added delay term in conventional Composite Nonlinear Feedback (CNF) control which improves robustness of the controller with fast transient response and better damping characteristics. Proposed control law is compared with conventional CNF to prove its superiority which is validated via computer simulation in MATLAB environment. Lyapunov-Krasovskii functional is proposed for the study of robust stability conditions which create boundaries of the closed-loop system.

Keywords**—**Composite Nonlinear Feedback Technique, Lyapunov-Krasovskii functional, Time Delay, Twin Rotor MIMO System.

I. Introduction

The control objective of most physical systems is to achieve desired output quickly and accurately. In recent years, development of several strategies for controlling the air vehicle has been studied frequently. Helicopters are application to flying air vehicle, which has ability to hover in a given place and fly in any directions. Rotors or blades give power to helicopters. The rotors blades when turn, air flows more rapidly (over the top of the blades) than below the blades, which creates the lift required for flight.

In recent years, modeling and control of TRMS is motivated because of its dynamics which is similar to helicopter in certain aspects. It has coupling effect and nonlinear dynamics that makes it unstable, hence need of controller design arises. Significant research efforts on controlling the helicopters are done in form of intelligent control [2], feedback linearization control [6-7], sliding mode control [9-13], robust adaptive control [5], composite nonlinear feedback control [3] and backstepping control [20]. To enhance the control performance, integration of different techniques are these days challenging in form of complexity.

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Composite nonlinear feedback (CNF) [3] is consist of linear feedback and nonlinear feedback part which has an advantage of improving damping and transient response in parallel. Once the linear-feedback part is fixed, the performance of the CNF control relies on the selection of the nonlinear function in the CNF control law. But it suffers from poor robustness.

In research era of time delay [10], it has proven that adding delay can improve robustness of nonlinear system. This paper deals with two term control strategy which is addition of a delay element in CNF control law. By adding delay in control law, performance of TRMS enhances in terms of fast transient response, less overshoot and robustness. Unlike previous reported controllers, it doesn't compromises between fast transient response and less overshoot.

In this paper two term controller is proposed which contains a delay term added with conventional CNF control law. The objective is to add robustness in conventional CNF by improving its transient response. In the simulation results, it can be seen that damping also improves with fast transient response which is duo improvement. Stability analysis is taken into account with Lyapunov-Krasovskii Functional which mathematically derives stability conditions for the proposed controller.

This paper is organized in following sections: Section II contains problem formulation, that contains dynamic model and linear model of TRMS. Section III contains introduction to conventional controller and designing of proposed controller, followed by its stability analysis which is required to prove that closed loop system is stable and also to get values of controller parameters. Section IV contains simulation results and also it contain values of proposed as well as conventional CNF controller parameters which are used throughout this paper. Last section is conclusion which states summary of the paper.

II. Problem Statement

The nonlinear mechanical system-TRMS [8] has two rotors(main rotor and tail rotor) placed on a beam together with a counterbalance arm with a fixed weight at its end. This determines a stable equilibrium position (shown in the Fig.1). The beam is pivoted on its base such that rotation of rotors will be in both the horizontal and vertical planes. The rotors are driven by dc motors where main rotor allows the beam to rise vertically and tail rotor makes the beam move horizontally. This device is a multivariable, nonlinear and strongly coupled system, with two degree-of-freedom (pitch and yaw angle).

The forces acting on TRMS are contributing for driving this nonlinear model and also for deriving its mathematical model for further analysis.

Fig. 1 Twin Rotor MIMO System

A. Nonlinear TRMS model:

The mathematical modeling of TRMS system can be represented as follows:

$$\frac{d}{dt}\psi = \dot{\psi}$$

$$\frac{d}{dt}\dot{\psi} = \frac{a_1}{I_1}\tau_1^2 + \frac{b_1}{I_1}\tau_1 - \frac{M_g}{I_1}\sin\psi - \frac{B_{1\psi}}{I_1}\dot{\psi} + \frac{0.0326}{2I_1}\sin(2\psi)\dot{\varphi}^2 - \frac{k_{gy}}{I_1}\cos(\psi)\dot{\varphi}\left[a_1\tau_1^2 - b_1\tau_1\right]$$

$$\frac{d}{dt}\varphi = \dot{\varphi}$$

$$\frac{d}{dt}\dot{\varphi} = \frac{a_2}{I_2}\tau_2^2 + \frac{b_2}{I_2}\tau_2 - \frac{B_{1\varphi}}{I_2}\dot{\varphi} - \frac{k_c a_1}{I_2}1.75\tau_1^2 - \frac{1.75}{I_2}k_c b_1 \tau_1$$

$$\frac{d}{dt}\tau_1 = -\frac{T_{10}}{T_{11}}\tau_1 + \frac{k_1}{T_{11}}u_1$$

$$\frac{d}{dt}\tau_2 = -\frac{T_{20}}{T_{21}}\tau_2 + \frac{k_2}{T_{21}}u_2 \qquad (1)$$

The output is given by

$$y = [\psi \quad \varphi]^T \qquad (2)$$

where,

ψ and $\dot{\psi}$ are pitch angle and velocity, φ and $\dot{\varphi}$ are yaw angle and velocity, τ_1 and τ_2 are momentum of main and tail rotor and u_1 and u_2 are control efforts.

Parameters of TRMS are mentioned in TABLE 1.

B. Linear TRMS Model:

The proposed controller in this paper deals with linear model, hence converting nonlinear TRMS model into linear model by Jacobian Linearization approach and putting value of parameters from Table 1, we will get:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bsat(u)\\ y(t) &= Cx(t)\end{aligned} \qquad (3)$$

where,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -4.7059 & -0.0882 & 0 & 0 & 1.3588 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -5 & 1.617 & 4.5 \\ 0 & 0 & 0 & 0 & -0.9091 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0.8 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$x = [\psi \quad \dot{\psi} \quad \varphi \quad \dot{\varphi} \quad \tau_1 \quad \tau_2]^T$$

$$u = [u_1 \quad u_2]^T$$

TABLE I. TRMS SYSTEM PARAMETERS

Symbol	Quantity	Values
I_1	Moment of inertia of vertical rotor	6.8×10^{-2} kg-m^2
I_2	Moment of inertia of horizontal rotor	2×10^{-2} kg-m^2
k_1	Motor 1 gain	1.1
k_2	Motor 2 gain	0.8
a_1	Static characteristic parameter	0.0135
b_1	Static characteristic parameter	0.0924
T_{11}	Motor 1 denominator parameter	1.1
T_{10}	Motor 1 denominator parameter	1
T_{21}	Motor 2 denominator parameter	1
T_{20}	Motor 2 denominator parameter	1
a_2	Static characteristic parameter	0.02
b_2	Static characteristic parameter	0.09
M_g	Gravity momentum	0.32 N-m
T_P	Cross reaction momentum parameter	2
T_0	Cross reaction momentum parameter	3.5
$B_{1\psi}$	Friction momentum parameter	6×10^{-3}N-m-s/rad
$B_{1\varphi}$	Friction momentum parameter	1×10^{-1}N-m-s/rad
k_c	Cross reaction momentum gain	-0.2
k_{gy}	Gyroscopic momentum parameter	0.05 s/rad
u_1, u_2	Input voltage applied to main and tail rotor are bounded	±2.5V

III. Two Term Control Strategy

A. Conventional CNF control Law

In CNF law some important assumptions are required i.e. (A,B) is controllable, (A,C) is observable and (A,B,C) is invertible without having any zero at $s=0$. If above assumptions are true, then CNF control law is designed as:

$$u_{cnf}(t) = u_l(t) + u_n(t) \tag{4}$$

where,

$u_l(t)$ is a linear feedback law which improves the damping ratio and $u_n(t)$ is a non-linear feedback law which reduces the overshoot as soon as output of the system reaches the reference value.

These two terms are expressed as:

$$\begin{aligned} u_l(t) &= K\tilde{x}(t) + Hr \\ u_n(t) &= \rho(\tilde{x}(t))B^T P\tilde{x}(t) \end{aligned} \tag{5}$$

where, r is the reference input, gain matrix $K \in R^{m\times n}$ and real symmetric matrix $P \in R^{n\times n}$ which are designed by LMI toolbox in order to make closed loop system asymptotically stable, H is a scalar quantity given in (8) which is taken from [13] and ρ is a non-positive function which is locally Lipschitz in $\tilde{x}(t)$ which plays a major role in changing the location of closed loop poles.

Here,

$$\tilde{x}(t) = x(t) - x_e \tag{6}$$

and

$$x_e = -(A+BK)^{-1}BGr \tag{7}$$

and

$$H = (1-K(A+BK)^{-1}B)G \tag{8}$$

Choice of x_e will be clear from (12) and (13).

A. Proposed Two term Control

In conventional CNF control law (4), a delay term is added as:

$$u_{cnf}(t) = u_l(t) + u_n(t) + K_d\tilde{x}(t-d) \tag{9}$$

where, K_d is delay gain matrix having same dimension as K. Delay d is introduced to make the controller robust. Value of d is chosen as very small value to avoid complexity.

Further $\tilde{x}(t-d)$ will be written as $\tilde{x}_d(t)$.

B. Stability analysis

Let closed loop system as new state vector (6) is written as:

$$\dot{\tilde{x}}(t) = \dot{x}(t) - 0$$

From (3) and (6),

$$\begin{aligned} \dot{\tilde{x}}(t) &= Ax(t) \pm Ax_e + Bsat(u) \pm BHr \pm BK\tilde{x}(t) \pm BK_d\tilde{x}_d(t) \\ &= (A+BK)\tilde{x}(t) + BK_d\tilde{x}_d(t) + Ax_e + BHr + B\omega \\ &= (A+BK)\tilde{x}(t) + BK_d\tilde{x}_d(t) + B\omega + p_1 \end{aligned} \tag{10}$$

where,

$$\omega = sat(u) - Hr - K\tilde{x}(t) - K_d\tilde{x}_d(t) \tag{11}$$

and

$$p_1 = Ax_e + BHr$$

putting value of x_e from (7) and H from (8) in p_1:

$$\begin{aligned} p_1 &= -A(A+BK)^{-1}BGr + B(1-K(A+BK)^{-1}B)Gr \\ &= -A(A+BK)^{-1}BGr + BGr - BK(A+BK)^{-1}BGr \\ &= -((A+BK)^{-1}\times(A+BK))BGr + BGr \\ &= -BGr + BGr = 0 \end{aligned} \tag{12}$$

Hence (10) will become:

$$\dot{\tilde{x}}(t) = (A+BK)\tilde{x}(t) + BK_d\tilde{x}_d(t) + B\omega \tag{13}$$

To verify the stability of controlled TRMS, following Lyapunov-Krasovskii functional is designed as:

$$V = \tilde{x}^T(t)P\tilde{x}(t) + \int_{t-d}^{t}\tilde{x}^T(\Omega)Q\tilde{x}(\Omega)d\Omega \tag{14}$$

Its derivative will be :

$$\begin{aligned} \dot{V} &= 2\tilde{x}^T(t)P\dot{\tilde{x}}(t) + \tilde{x}^T(t)Q\tilde{x}(t) - \tilde{x}_d^T(t)Q\tilde{x}_d(t) \\ &= 2\tilde{x}^T P(A+BK)\tilde{x} + 2\tilde{x}^T PBK_d\tilde{x}_d(t) + 2\tilde{x}^T PB\omega \\ &\quad + \tilde{x}^T(t)Q\tilde{x}(t) - \tilde{x}_d^T(t)Q\tilde{x}_d(t) \end{aligned}$$

It can also be written as:

$$\dot{V} = \begin{bmatrix} \tilde{x}(t) \\ \tilde{x}_d(t) \end{bmatrix}^T \begin{bmatrix} \Delta_1 & PBK_d \\ K_d^T B^T P & -Q \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \tilde{x}_d(t) \end{bmatrix} + 2\tilde{x}^T PB\omega \tag{15}$$

where,

$$\Delta_1 = P(A+BK) + (A+BK)^T P + Q$$

To prove system stability, $\dot{V} < 0$. This will only be satisfied if, first term of $\dot{V}$:

$$\begin{bmatrix} \Delta_1 & PBK_d \\ K_d^T B^T P & -Q \end{bmatrix} < 0,\ \Delta_1 < 0 \tag{16}$$

This is solved by LMI toolbox which calculate appropriate values of gains K and K_d and positive definite matrix P such that above inequality holds.

Second term of $\dot{V}$ is $2\tilde{x}^T(t)PB\omega$. Following section proves that $2\tilde{x}^T(t)PB\omega < 0$:

Because ω contains saturated input channels. The following investigations are done to make this term negative definite for the sake of stability:

- If input channels are unsaturated, i.e. $|u| \le u_{\max}$:

$$\omega = K\tilde{x}(t) + Hr + u_N + K_d \tilde{x}_d(t) - (K\tilde{x}(t) + Hr + K_d \tilde{x}_d(t)) \\ = u_n$$

hence,

$$\tilde{x}^T(t)PB\omega = \tilde{x}^T(t)PBu_n = \rho \tilde{x}^T(t)PBB^T P\tilde{x}(t)$$

As we know ρ is a non-positive function and $\tilde{x}^T(t)PBB^T P\tilde{x}(t)$ is positive function, hence $\tilde{x}^T(t)PB\omega < 0$.

- If input channels exceed their upper bound i.e. $u > u_{\max}$:

$$K\tilde{x}(t) + Hr + u_n + K_d \tilde{x}_d(t) > u_{\max}$$
$$u_n > u_{\max} - (K\tilde{x}(t) + Hr + K_d \tilde{x}_d(t))$$

$u_{\max}$ is saturated value of u . From (11), above inequality becomes :

$$\omega > 0 \text{ and } u_n = \rho B^T P\tilde{x}(t) > \omega > 0$$

ρ is non-positive function, hence to satisfy above inequality:

$$B^T P\tilde{x}(t) = \tilde{x}^T(t)PB < 0 .$$

As shown in above steps, $\omega > 0$ and $\tilde{x}^T(t)PB < 0$, second term of (14) will become:

$$\tilde{x}^T(t)PB\omega < 0 .$$

- If input channels exceed their lower bound i.e. $u < -u_{\max}$:

$$K\tilde{x}(t) + Hr + u_n + K_d \tilde{x}_d(t) < -u_{\max}$$
$$u_n < -u_{\max} - (K\tilde{x}(t) + Hr + K_d \tilde{x}_d(t))$$

$-u_{\max}$ is saturated value of u . From (11), above inequality becomes :

$$\omega < 0 \text{ and } u_n = \rho B^T P\tilde{x}(t) < \omega < 0$$

ρ is non-positive function, hence to satisfy above inequality:

$$B^T P\tilde{x}(t) = \tilde{x}^T(t)PB > 0$$

As shown in above steps, $\omega < 0$ and $\tilde{x}^T(t)PB > 0$ second term of (14) will become:

$$\tilde{x}^T(t)PB\omega < 0 .$$

Hence the three possible conditions of $\tilde{x}^T(t)PB\omega$ due to saturated term in ω are less than zero. Also if (16) exists, then $\dot{V}$ in (15) is always less than zero, which makes the closed loop system asymptotically stable.

C. Simulation Results

Here delay d is chosen as 0.5 seconds and reference r is taken as 0.5 radians for both pitch angle and yaw angle. By putting proposed controller in (1), we will get following results:

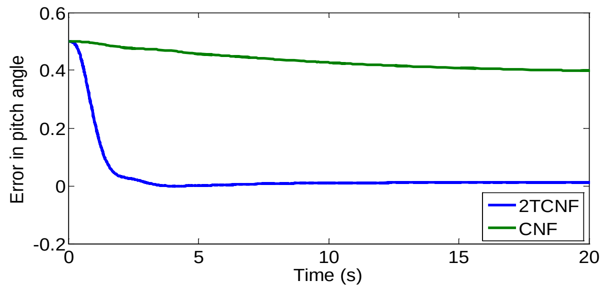

Fig. 1. *Tracking error in Pitch angle.*

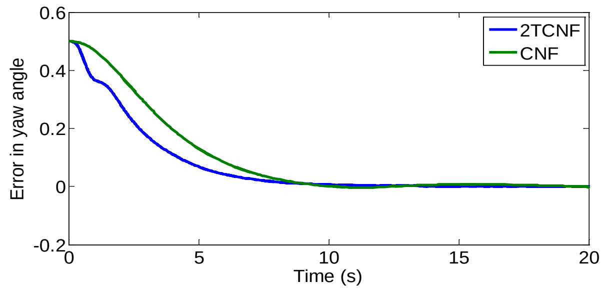

Fig. 2. *Tracking error in Yaw angle.*

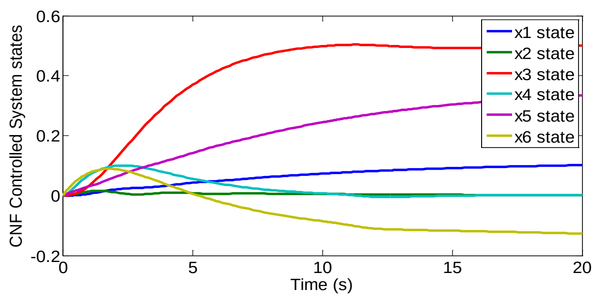

Fig. 3. *States of TRMS in CNF control*

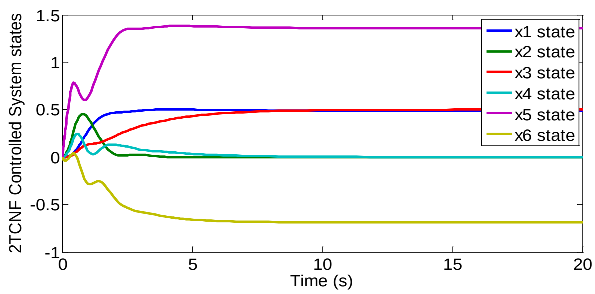

Fig. 4. *States of TRMS in 2TCNF control*

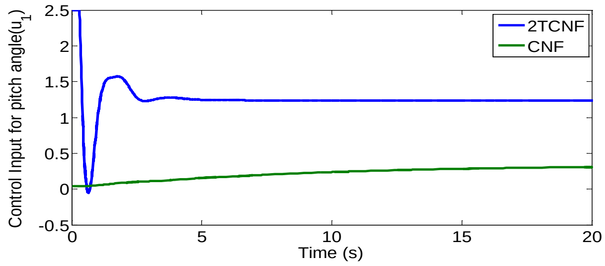

Fig. 5. *Control Efforts of Pitch angle*

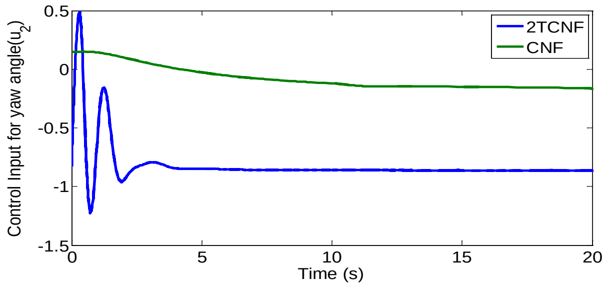

Fig. 6. *Control Efforts of Yaw angle*

From Fig.1 it can be seen that error of pitch angle in 2TCNF control, very quickly converges to zero whereas conventional CNF control doesn't track the reference hence its error is not zero. In Fig.2, yaw angle of both the controllers are tracking reference hence error is zero but proposed controller tracks the reference more quickly than conventional controller. Hence it is proven that proposed controller improves transient response better than conventional CNF controller.

Fig.3 and Fig.4 shows system states in conventional CNF and 2TCNF control. It can be seen that 2TCNF controlled states are bounded within 5 seconds whereas in conventional CNF, states are bounded in 15 seconds. Hence it can be concluded that 2TCNF control states are more robust and are quickly bounded than conventional CNF.

Fig.5 and Fig.6 shows control efforts of both the controllers. It can be said that adding delay can cause more control efforts.

Values of 2TCNF controller parameters are:

$$K = \begin{bmatrix} 2.1538 & -6.5940 & -2.7134 & -3.5474 & -3.0156 & -0.4532 \\ -3.4122 & 1.1641 & -4.9372 & -11.2291 & 3.2406 & 1.6037 \end{bmatrix}$$

$$K_d = \begin{bmatrix} -0.8938 & -0.2119 & -0.2428 & -0.1524 & -0.0129 & -0.0695 \\ 0.4956 & 0.3463 & 1.1447 & 0.6951 & 0.1638 & 0.3512 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.1303 & 0.0265 & 0.0160 & 0.0105 & -0.0011 & 0.0041 \\ 0.0265 & 0.0960 & 0.0112 & 0.0096 & 0.0469 & -0.0084 \\ 0.0160 & 0.0112 & 0.0369 & 0.0224 & 0.0053 & 0.0113 \\ 0.0105 & 0.0096 & 0.0224 & 0.0661 & -0.0041 & -0.0137 \\ -0.0011 & 0.0469 & 0.0053 & -0.0041 & 0.0402 & -0.0016 \\ 0.0041 & -0.0084 & 0.0113 & -0.0137 & -0.0016 & 0.0325 \end{bmatrix}$$

Values of CNF controller parameters are:

$$K = \begin{bmatrix} 2.5000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1.2025 & 0 & 0 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.0186 & -0.0075 & 0.0035 & 0.0017 & 0.0207 & -0.0036 \\ -0.0075 & 0.0587 & -0.0017 & -0.0085 & -0.0277 & -0.0002 \\ 0.0035 & -0.0017 & 0.0376 & -0.0075 & 0.0184 & -0.0192 \\ 0.0017 & -0.0085 & -0.0075 & 0.0193 & 0.0079 & 0.0169 \\ 0.0207 & -0.0277 & 0.0184 & 0.0079 & 0.0652 & -0.0140 \\ -0.0036 & -0.0002 & -0.0192 & 0.0169 & -0.0140 & 0.0260 \end{bmatrix}$$

Also in this paper stability analysis is done by Lyapunov-Krasovskii analysis, from which calculation of controller parameters are done by solving (15) in LMI toolbox. But in conventional CNF control, gain matrix is calculated by trial and error method. It can also be calculated by pole-placement method which does not always make the closed loop system stable. Hence it is easier to calculate the parameters by LMI instead of the time-consuming, effort intensive and inefficient trial and error and pole-placement methods.

IV. Conclusion

The proposed two term control for TRMS proves through simulation results that it gives robust performance with fast transient response small small overshoot. Proposed control law deals with the previous and present state information which improves the robustness of system. The calculated controller parameters by solving inequalities formed by Lyapunov–Krasovskii functional, with the help of LMI Toolbox, ensures the stability of the given closed loop TRMS. From simulation studies we can conclude that by adding information of past state in controller, transient response became fast and tracking error reduces quickly in comparison to present state controller.

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