Optimizing a Single Vendor-Multi Buyer Bi-Objective Supply Chain Problem with Stochastic Demand and Routing Under Storage Capacity and Transportation Constraints Using NSGA-II Algorithm

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Abstract: One of the most important factors for succession in a supply chain is decreasing their costs. Combining the decision making in inventory control and routing areas in distribution network may decrease the cost of supply chain and increase level of service. In this paper we work on a bi-objective supply chain consisting single vendor-multi buyer in infinite horizon. The vendor produces all products and presents them to the buyers through a heterogeneous fleet of transportation. The first objective of this problem deals with decreasing the cost of inventory and transportation and the second one with increasing customer satisfaction level. To draw the problem close to the real situations some practical constraints like storage capacity and transportation equipment are added to the problem. This model is a non-linear integer problem, so Non-dominated sorting genetic algorithm (NSGA-II) is used for solving the presented model.

Keywords: Single vendor-multi buyer, integrated inventory model, NSGA-II.

I. INTRODUCTION

Integrity and coordination in deferent sections is a basic need in a supply chain management (SCM). Most of the activities in a SCM are dependent together and changes in a section may effects on the performance of the other sections, so combining the sections of a SCM is a customary way for decreasing the cost and increasing the profit of supply chain.

Goyal [1] analyzed a single vendor-single buyer combined inventory model and presented framework has been used by many researchers after it. Pan and Yang [2] expand the Goyal model by considering lead time as a decision variable and minimized the cost of system.

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Lu [3] presented a single vendor-multi buyer inventory model and considered that each buyer order different items and minimized vendor cost. Yu-Jen [4] worked on a single vendor-single buyer with discount for backorders and considered lead-time as a decision variable. Bendaya and Hariga [5] presented a single vendor-single buyer for a mixed inventory model. The models that combined purchasing raw materials and productions called integrated procurement-production (IPP) [6]. Lee [7] considered purchase-product mixed model and vendor-buyer mixed model in the same time and decreased mean of inventory and produce cost.

This paper divided in to five parts. The second part is problem definitions and mathematical model. Solving methodology presented in third part and 4th part deals with a numerical example and the last part in conclusions and further studies.

II. PROBLEM DEFINITIONS

In this paper we works on a single vendor-multi buyer supply chain. In this model i th buyer order $\left(\sum_{p=1}^{P} Qip\, , i=1,2,\ldots,N\right)$ and vendor (producer) produce nQ_p units with constant rate $R_p\left(R_p > D_p\right)$ in one production time and send in n times with heterogeneous limited capacity fleet of transportation to the buyers.

A. Assumptions

- Storage capacity is limited.
- Buyers demand is stochastic and independent from other buyers.

- Production rate for all products are constant. $(R_p > D_p)$
- Planning horizon is limited.
- Fleets of transportation are heterogeneous and have limited capacity.
- Each route serviced by only one vehicle.
- In each period, each buyer meets only one time with each vehicle.

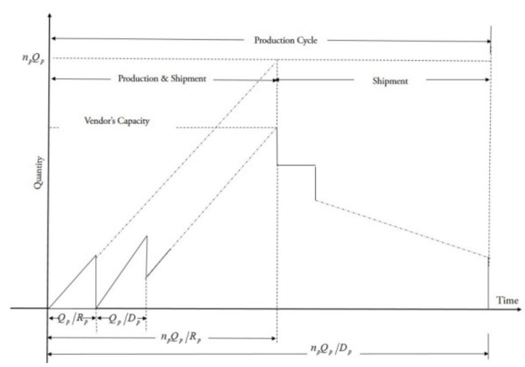
B. Parameters and decision variables

In an infinite horizon for buyers $(i = 1,2,\dots,N)$, products $(p = 1,2,\dots,P)$ and fleet of transportation $(k = 1,2,\dots,K)$ parameters are:

Q_p : Inventory of product p for vendor,
Q_{ip} : Value of product p ordered by buyer i,
D_p : Demand rate of product p
R_p : Production rate of product $p, (P_i \geq D_i)$
D_{ip} : Demand of product p ordered by buyer i, $\left(D_p = \sum_{i=1}^{p} D_{ip}\right)$
X_{ip} : Demand in lead-time, $X_{ip} \sim N\left\{D_{ip}L_{ip}, \left(\sigma_{ip}\sqrt{L_{ip}}\right)^2\right\}$
UL_{ip} : Upper bound for lead-time of product p for buyer i,
A_{ip} : Ordering cost of product p for buyer i,
A_{vp} : Set-up cost of product p,
C_p^v : Production cost of each unit of product p,
C_{ip}^B : purchasing cost of each unit of product p, $\left(C_p^v < C_{ip}^B \quad, \forall i, p\right)$
$C_{ip}^v(L_{ip})$: Violation cost of lead-time for product p for buyer i,
h_{ip} : Holding cost of product p for buyer i,
h_{vp} : Holding cost of product p for vendor
h'_{vp} : Safety coefficient of product p for buyer i,
K : Maximum capacity of transportation vehicles,
q_k : Maximum capacity of vehicle k,
t_{ij} : Travel time from vertex i to vertex j,
a_i : Receiving time to vertex i,
g_{ik} : Service time of vehicle k to vertex i,
w_{ik} : Lead-time of vehicle k in vertex i,
τ_k : Longest permitted route time for vehicle k,
f_{ik} : Servicing time of vehicle k in vertex i,
z_{0k} : Leaving time of vehicle k from purchasing storage,
e_i : Earliest time that buyer i received goods,
l_i : latest time that buyer i received goods,
f : Fix cost for using vehicles in the routes,
F^v : Maximum capacity of vendor capacity,
F_i^B : Maximum capacity of buyer i,
S_{ip} : Safety stock of product p for buyer i,
v_p : Ratio for volume of product p to basis product,
ct : Fix cost of each transportation time unit,
TEC_0^V : Expected total cost of each time unit for vendor,
TEC_0^B : Expected total cost of each time unit for buyer,
TEC_A^V : Expected fixed set-up cost of each time unit for vendor,
TEC_H^V : Expected holding cost of each time unit for vendor,
TEC_H^B : Expected holding cost of each time unit for buyer,
TEC_O^B : Expected ordering cost of each time unit for buyer,
TEC_T^B : Expected transportation cost of each time unit for buyer,
n : Number of transportations from vendor to buyer,
r_{ip} : Re-order point of product p,
L_{ip} : Lead-time of product p for buyer i,
x_{ijk} : Binary variable, if vehicle k travel from vertex i to vertex j is equal 1, else is equal zero, $(i, j \neq 0, i \neq j)$

C. Mathematical model

The inventory pattern of the vendor and buyer i for product p presented in figure 1.



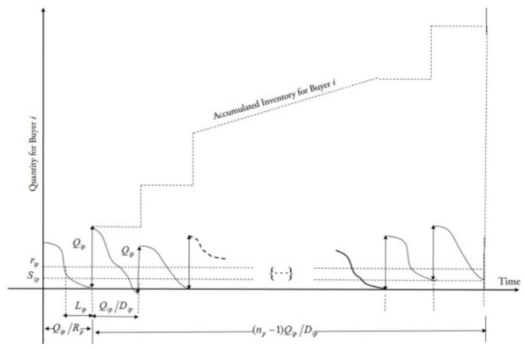


Fig. 1: The inventory pattern of the vendor and buyer i for product p.

And the mathematical model is as follows:

$$Z_1 = \min\left(\sum_{p=1}^{P}\frac{D_p}{Q_p}\left(\frac{A_{vp}}{n}+\sum_{i=1}^{N}(A_{ip}+C_{ip}(L_{ip}))\right)\right)+ \tag{1}$$

$$\sum_{i=1}^{N}\sum_{p=1}^{P}\left(h_{ip}C_{ip}^{B}(\frac{Q_p}{2R_p}D_{ip}+k'_{ip}\sigma_{ip}\sqrt{L_{ip}})\right)+$$

$$\sum_{p=1}^{P}\frac{Q_p}{2}h_{vp}C_p^V\left(n(1-\frac{D_p}{R_p})-1+\frac{2D_p}{R_p}\right)+\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{K}x_{ijk}t_{ij}ct_k$$

$$Z_2 = \min\max_i\{\sum_{p=1}^{P}\frac{D_p\sigma_{ip}\sqrt{L_{ip}}\psi(k'_{ip})}{D_{ip}Q_p}\} \tag{2}$$

$$\sum_{j=1,i\neq j}^{N} x_{ijk} = \sum_{j=1,i\neq j}^{N} x_{jik} \le 1,\ \forall i; i=\{0,1,\dots,N\}, \forall k; k=\{1,\dots,K\} \tag{3}$$

$$\sum_{k=1}^{K}\sum_{j=0,j\neq i}^{N} x_{ijk} = 1,\ \forall i; i=\{1,\dots,N\} \tag{4}$$

$$\sum_{k=1}^{K}\sum_{i=0,i\neq j}^{N} x_{ijk} = 1,\ \forall j; j=\{1,\dots,N\} \tag{5}$$

$$\sum_{i=1}^{N}\sum_{p=1}^{P}Q_{ip}v_p\sum_{j=0}^{N}x_{ijk} \le q_k\ ,\ \forall k; k=\{1,\dots,K\} \tag{6}$$

$$\sum_{i=0}^{N}\sum_{j=0,j\neq i}^{N} x_{ijk}(t_{ij}+g_{ik}+\omega_{ik}) \le \tau_k\ ,\ \forall k; k=\{0,1,\dots,K\} \tag{7}$$

$$z_{0k} = \omega_{0k} = g_{0k} = 0,\ \forall k; k=\{1,\dots,K\} \tag{8}$$

$$\sum_{k=1}^{K}\sum_{i=0,j\neq i}^{N} x_{ijk}(a_i+t_{ij}+g_{ik}+\omega_{ik}) \le a_j,\ \forall j; j=\{i,\dots,N\} \tag{9}$$

$$e_i \le (a_i + \omega_{ik}) \le l_i ,\ \forall i ; i = \{1,\dots,N\}, \forall k ; k = \{i,\dots,K\} \quad (10)$$

$$\sum_{p=1}^{P} (S_{ip} + Q_{ip}) V_p \le F_i^B ,\ \forall i ; i = \{1,\dots,N\} \quad (11)$$

$$\sum_{p=1}^{P} n V_p (Q_p - \frac{Q_p D_p}{R_p}) \le F^V \quad (12)$$

$$1 \le L_{ip} \le UL_{ip}$$

$$Q_{ip}, n \ge 0; x_{ijk} \in \{0,1\},\ \forall i, j ; i, j = \{1,\dots,N\}$$

III. SOLVING ALGORITHM

Murthy and deb [8] presented NSGA for improving the problems of multi-objective optimization algorithm. In this algorithm they used Goldberg non-dominant criterion for determining the rank of solutions. High sensitivity of NSGA to the parameters of share fitness prompted Deb et all., to introduce a better algorithm that called NSGA-II.

For the presented model the algorithm chromosome contains the value of production $\left(Q_p^{Chr}\right)$, production cycle of each product $\left(T_p^{Chr}\right)$, Lead-time $\left(L_{ip}^{Chr}\right)$, safety coefficient $\left(k'_{ip}\right)$ and the routes $\left(x_{ijk}^{Chr}\right)$. A schematic chromosome for 3 products, 5 buyers and 3 vehicles presented in figure 2.

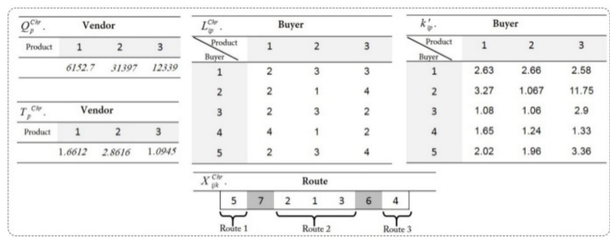


Fig. 2: Chromosome structure of presented model.

IV. NUMERICAL EXAMPLE

A numerical example with 8 buyers, 3 products and three vehicles presented in this section. All the parameters of this problem presented in tables 1 to 7 and the table 8 contain the results of NSGA-II algorithm. And the Pareto front of this problem presented in figure 3.

TABLE 1.

PARAMETERS OF PROBLEM

A_{ip}	Product		
Buyer	1	2	3
1	26	26	24
2	27	29	22
3	19	30	20

D_{ip}	Product		
Buyer	1	2	3
1	4117	3757	4100
2	4736	3993	4270
3	1519	2802	4475

C_{ip}^{B}	Product		
Buyer	1	2	3
1	25	20	27
2	28	39	40

Sigma	Buyer		
product	1	2	3
1	20	18	10
2	16	20	11
3	11	17	19

4	25	23	28
5	25	17	24
6	17	17	23
7	16	19	29
8	22	18	19
9	30	19	27
10	20	28	27
11	24	18	21
12	18	29	24
13	27	20	16
14	19	18	15
15	23	19	23

4	3275	1335	1337
5	2878	1916	2599
6	1047	4654	2039
7	2348	4304	2726
8	1648	4304	2726
9	4177	3153	4643
10	2245	4985	1727
11	3114	1312	2055
12	1662	2771	1582
13	3408	1426	1544
14	2052	4848	4478
15	3616	1018	3319

3	32	35	20
4	25	30	38
5	32	32	39
6	34	24	36
7	24	29	22
8	22	40	25
9	26	31	27
10	26	30	34
11	28	24	22
12	30	30	35
13	21	33	22
14	25	34	33
15	36	28	30

4	13	10	17
5	16	19	13
6	20	20	20
7	20	17	10
8	11	18	14
9	20	18	14
10	20	14	18
11	15	17	18
12	18	11	12
13	11	17	15
14	14	10	14
15	20	13	17

Table 2.

PARAMETERS OF PROBLEM

h_{ip}	Product		
Buyer	1	2	3
1	o.1567	0.3453	0.1691
2	0.3060	0.3384	0.3533
3	0.1551	0.2933	0.1584
4	0.2105	0.2136	0.1678
5	0.2877	0.3435	0.1512
6	0.3341	0.2598	0.1683
7	0.1243	0.2052	0.2307
8	0.3788	0.3817	0.1933
9	0.3327	0.3628	0.3770
10	0.2460	0.2650	0.2291
11	0.2308	0.2867	0.1554
12	0.2340	0.2761	0.3715
13	0.1919	0.1623	0.3939
14	0.2526	0.1904	0.2317
15	0.2532	0.2413	0.1333

w_{ik}	Product		
Buyer	1	2	3
1	4	3	2
2	4	2	4
3	3	5	5
4	4	3	1
5	1	3	1
6	2	5	5
7	3	2	4
8	2	3	2
9	4	5	3
10	5	1	1
11	2	5	2
12	4	1	2
13	1	4	2
14	5	3	1
15	1	4	2

g_{ik}	Product		
Buyer	1	2	3
1	2	4	4
2	4	10	8
3	1	2	5
4	6	8	5
5	5	7	7
6	10	9	10
7	9	4	4
8	5	8	9
9	9	9	8
10	2	4	10
11	4	6	1
12	10	10	4
13	10	6	7
14	8	4	3
15	7	7	3

TABLE 3.

PARAMETERS OF PROBLEM

S_{ip}	Product		
Buyer	1	2	3
1	324	220	309
2	207	476	249
3	400	493	475
4	302	214	432
5	359	421	440
6	223	459	249
7	155	339	337
8	290	454	449
9	245	478	474
10	416	320	368
11	412	392	182
12	368	331	362
13	153	110	128
14	108	279	263
15	324	359	367

UL_{ip}	Product		
Buyer	1	2	3
1	3	5	4
2	3	3	5
3	3	3	3
4	4	4	5
5	5	3	4
6	5	4	4
7	3	4	5
8	4	5	4
9	4	5	3
10	4	3	3
11	4	5	5
12	4	3	3
13	3	4	3
14	4	4	4
15	3	5	5

$C_{ip}^{V}(L_{ip})$	Product		
Buyer	1	2	3
1	16	20	16
2	11	15	10
3	19	15	12
4	16	13	13
5	13	19	19
6	15	14	10
7	14	11	10
8	10	18	11
9	12	14	17
10	11	12	18
11	12	14	17
12	12	11	14
13	14	11	16
14	10	20	13
15	19	20	18

TABLE 4.

PARAMETERS OF PROBLEM

t_{ij}	Buyer															
Buyer	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Depo
1	0	11	26	11	26	11	29	14	10	11	21	17	14	26	20	19
2	11	0	24	24	14	19	6	6	18	25	29	8	19	17	5	13
3	26	24	0	18	9	20	11	22	22	24	16	7	10	28	8	26
4	11	24	18	0	7	30	5	25	26	27	7	15	11	25	16	28
5	26	14	9	7	0	20	19	8	27	21	14	18	15	6	11	8
6	11	19	20	30	29	0	17	17	13	28	14	7	25	15	11	15
7	29	6	11	5	19	17	0	14	26	5	6	9	21	24	21	16
8	14	6	22	25	8	17	14	0	25	7	29	25	17	16	16	12
9	10	18	22	26	27	13	26	25	0	29	27	19	21	20	10	12
10	11	25	24	27	21	28	5	7	29	0	16	9	28	30	16	7
11	21	29	16	7	14	14	6	29	27	16	0	18	7	11	25	5
12	17	8	7	15	18	7	9	25	19	9	18	0	22	15	14	30
13	14	19	10	11	15	25	21	17	21	28	7	22	0	25	23	28
14	26	17	28	25	6	15	24	16	20	30	11	15	25	0	9	11
15	20	5	8	16	11	11	21	16	10	16	25	14	23	9	0	26
Depo	19	13	26	28	8	15	16	12	12	7	5	30	28	11	26	0

TABLE 5.

PARAMETERS OF PROBLEM

	Buyer														
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
e_i	19	13	15	14	6	8	5	11	12	8	10	13	9	4	7
l_i	190	133	150	140	60	80	52	111	120	82	103	135	90	46	70
F_i^B	96687	90548	74227	87838	70852	98590	99399	93208	69444	72737	62334	89221	94142	95686	77914

TABLE 6.

PARAMETERS OF PROBLEM

	Product		
	1	2	3
D_p	41842	43883	45095
R_p	42277	44223	45314
A_{vp}	2000	1500	2500
C_p^V	15	20	10
h_{vp}	0.2	0.15	0.25
V_p	3	1	2

TABLE 7.

PARAMETERS OF PROBLEM

	machine		
	1	2	3
q_k	50000	45000	55000
τ_k	100	90	110
ct	2	4	3

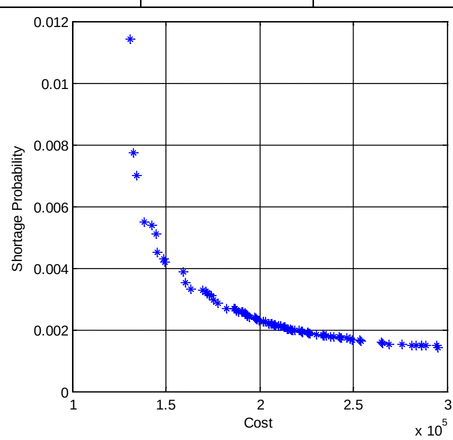


Fig. 3: Pareto front of NSGA-II.

TABLE 8.

RESULTS OF NSGA-II ALGORITHM.

	Time	Diversity	Spacing	Nos	MID
NSGA-II	85.23	1.6613e+05	1.0015e+03	100	**2.1197e+05**

V. CONCLUSION AND FURTHER STUDIES

In this paper we work on a single vendor-multi buyer bi-objectives inventory models. The first objective of this mode was minimized the cost of SCM ant the second one was maximizing the service level. Because this model is a non-linear integer programming and belongs to Np. Hard problems we used NSGA-II algorithm for solving the presented model. For further studies other multi-objective optimization algorithm like MOPSO can be used. Also a multi-vendor-multi-buyer model can be considered.

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