Almost Continuity in Soft Minimal Structure

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***Abstract*—The present paper, introduces a new class of soft mappings called soft almost M-continuous mappings in soft minimal structures and obtain some of its properties.**

***Index Terms*—Soft regular open set ,Soft minimal structure ,Soft m-open set, Soft almost continuous mappings, Soft M-continuous mappings.**

I. Introduction

In 1999, Molodtsov [14] introduced the concept of soft sets to deal with uncertainties while modelling the problems with incomplete information. A soft set is a collection of approximate descriptions of an object. He also showed how soft set theory is free from the parametrization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory and game theory. In 2011 Shabir and Naz [15] initiated the study of soft topological spaces.Theory of soft sets and soft topological spaces have been studied by some authors in [6], [8], [9], [10], [11], [14], [15], [25], [27], [28].Soft regular-open sets[5], soft semi-open sets[12],soft preopen sets [2], soft α-open sets [4],soft β-open sets [3],soft b-open sets [1] play an important role in generalizations of continuity in soft topological spaces.In the present paper,we introduce a new class of soft mappings called soft almost M-continuous mappings which contains all the classes of soft continuous mappings and investigate several properties and characterizations of this mappings.

II. Preliminaries

Let U is an initial universe set , E be a set of parameters , P(U) be the power set of U and A $\subseteq$ E.

Definition 2.1: [14] A pair (F, A) is called a soft set over U, where F is a mapping given by F: A $\rightarrow$ P(U). In other words, a soft set over U is a parameterized family of subsets of the universe U. For all e $\in$ A, F(e) may be considered as the set of e-approximate elements of the soft set (F, A).

Definition 2.2: [10] For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B), denoted by (F, A) $\subseteq$ (G,B), if:

(a) A $\subseteq$ B and
(b) F(e) $\subseteq$ G(e) for all e $\in$ E.

Definition 2.3: [10] Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal denoted by (F, A) = (G, B) If (F, A) $\subseteq$ (G, B) and (G, B) $\subseteq$ (F, A).

Definition 2.4: [11] The complement of a soft set (F, A), denoted by $(F,A)^c$, is defined by $(F,A)^c$ = (F^c, A), where F^c : A $\rightarrow$ P(U) is a mapping given by F^c(e) = U $-$ F(e), for all e $\in$ E.

Definition 2.5: [10] Let a soft set (F, A) over U.

(a) Null soft set denoted by ϕ if for all e $\in$ A, F (e) = ϕ.
(b) Absolute soft set denoted by $\widetilde{U}$, if for each e $\in$ A, F(e) = U.

Clearly, $\widetilde{U}^c$ = ϕ and ϕ^c = $\widetilde{U}$.

Definition 2.6: [6] Union of two sets (F, A) and (G, B) over the common universe U is the soft (H, C), where C = A $\cup$ B, and for all e $\in$ C,

$$H(e) = \begin{cases} F(e), & if e \in A - B \\ G(e), & if e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

Definition 2.7: [6] Intersection of two soft sets (F, A) and (G, B) over a common universe U, is the soft set (H, C) where C = A $\cap$ B and H(e) = F(e) $\cap$ G(e) for each e $\in$ E.

Let X and Y be an initial universe sets and E and K be the non empty sets of parameters, S(X, E) denotes the family of all soft sets over X and S(Y, K) denotes the family of all soft sets over Y.

Definition 2.8: [15] A subfamily τ of S(X,E) is called a soft topology on X if:

1) $\widetilde{\phi}$, $\widetilde{X}$ belong to τ.
2) The union of any number of soft sets in τ belongs to τ.
3) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. The members of τ are called soft open sets in X and their complements called soft closed sets in X.

Definition 2.9: If (X ,τ, E) is soft topological space and a soft set (F, E) over X.

(a) The soft closure of (F, E) is denoted by Cl(F,E) is defined as the intersection of all soft closed super sets of (F, E) [15].
(b) The soft interior of (F, E) is denoted by Int(F,E) is defined as the soft union of all soft open subsets of (F, E) [27].

Definition 2.10: [27] The soft set (F, E)$\in$ S(X, E) is called a soft point if there exist x $\in$ X and e $\in$ E such that F(e) = {x} and F(e') = ϕ for each e' $\in$ E – {e}, and the soft point (F, E) is denoted by $(x_e)_E$.

Definition 2.11: [27] The soft point $(x_e)_E$ is said to be in the soft set (G, E), denoted by $(x_e)_E \in$ (G, E) if $(x_e)_E \subset$ (G,E).

Definition 2.12: [25], [4], [7], [5], [1], [3] A soft set (F,E) in a soft topological space (X,τ,E) is said to be :

(a) Soft regular open if (F,E)= Int(Cl(F,E)).

(b) Soft α-open if (F,E) $\subseteq$ Int(Cl(Int(F,E))).

(c) Soft semi-open if (F,E) $\subseteq$ Cl(Int(F,E)).

(d) Soft pre-open if (F,E) $\subseteq$ Int(Cl(F,E)).

(e)Soft b-open if (A, E) $\subset$ Int(Cl(A, E)) $\cup$ Cl(Int (A, E)).

(f)Soft β-open if (A, E) $\subset$ Cl(Int(Cl(A, E))).

The complement of soft α-open set (resp. soft semi-open set, soft pre-open,soft b-open, soft β-open) set is called soft α-closed(resp. soft semi-closed ,soft pre-closed,soft b-closed, soft β-closed) set.
[18]

Remark 2.13: [4], [25], [1] (a) Every soft regular open (resp. soft regular closed) set is soft open (resp. closed), every soft open (resp. soft closed) set is soft α-open (resp. soft α-closed) ,every soft α-open (resp. soft α-closed) set is soft pre-open (resp. pre-closed) and soft semi-open (resp. semi-closed) but the converses may not be true.

(b)The concepts of soft semi-open (resp. soft semi-closed) and soft pre-open (resp.soft pre-closed)sets are independent to each other.

(c)Every soft pre-open (resp. pre-closed) and soft semi-open (resp. semi-closed) is soft b-open(resp. soft b-closed) set and every soft b-open(resp. soft b-closed) set is soft β-open(resp. soft β-closed) set but the converses may not be true.

Definition 2.14: [9] Let S(X,E) and S(Y,K) be families of soft sets. Let u: X $\to$ Y and p: E $\to$ K be mappings. Then a mapping f_{pu}: S(X, E) $\to$ S(Y, K) is defined as :

(i)Let (F, A) be a soft set in S(X, E). The image of (F, A) under f_{pu}, written as f_{pu} (F, A) = (f_{pu}(F), p(A)), is a soft set in S(Y,K) such that

$$f_{pu}(F)(k) = \begin{cases} \bigcup_{e\in p^{-1}(k)\bigcap A} u(F(e)), & p^{-1}(k)\bigcap A \neq\phi \\ \phi, & p^{-1}(k)\bigcap A =\phi \end{cases}$$

For all k $\in$ K.

(ii) Let (G , B) be a soft set in S(Y , K). The inverse image of (G , B) under f_{pu}, written as

$$f^{-1}_{pu}(G)(e) = \begin{cases} u^{-1}G(p(e)), & p(e)\in B \\ \phi, & otherwise \end{cases}$$

For all e $\in$ E.

Definition 2.15: [25], [4], [7], [5], [1], [3] Let (X,τ,E) and (Y,υ,K) be a soft topological spaces. A soft mapping f_{pu} : (X,τ,E)$\to$ (Y,υ,K) is said to be soft continuous (resp. soft α-continuous,soft semi-continuous ,soft pre-continuous,soft b-continuous,soft β-continuous) mapping if f^{-1}_{pu}(G, K) is soft open(resp. soft α-open ,soft semi-open,soft pre-open,soft b-open,soft β-open) over X , for all soft open set(G,K) over Y.

Definition 2.16: [25], [4], [7], [5], [1], [3] Let (X,τ,E) and (Y,υ,K) be two soft topological spaces. A soft mapping f_{pu} : (X,τ,E)$\to$ (Y,υ,K) is said to be soft open(resp. soft α-open,soft semi-open,soft pre-open,soft b-open,soft β-open) mapping if f_{pu} (F, E) is soft open(resp. soft α-open ,soft semi-open,soft pre-open,soft b-open,soft β-open) over Y, for all soft open set (F, E) over X.

Remark 2.17: [4], [3], [1] (a)Every soft continuous (resp. soft open) mapping is soft α-continuous(resp. soft α-open) mapping ,every soft α-continuous (resp. soft α-open) mapping is soft pre-continuous (resp. soft pre-open) and soft semi-continuous (resp. soft semi-open) mapping but the converse may not be true.

(b)The concepts of soft semi-continuous and soft pre-continuous (resp. soft semi-open and soft pre-open) mappings are independent.

(c)Every soft pre-continuous (resp. soft pre-open) and soft semi-continuous (resp. soft semi-open) mappings are soft b-continuous and every soft b-continuous mapping is soft β-continuous mapping but the converse may not be true.

Definition 2.18: [19], [20], [21], [22], [24] A soft mapping f_{pu} : (X,τ,E) $\to$ (Y,ϑ,K) is said to be soft almost(resp. α-continuous,semi-continuous,pre-continuous,β-continuous , b-continuous) mapping if the inverse image of every soft regular open set over Y is soft open(soft α-open,soft semi-open,soft pre-open,soft β-open,b-open) over X.

Remark 2.19: [19], [20], [21], [22], [24] (a)Every soft continuous mapping is soft almost continuous but the converse may not be true.

(b)Every soft α-continuous mapping is soft almost α-continuous but the converse may not be true.

(c)Every soft almost continuous(resp. soft almost-open) mapping is soft almost α-continuous(resp. soft almost α-open) but the converse may not be true.

(d)Every soft almost α-continuous(resp. soft almost α-open) mapping is almost pre-continuous(resp. soft almost pre-open) and almost semi-continuous(resp. soft almost semi-open) but the converse may not be true.

(e) Every soft semi-continuous mapping (resp. soft semi-open) is soft almost semi-continuous(resp. soft almost semi-open) but the converse may not be true.

(f) Every soft pre-continuous(resp. soft pre-open) mapping is soft almost pre-continuous(resp. soft almost pre-open) but the converse may not be true.

(g) The concepts of soft almost semi-continuous and soft almost pre-continuous (resp. soft almost semi-open and soft almost pre-open) mappings are independent.

(h)Every soft β-continuous(resp. soft β-open) mapping is soft almost β-continuous(resp. soft almost β-open) but the converse may not be true.

(i)Every soft almost pre-continuous(resp. soft almost pre-open) and soft almost semi-continuous(resp. soft almost semi-open) mapping is soft almost β-continuous(resp. soft almost β-open) but the converse may not be true.

Definition 2.20: [16] A soft subfamily $m_{(X,E)}$ of S(X,E) over X is called a soft minimal structure (briefly soft m-structure) on X if $\phi \in m_{(X,E)}$ and $\tilde{X} \in m_{(X,E)}$.

Each member of $m_{(X,E)}$ is called a soft m-open set and complement of a soft m-open set is called a soft m-closed set.

Remark 2.21: [16] Let (X,τ,E) be a soft topological space. Then the families τ , SO(X,E), SPO(X,E),SαO(X,E), SβO(X,E), SbO(X,E), SRO(X,E), are all soft m-structures on X.

Definition 2.22: [16] Let X be a nonempty set, E be set of parameters and $m_{(X,E)}$ be a soft m-structure over X .The soft $m_{(X,E)}$-closure and the soft $m_{(X,E)}$-interior of a soft set (A,E) over X are defined as follows :

(1) $m_{(X,E)}$-Cl(A,E) = $\cap$ {(F,E) : (A,E) $\subset$ (F,E) ,$(F,E)^c \in m_{(X,E)}$ }

(2)$m_{(X,E)}$-Int(A,E) = $\cup$ {(F,E) : (F,E) $\subset$ (A,E) ,(F,E) $\in m_{(X,E)}$ }.

Remark 2.23: [16] Let (X,τ,E) be a soft topological space and (A,E) be a soft set over X. If $m_{(X,E)} = \tau$ (respectively SO(X,E), SPO(X,E), SαO(X,E), SβO(X,E), SbO(X,E), SRO(X,E)), then we have:

(1) $m_{(X,E)}$-Cl(A,E) = Cl(A,E) (resp. SCl(A,E) ,PCl(A,E),αCl(A,E),β Cl(A,E), bCl(A,E),S_θCl(A,E)),

(2) $m_{(X,E)}$-Int(A,E)= Int(A,E) (resp. SInt(A,E) ,P Int(A,E),αInt(A,E),βInt(A,E), bInt(A,E),S_θInt(A,E)).

Theorem 2.24: [16] Let S(X,E) be a family of soft sets and $m_{(X,E)}$ a soft minimal structure on X.
For soft sets (A,E) and (B,E) of X, the following holds:

(a) (i): $m_{(X,E)}$-Int$(A,E)^c$ = $(m_{(X,E)} - Cl(A,E))^c$ and (ii) : $m_{(X,E)}$-Cl$(A,E)^c$ = $(m_{(X,E)} - Int(A,E))^c$

(b) If $(A,E)^c \in m_{(X,E)}$, then $m_{(X,E)}$-Cl(A,E) = (A,E) and if (A,E) $\in m_{(X,E)}$,then $m_{(X,E)}$-Int(A,E) = (A,E).

(c) $m_{(X,E)}$-Cl(ϕ) = ϕ ,$m_{(X,E)}$-Cl($\tilde{X}$) = $\tilde{X}$, $m_{(X,E)}$-Int(ϕ) = ϕ ,$m_{(X,E)}$-Int($\tilde{X}$) = $\tilde{X}$.

(d) If (A,E) $\subset$ (B,E), then $m_{(X,E)}$-Cl(A,E) $\subset$ $m_{(X,E)}$-Cl(B,E) ,$m_{(X,E)}$-Int(A,E) $\subset$ $m_{(X,E)}$-Int(B,E).

(e) (A,E) $\subset$ $m_{(X,E)}$-Cl(A,E) and $m_{(X,E)}$-Int(A,E) $\subset$ (A,E)

(f) $m_{(X,E)}$-Cl($m_{(X,E)}$-Cl(A,E)) = $m_{(X,E)}$-Cl(A,E) and $m_{(X,E)}$-Int($m_{(X,E)}$-Int(A,E)) = $m_{(X,E)}$-Int(A,E)

III. Soft Almost M-Continuous Mappings

Definition 3.1: A soft mapping f_{pu} : (X,$m_{(X,E)}$) $\rightarrow$ (Y,ϑ,K) soft almost M-continuous for each soft point $(x_e)_E$ over X and each soft regular open set (V,K) over Y containing $f_{pu}((x_e)_E)$, there exists soft m-open set(U,E) over X containing $(x_e)_E$ such that f_{pu}(U,E) $\subset$ (V,K).

Remark 3.2: Let (X,τ,E) and (Y,ϑ,K) be two soft topological spaces over X and Y respectively. If $m_{(X,E)} = \tau$ (resp. SαO(X,E), SSO(X,E),SPO(X,E),SβO(X,E),SbO(X,E)) . A soft mapping f_{pu} : (X ,τ,E) $\rightarrow$ (Y,ϑ,K) soft almost α-continuous (resp. soft almost semi-continuous, soft almost pre-continuous,soft almost β-continuous, soft almost b-continuous) for each soft point $(x_e)_E$ over X and each soft regular open set (V,K) over Y containing $f_{pu}((x_e)_E)$, there exists soft α-open(resp. soft semi-open ,soft pre-open, soft β-open,soft b-open) set(U,E) over X containing $(x_e)_E$ such that f_{pu}(U,E) $\subset$ (V,K).

Theorem 3.3: Let f_{pu} : (X,$m_{(X,E)}$) $\rightarrow$ (Y,ϑ,K) be a soft mapping. Then the following conditions are equivalent:

(a)f_{pu} soft almost M-continuous.

(b)For each soft point $(x_e)_E$ over X and each soft open set (V,K) over Y containing $f_{pu}((x_e)_E)$, there exists soft m-open set (U,E) over X containing $(x_e)_E$ such that f_{pu}(U,E) $\subset$ Int(Cl(V,K)).

(c) f_{pu}^{-1}(V,K) be a soft m-open set over X, for every soft regular open set (V,K) over Y.

Proof: It is obvious.

Corollary 3.4: Let (X,τ,E) and (Y,ϑ,K) be two soft topological spaces over X and Y respectively. Let f_{pu} : (X,τ,E) $\rightarrow$ (Y,ϑ,K) be a soft mapping. Then the following conditions are equivalent:

(a)f_{pu} soft almost α-continuous (resp. soft almost semi-continuous, soft almost pre-continuous,soft almost β-continuous, soft almost b-continuous)

(b)For each soft point $(x_e)_E$ over X and each soft open set (V,K) over Y containing $f_{pu}((x_e)_E)$, there exists soft α-open(resp. soft semi-open ,soft pre-open, soft β-open,soft b-open) set (U,E) over X containing $(x_e)_E$ such that f_{pu}(U,E) $\subset$ Int(Cl(V,K)).

(c) f_{pu}^{-1}(V,K) be a soft α-open(resp. soft semi-open ,soft pre-open, soft β-open,soft b-open) set over X, for every soft regular open set (V,K) over Y.

Remark 3.5: Every soft M-continuous mapping is soft almost M-continuous but the converse may not be true.

Theorem 3.6: Let f_{pu} : (X,$m_{(X,E)}$) $\rightarrow$ (Y,ϑ,K) be a soft mapping. Then the following conditions are equivalent:

(a) f_{pu} is soft almost M-continuous.

(b) f_{pu}^{-1} (G,K) is soft m-closed set in X for every soft regular closed set (G,K) over Y.

(c) f_{pu}^{-1} (A,K) $\subset$ $m_{(X,E)}$-Int(f_{pu}^{-1}(Int(Cl(A,K)))) for every soft open set (A,K) over Y.

(d) $m_{(X,E)}$-Cl(f_{pu}^{-1}(Cl(Int(G,K)))) $\subset$ f_{pu}^{-1}(G,K) for every soft closed set (G,K) over Y.

(e) For each soft point $(x_e)_E$ over X and each soft regular open set (G,K) over Y containing f_{pu} $((x_e)_E)$,there exists a soft m-open set (F,E) over X such that $(x_e)_E \in$ (F,E) and (F,E) $\subset$ f_{pu}^{-1}(G,K).

(f) For each soft point $(x_e)_E$ over X and each soft regular open set (G,K) over Y containing $f_{pu}((x_e)_E)$,there exists a soft m-open set (F,E) over X such that $(x_e)_E \in$ (F,E) and f_{pu}(F,E) $\subset$ (G,K).

Proof: (a) $\Leftrightarrow$ (b) Since f_{pu}^{-1} $((G,K)^C)$ = $(f_{pu}^{-1}(G,K))^C$ for every soft set (G,K) over Y.

(a)$\Rightarrow$(c) Since (A,K) is soft open set over Y, (A,K) $\subset$ Int(Cl(A,K)) and hence, f_{pu}^{-1} (A,K) $\subset$ f_{pu}^{-1}(Int(Cl(A,K))). Now Int(Cl(A,K)) is a soft regular open set over Y. By (a), f_{pu}^{-1} (Int(Cl(A,K))) is soft m-open set over X. Thus, f_{pu}^{-1} (A,K) $\subset$ f_{pu}^{-1} (Int(Cl(A,K))) = $m_{(X,E)}$-Int(f_{pu}^{-1}(Int(Cl(A,K)))).

(c)$\Rightarrow$(a) Let (A,K) be a soft regular open set over Y, then we have f_{pu}^{-1} (A,K) $\subset$ $m_{(X,E)}$-Int(f_{pu}^{-1}(Int(Cl(A,K)))) = $m_{(X,E)}$-Int(f_{pu}^{-1}(A,K)). Thus, f_{pu}^{-1} (A,K) = $m_{(X,E)}$-Int(f_{pu}^{-1}(A,K)) shows that f_{pu}^{-1} (A,K) is a soft m-open set over X.

(b)⇒(d) Since (G,K) is soft closed set over Y, Cl(Int(G,K)) $\subset$ (G,K) and f_{pu}^{-1}(Cl(Int (G,K))) $\subset$ f_{pu}^{-1} (G,K). Cl(Int(G,K)) is soft regular closed set over Y. Hence, f_{pu}^{-1}(Cl(Int(G,K) is soft m-closed set over X.Thus, $m_{(X,E)}$-Cl(f_{pu}^{-1}(Cl(Int(G,K)))) = f_{pu}^{-1}(Cl(Int(G,K))) $\subset$ f_{pu}^{-1} (G,K).

(d)⇒(b) Let (G,K) be a soft regular closed set over Y,then we have $m_{(X,E)}$-Cl(f_{pu}^{-1}(G,K)) = $m_{(X,E)}$-Cl(f_{pu}^{-1}(Cl(Int(G,K)))) $\subset$ f_{pu}^{-1} (G,K) .Thus, $m_{(X,E)}$-Cl(f_{pu}^{-1}(G,K)) $\subset$ f_{pu}^{-1} (G,K), shows that f_{pu}^{-1} (G,K) is soft m-closed set over X.

(a)⇒(e) Let $(x_e)_E$ be a soft point over X and (G,K) be a soft regular open set over Y such that $f_{pu}((x_e)_E) \in$ (G,K), Put (F,E) = f_{pu}^{-1}(G,K) .Then by (a), (F,E) is soft m-open set, $(x_e)_E \in$ (F,E) and (F,E) $\subset$ f_{pu}^{-1}(G,K).

(e)⇒(f) Let $(x_e)_E$ be a soft point over X and (G,K) be a soft regular open set over Y such that $f_{pu}((x_e)_E)$. By (e) there exists a soft m-open set (F,E) such that $(x_e)_E \in$ (F,E), (F,E) $\subset$ f_{pu}^{-1}(G,K). And so , we have $(x_e)_E \in$ (F,E), f_{pu}(F,E) $\subset$ f_{pu}(f_{pu}^{-1}(G,K)) $\subset$ (G,K).

(f)⇒(a) Let (G,K) be a soft regular open set over Y and $(x_e)_E$ be a soft point over X such that $(x_e)_E \in f_{pu}^{-1}$(G,K). Then $f_{pu}((x_e)_E) \in f_{pu}($ f_{pu}^{-1}(G,K)) $\subset$(G,K). By (f) ,there exists a soft m-open set (F,E) such that $(x_e)_E \in$ (F,E) and f_{pu}(F,E) $\subset$(G,K) .This shows that $(x_e)_E \in$ (F,E) $\subset$ f_{pu}^{-1}(G,K). It follows that f_{pu}^{-1}(G,K) is soft m-open set and hence f_{pu}^{-1} is soft almost M-continuous.

Corollary 3.7: Let (X,τ,E) and (Y,ϑ,K) be two soft topological spaces over X and Y respectively. Let f_{pu} : (X,τ,E) $\rightarrow$ (Y,ϑ,K) be a soft mapping. Then the following conditions are equivalent: (a) f_{pu} is soft almost α-continuous (resp. soft almost semi-continuous, soft almost pre-continuous,soft almost β-continuous, soft almost b-continuous)

(b) f_{pu}^{-1} (G,K) is soft soft α-closed(resp. soft semi-closed ,soft pre-closed, soft β-closed,soft b-closed)set in X for every soft regular closed set (G,K) over Y.

(c) f_{pu}^{-1} (A,K) $\subset$ αInt(f_{pu}^{-1}(Int(Cl(A,K))))(resp. sInt(f_{pu}^{-1}(Int(Cl(A,K)))),pInt(f_{pu}^{-1}(Int(Cl(A,K)))),βInt(f_{pu}^{-1}(Int(Cl(A,K)))),bInt(f_{pu}^{-1}(Int(Cl(A,K)))))for every soft soft α-open(resp. soft semi-open ,soft pre-open, soft β-open,soft b-open) set (A,K) over Y.

(d) mCl(f_{pu}^{-1}(Cl(Int(G,K))))(resp. αCl(f_{pu}^{-1}(Cl(Int(G,K)))), sCl(f_{pu}^{-1}(Cl(Int(G,K)))), pCl(f_{pu}^{-1}(Cl(Int(G,K)))) , βCl(f_{pu}^{-1}(Cl(Int(G,K)))), bCl(f_{pu}^{-1}(Cl(Int(G,K))))) $\subset$ f_{pu}^{-1}(G,K) for every soft soft α-open(resp. soft semi-open ,soft pre-open, soft β-open,soft b-open) set (G,K) over Y.

(e) For each soft point $(x_e)_E$ over X and each soft regular open set (G,K) over Y containing f_{pu} $((x_e)_E)$,there exists a soft α-open(resp. soft semi-open ,soft pre-open, soft β-open,soft b-open) set (F,E) over X such that $(x_e)_E \in$ (F,E) and (F,E) $\subset$ f_{pu}^{-1}(G,K).

(f) For each soft point $(x_e)_E$ over X and each soft regular open set (G,K) over Y containing $f_{pu}((x_e)_E)$,there exists a soft soft α-open(resp. soft semi-open ,soft pre-open, soft β-open,soft b-open) set (F,E) over X such that $(x_e)_E \in$ (F,E) and f_{pu}(F,E) $\subset$ (G,K).

IV. Soft Almost M-Open Mappings

Definition 4.1: A soft mapping f_{pu} : (X,τ,E) $\rightarrow$ (Y,$m_{(Y,K)}$) is said to be soft almost m-open if for each soft regular open set (F,E) over X, f_{pu}(F,E) is soft m-open in Y.

Remark 4.2: Let (X,τ,E) and (Y,ϑ,K) be two soft topological spaces over X and Y respectively. If $m_{(Y,K)} = \vartheta$ (resp. SαO(Y,K), SSO(Y,K),SPO(Y,K),SβO(Y,K),SbO(Y,K)). A soft mapping f_{pu} : (X ,τ,E) $\rightarrow$ (Y,ϑ,K) is said to be soft almost α-open(resp. soft almost semi-open, soft almost pre-open,soft almost β-open, soft almost b-open) if for each soft regular open set (F,E) over X, f_{pu}(F,E) is soft α-open(resp. soft semi-open, soft pre-open,soft β-open, soft b-open) in Y.

Remark 4.3: Every soft M-open mapping is soft almost M-open but the converse may not be true.

Theorem 4.4: Let soft mapping f_{pu} : (X,τ,E) $\rightarrow$ (Y,$m_{(Y,K)}$) be soft almost M-open mapping. If (G,K) is soft set over Y and (F,E) is soft regular closed set of X containing f_{pu}^{-1} (G,K) then there is a soft m-closed set (A,K) over Y containing (G,K) such that f_{pu}^{-1} (A,K) $\subset$ (F,E).

Proof: Let (A, K) = $(f_{pu}(F,E)^C)^C$. Since f_{pu}^{-1} (G,K) $\subset$ (F,E) we have $f_{pu}(F,E)^C$ $\subset$ (G,K). Since f_{pu} is soft almost M-open then (A,K) is soft m-closed set of Y and f_{pu}^{-1} (A,K) = (f_{pu}^{-1} $(f_{pu}(F,E)^C)^C$ $\subset$ $((F,E)^C)^C$ = (F,E). Thus, f_{pu}^{-1} (A,K) $\subset$ (F,E).

Corollary 4.5: Let (X,τ,E) and (Y,ϑ,K) be two soft topological spaces over X and Y respectively. Let f_{pu} : (X,τ,E) $\rightarrow$ (Y,ϑ,K) be a soft almost α-open(resp. soft almost semi-open, soft almost pre-open,soft almost β-open, soft almost b-open) .If (G,K) is soft set over Y and (F,E) is soft regular closed set of X containing f_{pu}^{-1} (G,K) then there is a soft α-closed(resp. soft semi-closed, soft pre-closed,soft β-closed, soft b-closed) set (A,K) over Y containing (G,K) such that f_{pu}^{-1} (A,K) $\subset$ (F,E).

References

[1] M. Akdag , A. Ozkan, soft b-open sets and soft b-continuous functions ,Math Sci 8:124 DOI 10.1007/s40096-014-0124-7 (2014).

[2] *M. Akdag , A Ozkan* : On Soft Preopen Sets and Soft Pre Separation Axioms,Gazi University Journal of Science GU J Sci ,**27**(4) (2014) 1077–1083.

[3] M. Akdag ,A Ozkan , On Soft $\beta-$Open Sets and Soft $\beta-$continuous Functions ,The Scientific World Journal, Article ID 843456,6 pages(2014).

[4] M. Akdag,A. Ozkan , On soft α-open sets and soft α-continuous functions, Abstract and Applied Analysis http//dx.doi.org/101155/2014/891341 2014 Article ID 891341 ,(2014) ,7 pages.

[5] I. Arockiarani and A. A. Lancy, Generalized soft gβ- closed sets and soft gsβ-closed sets in soft topological spaces. International Journal Of Mathematical Archive, 4(2), (2013) 1-7.

[6] M. Irfan Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Comput. Math. Appl. 57, 2009, 1547-1553.

[7] Bin Chen , Soft semi-open sets and related properties in soft topological spaces, Appl.Math.Inf.Sci. 7, (1) (2013), 287–294.

[8] S. Hussain and B. Ahmad, Some properties of soft topological spaces, Comput. Math. Appl.,62 ,2011, 4058-4067.

[9] A. Kharral and B. Ahmad, Mappings on soft classes, New Math. Nat. Comput. ,7(3),2011, 471-481.

[10] P.K. Maji, R. Biswas, R. Roy, Soft set theory, Comput. Math. Appl., 45 ,2003, 555-562.

[11] P. Majumdar and S. K. Samanta, Similarity measure of soft sets, New Math. Nat. Comput. 4(1) (2008) 1-12.

[12] J. Mahanta and P. K. Das , On soft topological space via semi open and semi closed soft sets , Kyungpook Math. J . 54, (2014), 221–236.

[13] W. K. Min, A note on soft topological spaces, Comput. Math. Appl.,62 ,2011,3524-3528.

[14] D. Molodtsov, Soft set theory first results, Comput. Math. Appl. ,37 ,1999, 19-31. Gnanambal Ilango and Mrudula Ravindran, On Soft Preopen Sets in Soft Topological Spaces.International Journal of Mathematics Research ISSN 0976- 5840 Volume 5 Number 4 (2013),pp. 399-499.

[15] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl.,61 ,2011, 1786-1799.

[16] S. S. Thakur and Alpa Singh Rajput, On soft M-continuous mappings, The Journal of Fuzzy Mathematics, 25(2)(2017), 313-326.

[17] S. S. Thakur and Alpa Singh Rajput, Connectedness between soft sets, New Mathematics and Natural Computation, 14(1) (2018),1-19.

[18] S.S.Thakur and Alpa Singh Rajput,Extremally Disconnectedness in Soft Topological Spaces,(Submitted).

[19] S.S.Thakur and Alpa Singh Rajput, Soft Almost Continuous Mappings ,International Journal of Advances in Mathematics,2017 (1), 2017, 22-29.

[20] S. S. Thakur and Alpa Singh Rajput, Soft almost alpha-continuous mappings, Journal of Advanced Studies in Topology, (Published).

[21] S. S. Thakur, Alpa Singh Rajput and M.R. Dhakad, Soft almost semi-continuous mappings ,Malaya J. Mat., 5(2)(2017), 395-400.

[22] S. S. Thakur and Alpa Singh Rajput, Soft almost Pre-continuous mappings,The Journal of Fuzzy Mathematics, 26(2)(2018),439-449.

[23] S.S. Thakur, Alpa Singh Rajput, Soft almost b-continuous mappings,Journal of New Theory,23 ,93-104,2018.

[24] S.S.Thakur and Alpa Singh Rajput, Soft Almost β-Continuous Mappings,(Submitted).

[25] Saziye Yuksel.,Soft Regular Generalized Closed Sets in Soft Topological Spaces Int. Journal of Math. Analysis, 8(8), 2014 , 355-367.

[26] Y.Yumak, A.K. Kayamakci, Soft β-open sets and their application, arXiv:1312.6964(2013).

[27] I. Zorlutana, N. Akdag and W.K. Min, Remarks on soft topological spaces, Ann. Fuzzy Math. Inf., 3(2) ,2012, 171-185.

[28] Idris Zorlutuna, Hatice Cakir, On Continuity of Soft Mappings, Appl.Math. Inf. Sci. 9(1), 2015, 403-409.