Theory of FSMs Sharing Internal States

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***Abstract*—**The paper presents theory of two FSMs that share internal states or input symbols. Modified definitions of FSMs are presented and two reciprocal meaningful transformations between the pair of FSMs sharing input symbols and the pair sharing internal states are constructed. Transformations are based on Mealy to Moore FSM and Moore to Mealy FSM transformations. Conditions of transformations are discussed and minimization of FSMs is shown. Practical aspects are discussed.

***Keywords*—** Finite State Machines, sharing inputs, transformation, sharing states.

I. Introduction

FSMs are used as general models of behavior on the system analysis level, not only in digital design. FSMs model behavior of interfaces between two members of alliance [1][2][3]. We consider a pair of FSMs, each of them describes one direction of communication (Fig. 1). We suppose the standard definition of Mealy FSM:

$$A = \langle Q, X, Y, \delta, \lambda, Q_0 \rangle \tag{1}$$

where Q is an internal state set, X is an input symbol set, Y is an output symbol set, $\delta : Q \times X \to Q$ is a transition function, $\lambda : Q \times X \to Y$ is an output function, Q_0 is an initial state.

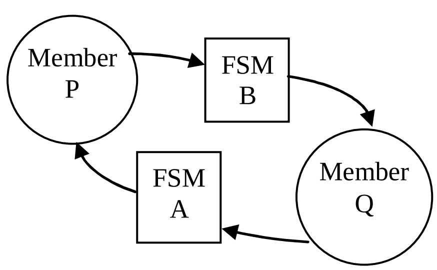

Fig. 1 FSMs model interfaces in system alliance

FSMs are generally separated, but we consider the situations when two machines share some elements (internal states, input symbols, transition functions etc.). It may be necessary to adapt the definition (1) of the FSM if spoken about sharing. Firstly, we must specify the meaning of the term "sharing" and define which types of sharing make sense. We narrow our reflection on the case of sharing inputs and internal states. The aim is to find the algorithm that transforms two "independent" FSMs to the pair sharing internal states. But what does it mean that two FSMs share their internal states? We see the sense of "sharing" such the one FSM has knowledge about the state of the second one and it uses this knowledge. If the FSM A knows the internal state of FSM B, it has knowledge about the previous input of FSM B. So, we can formulate the task: to replace the pair of independent FSMs which has two same inputs (each one from P and Q members) by the pair with independent inputs and sharing internal states. We try to find the reciprocal transformations between pairs of FSMs.

We consider two Mealy FSMs – A and B that share input symbols, Fig. 2 and 3(a). The X_A and X_B are set of input symbols related formally to the A and B FSMs. The definitions of the FSMs A, B are

$$\begin{aligned} A &= \langle Q_A, X_A, X_B, Y_A, \delta_A, \lambda_A, Q_{A0} \rangle \\ B &= \langle Q_B, X_A, X_B, Y_B, \delta_B, \lambda_B, Q_{B0} \rangle \end{aligned} \tag{2}$$

Transition and output functions must be extended and they are defined:

$$\begin{aligned} \delta_A &: Q_A \times X_A \times X_B \to Q_A \\ \delta_B &: Q_B \times X_A \times X_B \to Q_B \\ \lambda_A &: Q_A \times X_A \times X_B \to Y_A \\ \lambda_B &: Q_B \times X_A \times X_B \to Y_B \end{aligned} \tag{3}$$

We suppose indices of all symbols start from 0, for example the set of internal states contains elements

$$Q_A = \{Q_{A0}, \dots, Q_{An-1}\}, n = |Q_A| \ . \tag{4}$$

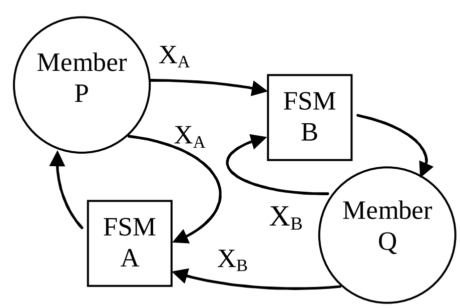

Fig. 2 FSMs with sharing inputs

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Each of transformed FSMs (A’ and B’) has its own input symbol set. Transformed FSMs share internal states that is FSM A’ has a knowledge about internal state of *B´* one and vice versa.

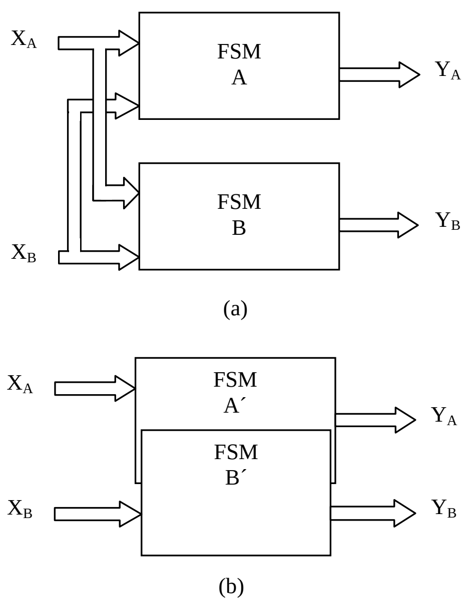

Fig. 3 replacement of FSM pair

The extended definition of FSMs A´ and B’ are

$$A'=\langle Q_A', Q_B', X_A, Y_A, \delta_A', \lambda_A', \mathrm{Q}_{A0}\rangle$$
$$B'=\langle Q_B', Q_A', X_B, Y_B, \delta_B', \lambda_B', \mathrm{Q}_{B0}\rangle \tag{5}$$

Transition and output functions of transformed FSMs have the second internal state as a new argument instead of adjacent input symbol:

$$\delta_A' : Q_A' \times Q_B' \times X_A \to Q_A'$$
$$\delta_B' : Q_B' \times Q_A' \times X_B \to Q_B'$$
$$\lambda_A' : Q_A' \times Q_B' \times X_A \to Y_A$$
$$\lambda_B' : Q_B' \times Q_A' \times X_B \to Y_B \tag{6}$$

II. Transformations of Pair of FSMs Sharing Input Symbols

The transformation is derived from the principle of the conversion of Mealy FSM to the Moore one. There are two well known algorithms of this transformation, the first one uses the disruption of current states [5] and the second one makes a disintegration of target states. We have inspired by the former method.

A. Transformation of Mealy to Moore FSM

We have a Mealy FSM with Q_A set of internal states and an equivalent transformed Moore FSM with the set Q_A’. The new set of internal states contains states Q_{Ai} which are marked with $[Q_i,X_i]$ symbols ($Q_{Ai} = [Q_i,X_i]$). In other words, we insert a new state $[Q_i,X_i]$ into Q_A’ set if a transition from Q_i conditioned by X_i input symbol exists in the original Mealy FSM. The output function of Moore FSM is transformed by the formula:

$$\lambda_{Moore}([\mathrm{Q_i}, \mathrm{X_i}]) = \lambda_{Mealy}(\mathrm{Q_i}, \mathrm{X_i}) \; . \tag{7}$$

The conversion of the transition function is given after this manner:

$$\delta_{Moore}([\mathrm{Q_i}, \mathrm{X_i}], \mathrm{X_j}) = [\delta_{Mealy}(\mathrm{Q_i}, \mathrm{X_i}), \mathrm{X_j}] \tag{8}$$

B. Transformation of pair of FSMs

Two FSMs A and B are given with shared inputs, according to the definitions (2) and (3). Our assumption is the both ones are fully defined and synchronized by the common clock signal. Transformed pair of FSMs A’, B’ has a form according to the definition (5), (6).

The transformation is built on disintegration of present states. Steps are related to the FSM A, they are analogical if we make a transformation of FSM B.

Output and transition functions of FSM A are dependent on the present internal state Q_{Ai} and on the present input symbols $\mathrm{X_A} \in X_A$ and $\mathrm{X_B} \in X_B$. The transformed FSM A’ lost a piece of information about the present (actual) input symbol X_B as a consequence of separation of input symbols. But it is able to obtain this knowledge by analyzing the target (next) internal state after the transition of the both FSMs (after the clock transition). We have to apply the algorithm of the disruption of source states of FSM B on that account so that the present state of B’ implies the previous input symbol X_{Bi}. We insert a new state $[Q_{Bi},X_{Bi}]$ into Q_B’ for each transition from the state $\mathrm{Q_{Bi}} \in Q_B$ initiated by input symbol $\mathrm{X_{Bi}} \in X_B$. So if the FSM B’ is in the present state $[Q_{Bi},X_{Bi}]$ it means the previous input symbol of B’ was exactly X_{Bi}. The same disruption is applied to the FSM A.

The output function of FSM A assigns output symbol by $\lambda_A(\mathrm{Q_{Ai}}, \mathrm{X_{Ai}}, \mathrm{X_{Bi}}) = \mathrm{Y_{Ai}}$. Remember that the information about input symbol X_{Bi} is known after the transition of B’ to the $[Q_{Bi},X_{Bi}]$. Because the both FSMs A’ and B’ have common synchronization the FSM A’ makes transition simultaneously to the new state $[Q_{Aj},X_{Ai}]$. This state $[Q_{Aj},X_{Ai}]$ of the A’ implicates previous input symbol X_{Ai} and the previous state Q_{Aj}. Due to these facts, both states $[Q_{Aj},X_{Ai}]$ and $[Q_{Bi},X_{Bi}]$ implicate commonly the output symbol Y_{Ai} of the FSM A´. Hence, the position of the correct output symbol Y_{Ai} is shifted to the right in the output sequence, analogous to the case if the Mealy FSM is transformed to the Moore one. So, the outputs of FSM A´ and B´ are independent on the current input symbols and they are like Moore machines. The output functions are reduced:

$$\begin{aligned} &\lambda_A{}': Q_A{}' \times Q_B{}' \rightarrow Y_A \\ &\lambda_B{}': Q_B{}' \times Q_A{}' \rightarrow Y_B \end{aligned} \tag{9}$$

If the FSMs are in the initial states the output is empty because they do not know the input symbol of the second FSM.

The internal state set of FSM A' is

$$\begin{aligned} &Q_A{}' = \{ Q_{A0} \cup \bigcup [Q_{Ai}, X_{Aj}], \\ &i = 0 \ldots |Q_A| - 1,\ j = 0 \ldots |X_A| - 1 \} \end{aligned} \tag{10}$$

where the Q_{A0} is the initial state.

The output of the A' is determined by equation:

$$\begin{aligned} &\lambda_A{}'([Q_{Ak}, X_{Ai}], [Q_{Bl}, X_{Bj}]) = \lambda_A(Q_{Ak}, X_{Ai}, X_{Bj}) \\ &\lambda_A{}'(Q_{AO}, Q_{B0}) = \varepsilon \text{ (ε is an empty symbol)} \end{aligned} \tag{11}$$

Now we construct transition function. It is clear that a transition from initial states is defined:

$$\delta_{A'}(Q_{A0}, Q_{B0}, X_{Ai}) = [Q_{A0}, X_{Ai}] \tag{12}$$

We must derive transitions from the other states. It is evident that if the transition is initiated by X_{Au} input symbol the next state contains X_{Au} symbol in its marking:

$$\delta_{A'}([Q_{Ai}, X_{Aj}], [Q_{Bm}, X_{Bn}], \mathbf{X_{Au}}) = [Q_{Aj}, \mathbf{X_{Au}}] \tag{13}$$

The remaining part of the next state is derived as follows. The original FSMs accept input sequences (X_{Ai},X_{Bm}), (X_{Aj},X_{Bn}) , …, (X_{Ak},X_{Bo}), (X_{Al},X_{Bp}). The sequence of internal states of FSM A and FSM B are:

Q_{A0}, Q_{Ai}, Q_{Aj}, …,Q_{Ak}, Q_{Al},
Q_{B0}, Q_{Bm}, Q_{Bn}, …,Q_{Bo}, Q_{Bp}.

The transformed FSMs accept the same input sequences, each accepts a part belonging to its input symbol set, ie. FSM A' accepts X_{Ai}, X_{Aj}, …, X_{Ak}, FSM B' accepts X_{Bm}, X_{Bn}, …, X_{Bo}. We wrote shorter sequences by intent. New sequences of internal states of FSM A' and FSM B' must be:

Q_{A0}, $[Q_{A0},X_{Ai}]$, $[Q_{Ai},X_{Aj}]$, $[Q_{Aj},X_{Au}]$, …, $[Q_{Ak},X_{Ak}]$
Q_{B0}, $[Q_{B0},X_{Bm}]$, $[Q_{Bm},X_{Bn}]$, $[Q_{Bn},X_{Bu}]$, …, $[Q_{Bo},X_{Bo}]$

The situation is shown on Fig. 4 and in previous sequence. The Q_{Aj} is determined with the transition function of original FSM A with arguments Q_{Ai},X_{Aj}, X_{Bn}. Thus, the transitions are transformed

$$\begin{aligned} &\delta_{A'}([Q_{Ai}, X_{Aj}], [Q_{Bm}, X_{Bn}], \mathbf{X_{Au}}) = \\ &= [\delta_A(Q_{Ai}, X_{Aj}, X_{Bn}), \mathbf{X_{Au}}] \end{aligned} \tag{14}$$

The transition and output functions do not depend on the element Q_{Bm} of the state of the FSM B´.

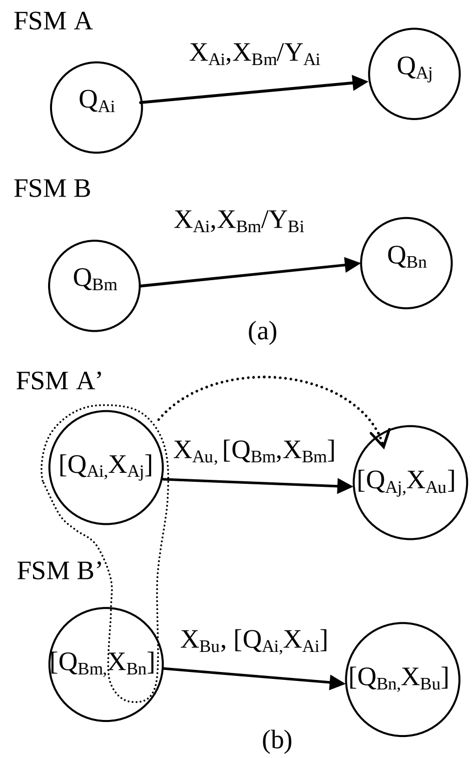

Fig. 4 transformation of transitions

Example:

Transform two FSMs A and B. Sets of input and output symbols are given: X_A = {a,b}, X_B = {c,d}, Y_A = {X,Y,Z}, Y_B = {U,V,W}. The transition graphs are on Fig. 5.

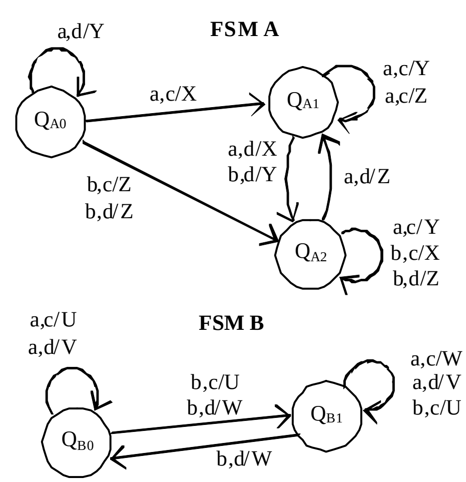

Fig. 5 an example of the transformation

The transformed FSM A’ has seven states Q_{A0}, $[Q_{A0},a]$, $[Q_{A0},b]$, $[Q_{A1},a]$, $[Q_{A1},b]$, $[Q_{A2},a]$, $[Q_{A2},b]$. The transformed FSM B’ has five states Q_{B0}, $[Q_{B0},c]$, $[Q_{B0},d]$, $[Q_{B1},c]$, $[Q_{B1},d]$.

The calculation of one output and one transition of each transformed FSM is shown:

$$\lambda_A\acute{}([Q_{A0},a],[Q_{B0},c]) = \lambda_A(Q_{A0},a,c) = X$$
$$\delta_A\acute{}([Q_{A0},a],[Q_{B0},c],a) = [\delta_A(Q_{A0},a,c),a] = [Q_{A1},a]$$
$$\lambda_B\acute{}([Q_{B0},c],[Q_{A0},a]) = \lambda_B(Q_{B0},a,c) = U$$
$$\delta_B\acute{}([Q_{B0},c],[Q_{A0},a],c) = [\delta_B(Q_{B0},a,c),c] = [Q_{B0},c]$$

The transformed FSM B´ is on Fig. 6. FSM A’ is not shown because the picture is large.

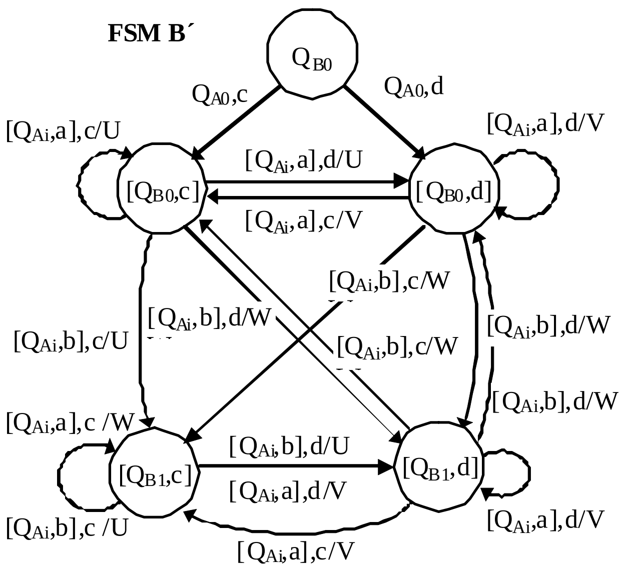

Fig. 6 transformed FSM B´

The functionality of the transformed machines was verified by the simulation. An example of output is printed in Table I.

Table I an example of output

Input A:	aaaabbb
Input B:	ddcccdd
Output of FSM A:	YYXYZYZ
Output of FSM B:	VVUUUWW
Transformed FSMs	
Output of FSM A´:	εYYXYZY
Output of FSM B´:	εVVUUUW

III. Back transformation of FSMs sharing internal states

The back transformation can be based on the Moore machine to Mealy machine conversion if we assume the fact that these machines are like Moore automata. It means output functions only depend on internal states (9).

Transition functions $\delta_A\acute{}, \delta_B\acute{}$ have the form according to (6). Other characteristics of FSMs were discussed in section II.

A. Transformation of Moore to Mealy FSM

Transformation of Moore to Mealy FSM is simpler than the reverse transformation. The set of internal state does not change during the transformation process, the transition function is the same. Output function of transformed Mealy FSM is constructed using this rule:

$$\begin{aligned} &\lambda_{Mealy}(Q_i, X_i) = \lambda_{Moore}(\delta_{Moore}(Q_i, X_i)) \\ &\delta_{Mealy}(Q_i, X_i) = \delta_{Moore}(Q_i, X_i) = Qj \end{aligned} \qquad (15)$$

In other words an output assigned to the pair Q_i, X_i of Mealy FSM is an output symbol assigned to the Q_j state where Q_i is a predecessor of Q_j.

B. Back Transformation of pair of FSMs

Let us consider two machines FSM A’ and FSM B’ share internal states. Internal states are marked like in (10) . Q_{A0}, Q_{B0} are initial states.

We transform them back to the pair FSM A, FSM B sharing input symbols. Machines FSM A and FSM B have identical set of internal states as FSM A’, FSM B’, $|Q_A| = |Q_A'|$, $|Q_B| = |Q_B'|$.

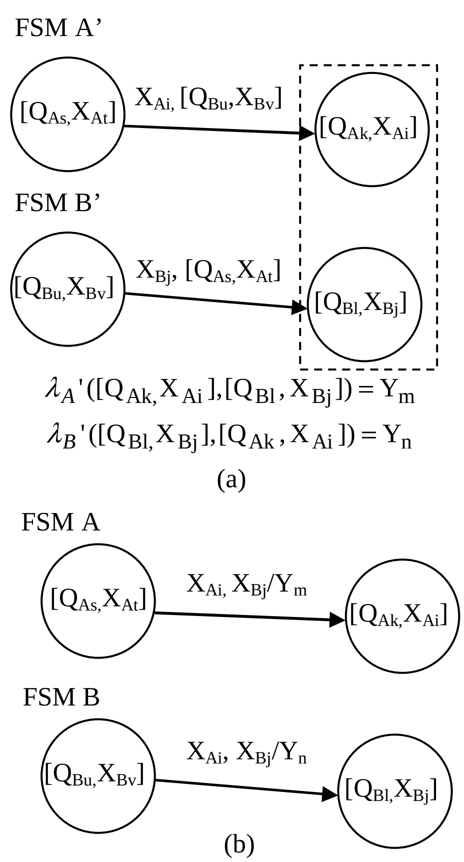

Fig. 7 derivation of the transformation

Fig. 7 shows the principle how to construct the transformation. Let us suppose the FSM A' is in the state $[Q_{As},X_{At}]$. We must change the transition of FSM A'

$\delta_A'([Q_{As},X_{At}],[Q_{Bu},X_{Bv}],X_{Ai})$

to the transition

$\delta_A([Q_{As},X_{At}],X_{Ai},X_{Bj})$

of the back transformed FSM A and to find its output function $\lambda_A([Q_{As},X_{At}],X_{Ai},X_{Bj})$. The target state of $[Q_{As},X_{At}]$ is the same, hence the transition function is defined:

$$\begin{aligned}&\delta_A([Q_{As},X_{At}],X_{Ai},X_{Bj})=\\&\delta_A'(([Q_{As},X_{At}],[Q_{Bu},X_{Bv}],X_{Ai})\end{aligned} \tag{16}$$

The construction of the output function is more complicated. We must find correspondent transition of FSM B' from $[Q_{Bu},X_{Bv}]$ state which is conditioned by X_{Bj} input and $[Q_{As},X_{At}]$ state. Let be the target state $[Q_{Bl},X_{Bj}]$. Output function is defined

$$\begin{aligned}&\lambda_A([Q_{As},X_{At}],X_{Ai},X_{Bj})=\\&\lambda_A'([Q_{Ak},X_{Ai}],[Q_{Bl},X_{Bj}])\end{aligned}. \tag{17}$$

But, $\delta_A'([Q_{As},X_{At}],[Q_{Bu},X_{Bv}],X_{Ai})$ transition is replaced with multiple transitions for all X_{Bj}. One transition for X_{Bv} is selected because previous input symbol was X_{Bv}.

Outputs and transition from the initial state are transformed with extra formula:

$$\begin{aligned}&\lambda_A(Q_{A0},X_{Ai},X_{Bj})=\\&\lambda_A'(\delta_A'(Q_{A0},Q_{B0},X_{Ai}),\\&\qquad\delta_B'(Q_{B0},Q_{A0},X_{Bj}))\end{aligned} \tag{18}$$

The transition from the initial state is defined:

$$\begin{aligned}&\delta_A(Q_{A0},X_{Ai},X_{Bj})=\\&\delta_A'(Q_{A0},Q_{B0},X_{Ai})\end{aligned} \tag{19}$$

for all X_{Bj}.

If transformed FSMs A' and B' are constructed using previous algorithm, machines FSM A and FSM B are deterministic. If states of FSM A', FSM B' are not marked with pairs $[Q_{Bi}, X_{Bi}]$ but more generally like Q_{Bi}' the necessary condition is that all transition to Q_{Bi}' state are initiated only by one input symbol. Then the transformation can be proceeded.

We show two steps of transformations of FSM B'. For initial states:

$\lambda_B(Q_{B0},a,c)=\lambda_B'([Q_{B0},c],[Q_{A0},a])=U$

$\delta_B(Q_{B0},a,c)=[\delta_B'(Q_{B0},Q_{A0},c]=[Q_{B0},c]$

We transform the transition

$\delta_B'([Q_{B0},c],[Q_{A0},a],d)$

Corresponding transition of FSM A' is

$\delta_A'([Q_{A0},a],[Q_{B0},c],a)=[Q_{A1},a]$

Hence,

$\lambda_B([Q_{B0},c],a,d)=\lambda_B'([Q_{B0},d],[Q_{A1},a])=V$

$\delta_B([Q_{B0},c],a,d)=[Q_{B0},d]$

Back transformation of FSM B' is on Fig. 8.

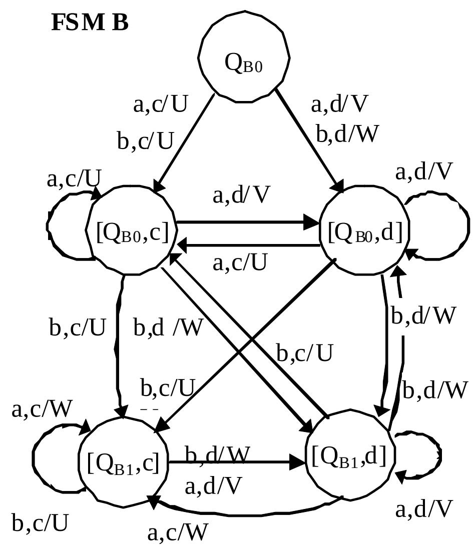

Fig. 8 back transformation of the FSM B'

C. Minimization

The back transformed FSM is not minimized. We can use the same techniques if classical FSM is minimized. We look at equivalent states which have the same output table (Table II). Three states Q_{B0}, $[Q_{B0},c]$ $[Q_{B0},d]$ can be equivalent and two states $[Q_{B1},c]$ $[Q_{B1},d]$ because their outputs are the same. States $[Q_{B0},c]$ $[Q_{B0},d]$ are equivalent because their transitions are the same and they can be combined (Table III). States $[Q_{B1},c]$ $[Q_{B1},d]$ could be combined if $[Q_{B1},c]$ $[Q_{B0},c]$ were equivalent and $[Q_{B1},d]$ $[Q_{B0},d]$ were equivalent. But mentioned pairs have different outputs so these pairs can not be combined.

Table II non-minimized FSM B

	Transitions				Outputs			
	a,c	a,d	b,c	b,d	a,c	a,d	b,c	b,d
Q_{B0}	$[Q_{B0},c]$	$[Q_{B0},d]$	$[Q_{B0},c]$	$[Q_{B0},d]$	U	V	U	W
$[Q_{B0},c]$	$[Q_{B0},c]$	$[Q_{B0},d]$	$[Q_{B1},c]$	$[Q_{B1},d]$	U	V	U	W
$[Q_{B0},d]$	$[Q_{B0},c]$	$[Q_{B0},d]$	$[Q_{B1},c]$	$[Q_{B1},d]$	U	V	U	W
$[Q_{B1},c]$	$[Q_{B1},c]$	$[Q_{B1},d]$	$[Q_{B1},c]$	$[Q_{B1},d]$	W	V	U	W
$[Q_{B1},d]$	$[Q_{B1},c]$	$[Q_{B1},d]$	$[Q_{B0},c]$	$[Q_{B0},d]$	W	V	U	W

Table III minimized FSM B

	Transitions				Outputs			
	a,c	a,d	b,c	b,d	a,c	a,d	b,c	b,d
Q_{B0}	[Q_{B0},c]	[Q_{B0},c]	[Q_{B0},c]	[Q_{B0},c]	U	V	U	W
[Q_{B0},c]	[Q_{B0},c]	[Q_{B0},c]	[Q_{B1},c]	[Q_{B1},d]	U	V	U	W
[Q_{B1},c]	[Q_{B1},c]	[Q_{B1},d]	[Q_{B1},c]	[Q_{B1},d]	W	V	U	W
[Q_{B1},d]	[Q_{B1},c]	[Q_{B1},d]	[Q_{B0},c]	[Q_{B0},c]	W	V	U	W

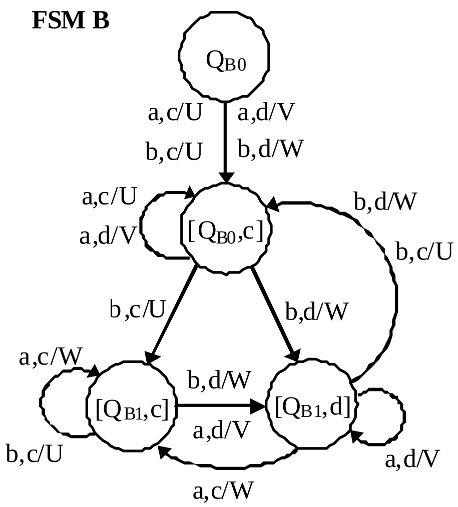

Fig. 9 minimized FSM B

IV. Practical aspects

Real application of this approach has not been constructed yet. Practical aspect of this transformation can be viewed in dynamics system. Let us suppose two machines according to Fig. 2 and Fig. 3(a). Machines are implemented in one reconfigurable chip. If one connection between one member and one FSM is corrupted, the pair of FSM sharing input symbol is *dynamically* transformed to the pair sharing internal states and this fault is masked. The approach can increase the reliability of interfaces in alliances.

V. Conclusion

The transformation of pair of FSMs with common inputs to the pair of FSMs having separated inputs and sharing internal states and the back transformation are presented. The effect is that the coupling between FSMs is shifted from the outside of FSMs into inside in the former transformation or from the inside into outside in the second transformation. The disadvantage of the transformations is the growth of count of internal states.

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