Theoretical Analysis of BER Performance of Optical ZCZ-CDMA System

Takahiro MATSUMOTO, Hideyuki TORII and Shinya MATSUFUJI

Abstract**—** In this paper, we analyze the bit error rate (BER) performance of optical code division multiple access (CDMA) system based on the optical zero-correlation zone (ZCZ) code with zero-correlation zone $4n-2$ or 1 which is called the optical ZCZ-CDMA system, by the electrical and optical processing over the optical fiber and space by computer simulation and theoretical formula. In the optical fiber transmission of this system, the effect of the avalanche photodiode (APD) noise, thermal noise and co-channel interference are considered. Additionally, the effects of the scintillation and background noise are considered in the optical space transmission of this system. The optical ZCZ code, which is a set of pairs of binary and biphase sequences consisting of 1 or -1 with zero-correlation zone, may be able to provide optical CDMA communication system suppressed co-channel interference, and we have proposed the compact construction of a code generator and a bank of matched filters for this code. As a result, the BER performance of optical ZCZ-CDMA system over the optical space go down compared to that over the optical fiber, and the BER performance of this system by the electrical processing go down compared to that by the optical processing, and this system can't remove completely co-channel interference.

Keywords**—** optical communication, optical code division multiple access (CDMA), optical ZCZ code, scintillation, avalanche photodiode (APD) noise, co-channel interference.

I. Introduction

THE optical code division multiple access (CDMA) communication system can expect a high speed communication because this system is able to use a wide band [1], [2], [3]. In addition, this system can give the multiple access without changing the wave length. In the optical CDMA system, a pseudo-noise (PN) sequence used to this system decides the communication performance [4], [5], [6], [7], [8], and an optical orthogonal code is often generally used as a PN sequence. The optical orthogonal code [9] is a set of binary sequences consisting of 1 or 0, and its crosscorrelation property has low correlation value. Therefore, the optical CDMA communication system based on optical orthogonal code goes down the bit error rate (BER) performance. On the other hand, the optical zero-correlation zone (ZCZ) code [10] is a set of pairs of a binary sequence consisting of 1 or 0 and a biphase sequence consisting of 1 or -1 with zero-correlation zone. Therefore, this code may be able to provide optical CDMA communication system suppressed co-channel interference [11], [12], [13], and we have proposed the compact construction of a code generator [14] and a bank of matched filters [15], [16] for this code. In the low- and hi-speed optical communications, the receiver is processed by electrical and optical signals, respectively. The former and latter are called the electrical and optical processing, respectively. The communication performance of this system is influenced from these processing.

In this paper, we analyze the bit error rate (BER) performance of optical CDMA system based on the optical ZCZ code with zero-correlation zone $4n-2$ or 1 which is called the optical ZCZ-CDMA system, by the electrical and optical processing over the optical fiber and space by computer simulation and theoretical formula. In the optical fiber transmission of this system, the effect of the avalanche photodiode (APD) noise, thermal noise and co-channel interference are considered. Additionally, in the optical space transmission of this system, the effects of the scintillation and background noise are considered.

II. Optical ZCZ Code

A. Definition of optical ZCZ code

Let a_N^j be a biphase sequence of length N whose elements take 1 or -1, written as

$$a_N^j = (a_{N,0}^j, a_{N,1}^j, \cdots, a_{N,i}^j, \cdots, a_{N,N-1}^j), \quad a_{N,i}^j \in \{1,-1\}. \tag{1}$$

Similarly, let $\hat{a}_N^{j,d}$ be a binary sequence of length N whose elements take 1 or 0, written as

$$\hat{a}_N^{j,d} = (\hat{a}_{N,0}^{j,d}, \hat{a}_{N,1}^{j,d}, \cdots, \hat{a}_{N,i}^{j,d}, \cdots, \hat{a}_{N,N-1}^{j,d}), \quad \hat{a}_{N,i}^{j,d} \in \{1,0\}, \quad d \in \{1,0\}, \tag{2}$$

where i denotes $i \bmod N$. Let A be a set of M pairs consisting of a biphase sequence a_N^j and a binary sequence $\hat{a}_N^{j,d}$, written as

$$A = \{(a_N^1, \hat{a}_N^{1,d}), (a_N^2, \hat{a}_N^{2,d}), \cdots, (a_N^j, \hat{a}_N^{j,d}), \cdots, (a_N^M, \hat{a}_N^{M,d})\}, \tag{3}$$

where M is the number of sequences in a sequence family and is called family size.

A periodic correlation function between sequences a_N^j and $\hat{a}_N^{j',d}$ at shift i' is defined by

$$\rho_{a_N^j, \hat{a}_N^{j',d}, i'} = \sum_{i=0}^{N-1} a_{N,i}^j \hat{a}_{N,(i+i') \bmod N}^{j',d}. \tag{4}$$

Manuscript received December 30, 2012.

T. Matsumoto is associate professor with University Evaluate Department, Yamaguchi University, Yamaguchi-shi, Yamaguchi, Japan e-mail: matugen@yamaguchi-u.ac.jp.

H. Torii is associate professor with Department of Information Network and Communication, Kanagawa Institute of Technology, Atsugi-shi, Kanagawa, Japan e-mail:torii@nw.kanagawa-it.ac.jp.

S. Matsufuji is professor with Graduate School of Science and Engineering, Yamaguchi University, Ube-shi, Yamaguchi, Japan e-mail: s-matsu@yamaguchi-u.ac.jp.

In this paper, the above correlation function $\rho_{a_N^j,\hat{a}_N^{j',d},i'}$ is called the auto-correlation function for $j=j'$ and the cross-correlation function for $j \neq j'$. The set is called an optical ZCZ code [10] if the periodic auto- and cross-correlation functions satisfy

$$\rho_{a_N^j,\hat{a}_N^{j',d},i'} = \begin{cases} w & ; i'=0, j=j', d=0, \\ -w & ; i'=0, j=j', d=1, \\ 0 & ; i'=0, j\neq j', \\ 0 & ; 1\le |i'| \le z, \end{cases} \quad (5)$$

$$w = \sum_{i=0}^{N-1} \hat{a}_{N,i}^{j',d} < N, \quad (6)$$

where w is the number of the nonzero elements in each sequence and z is a zero-correlation zone. The optical ZCZ codes are bounded by

$$M \le \frac{N}{z+1}. \quad (7)$$

B. Optical ZCZ code with $z = 4n-2$

Let b_{N_1} be a Legendre sequence [9] of length $N_1 = 4n_1 - 1$ with positive n_1 or an M-sequence of length $N_1 = 2^{n_1} - 1$ with $n_1 \ge 2$, whose elements take values of 1 or -1, written as

$$b_{N_1} = (b_{N_1,0}, b_{N_1,1}, \cdots, b_{N_1,i}, \cdots, b_{N_1,N_1-1}), \quad b_{N_1,i} \in \{1,-1\} \quad (8)$$

with $\sum_{i=0}^{N_1-1} b_{N_1,i} = 1$. A binary sequence $\hat{b}_{N_1,i} \in \{1,0\}$ of length N_1 is given by

$$\hat{b}_{N_1,i} = \frac{1+b_{N_1,i}}{2}. \quad (9)$$

On the other hand, let $\mathbf{H}_{N_2}$ be an Hadamard matrix of order $N_2 = 2^{n_2}$ with $n_2 \ge 2$, written as

$$\mathbf{H}_{N_2} = [h_{N_2}^0, h_{N_2}^1, \cdots, h_{N_2}^j, \cdots, h_{N_2}^{N_2-1}]^T, \quad (10)$$

$$h_{N_2}^j = (h_{N_2,0}^j, h_{N_2,1}^j, \cdots, h_{N_2,i}^j, \cdots, h_{N_2,N_2-1}^j), \quad h_{N_2,i}^j \in \{1,-1\}, \quad (11)$$

where T denotes the matrix transposition, $h_{N_2}^j$ is called an Hadamard sequence and $h_{N_2,i}^0 = 1, 0 \le i \le N_2-1$. A binary sequence $\hat{h}_{N_2,i}^{j,d} \in \{1,0\}$ of length N_2 is given by

$$\hat{h}_{N_2,i}^{j,d} = \frac{1+(-1)^d h_{N_2,i}^j}{2}. \quad (12)$$

If N_1 and N_2 are relatively prime, i.e., $gcd(N_1,N_2)=1$, a biphase sequence a_N^j of length $N = N_1N_2$ is produced by

$$a_{N,i}^j = b_{N_1, i \bmod N_1} \cdot h_{N_2, i \bmod N_2}^j. \quad (13)$$

Similarly, a binary sequence $\hat{a}_N^{j,d}$ of length $N = N_1N_2$ is produced by

$$\hat{a}_{N,i}^{j,d} = \hat{b}_{N_1, i \bmod N_1} \cdot \hat{h}_{N_2, i \bmod N_2}^{j,d}. \quad (14)$$

Let A be a set of $M = N_2 - 1$ pairs consisting of a biphase sequence a_N^j and a binary sequence $\hat{a}_N^{j,d}$ of length $N = N_1N_2$, written as

$$A = \Big\{(a_N^1, \hat{a}_N^{1,d}), (a_N^2, \hat{a}_N^{2,d}), \cdots, (a_N^j, \hat{a}_N^{j,d}), \cdots, (a_N^{N_2-1}, \hat{a}_N^{N_2-1,d})\Big\}. \quad (15)$$

The periodic correlation function between a_N^j and $\hat{a}_N^{j',d}$ except when $j=j'=0$ is given by

$$\rho_{a_N^j,\hat{a}_N^{j',d},i'} = \begin{cases} \frac{(N_1+1)N_2}{4} & ; i'=0, j=j', d=0, \\ \frac{-(N_1+1)N_2}{4} & ; i'=0, j=j', d=1, \\ 0 & ; i'=0, j\neq j', \\ 0 & ; 1\le |i'| \le z = N_1 - 1, \end{cases} \quad (16)$$

where $N/4 < w = (N_1+1)N_2/4 \le N/3$. Therefore, the above set of $M = N_2 - 1$ pairs consisting of a biphase sequence a_N^j and a binary sequence $\hat{a}_N^{j',d}$ is an optical ZCZ code [10] with $z = N_1 - 1 = 4n_1 - 2$ and $M = N_2 - 1 = N/(z+1) - 1$.

As an example, we generate an optical ZCZ code of $N = N_1N_2 = 3\times 4 = 12$, $z = N_1 - 1 = 2$ and $M = N_2 - 1 = 3$. Let

$$b_3 = (+,+,-)$$

and

$$\mathbf{H}_4 = \begin{bmatrix} h_4^0 \\ h_4^1 \\ h_4^2 \\ h_4^3 \end{bmatrix} = \begin{bmatrix} + & + & + & + \\ + & - & + & - \\ + & + & - & - \\ + & - & - & + \end{bmatrix}, \quad (17)$$

where $\mathbf{H}_4$ is a Sylvester-type Hadamard matrix, $+$ and $-$ denote $+1$ and -1, respectively. From Eq. (13), we can generate bi-phase sequences a_{12}^j, as follows, respectively.

$$\begin{aligned} a_{12}^1 &= (+,-,-,-,+,+,+,-,-,-,+,+), \\ a_{12}^2 &= (+,+,+,-,+,-,-,-,-,+,-,+), \\ a_{12}^3 &= (+,-,+,+,+,+,-,+,-,-,-,-). \end{aligned}$$

Similarly, let

$$\hat{b}_3 = (+,+,0)$$

and

$$\begin{bmatrix} \hat{h}_4^{0,0} \\ \hat{h}_4^{1,0} \\ \hat{h}_4^{2,0} \\ \hat{h}_4^{3,0} \end{bmatrix} = \begin{bmatrix} + & + & + & + \\ + & 0 & + & 0 \\ + & + & 0 & 0 \\ + & 0 & 0 & + \end{bmatrix},$$

$$\begin{bmatrix} \hat{h}_4^{0,1} \\ \hat{h}_4^{1,1} \\ \hat{h}_4^{2,1} \\ \hat{h}_4^{3,1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & + & 0 & + \\ 0 & 0 & + & + \\ 0 & + & + & 0 \end{bmatrix},$$

where $+$ denotes 1. From Eq. (14), we can generate binary sequences $\hat{a}_{12}^{j}$, as follows, respectively.

$$
\begin{aligned}
\hat{a}_{12}^{1,0} &= (+,0,0,0,+,0,+,0,0,0,+,0),\\
\hat{a}_{12}^{2,0} &= (+,+,0,0,+,0,0,0,0,+,0,0),\\
\hat{a}_{12}^{3,0} &= (+,0,0,+,+,0,0,+,0,0,0,0),\\
\hat{a}_{12}^{1,1} &= (0,+,0,+,0,0,0,+,0,+,0,0),\\
\hat{a}_{12}^{2,1} &= (0,0,0,+,0,0,+,+,0,0,+,0),\\
\hat{a}_{12}^{3,1} &= (0,+,0,0,0,0,+,0,0,+,+,0).
\end{aligned}
$$

A set consisting of a bi-phase sequence a_{12}^{j} and a binary sequence $\hat{a}_{12}^{j,d}$, is an optical ZCZ code with $N=12$, $z=2$ and $M=3$.

Its auto-correlation functions are given by

$$
\begin{aligned}
\rho_{a_{12}^{1},\hat{a}_{12}^{1,0},i'} &= (4,0,0,-4,0,0,4,0,0,-4,0,0),\\
\rho_{a_{12}^{2},\hat{a}_{12}^{2,0},i'} &= (4,0,0,0,0,0,-4,0,0,0,0,0),\\
\rho_{a_{12}^{3},\hat{a}_{12}^{3,0},i'} &= (4,0,0,0,0,0,-4,0,0,0,0,0),\\
\rho_{a_{12}^{1},\hat{a}_{12}^{1,1},i'} &= (-4,0,0,4,0,0,-4,0,0,4,0,0),\\
\rho_{a_{12}^{2},\hat{a}_{12}^{2,1},i'} &= (-4,0,0,0,0,0,4,0,0,0,0,0),\\
\rho_{a_{12}^{3},\hat{a}_{12}^{3,1},i'} &= (-4,0,0,0,0,0,4,0,0,0,0,0)
\end{aligned}
$$

and its cross-correlation functions

$$
\begin{aligned}
\rho_{a_{12}^{1},\hat{a}_{12}^{2,0},i'} &= (0,0,0,0,0,0,0,0,0,0,0,0),\\
\rho_{a_{12}^{2},\hat{a}_{12}^{3,0},i'} &= (0,0,0,4,0,0,0,0,0,-4,0,0),\\
\rho_{a_{12}^{3},\hat{a}_{12}^{1,0},i'} &= (0,0,0,0,0,0,0,0,0,0,0,0),\\
\rho_{a_{12}^{1},\hat{a}_{12}^{2,1},i'} &= (0,0,0,0,0,0,0,0,0,0,0,0),\\
\rho_{a_{12}^{2},\hat{a}_{12}^{3,1},i'} &= (0,0,0,-4,0,0,0,0,0,4,0,0),\\
\rho_{a_{12}^{3},\hat{a}_{12}^{1,1},i'} &= (0,0,0,0,0,0,0,0,0,0,0,0).
\end{aligned}
$$

C. Optical ZCZ code with $z=1$

Let $\mathbf{S}_{N_2}$ be a Sylvester-type Hadamard matrix of order $N_2 = 2^{n_2}$ with $n_2 \geq 2$, written as

$$
\begin{aligned}
\mathbf{S}_{N_2} &= [s_{N_2}^{0}, s_{N_2}^{1}, \cdots, s_{N_2}^{j}, \cdots, s_{N_2}^{N_2-1}]^T, &(18)\\
s_{N_2}^{j} &= (s_{N_2,0}^{j}, s_{N_2,1}^{j}, \cdots, s_{N_2,i}^{j}, \cdots, s_{N_2,N_2-1}^{j}), &(19)\\
& \quad s_{N_2,i}^{j} \in \{1,-1\},
\end{aligned}
$$

where T denotes the matrix transposition. It can also be expressed as

$$
\begin{aligned}
\mathbf{S}_{N_2} &= \mathbf{S}_{\frac{N_2}{2}} \otimes \mathbf{S}_2, &(20)\\
\mathbf{S}_2 &= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, &(21)
\end{aligned}
$$

where the operation $\otimes$ denotes the Kronecker product. Each row $s_{N_2}^{j}$ is called a Sylvester-type Hadamard sequence.

A biphase sequence a_N^j with length $N = 2N_2$ is given by

$$
\begin{aligned}
a_{N,i}^{j} &= \alpha_{N,i} \cdot s_{N_2, i \bmod N_2}^{j}, &(22)\\
\alpha_{N,i} &= \begin{cases} s_{N_2,i \bmod N_2}^{0} = 1 & ;0 \leq i \leq \frac{N}{2}-1, \\ -s_{N_2,i \bmod N_2}^{1} = (-1)^{i+1} & ;\frac{N}{2} \leq i \leq N-1. \end{cases} &(23)
\end{aligned}
$$

From Eqs. (22) and (23), the mean value of this sequence a_N^j is given by

$$
\sum_{i=0}^{N-1} a_{N,i}^{j} = \sum_{i=0}^{N_2-1} s_{N_2,i}^{0} s_{N_2,i}^{j} - \sum_{i=0}^{N_2-1} s_{N_2,i}^{1} s_{N_2,i}^{j} = 0, \quad (24)
$$

where $j \neq 0, 1$. Therefore, a biphase sequence a_N^j is called a biphase balanced sequence.

A binary sequence $\hat{a}_N^{j,d}$ of length N is given by

$$
\hat{a}_{N,i}^{j,d} = \frac{1+(-1)^d a_{N,i}^{j}}{2}. \quad (25)
$$

Let A be a set of $M = N/2-2$ pairs consisting of a biphase sequence a_N^j and a binary sequence $\hat{a}_N^{j,d}$ of length $N = 2N_2$, written as

$$
\begin{aligned}
A &= \Big\{(a_N^2, \hat{a}_N^{2,d}), (a_N^3, \hat{a}_N^{3,d}), \cdots, (a_N^j, \hat{a}_N^{j,d}),\\
&\quad \cdots, (a_N^{\frac{N}{2}-1}, \hat{a}_N^{\frac{N}{2}-1,d})\Big\}. &(26)
\end{aligned}
$$

The periodic correlation function between a_N^j and $\hat{a}_N^{j',d}$ except when $j, j' \leq 1$ is given by

$$
\rho_{a_N^j, \hat{a}_N^{j',d}, i'} = \begin{cases} \frac{N}{2} & ; i'=0, j=j', d=0, \\ \frac{-N}{2} & ; i'=0, j=j', d=1, \\ 0 & ; i'=0, j \neq j', \\ 0 & ; i' = \pm z = \pm 1, \end{cases} \quad (27)
$$

where $w = N/2$. Therefore, the above set of $M = N/2 - 2$ pairs consisting of a biphase sequence a_N^j and a binary sequence $\hat{a}_N^{j',d}$ is an optical ZCZ code with $z=1$ and $M = N/2-2 = N/(z+1)-2$.

As an example, we generate an optical ZCZ code of $N = 2N_2 = 2 \times 4 = 8$, $z=1$ and $M = N/2-2 = 2$. From Eqs. (17), (22) and (23), we can generate bi-phase sequences a_8^j, as follows, respectively.

$$
\begin{aligned}
a_8^2 &= (+,+,-,-,-,+,+,-),\\
a_8^3 &= (+,-,-,+,-,-,+,+).
\end{aligned}
$$

From Eq. (25), we can generate binary sequences $\hat{a}_8^{j,d}$, as follows, respectively.

$$
\begin{aligned}
\hat{a}_8^{2,0} &= (+,+,0,0,0,+,+,0),\\
\hat{a}_8^{3,0} &= (+,0,0,+,0,0,+,+),\\
\hat{a}_8^{2,1} &= (0,0,+,+,+,0,0,+),\\
\hat{a}_8^{3,1} &= (0,+,+,0,+,+,0,0).
\end{aligned}
$$

A set of pairs consisting of a bi-phase sequence a_8^j and a binary sequence $\hat{a}_8^{j,d}$ is an optical ZCZ code with $N=8$, $z=1$ and $M=2$.

Its auto-correlation functions are given by

$$
\begin{aligned}
\rho_{a_8^2,\hat{a}_8^{2,0},i'} = \rho_{a_8^3,\hat{a}_8^{3,0},i'} &= (4,0,-2,0,0,0,-2,0),\\
\rho_{a_8^2,\hat{a}_8^{2,1},i'} = \rho_{a_8^3,\hat{a}_8^{3,1},i'} &= (-4,0,2,0,0,0,2,0)
\end{aligned}
$$

and its cross-correlation functions are given by

$$
\begin{aligned}
\rho_{a_8^2,\hat{a}_8^{3,0},i'} = \rho_{a_8^3,\hat{a}_8^{2,0},i'} &= (0,0,2,0,-4,0,2,0),\\
\rho_{a_8^2,\hat{a}_8^{3,1},i'} = \rho_{a_8^3,\hat{a}_8^{2,1},i'} &= (0,0,-2,0,4,0,-2,0).
\end{aligned}
$$

III. Optical ZCZ-CDMA System

The optical CDMA system based on the optical ZCZ code is called the optical ZCZ-CDMA system. In the transmitter, the binary sequence is used to send optical signal corresponding to a short pulse and the absence of light. A transmitter sends a binary sequence $\hat{a}_N^{j,d_j}$ in according to input data $d_j \in \{1, 0\}$ as optical signal. On the other hand, in the receiver, the biphase sequence is used as the reference sequence.

In a low-speed communication, the receiver is processed by an electrical signal, which is called the electrical processing. This system needs one avalanche photodiode (APD) element. Figure 1 shows optical ZCZ-CDMA system by the electrical processing. The received signal r_i is passed through a filter matched to the biphase sequence, a_N^j, which is called the electrical correlator, and is recovered to the bit data in the detector. From Eq. (4), the input of a detector R_0 is given by

$$R_0 = \sum_{i=0}^{N-1} a_{N,i}^j r_i. \tag{28}$$

Similarly, in a high-speed communication, the receiver is processed by an optical signal, which is called the optical processing. This system needs two avalanche photodiode (APD) elements because the reference sequence is the biphase sequence which takes as 1 or -1. Figure 2 shows optical ZCZ-CDMA system by the optical processing. In splitter, the received signal is switched to the optical correlator 0 and 1 according to element values 1 and -1 of the reference sequence a_N^j, respectively, and is added as $\hat{R}_0^0$ and $\hat{R}_0^1$ in the optical correlator 0 and 1, respectively. The difference output signal of APD 0 and APD 1 is recovered to the bit data in the detector. From Eq. (28), the input of a detector R_0 is given by

$$\begin{aligned} R_0 &= \sum_{i=0}^{N-1}\left(\frac{1+a_{N,i}^j}{2}\right) r_i - \sum_{i=0}^{N-1}\left(\frac{1-a_{N,i}^j}{2}\right) r_i \\ &= \hat{R}_0^0 - \hat{R}_0^1. \end{aligned} \tag{29}$$

Note that if $z = 1$ then $(1 + a_{N,i}^j)/2$ and $(1 - a_{N,i}^j)/2$ are $\hat{a}_{N,i}^{j,0}$ and $\hat{a}_{N,i}^{j,1}$ from Eq. (25), respectively.

IV. Theoretical analysis of BER performance

We theoretically analyse the BER performance of optical ZCZ-CDMA systems by the electrical and optical processing, and evaluate it by computer simulation. Table I shows specifications of theoretical analysis and computer simulation.

A. BER performance in optical fiber transmission

The theoretical formula of BER characteristics of the optical ZCZ-CDMA system by the electrical processing over the optical fiber is given by

$$\begin{aligned} p_{BER} &= \frac{1}{\sqrt{2\pi\sigma^2(m)}} \int_{-\infty}^{0} exp\left\{\frac{-(a-\mu)^2}{2\sigma^2(m)}\right\} da \\ &= \frac{1}{2} erfc\left(\frac{\mu}{\sqrt{2\sigma^2(m)}}\right), \end{aligned} \tag{30}$$

$$\mu = wGT_c\lambda_s\left(\frac{M_e - 1}{M_e}\right), \tag{31}$$

TABLE I

Specifications of theoretical analysis and computer simulation.

Spreading sequence	optical ZCZ code	
Sequence length N	24	16
Family size M	7	6
Zero correlation zone z	2	1
Num. of the nonzero elements w	4	
Num. of trials	10^6	
Bit rate	156Mbps	234Mbps
Chip rate	3,744Mcps	
Laser wavelength λ	830nm	
APD quantum efficiency η	0.6	
APD gain G	100	
APD effective ionization ratio K_{eff}	0.02	
Bulk leakage current I_b	0.1nA	
Surface leakage current I_s	10nA	
Modulation extinction ratio M_e	100	
Receiver noise temperature T_r	1,100K	
Receiver load resistor R_L	1,030 Ω	
Background noise P_b	-45 dBm	
Scintillation logarithm variance σ_s^2	0.1	

$$\begin{aligned} \sigma^2(m) &= G^2F_eT_c\left\{\lambda_s\left(w + \frac{N-w}{M_e}\right)m + N\frac{I_b}{e}\right\} \\ &\quad + N\frac{I_sT_c}{e} + N\sigma_{th}^2, \end{aligned} \tag{32}$$

$$\lambda_s = \frac{\eta P_w}{hf}, \tag{33}$$

$$P_w = \frac{1}{w}P_{bit}, \tag{34}$$

$$F_e = K_{eff}G + (1 - K_{eff})\left(\frac{2G-1}{G}\right), \tag{35}$$

$$\sigma_{th}^2 = \frac{2k_BT_rT_c}{e^2R_L}, \tag{36}$$

$$f = \frac{c}{\lambda}, \tag{37}$$

$$w = \begin{cases} \frac{(z+2)N}{4(z+1)} & ; z = 4n-2, \\ \frac{N}{2} & ; z = 1, \end{cases} \tag{38}$$

where μ and $\sigma^2(m)$ are the average and variance of the correlation output, m is the number of users, G is the APD gain, T_c is the chip duration, N is the sequence length, M_e is the modulation extinction ratio, λ_s is the average number of absorbed photons over T_c, w is the number of the nonzero elements in each sequence, F_e is the excess noise factor, I_b is the bulk leakage current, I_s is the surface leakage current, e is the elementary electric charge, σ_{th}^2 is the variance of thermal noise, η is the APD quantum efficiency, P_w is the received laser power per chip without scintillation, P_{bit} is the average received laser power per bit without scintillation, P_b is the background noise per chip duration T_c, h is Planck's constant, f is the optical frequency, K_{eff} is the APD effective ionization ratio, k_B is Boltzmann constant, T_r is the receiver noise temperature, R_L is the receiver load resistor, c is velocity of light, and λ is the laser wavelength. Note that the background noise isn't considered over the optical fiber. Figures 3 and 4 show BER performance of optical ZCZ-CDMA system with $z = 2$ and 1 by the electrical processing

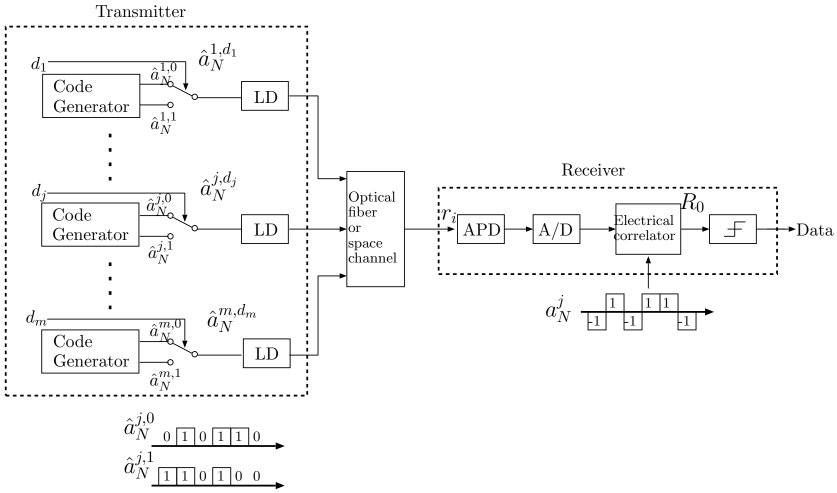

Fig. 1. Optical ZCZ-CDMA system by the electrical processing.

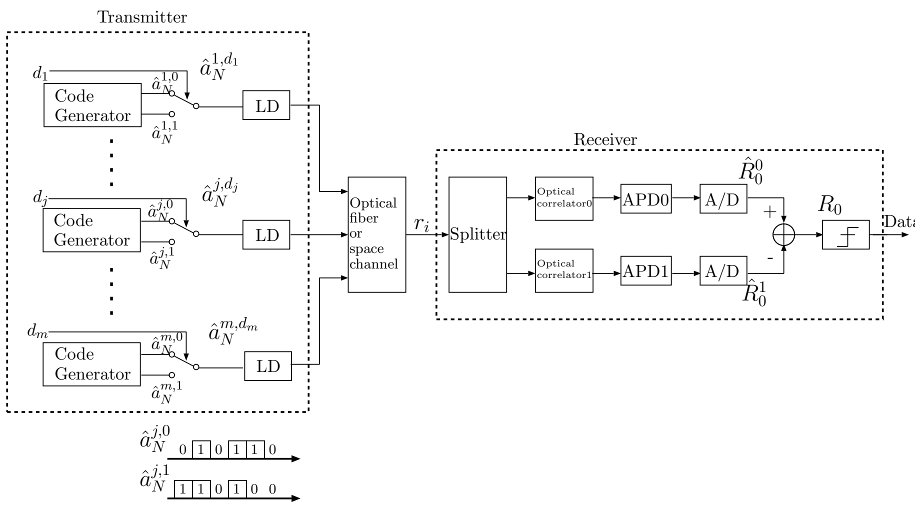

Fig. 2. Optical ZCZ-CDMA system by the optical processing.

over the optical fiber, respectively.

Similarly, the theoretical formula of BER characteristics of the optical ZCZ-CDMA system by the optical processing over the optical fiber is given by

$$\tilde{p}_{BER} = \frac{1}{2} erfc\left(\frac{\tilde{\mu}}{\sqrt{2\tilde{\sigma}^2(m)}}\right), \quad (39)$$

$$\tilde{\mu} = wGT_c\lambda_s\left(\frac{M_e-1}{M_e}\right) = \mu, \quad (40)$$

$$\tilde{\sigma}^2(m) = G^2F_eT_c\left\{\lambda_s\left(w+\frac{N-w}{M_e}\right)m + 2\frac{I_b}{e}\right\} + 2\frac{I_sT_c}{e} + 2\sigma_{th}^2. \quad (41)$$

Figures 5 and 6 show BER performance of optical ZCZ-CDMA system with $z = 2$ and 1 by the optical processing over the optical fiber, respectively.

From Figs. 3, 4, 5 and 6, the optical ZCZ-CDMA system by the electrical and optical processing over the optical fiber can't remove completely co-channel interference, and the BER performance of this system by the electrical processing over the optical fiber go down compared to that by the optical processing.

B. BER performance in optical space transmission

The theoretical formula of BER characteristics of the optical ZCZ-CDMA system by the electrical processing over the

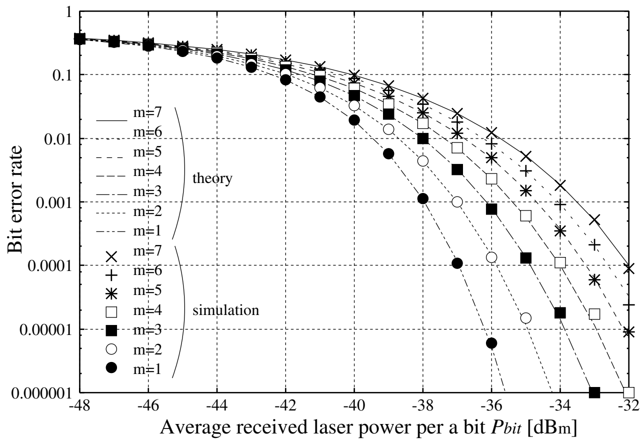

Fig. 3. BER performance of optical ZCZ-CDMA system with $z = 2$ by the electrical processing over the optical fiber.

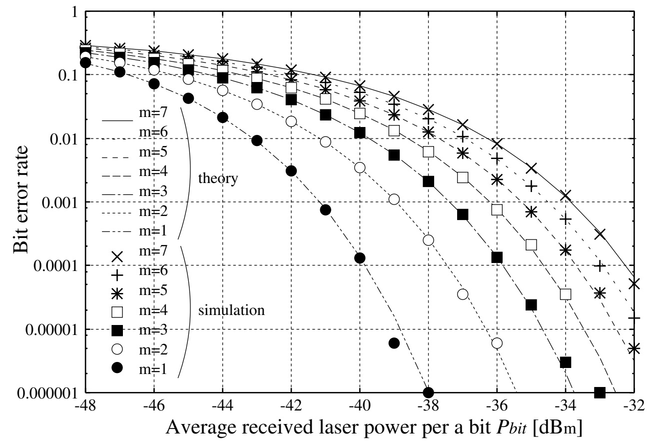

Fig. 5. BER performance of optical ZCZ-CDMA system with $z = 2$ by the optical processing over the optical fiber.

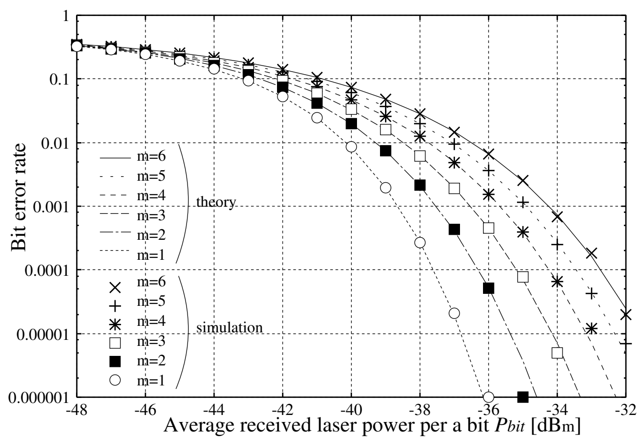

Fig. 4. BER performance of optical ZCZ-CDMA system with $z = 1$ by the electrical processing over the optical fiber.

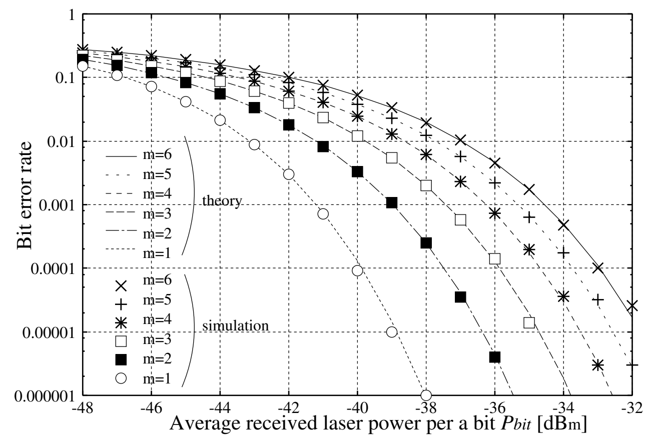

Fig. 6. BER performance of optical ZCZ-CDMA system with $z = 1$ by the optical processing over the optical fiber.

optical space is given by

$$
\begin{aligned}
P_{BER} &= \frac{1}{2}\int_0^\infty p(A_1)\int_0^\infty p(A_2)\cdots\int_0^\infty p(A_m) \\
&\quad \cdot erfc\left(\frac{\mu(A_1)}{\sqrt{2\sigma^2(A_1, A_2, \cdots, A_m)}}\right)dA_1 \\
&\quad dA_2\cdots dA_m \\
&= \frac{1}{2}\int_0^\infty p(A_1)\int_0^\infty p(A) \\
&\quad \cdot erfc\left(\frac{\mu(A_1)}{\sqrt{2\sigma^2(A_1, A)}}\right)dA_1 dA, \qquad (42)
\end{aligned}
$$

$$A = \sum_{j=2}^{m} A_j, \qquad (43)$$

$$p(A_j) = \frac{1}{\sqrt{2\pi\sigma_s^2}A_j} exp\left\{\frac{-\left(\ln A_j + \frac{\sigma_s^2}{2}\right)^2}{2\sigma_s^2}\right\}, \qquad (44)$$

$$p(A) = p(A_2) * \cdots * p(A_m), \qquad (45)$$

$$\mu(A_1) = wGT_c\lambda_s\left(\frac{M_e - 1}{M_e}\right)A_1, \qquad (46)$$

$$
\begin{aligned}
\sigma^2(A_1, A) &= G^2F_eT_c\left\{\lambda_s\left(w + \frac{N-w}{M_e}\right)(A_1 + A)\right. \\
&\quad \left. + N\lambda_b + N\frac{I_b}{e}\right\} + N\frac{I_sT_c}{e} + N\sigma_{th}^2, \qquad (47)
\end{aligned}
$$

where $p(A_j)$ is the probability density function of the scintillation A_j for jth $(1 \leq j \leq m)$ user, σ_s^2 is the scintillation logarithm variance, $\mu(A_1)$ and $\sigma^2(A_1, A)$ are the average and variance of the correlation output, respectively and $*$ denotes the convolution as

$$p(a) * p(a) = \sum_{a'=0}^{\infty} p(a')p(a - a'). \qquad (48)$$

Figures 7 and 8 show BER performance of optical ZCZ-CDMA system with $z = 2$ and 1 by the electrical processing over the optical space, respectively.

Similarly, the theoretical formula of BER characteristics of the optical ZCZ-CDMA system by the optical processing over

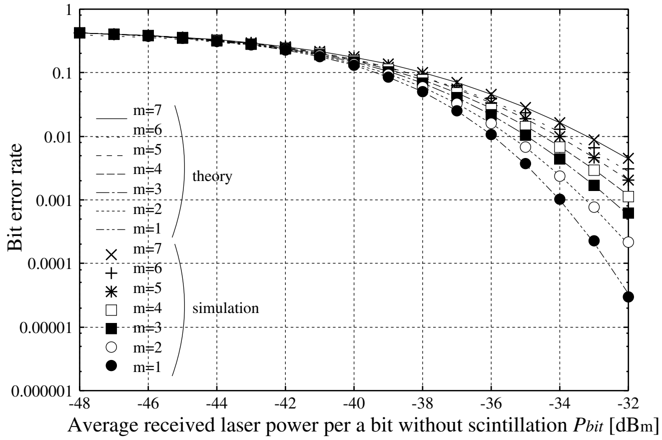

Fig. 7. BER performance of optical ZCZ-CDMA system with $z = 2$ by the electrical processing over the optical space.

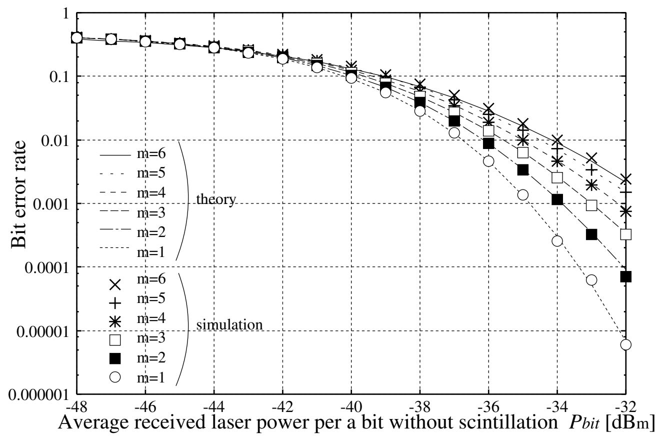

Fig. 8. BER performance of optical ZCZ-CDMA system with $z = 1$ by the electrical processing over the optical space.

the optical space is given by

$$\begin{aligned}\tilde{P}_{BER} &= \frac{1}{2}\int_0^\infty p(A_1)\int_0^\infty p(A_2)\cdots\int_0^\infty p(A_m) \\ &\quad \cdot erfc\left(\frac{\tilde{\mu}(A_1)}{\sqrt{2\tilde{\sigma}^2(A_1, A_2, \cdots, A_m)}}\right)dA_1 \\ &\quad dA_2\cdots dA_m \\ &= \frac{1}{2}\int_0^\infty p(A_1)\int_0^\infty p(A) \\ &\quad \cdot erfc\left(\frac{\mu(\tilde{A}_1)}{\sqrt{2\tilde{\sigma}^2(A_1, A)}}\right)dA_1 dA, \qquad (49)\end{aligned}$$

$$\tilde{\mu}(A_1) = wGT_c\lambda_s\left(\frac{M_e - 1}{M_e}\right)A_1 = \mu(A_1), \qquad (50)$$

$$\begin{aligned}\tilde{\sigma}^2(A_1, A) &= G^2F_eT_c\left\{\lambda_s\left(w + \frac{N-w}{M_e}\right)(A_1 + A)\right. \\ &\quad \left. + N\lambda_b + 2\frac{I_b}{e}\right\} + 2\frac{I_sT_c}{e} + 2\sigma_{th}^2. \qquad (51)\end{aligned}$$

Figures 9 and 10 show BER performance of optical ZCZ-CDMA system with $z = 2$ and 1 by the optical processing over the optical space, respectively.

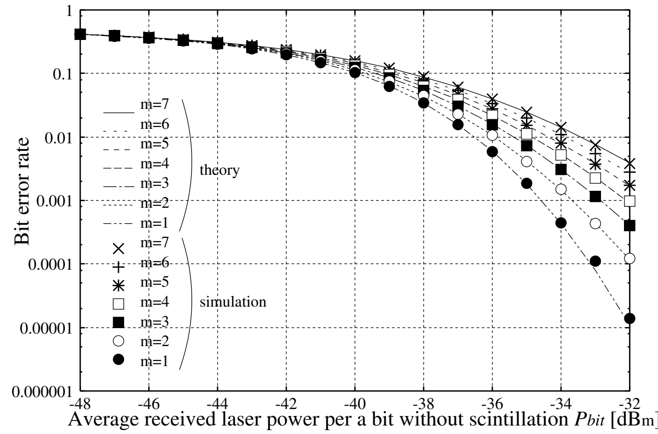

Fig. 9. BER performance of optical ZCZ-CDMA system with $z = 2$ by the optical processing over the optical space.

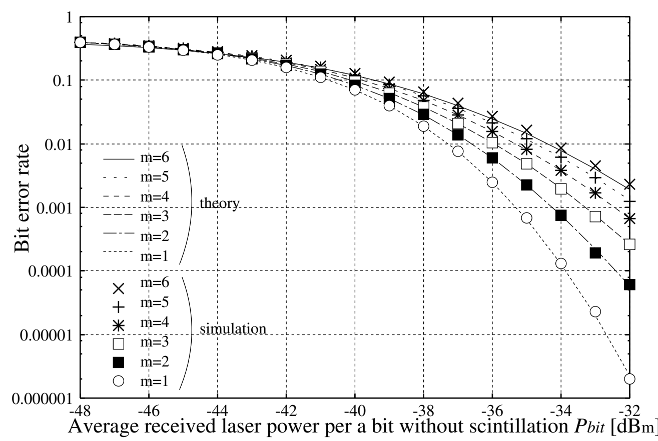

Fig. 10. BER performance of optical ZCZ-CDMA system with $z = 1$ by the optical processing over the optical space.

From Figs. 7, 8, 9 and 10, the optical ZCZ-CDMA system by the electrical and optical processing over the optical space can't remove completely co-channel interference, and the BER performance of this system by the electrical processing over the optical space go down compared to that by the optical processing. From Figs. 3, 4, 5, 6, 7, 8, 9 and 10, the BER performance of this system over the optical space go down compared to that over the optical fiber.

V. Conclusions

In this paper, we analyze the BER performance of optical ZCZ-CDMA system with zero-correlation zone $4n-2$ or 1 by the electrical and optical processing over the optical fiber and space with considering the effects of the scintillation, APD noise, thermal noise and co-channel interference by computer simulation and theoretical formula.

As a result, the BER performance of this system over the optical space go down compared to that over the optical fiber, and the BER performance of this system by the electrical processing go down compared to that by the optical processing.

The optical ZCZ-CDMA system by the electrical and optical processing can't remove completely co-channel interference.

ACKNOWLEDGMENT

This research was supported by the Ministry of Education,Culture, Sports, Science and Technology (MEXT), Grant-in-Aids for Young Scientists (B)(21760294) and challenging Exploratory Research (23651167).

REFERENCES

[1] J. G. Zhang and G. Picchi, "Tunable prime-code encoder/decoder for all-optical CDMA applications," *IEE Electron. Lett.* , vol. 29, no. 13, 1993, pp. 1211–1212.

[2] J. A. Salehi, A. M. Weiner and J. P. Heritage, "Coherent ultrashort light pulse code-division multiple access communication systems," *IEEE J. Lightwave Technol.* , vol. 8, no. 3, 1990, pp. 478–491.

[3] H. Fathallah, L. A. Rusch and S. LaRochelle, "Passive optical fast frequency-hop CDMA communications system," *IEEE J. Lightwave Technol.* , vol. 17, no. 3, 1999, pp. 397–405.

[4] T. Otsuki, "Performance analysis of atmospheric optical PPM CDMA systems," *IEEE J. Lightwave Technol.* , vol. 21, no. 2, 1994, pp. 2470–2479.

[5] Y. Kozawa and H. Habuchi, "Theoretical analysis of optical wireless CDMA with modified pseudo orthogonal M-sequence sets," *Proc. of IEEE Global Communications Conference, Exhibition and Industry Forum Co-located with WTC(GLOBECOM2009)*, 2009.

[6] Y. Kozawa and H. Habuchi, "A comparison of wireless optical CDMA systems using chip-level detection," *IEICE Trans. Fundam.* , vol. E93-A, no. 11, 2010, pp. 2291–2298.

[7] A. T. Pham and H. Yashima, "Performance Analysis of Reed-Solomon Coded Spectral Amplitude Encoding OCDMA System," *WSEAS Trans. on Electronics*, Issue 4, vol. 1, 2004, pp. 645–649.

[8] Y. Mizuno and H. Yashima, "Optical CDMA systems using extended chaotic binary codes," *J. Signal Processing*, vol. 14, no.4, 2010, pp. 309–312.

[9] P. Z. Fan and M. Darnell, Sequence design for communications applications, *Research Studies Press*, 1996.

[10] S. Matsufuji, T. Matsumoto, Y. Tanada and N. Kuroyanagi, "ZCZ codes for ASK-CDMA system," *IEICE Trans. Fundam.* , vol. E89-A, no. 9, 2006, pp. 2268–2274.

[11] T. Matsumoto, M. Inoue, H. Torii and S. Matsufuji, "Study on BER Performance of Atmospheric Optical CDMA System Using Optical ZCZ Code," *Proc. of the 2012 RISP International Workshop on Nonlinear Circuits, Communications and Signal Processing*, 2012, pp. 792–795.

[12] T. Matsumoto, H. Torii and S. Matsufuji, "Theoretical Analysis of BER Performance of Optical ZCZ-CDMA System With Smallest Zero Correlation Zone Using Optical Correlator," *Proc. of the 2012 IEEE International Symposium on Intelligent Signal Processing and Communication Systems*, 2012, pp. 549–552.

[13] T. Matsumoto, H. Torii and S. Matsufuji, "Comparison of Optical ZCZ-CDMA System with Zero Correlation Zone $4n - 2$ by Electrical and Optical Processing," *Proc. of the 3rd European Conference of Communications*, 2012 pp. 268–273.

[14] T. Matsumoto and S. Matsufuji, "Optical ZCZ code generators using Sylvester-type Hadamard matrix," *International J. of Communications*, Issue 1, vol. 4, 2010, pp. 22–29.

[15] T. Matsumoto, S. Tsukiashi, S. Matsufuji and Y. Tanada, "The bank of matched filters for an optical ZCZ code using a Sylvester type Hadamard matrix," *IEICE Trans. Fundam.* , vol. E89-A, no. 9, 2006, pp. 2292–2298.

[16] T. Matsumoto and S. Matsufuji, "Compact matched filter banks of optical ZCZ codes using fast algorithm for M-sequence type Hadamard matrix," *J. Signal Processing*, vol. 14, no. 6, 2010, pp. 427–432.

Takahiro Matsumoto received his B. Eng. and M. Eng. degrees in Information and Computer Science from Kagoshima University, Japan, in 1996 and 1998, respectively, and his Ph. D. degree in Engineering from Yamaguchi University, Japan, in 2007. He was a Research Associate from 1998 to 2007 and an Assistant Professor from 2007 to 2012 at Yamaguchi University, Japan. Since 2012, he has been an Associate Professor of the University Evaluation Department at Yamaguchi University. From 2010 to 2011, he was a visiting researcher at the University of Melbourne, Australia. His current research interests include spread spectrum systems and their applications. He received the Best Paper Award in the 10th WSEAS Intenational Conference on Applied Informatics and Communications in 2010. He is a member of IEEE and IEICE.

Hideyuki Torii received the B. Eng., M. Eng., and Ph. D. degrees from the University of Tsukuba, Tsukuba, Japan in 1995, 1997, and 2000 respectively. In 2000, he joined the Department of Network Engineering, Kanagawa Institute of Technology as a Research Associate. He is currently an Associate Professor in the Department of Information Network and Communication at the same university. His research interests include spreading sequences, CDMA systems, and mobile communication systems. He is a member of IEEE and IEICE.

Shinya Matsufuji graduated from the Department of Electronic Engineering at Fukuoka University in 1977. He received the Dr. Eng. in Computer Science and Communication Engineering from Kyushu University, Fukuoka, Japan in 1993. From 1977 to 1984, he was a technical official at Saga University, Saga, Japan. From 1984 to 2002, he was a research associate in the Department of Information Science at Saga University. From 2002 to 2012, he was an associate professor of the Department of Computer Science and Systems Engineering at Yamaguchi University. Since 2012, he has been a professor of the Graduate School of Science and Engineering at Yamaguchi University. His current research interests include sequence design and spread spectrum systems. He received the Excellent Paper Award in IET International Communication Conference on Wireless Mobile and Computing, and the Best Paper Award in the 10th WSEAS Intenational Conference on Applied Informatics and Communications in 2009 and 2010, respectively. He is a member of IEEE, IEICE and IEEJ.