

# An efficient multi-stages algorithm for the determination of communication network reliability

Musaria K. Mahmood, Fawzi M. Al-Naima, and Lujain S. Abdulla

**Abstract**— A new method for calculating network reliability is presented in this paper. This method allows the analysis of complex communication networks by the use of multistage algorithm based on the hybrid combination of reduction technique and tie set methods. The algorithm begins with the application of reduction technique to simplify the network topology by elimination all series and parallel connection of communication links. The second stage is the application of an algorithm consisting of finding all tie-sets of the reduced network. The simplifications of the network in the first stage will affect positively the number and the complexity of tie sets, keeping the final result of the network reliability unchanged.

Links probabilities are used to generate the network topologies with imperfect communication nodes, and bidirectional links which will yield to a more general situation. Algorithms and calculation are executed on MATLAB, where all the above assumptions are taken into account.

**Keywords**— Hybrid algorithm, graph reduction, network reliability, tie-sets.

## I. INTRODUCTION

THE reliability of a communication network can be defined as the probability that the system *never fails* over some defined time interval [1]. The goal of this work is to develop a new algorithm to estimate the network reliability quickly enough for communication networks. The algorithm can be used to find the reliability of all systems which can be modeled by a graph such as communication networks [2], computer network [3], mobile phone networks [4], transportation system networks [5], wireless sensor network (WSN) [6], mobile ad hoc networks [7], and electrical power networks [8]. In certain applications, it is vital to determine the network reliability quickly as in the case of topological changes due to failure of network components or change in the geographical location of nodes in the case of mobile nodes. Such analysis is usually quite complex and time consuming [9].

Musaria K. Mahmood is with the Department of Electrical Engineering, Tikrit University, Tikrit, Iraq ( email: [musariaoja@yahoo.com](mailto:musariaoja@yahoo.com)).  
Fawzi M. Al-Naima is with Al-Esraa University College, and Department of Computer Engineering, Al-Nahrain University, Baghdad, Iraq (corresponding author, phone 009647810314812 email: [fawzi.alnaima@ieec.org](mailto:fawzi.alnaima@ieec.org)).  
Lujain S. Abdulla is with the Department of Electrical Engineering, Tikrit University, Tikrit, Iraq ( email: [lujainsabah@yahoo.com](mailto:lujainsabah@yahoo.com)).

Various methods are used to estimate network reliability based on different techniques such as a State Space Enumeration method (SSE) [10], Graph Reduction Technique (GRT) [11], Tie-Sets (TS) [12], Cut-Sets (CS) [13], and genetic algorithms (GA) [14]. Selection of the best method to be used is a function of network complexity, time constraint, and computing facilities. Several researchers have proposed algorithms for estimating reliability based on one of the previous methods, which has led to a huge body of literature. Researchers usually make a choice of one of classical reliability methods such as SSE, GRT, TS, CS and GA to develop new algorithms for calculating reliability.

Reliability estimation methods are presented in different works where communication nodes are considered as perfect [15] -[20]. On the other hand the case where nodes (vertices) can fail was analyzed in [21]. Imperfect vertices can be factored like links, but the complexity increases exponentially with their number. The effect of using network reductions before estimating network reliability using a simulation was discussed in [11]. This method had shown that applying network reductions before simulation provides significant computing time reduction.

In this work a hybrid algorithm is presented. It begins by a network topological simplification of the application of GRT method followed by the application of TS method toward the reliability estimation. The main difficulties in the TS method are the complexity of finding all TS groups and the application of *inclusion exclusion expansion function*, [22]. GRT has also its own complexity related to finding the appropriate algorithm for identifying, and simplifying networks conform to developed reduction procedure.

The proposed algorithm takes into account the worst case with, link possible failure, imperfect nodes, unidirectional, and bidirectional links. It is used to calculate the two terminal reliability but can be generalized to find the overall terminal reliability.

Section II deals with the terminology and modeling used to present a network by a directed graph. The theory of network reliability evaluation used in this work is presented in section III. The proposed algorithm is discussed in section IV. In section V the implementation of the developed algorithm by

MATLAB is analyzed with a general case study. Finally a conclusion of the work is given in VI.

## II. MODELING

Communication networks, computer networks, piping systems, transportation networks, and power grids, can be modeled by graphical models. In the context of this work, a graph is representing a connected communication network modeled by  $G(N,L)$ , where vertices or nodes ( $N$ ) represent the group of communication centers, and ( $L$ ) is the set of links connecting these nodes. Consider each link  $l_{ik}$  between node  $n_i$  and node  $n_k$ , has a probability  $p_{ik}$  which describes that this link is UP.  $q_{ik}$  Is the probability that a link is DOWN and equal to  $(1-p_{ik})$ . The probability of the node ( $n_i$ ) is given by  $p_{ii}$  (or  $p_i$ ), where it is equal to 1 (100%) in the case of perfect node. The reliability between two terminal nodes ( $n_s$ , and  $n_d$ ) represents the two terminal reliability  $R_{sd}$ .  $n_s$  and  $n_d$  are the first (source) and last (destination) elements of a path. A path between node  $n_i$  and  $n_k$  is a subset of  $L$ , and is a sequence of links and nodes, links being followed by nodes, and vice versa. A path whose terminal nodes are equal is called a loop. In this work the assumption of loop-free is considered to compute all tie-sets.

The primitive network topology is represented by the matrix  $T_p$ , defined to be a three dimensions matrix  $T_p(N \times N \times D)$ , where  $D$  is the number of the maximum links in parallel between any two nodes of the graph.  $D$  Can take any value by allowing the topology to have more than two parallel links between two fixed nodes.  $D$  must be at least equal to two even when there are no parallel links in the primitive topology because applying series connection reduction will yield 'some time' a newly parallel connection.

The matrix  $T_p$ , is reduced after the application of the procedures parallel links, nodes imperfection, and series connection reductions, yields to a matrix with only two dimensions  $T(N \times N)$ , describing the connectivity of the graph after simplification stages. Elements of this matrix can have three values,  $t_{ik}$  if there is a connected link between  $n_i$  and  $n_k$ ,  $t_{ii}$  describing the diagonal elements representing the probability of the node  $n_i$  equal to (1), and (0) if there is no connected link between  $n_i$  and  $n_k$ .

$T$  is reduced to the matrix  $T_{red}$  of order  $(S \times S)$  with  $(S)$  equal to the number of nodes in the reduced topology with  $S \leq N$ . Elements of  $T_{red}$  ( $tr_{ik}$ ) represent the new topology after application of all reduction techniques used in this work.  $T_{red}$  is used as a base for the application of tie set algorithm to evaluate network reliability.

Two groups of matrices are used to determine tie sets of the network. The first is the route matrix group  $(R)_m$ , and the second is the node matrix group  $(Nd)_m$ , where  $(m)$  is the tie-set number.  $(R)_m$  is a  $S \times S$  matrix represents the set of path of ' $m^{th}$ ' tie set. Elements  $r_{kj}$  of the selected  $(R)_m$  matrix representing the use or not of the corresponding link between node  $n_k$  and node  $n_j$ , by the tie set number  $(m)$ . Elements are set to be (1) when it represents a link used by the selected tie sets. The number of matrices  $(R)_m$ , is variable related to the

number of tie sets which depends on the topology and commodity.

$(Nd)_m$  Is a group of row matrices of dimension  $(I \times S)$ , where all elements are initialized to be (0) except  $(nd_s)$  representing the source node  $n_s$  set to be (1). The element  $(nd_k)$  represents the state of node  $n_k$ , and used to find the set of nodes in the network which, traces the route from the source node  $n_s$  to the destination node  $n_d$  for the tie-set number  $(m)$ . If a node  $n_i$  is traversed by tie set  $(m)$ , then the associated element  $nd_i$  is set to be (1).  $(R)_m$ , and  $(Nd)_m$  are used together to generate all possible tie-sets for the selected network. The states of nodes and links are mutually statistically-independent and can only take one of two states- working or failed with corresponding probability.

## III. RELIABILITY DETERMINATION METHODS

### A. Graph Reduction Technique (GRT)

Graphs representing communication networks are related to the geographical location of network nodes and the connectivity between these nodes. Networks can have many connected nodes in series and in parallel. Fig. 1 proposes a technique for simplifying topologies based on series and parallel reduction technique.

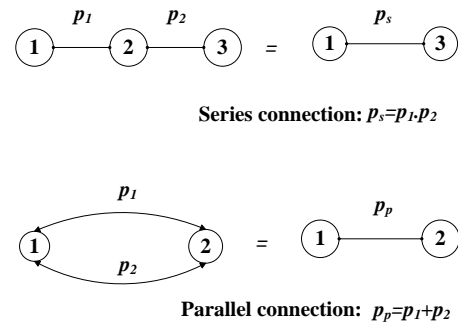


Fig. 1 Series and parallel simplification

Series transformation indicates that two branches in series are replaced by a single branch that is denoted by the intersection of the two original branches with probability equals to :

$$p_s = p_1 \cdot p_2 \quad (1)$$

Two parallel branches are reduced to one branch that is denoted by the union of the two parallel branches. Assume that any failures of edges are independent then:

$$p_p = p_1 + p_2 - p_1 p_2 = 1 - q_1 q_2 \quad (2)$$

Where  $q_i$  is the probability of failure of edge  $(i)$ , that is,  $q_i = 1 - p_i$ .

### B. The Tie- Set Technique (TST)

The tie-set technique is used to find the reliability for medium to large communication network both for two or all-terminals models [9]. Tie-sets are the groups of edges that form a path between source nodes ( $n_s$ ) and destination node ( $n_d$ ). The term minimal tie-set implies that no node or link is

traversed more than once (loop free). Also a minimal TS is one in which all the components within the set must function for the system correct operation, otherwise if any one element does not function then the entire system fails [18].

If there is ( $m$ ) TS between ( $n_s$ ) and ( $n_d$ ), then the reliability expression is given by the following expansion [1]:

$$R_{sd} = P(ts_1 \cup ts_2 \dots \cup ts_m) \quad (3)$$

Where,  $ts_i$ , is the TS number ( $i$ ),  $P$  is the probability of the union of all TS.

If ( $ts_1, ts_2, \dots, ts_m$ ) are all disjoint (or mutually exclusive) then (3) can be written as:

$$R_{sd} = P(ts_1) + P(ts_2) + \dots + P(ts_m) \quad (4)$$

But usually different TS (paths) are not disjoint. Two TS can have the same links in part. So (4) cannot be used to develop the reliability. Generally the reliability is found by applying the inclusion, exclusion expansion function used to evaluate the reliability of TS methods:

$$\begin{aligned} R_{sd} = & P(ts_1 + ts_2 + \dots + ts_m) = [P(ts_1) + P(ts_2) + \dots + \\ & P(ts_m)] - [P(ts_1 ts_2) + P(ts_1 ts_3) + \dots + P(ts_r ts_k)_{r \neq k}] + \\ & [P(ts_1 ts_2 ts_3) + P(ts_1 ts_2 ts_4) + \dots + P(ts_r ts_k ts_j)_{r \neq k \neq j}] + \\ & \dots + (-1)^{i-1} [P(ts_1 ts_2 ts_3 \dots ts_m)] \end{aligned} \quad (5)$$

#### IV. PROPOSED ALGORITHM

The developed multi-stages algorithm begins by the initialization, followed by two stages where each one is composed of many sub-stages. Initialization of all inputs describes the primitive connectivity of the communication network given as  $T_p$  matrix. The first stage is the application of reduction techniques composed of three sub-stages. The first sub-stage concerns the removal of all parallel connections by applying simplification rule as given in (2). Nodes imperfection is removed in the second sub-stage. Series sub-stage simplification reduces the network complexity by decreasing network node numbers with new corresponding links probability by applying (1).

The second stage is the application of TS algorithm based on the reduced matrix  $T_{red}$  of order ( $S \times S$ ).  $T_p$  is re-computed based on previous simplifications with new links and nodes probability values given as square matrix  $T_{red}$ . Finally an original procedure is developed to enumerating all possible TS based on the reduced network, and the application of (5) to find the network reliability.

##### A. Initialization

The network topology is given by cube matrix  $T_p(N \times N \times D)$  representing the primitive network before any changes, whose elements describing the probabilities that a link or a node is UP and defined as:

- $T_{p_{ik1}} = p_{ik}$ ,
- If there are no parallel links between  $n_i$  and  $n_k$ , then  $(T_{p_{ikj}})_{j \neq 1} = 0$

- There is ( $j$ ) parallel links between  $n_i$  and  $n_k$ , then  $T_{p_{ikx}} = (p_{ikx})_{x=1 \text{ to } j}$ , where  $p_{ikx}$ , is the probability of the  $x^{\text{th}}$  link connecting  $n_i$  to  $n_k$ , with  $1 \leq j \leq D$ .
- If  $i=k$ , diagonal elements representing the probability of node  $n_i$  ( $p_i \leq 1$ ).

All elements of the two groups of matrices  $(R)_m$ , and  $(Nd)_m$ , are set to be zero at the beginning of the application of the algorithm.

##### B. Parallel Simplification

Matrix  $T_p$  presents the general case with possible links in parallel, nodes in series connection, and node with imperfect behavior. The developed algorithm begins by a quick simplification of the topology by removing links in parallel as presented in the first stage of Fig. 2. The procedure starts with a search of all links in parallel by reading values of elements  $T_{p_{ikj}}$  and replacing two links in parallel by a single link with modified probability as given in (2). The simplification continues till all links in parallel are removed ( $D$  simplification rounds), one simplification of a pair of parallel links at a time. Elements  $T_{p_{ik1}}$  present the correct values of the updated links probabilities, but the dimension of the  $T_p$  matrix remains unchanged to be used for other simplification procedures.

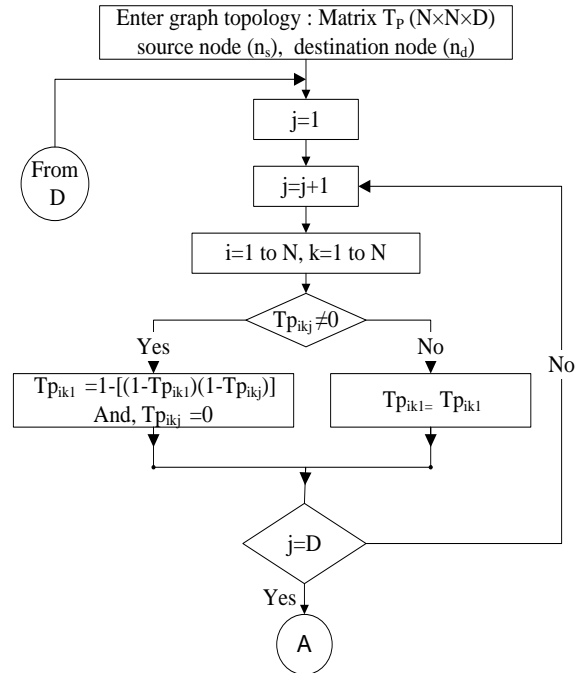


Fig. 2 Parallel simplifications Procedure

##### C. Node Imperfection

Nodes imperfection related to deterioration must be taken into account for a correct computation of network reliability. Fig. 3 explains the method used to remove nodes imperfection. The algorithm starts from the end of the parallel simplification procedure.

It will discover nodes ( $n_i$ ), with  $0 < p_i < 1$ , which represents imperfect nodes. This is accomplished by reading diagonal elements  $T_{p_{i1}}$  representing the probabilities of node ( $n_i$ ). The

value  $p_i = 0$  is rejected because after the application of the series connection simplification procedure some nodes will be removed, which will set the diagonal element corresponding to this node to (0). Perfect nodes with probability of success ( $p_i = T_{P_{ii1}} = 1$ ) are also excluded.

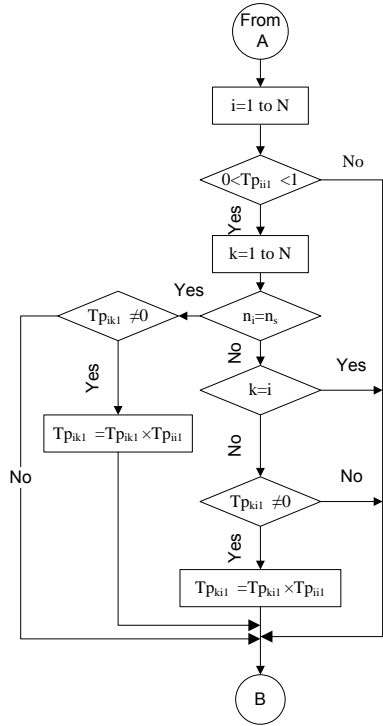


Fig. 3 Imperfect node simplification procedure

After scanning all nodes for imperfection, a procedure of simplification is executed first by localizing imperfect node ( $n_i$ ), and updating all ingoing links connected to node ( $n_i$ ) by changing their probabilities( $T_{P_{kii}}$ ). Each imperfect node is considered as the connection of two nodes in series with link between them equal to the probability of the node ( $n_i$ ) taking from the matrix topology ( $T_{P_{ii1}}$ ) as presented in Fig. 4-b. A procedure is developed to account this nodes imperfection based on the matrix  $T_p$  given the new calculated values as ( $T_{P_{kii1}} = T_{P_{kii}} \times T_{P_{ii1}}$ ) as in Fig. 4-c. If the source node ( $n_s$ ) is found to be imperfect, the same procedure is executed with outgoing links instead of ingoing( $T_{P_{ik1}} = T_{P_{ik1}} \times T_{P_{ii1}}$ ). At the end of the procedure, a node is considered as perfect by setting ( $p_i = T_{P_{ii1}} = 1$ ).

D. Series Connection Simplification

Series node simplification sub-stage starts with the identification of nodes in series by searching nodes connected to 'two and only two' neighboring nodes. This is accomplished by inspecting node connectivity given by  $T_p$ , and tagging the reducible node ( $n_i$ ) with the two neighboring nodes ( $n_{r1}$ , and

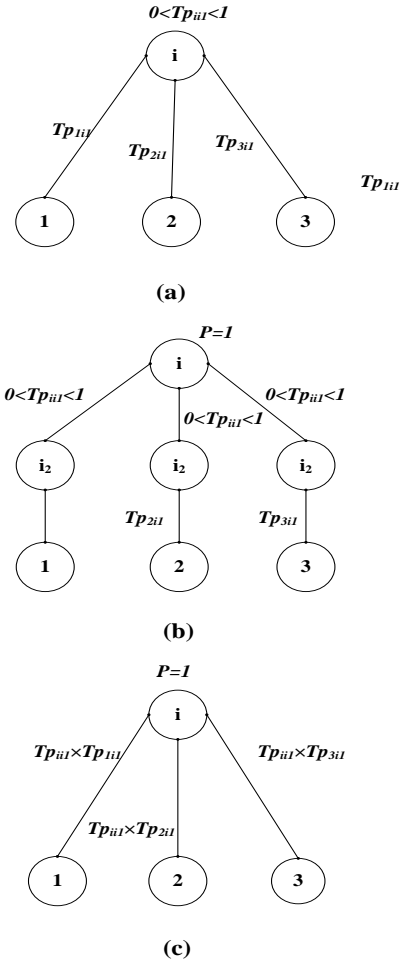


Fig. 4 Imperfect nodes simplification

$n_{r2}$ ) as presented in Fig. 5. Values  $T_{P_{r1,r2,1}}$  and  $T_{P_{r2,r1,1}}$  are inspected to check the possible direct connection between  $n_{r1}$ , and  $n_{r2}$ . If there is a connection between these two nodes in any direction, a new parallel connection will appear which must be resolved by applying the parallel simplification as presented at the end of the flow chart of Fig. 5 (in the case where  $p_2=1$ ).

Nodes ( $n_{r1}$ ,  $n_i$ ,  $n_{r2}$ ) are subject to node reduction procedure by removing the middle node ( $n_i$ ), setting the diagonal elements  $T_{P_{ii1}}$  to be zero, and updating  $T_{P_{r1,r2,1}}$  and  $T_{P_{r2,r1,1}}$  regarding the existence or not of the newly parallel connection. If there is a new parallel links composed between a pair of nodes ( $n_{r1}$ ,  $n_{r2}$ ) the procedure given in Fig. 2 is repeated otherwise matrix  $T_p$  will be directed to the next step of the reduction.

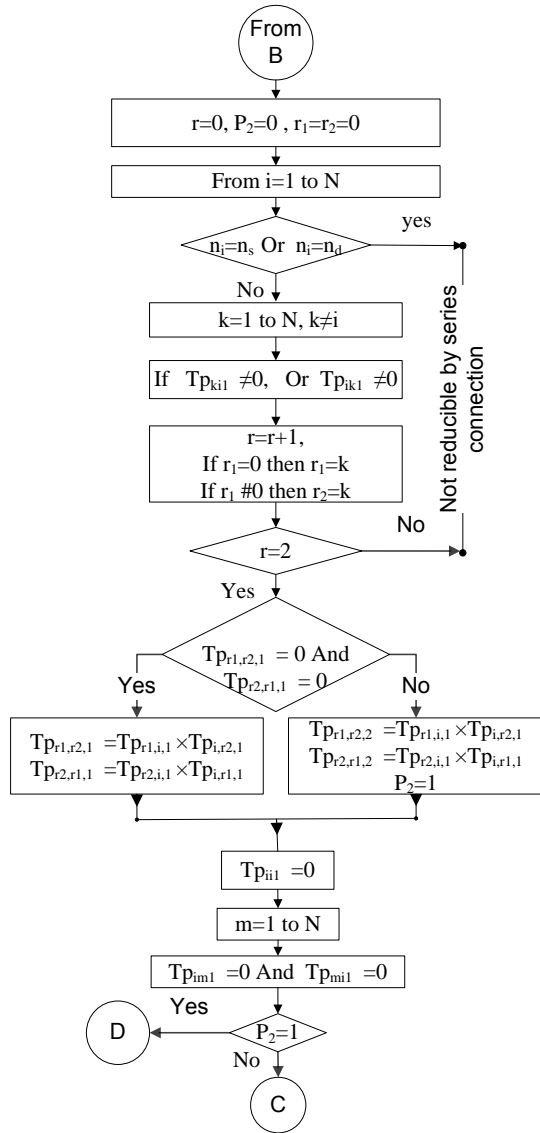


Fig. 5 Series simplification procedure

### E. Matrix Reduction

Matrix  $T_p$  is reduced in dimensions starting from its original size  $(N \times N \times D)$  to a two dimensions matrix  $T_{red}(S \times S)$ , with  $S \leq N$ . First  $T_p$  is transferred to  $T$  matrix by keeping the principal elements of  $T_p$  as  $T_{i,k} = T_{p_{ik1}}$ . By checking all diagonal elements of  $T$ , removed nodes by series reduction procedure are localized with  $T_{ii} = 0$ . If the number of removed nodes is equal to  $N_r$ , then the new dimension of matrix  $T_{red}$  is  $S=N-N_r$ . The reduced matrix is used as a base for the application of tie set method for reliability evaluation.

### F. Tie sets Generation

Tie sets generation method is considered as: Known the source node  $n_s$  and the destination node  $n_d$ , looking for all possible routes between  $n_s$  and  $n_d$ , where each one is represented as set of links. Matrix groups  $(R)_m$ , and  $(Nd)_m$ , are used together to define Tie sets as shown in the flow chart of Fig. 6.

Starting from  $n_s$  the matrix  $Nd$ , forwarded to the next node until the destination node  $n_d$  is reached presenting one TS. The developed algorithm uses the property of flooding (forwarding  $Nd$ ) to implement a procedure to find all  $(m)$  tie-sets for a graph. To avoid the possibility of a deadlock situation, the flooding procedure is modified to be a loop free algorithm. The modification can be accomplished via a test in the program to see if the location of the new arriving node  $(n_k)$  is equal to 1 in  $Nd$ , that is this node is traversed previously “d’jà vu”, and the route is cancelled.

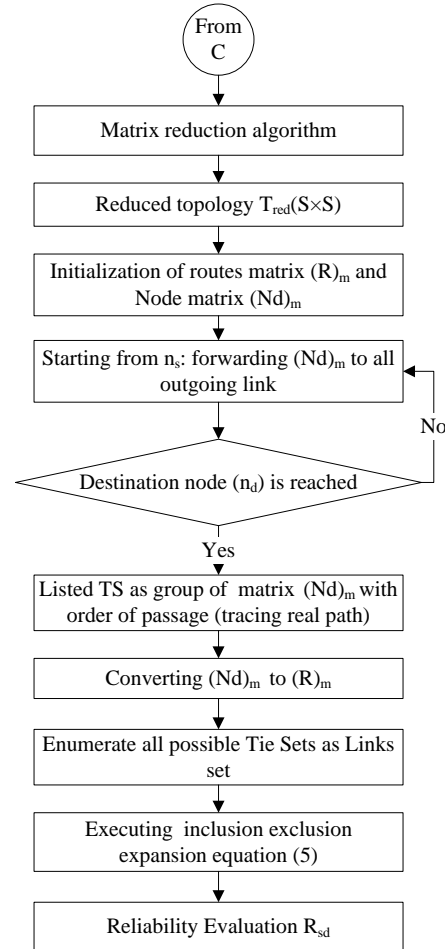


Fig. 6 Tie sets generation procedure

Arriving at  $n_d$ , the matrix  $Nd$  represents the group of nodes of passage between  $n_s$  to  $n_d$  for a selected tie set. These nodes have their indices (corresponding locations in  $Nd$ ) equal to 1. A problem arises that two tie sets can have the same matrix  $Nd$ , but differ by the order of passage. The two tie sets must be taken into account for a correct evaluation of reliability. A procedure is developed to resolve this problem by introducing a binary code of order of passage. The code will assign to each passage of new node a binary code to identify the exact path. The binary code will begin by setting the order or “code” of  $n_s$  to (1-binary), and continue by increasing order to assign a code for each new node. The number of bits used in this code is related to the dimension of  $T_{red}$  matrix. This code is used to construct the matrix  $R$  which defines the set of links for each TS, given a total of  $(m)$  different matrix  $(R)_m$  calculated for

(m) TS. These  $(R)_m$  are used in the inclusion expansion tie set equation defined in (5) to calculate the reliability.

## V. ALGORITHM IMPLEMENTATION

The algorithm is explained using a simple network, but can be applied to any system with higher complexity. Fig. 7-a, shows a 9-node network topology with unidirectional and bidirectional links. This topology is presented by the topology matrix  $T_p$  with dimensions  $(9 \times 9 \times 2)$ , where each element  $T_{p_{ik1}}$  of the first slide, is the probability of the link connecting  $n_i$  to  $n_k$ . There are no parallel links in the topology given in Fig. 7-a, so all elements  $T_{p_{ik2}}$  of the second slide are equal to zero. All existing links are considered with unified probability ( $p = 0.9$ ) as presented below.

$$T_{p_{ik1}} = \begin{bmatrix} 1.0 & 0.9 & 0.0 & 0.0 & 0.9 & 0.0 & 0.9 & 0.9 & 0.0 \\ 0.9 & 1.0 & 0.9 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.9 & 0.9 & 0.0 & 0.0 & 0.0 & 0.9 \\ 0.0 & 0.0 & 0.9 & 1.0 & 0.0 & 0.9 & 0.0 & 0.0 & 0.9 \\ 0.9 & 0.0 & 0.9 & 0.0 & 1.0 & 0.9 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.9 & 0.9 & 1.0 & 0.9 & 0.0 & 0.9 \\ 0.9 & 0.0 & 0.0 & 0.0 & 0.0 & 0.9 & 0.95 & 0.0 & 0.0 \\ 0.9 & 0.0 & 0.9 & 0.0 & 0.0 & 0.0 & 0.0 & 0.9 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.9 & 0.0 & 0.9 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

$$T_{p_{ik2}} = [0], \text{ For all } i, k.$$

### A. Application of Algorithm Stages

The algorithm is applied by successive steps, starting with the initialization, parallel reduction, nodes imperfection, and series reduction. This procedure is repeated until the fully simplified network is reached.  $T_p$  Is transferred to  $T_{red}$ , TS are calculated, and the algorithm is terminated by the application of (5).

**Step 1- Parallel reduction:** no parallel links

**Step 2- Node imperfection reduction:** Two nodes ( $n_7$ , and  $n_8$ ) are imperfects, and tagged as reducible nodes.  $T_p$  is updated as:

$$T_{p_{171}} = T_{p_{671}} = 0.9 \times 0.95 = 0.855,$$

$$T_{p_{181}} = T_{p_{381}} = 0.9 \times 0.9 = 0.81,$$

$$T_{p_{771}} = T_{p_{881}} = 1.$$

**Step3- Series reduction:** Three nodes ( $n_2$ ,  $n_7$ , and  $n_8$ ) are localized as in series connection with other nodes.  $T_p$  is updating regarding the series connection reduction procedure given in Fig. 5 by setting neighboring links to newly calculated values.

Removing  $n_2$  yields:

$$T_{p_{131}} = 0.9 \times 0.9 = 0.81, \quad T_{p_{311}} = 0.9 \times 0 = 0.$$

Removing  $n_7$  yields:

$$T_{p_{161}} = 0.855 \times 0.9 = 0.7695, \quad T_{p_{611}} = 0.855 \times 0.9 = 0.7695.$$

Removing  $n_8$  updates the second  $T_p$  slide because another link between ( $n_1$ , and  $n_3$ ) exists, then:

$$T_{p_{132}} = T_{p_{312}} = 0.81 \times 0.9 = 0.729.$$

Finally, elements corresponding to reducible nodes in  $T_p$  are set to be zero as:

$$T_{p_{221}} = T_{p_{771}} = T_{p_{881}} = 0.$$

For any node ( $n_i$ ), links:

$$T_{p_{2i1}} = T_{p_{i21}} = T_{p_{7i1}} = T_{p_{i71}} = T_{p_{8i1}} = T_{p_{i81}} = 0.$$

At the end of Fig. 5, the parallelism test succeeds, so step 1 is repeated by sending  $T_p$  to (D).

**Step 4- Repeating parallel reduction:** Two links are found to be in parallel connecting ( $n_1$ , to  $n_3$ ), corresponding links are updated as:

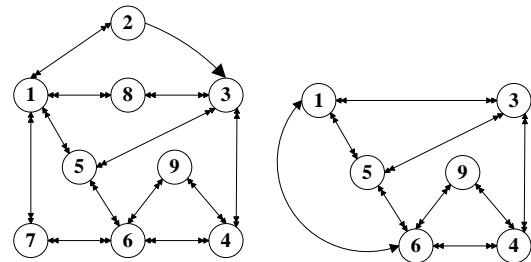
$$T_{p_{131}} = 1 - (1 - T_{p_{131}})(1 - T_{p_{132}}) = 1 - (0.19)(0.271) = 0.949$$

$$T_{p_{311}} = 1 - (1 - T_{p_{311}})(1 - T_{p_{312}}) = 1 - (1)(0.271) = 0.729.$$

At the end of the simplification the matrix  $T_p$  is transferred to matrix  $T$ , and to  $T_{red}$  ( $7 \times 7$ ) given as:

$$T_{red} = \begin{bmatrix} 1.000 & 0.949 & 0.000 & 0.900 & 0.769 & 0.000 \\ 0.729 & 1.000 & 0.900 & 0.900 & 0.000 & 0.900 \\ 0.000 & 0.900 & 1.000 & 0.000 & 0.900 & 0.900 \\ 0.900 & 0.900 & 0.000 & 1.000 & 0.900 & 0.000 \\ 0.769 & 0.000 & 0.900 & 0.900 & 1.000 & 0.900 \\ 0.000 & 0.000 & 0.900 & 0.000 & 0.900 & 1.000 \end{bmatrix}$$

**Step 5- Reliability evaluation:** By applying the procedure given in Fig. 6, followed by the calculation of the reliability using (5) based on the simplified graph given in Fig. 7-b, Table I gives all tie-sets between node  $n_1$ , and node  $n_9$ .



(a)  $N_1$  (b) Simplified graph

Fig. 7 A 9-node network

Table I Tie-sets list for the commodity  $R_{1-9}$

Tie sets	Links groups
TS1	1-3-4-9
TS2	1-3-4-6-9
TS3	1-3-5-6-9
TS4	1-3-5-6-4-9
TS5	1-5-3-4-9
TS6	1-5-3-4-6-9
TS7	1-5-6-9
TS8	1-5-6-4-9
TS9	1-6-4-9
TS10	1-6-9
TS11	1-6-5-3-4-9

### B. Evaluation of the New Algorithm

To illustrate the efficiency of the designed algorithm, four networks given in Fig. 8 with increasing complexity are tested to evaluate the gain in delay. The reliability of each network is calculated by applying TS method only as in [9], [16], and by applying the new multi-stages algorithm with network reduction followed by TS method developed in the present work. The simulation is accomplished using MATLAB, and PC (Intel I3), to observe the resulting improvement in the new algorithm by bringing down TS number which will have a positive impact by decreasing the calculation time of the reliability.

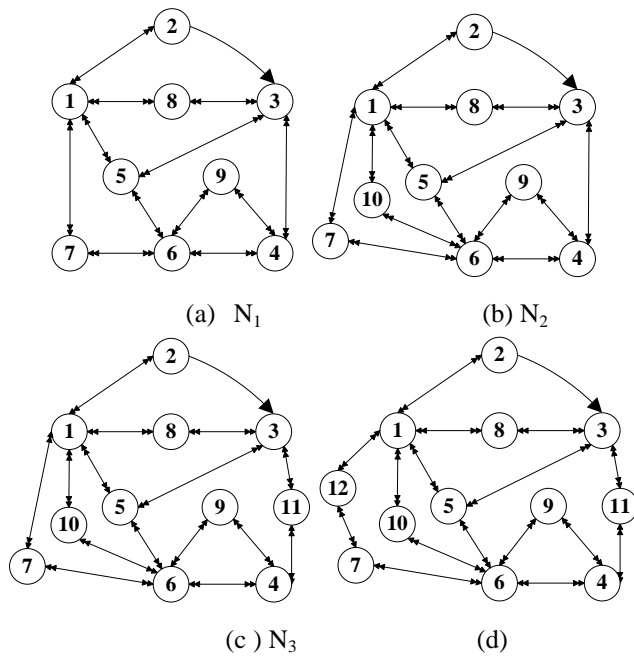


Fig. 8 Network topologies

As presented in Table II, networks reliabilities are the same regardless the algorithm used for calculation, because the two are exact reliability algorithms. Run time (reliability calculation time) related to the TS number, having direct impact to the execution of the inclusion equation (5).

Table II Comparison results

Network	$N_1$	$N_2$	$N_3$	$N_4$
Nodes no.	9	10	11	12
Links no.	25	29	31	33
Run time (sec) TS algorithm	3.070	418.9	long	long
TS no. TS algorithm	15	18	22	26
Run time (sec) New algorithm	0.117	0.141	0.120	1.064
TS no. New algorithm	11	11	11	11
Reliability	0.986	0.988	0.989	0.989
	2	5	7	7

The implementation of the two algorithms shows an improvement in run time in the case of new algorithm compared to the original algorithm. New algorithm run time is maximum equal to 1.064 sec for network  $N_4$ , while in the case of the TS algorithm the calculation takes more time, and may fail like in the cases of  $N_3$ , and  $N_4$ . This can be explained by the simplification of the topology in the graph reduction stage, which has a positive effect on the run time.

### VI. CONCLUSION

This paper presents a new algorithm for network reliability evaluation for any system that can be modeled by a graph representation. It is based on hybrid combination of two classical methods: graph transformation, and tie set. The developed algorithm covers all possible cases such heterogeneous links, imperfect nodes bidirectional links, and complex random topology. Results of MATLAB simulations show the marked improvement of the new algorithm compared to the classical algorithm. The first stage simplifies topologies by removing all series- parallel connection and resolving the node imperfection problem which decreases the number of tie-sets, and the time required for reliability evaluation.

The algorithm is programmed using MATLAB and showed a simple implementation of different procedures.

### REFERENCES

- [1] M. L. Shooman, Reliability of Computer Systems and Networks: Fault Tolerance, Analysis, and Design, John Wiley & Sons, Inc., 2002, pp. 283-324.
- [2] S. Zarezadeh, and M. Asadi, "Network Reliability Modeling Under Stochastic Process of Component Failures", *IEEE Trans. Reliab.*, Vol. 62, no. 4, Dec. 2013, pp 917-927.
- [3] J. Yu, "Computer Network Reliability Optimization Calculation Based on Genetic Algorithm", *TELKOMNIKA*, Vol. 11, No.3, Sept. 2013, pp. 603-610.
- [4] Y. Şahin, "Algorithm design for reliability analysis in mobile communication networks", *Journal of aeronautics and space technologies*, Jan. 2007, Vol. 3, No. 1, pp. 29-39.
- [5] M. Hassan, R. M. Daoud, and H. H. Amer, "Reliable Train Network with Active Supervisor", *Journal of Transportation Technologies*, Vol. 3, 2013, pp. 214-219.
- [6] P. Maktedar, V. S. Deshpande, J. B. Helonde, and V.M. Wadhai, "Performance Analysis of Reliability in Wireless Sensor Network", *International Journal of Innovative Technology and Exploring Engineering*, Vol. 2, Issue 4, March 2013, pp. 299- 302.
- [7] N. Padmavathy, Sanjay K. Chaturvedi, "Evaluation of mobile ad hoc network reliability using propagation-based link reliability model", *Elsevier, Reliability Engineering & System Safety*, Vol. 115, July 2013, pp. 1-9.
- [8] M. Al-Muhaini, and G. T. Heydt, "A Novel Method for Evaluating Future Power Distribution System Reliability", *IEEE Trans. Power Syst.*, Vol. 28, No. 3, Aug. 2013, pp. 3018-3027.
- [9] Musaria K. Mahmood, "Development of new algorithm for communication networks reliability based on Tie Set method combined with a modified flooding algorithm", *Tikrit Journal of Engineering Sciences*, Vol. 20, No.1, March 2013, pp. 10-20.
- [10] A. Gaun, G. Rechberger, H. Renner, and M. Lehtonen, "Enumeration Based Reliability Assessment Algorithm Considering Nodal Uncertainties", *IEEE Power and Energy Society General Meeting*, Minneapolis July 2010, pp. 1-8.

- [11] K. Abdullah, "Combining Network Reductions and Simulation to Estimate Network Reliability", *Proceedings of the 2007 Winter simulation conf.*, pp. 2301-2305.
- [12] H. Cancela, F. Robledo, G. Rubino, and P. Sartor, "Monte Carlo estimation of diameter-constrained network reliability conditioned by path sets and cut sets", *Elsevier, Computer communications*, vol. 36, 2013, pp. 611-620.
- [13] M. Al-Muhaini, and G. T. Heydt, "Minimal Cut Sets, Petri Nets, and Prime Number Encoding in Distribution System Reliability Evaluation", *IEEE Transmission and Distribution Conference and Exposition*, Orlando May 2012, pp. 1-8.
- [14] N. Gupta, A. Swarnkar, and K.R. Niazi, "Distribution network reconfiguration for power quality and reliability improvement using Genetic Algorithms", *Elsevier, Electrical Power and Energy Systems*, vol. 54, 2014, pp. 664-670.
- [15] M. Fotuhi, R. Billinton, T. S. Munian, and B. Vinayagam, "A Novel approach to determine minimal Tie-Sets of complex network", *IEEE Trans. on Reliability*, Vol. 53, NO. 1, March 2004, pp. 61-70.
- [16] S. Sieteng, and Suresh R., 'An efficient Cut-Set approach for evaluating communication network reliability with heterogeneous link capacities', *IEEE Trans. on Reliability*, Vol. 54, No. 1, March 2005, pp.133-144.
- [17] J. Malinowski, 'A new efficient algorithm for generating all minimal Tie-Sets connecting selected nodes in a mesh-structured network', *IEEE Trans. on Reliability*, Vol. 59, No. 1, March 2010, pp. 203-211.
- [18] Kiyoto K., Kiyoshi N., and Norihiko S., "Network failure recovery with Tie-Sets", *Workshops of Int. Conf. on Advanced Information Networking and Applications*, 2011, pp. 467-472.
- [19] S. Yunlin, and L. Min, "Research for slope structural reliability and the application of MATLAB program software in its computation", 9<sup>th</sup> International Conference on Reliability, Maintainability and Safety (ICRMS), Guiyang 2011, pp. 41-45.
- [20] V. Nitika, K. Durgesh, and T. Sanjeev, "Reliability analysis for wireless sensor networks considering environmental parameters using MATLAB", *Third Int. Conf. on computational intelligence, communication systems and networks*, 2011, pp. 99-102.
- [21] Y. K. Sy, M. Y. Fu, and Y. L. Hung, "Efficient and exact reliability evaluation for networks with imperfect vertices", *IEEE Tran. on Reliability*, Vol. 56, No. 2, June 2007, pp. 288-300.
- [22] S. Soh, and S. Rai, "An efficient Cut-set approach for evaluating communication network reliability with heterogeneous link capacities", *IEEE Trans. on Reliability*, Vol. 54, No. 1, March 2005, pp.133-144.