Abstract—This paper considers recursive tracking of one mobile target using a sequence of time difference of arrival (TDOA) and frequency difference of arrival (FDOA) measurement pairs obtained by distributed sensor network in a three dimension situation. As the conventional target tracking using TDOA measurement is not accurate enough to estimate the target location, we use the TDOA and FDOA measurement signals together to estimate the location and the velocity of a target at discrete times. Although, the Kalman filter shows remarkable performance in calculation and location estimation, the estimation error can be large when the priori noise covariances are assumed with improper values. We proposed an adaptive extended Kalman filter (AEKF) to update the noise covariance at each TDOA/FDOA measurement and estimation process. Although many methods derive the estimates of position and velocity with iterative numerical techniques, the proposed AEKF method can be a good alternative to update the noise covariance guess under conditions of measurement error. The simulation results show that the algorithm efficiently reduces the position error and it also greatly improves the accuracy of target tracking. It is proven that the AEKF algorithm deals with the nonlinear nature of the mobile target tracking problem successfully.

Keywords—Kalman filter; Target tracking; Time Difference of Arrival; Frequency Difference of Arrival.

I. INTRODUCTION

PASSIVE target location and tracking have been of considerable interest in many fields, including radar, sonar, microphone arrays, sensor network, and wireless communication [1]. The target location and tracking system intercepts the electromagnetic signals radiated from an unknown target and analyses the intercepted signals to identify the type of an unknown target and estimate its position and/or velocity. Recently, demand for more accurate mobile target location has grown for establishing appropriate methods for location and tracking activities. Although various physical quantities can be utilized for estimating the unknown position of a target such as range, angle, and Doppler shift, most of them are mainly based on different measurement information including Time of Arrival (TOA), Time Difference of Arrival (TDOA), Angle of Arrival (AOA), Received Signal Strength (RSS), and in various combinations [2]. Both time based (e.g., TOA and TDOA) and angle based (e.g., AOA) schemes have their own advantages and limitations [3]. However, other, more advanced methods, require joint processing of the signals intercepted at two or more sensors - these are methods based, for example, TOA/TDOA methods require at least three non-collinearly located Base Stations (BSs) to produce a two-dimensional fix, while AOA schemes need only a minimum of two BSs. TOA/TDOA schemes generally have better accuracy while AOA schemes are highly range dependent when the mobile station is far away from the BS, a small AOA measurement error will result in a large localization error [4]. When there is relative motion between the sensors and the target, frequency difference of arrival (FDOA) can be used to estimate the velocity of a moving target as well as the target position. Hence, TDOA and FDOA measurements have been jointly utilized to simultaneously estimate target position and velocity [5].

Most importantly, target tracking uses only TDOA measurements which are not accurate to estimate the target location when the number of receivers is not enough [6,7,8]. To solve this problem, we use the TDOA and FDOA measurement signals together to estimate the location and the velocity of the target. The Kalman filter is well-known for solving the problem of the target location. To overcome the nonlinear problems, the extended Kalman filter (EKF) estimates the state through a linearization process [9,10]. The EKF uses priori guess to estimate the process and measurement noise covariance. As the circumstances change at different times, it’s difficult to track the position precisely when the priori values are estimated with too much error from the real values [11,12]. In order to overcome these problems, we propose an adaptive extended Kalman filter (AEKF) for precise
position tracking. Using the adaptive factor, the process and measurement error covariance can be modified to approach the real values, and then is used to deal with the TDOA and FDOA measurement signals together to estimate the location and the velocity of the target. The remainder of the paper is organized as follows. Section II provides problems of conventional methods. In section III, we introduce the system modeling for target localization. Section IV designs the adaptive extended Kalman filter (AEKF) algorithm for geolocation. The simulation results show the improved tracking accuracy in section V. Finally, the conclusions are given.

II. PROBLEMS OF CONVENTIONAL METHODS

Target localization is a nontrivial problem because the measurements are nonlinearly related to the target location parameters. Due to this problem, the mean square error (MSE) is composed of two parts: the variance and the bias square. When the noise level is low and the observation period is short, the bias is not significant compared with the variance of the target position estimation [13]. Regardless of the localization algorithms used, the target location accuracy can be very sensitive to the accurate knowledge of the sensor positions and velocities. A slight error in receiver locations can lead to a big error in source location estimate. In application of target tracking, the location deviation has a great influence on performance of target tracking. Over the years, many algorithms have been proposed for the problem, including the iterative Taylor-series method, which a linearizing method is used to convert the system model to a linear least squares estimator with a nonlinear constraint. The Gauss-Newton iteration method is used to conquer the source localization problem. The conventional method cannot position target when the number of sensors is not enough for localization. In order to reduce this influence, B. Hao et al. put forward a bias reduction method for target localization using TDOA and gain ratios of arrival (GROA) [14]. However, the methods proposed by the above research can only be applied to the stationary target. For moving target, a new bias reduction algorithm using both TDOA and FDOA is proposed by H. W. Wei et al. [15]. It can reduce the estimation error by adding new constraints to the original position equation. Motivated by the above short comings of algorithms based on TDOA and FDOA measurements, an update the noise covariance method based on extend Kalman filter for moving target localization and tracking using TDOA and FDOA is proposed in this paper.

III. SYSTEM MODELING FOR TARGET LOCALIZATION

The localization method is based on using the time difference of arrival (TDOA) and the frequency difference of arrival (FDOA) signals collected from receiver sensors or UAVs receiver equipped with sensors under the interference noise. We assume a 3D Euclidean space. Multiple sensors are located to determine the position and the velocity of a moving target. The state of the target $x_k \in \mathbb{R}^{3n}$ at $t_k$ is given as

$$ x_k = \Phi x_{k-1} + \Gamma w_{k-1}, $$

where $x_k = [u^T_k, \hat{u}^T_k, \hat{u}_a^T_k]$ consists of the 3D position vector $u_k$, the velocity vector $\hat{u}_k$, and the acceleration vector $\hat{u}_a$:

$$ u_k = [x_k y_k z_k]^T, \quad \hat{u}_k = [\dot{x}_k \dot{y}_k \dot{z}_k]^T, \quad \hat{u}_a = [\ddot{x}_k \ddot{y}_k \ddot{z}_k]^T. $$

The state transition matrix $\Phi$ is

$$ \Phi = \begin{bmatrix} I_{3 \times 3} & \Delta I_{3 \times 3} & \Delta^2 / 21_{3 \times 3} \\ 0_{3 \times 3} & I_{3 \times 3} & \Delta I_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}, $$

where $I_{n \times n}$ is an $n \times n$ identity matrix, $0_{n \times n}$ is an $n \times n$ zero matrix, and $\alpha$ is a constant acceleration parameter. The transformation matrix of the process noise, $\Gamma$, is

$$ \Gamma = \begin{bmatrix} \Delta^2 / 2 I_{3 \times 3} \\ \Delta I_{3 \times 3} \\ I_{3 \times 3} \end{bmatrix}, $$

where $\Delta = t_k - t_{k-1}$ is a fixed time step and $w_{k-1}$ is the following white Gaussian noise process

$$ w_{k-1} = \begin{bmatrix} w_x, w_y, w_z \end{bmatrix}^T, $$

$$ E[w_{k-1}] = 0, \quad E[w_{k-1}w_{k-1}^T] = Q_w, $$

where the covariance matrix $Q_w = \sigma_w^2 I_{3 \times 3}$ and $\sigma_w$ is the standard deviation of the process noise.

A signal is transmitted from the moving target, and the sensors receive the signal. Let $m$ be the number of sensors. In this circumstance, the distance between the target and the i-th sensor $(s_i)$, $r_i(x_k)$ is given as

$$ r_i(x_k) = |u_k - s_i|, $$

When $u_k$ is the position vector of the target and $s_i$ is the known position vector of $s_i$. Let $T_{ij}$ be the TDOA measurement between $s_i$ and $s_j$. If $c$ is the signal propagation speed, then the range difference of arrivals between $s_i$ and $s_j$ at time $t_k$ is

$$ r_{ij}(x_k) = cT_{ij}(x_k) = r_i(x_k) - r_j(x_k), $$

where $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, m$. To improve the tracking performance, we exploit not only the TDOA measurements but also the FDOA measurements. From the time derivative of (7), we can obtain

$$ f_i(x_k) = \frac{(u_k - s_i)\tau(x_k)}{n(x_k)}, \quad i = 1, 2, \ldots, m, $$

where $\tau(x_k)$ is the time of arrival (TOA) at $x_k$.
where \( \mathbf{u}_k \) is the velocity vector of the target and \( \mathbf{s}_t \) is the known velocity vector of \( s_t \). The FDOA measurement is then

\[
\hat{r}_k(x_k) = \left( \frac{(u_k - e_k)^T (u_k - s_t)}{|u_k - s_t|} \right)^{-1} \left( \frac{(u_k - e_k)^T (u_k - s_t)}{|u_k - s_t|} \right)
\]

(9)

The first sensor \( s_1 \) acts as a reference sensor, and the TDOA/FDOA measurements between \( s_1 \) and \( s_l \), \( l = 2, \ldots, m \), are accumulated a full measurement vector \( Z_k \) at time \( t_k \) :

\[
Z_k = H_k(x_k) + V_k,
\]

(10)

where

\[
H_k(x_k) = [h_{21}(x_k) h_{31}(x_k) \ldots h_{m1}(x_k)]^T
\]

(11)

and \( h_j(x_k) \) is the true TDOA/FDOA measurement between \( s_i \) and \( s_j \) at \( k \) as

\[
h_j(x_k) = \left[ \frac{r_j(y_k)}{r_j(y_k)} \right].
\]

(12)

The white Gaussian noise process \( V_k \) is

\[
E[V_k] = 0,
\]

(13a)

\[
E[V_k(V_k)^T] = \text{diag}[\sigma_y^2 \sigma_z^2] \otimes I_{m-1 \times m-1},
\]

(13b)

where the variances of TDOA measurement noise and FDOA measurement noise are denoted as \( \sigma_y \) and \( \sigma_z \), respectively, and \( \otimes \) represents the Kronecker product.

IV. LOCALIZATION USING ADAPTIVE EXTENDED KALMAN FILTER

In this section, a brief introduction to the extended Kalman filter (EKF) will be given [16, 17], and then adaptive EKF (AEKF) will be proposed.

A. Extended Kalman Filter

The traditional EKF algorithm is utilizing a set of equations as follows [18, 19]:

\[
X_{k|k-1} = \Phi X_{k-1|k-1},
\]

(14)

\[
P_{k|k-1} = F_{k|k-1}^T P_{k-1|k-1} F_{k|k-1} + Q_k,
\]

(15)

\[
K_k = P_{k|k-1} H_k^T \left[ H_k P_{k|k-1} H_k^T + R_k \right]^{-1},
\]

(16)

\[
X_k = X_{k|k-1} + K_k (z_k - H_k X_{k|k-1}),
\]

(17)

\[
X_{k|k} = X_{k|k-1} + K_k (z_k - H_k X_{k|k-1}),
\]

(18)

\[
P_{k|k} = (I - K_k H_k) P_{k|k-1},
\]

(19)

\[
F_{k|k-1} = \frac{\partial f(X_{k|k-1})}{\partial X_{k|k-1}}
\]

(20)

\[
H_k = \frac{\partial h(X_{k|k-1})}{\partial X_{k|k-1}}
\]

(21)

B. Adaptive Extended Kalman Filter

Compared with the EKF, the adaptive EKF employs a few simple iterative operations to reduce the bias and the estimation error after getting \( X_k \) in (14) and \( P_k \) in (15). \( R_k \) is computed by the time-varying noise statistics with adaptive factor. The corresponding recursive relations are

\[
x_{k|k-1} = \Phi x_{k-1|k-1},
\]

(22)

\[
P_{k|k-1} = A_{k|k-1} P_{k-1|k-1} A_{k|k-1}^T + Q_{k-1},
\]

(23)

\[
X_{k|k} = X_{k|k-1},
\]

(24)

\[
F_{k|k-1} = P_{k|k-1}.
\]

(25)

For \( n = 1, 2, \ldots, N \),

\[
H_{k}^{(n)} = \frac{\partial h(X_{k}^{(n)})}{\partial X_{k}^{(n)}}
\]

(26)

\[
K_{k}^{(n)} = P_{k|k-1} H_{k}^{(n)} \left[ H_{k}^{(n)} P_{k|k-1} H_{k}^{(n)} + R_{k}^{(n)} \right]^{-1}
\]

(27)

\[
R_{k}^{(n)} = (1 - (1 - \delta) / (1 - \delta^k)) R_{k}^{(n-1)} (1 - \delta) / (1 - \delta^k)
\]

(28)

\[
X_{k|k}^{(n+1)} = X_{k|k}^{(n)} + K_{k}^{(n)} (z_k - H_{k}^{(n)} X_{k|k}^{(n)} - H_{k}^{(n)} X_{k|k-1}^{(n)}),
\]

(29)

\[
P_{k|k}^{(n+1)} = (I - K_{k}^{(n)} H_{k}^{(n)}) P_{k|k}^{(n)}.
\]

(30)
where adaptive factor $0 < \delta < 1$.

End for,

$$X_{k|k}^{(n+1)},$$ \hspace{1cm} (31)

$$P_{k|k}^{(n+1)}.$$ \hspace{1cm} (32)

Here, $N$ is the maximum iterative number. The update algorithm of the AEKF reduces to that of the EKF in the case of a single iteration. Inevitably, the iteration will increase the filter time and improve the tracking precision. Compromise always has to be made between the tracking precision and computation cost.

V. SIMULATION TESTS AND DISCUSSION

In this section, through some simulation results, we demonstrate the effectiveness of the proposed TDOA/FDOA location finding using AEKF algorithm. We collected a dataset in city Taoyuan, Taiwan using a commercially available Bumblebee radars that compose a sensor network of low-power Doppler radars that actively measure the target’s radial velocity. Each radar independently measures the Doppler velocity of a mobile target and transmits it to a PC-class base station then collects the estimated Doppler velocities by at least four radars and estimates the position and velocity of the mobile target by solving a system of nonlinear equation (10).

The target tracking algorithm (AEKF) proposal described in this paper is one of the main modules of a complete framework designed to detect and track mobile target. This framework (Fig. 1) is composed by:

- A mobile target detector that analyses TDOA/FDOA data stream detecting the presence of new target.
- A target location that calculates TDOA/FDOA measurements and adds it to the AEKF algorithm.
- The mobile target tracking module using the proposed AEKF in this paper.
- A control module that takes care of suppressing tracked targets which have no longer interest (are mis-tracked, or out of the detection field of view).

![Fig. 1. The TDOA/FDOA location and tracking framework to be considered](image1)

This section examines the accuracy of the proposed AEKF algorithm in measurements of TDOA/FDOA. After collecting a set of data from Doppler radars (a bird-eye-view perspective overlayd on Google Maps in Fig. 2 labeled with red star symbol), we use TDOA/FDOA measurements and the proposed AEKF algorithm to estimate the mobile target’s position. The mobile target’ trajectory starts from position $\mathbf{u} = [850, 10000, 1000]^T$, and velocity $\dot{\mathbf{u}} = [50, 50, 50]^T$, with constant acceleration, $a=200$ meters per second squared, then turned around and finally along the line with constant acceleration. Fig. 3 shows trajectory measurements of the altitude above the ground.

![Fig. 2. Doppler radars with red star overlaid on Taoyuan city Google Maps](image2)

![Fig. 3. Trajectory of the mobile target in 3D](image3)

The maximum iterative number is $n = 5$ and $\delta=0.7$. The trajectory of the mobile target of show the position estimation of the AEKF algorithm in the directions $x$, $y$, and $z$, respectively. Fig. 4 shown the position estimation (with red circles) and the true trajectory (with black circles). Fig. 4 shows that the proposed algorithm can estimate more closely to the real position with mobile target. The red circle line is the AEKF algorithm estimate which updates the system and measurement noise covariance in the process.
Monte Carlo simulation results are presented here in order to demonstrate the tracking performance of the AEKF algorithm, and 100 runs were performed. The standard deviation of TDOA and FDOA measurement noise are set to be 18.5 m and 0.185 m/s respectively. TDOA and FDOA measurements are taken every 1 second during a period of 100 seconds, amounting to 9 Doppler radars. Fig. 5(a) show created tracks in XYZ coordinates for TDOA/FDOA measurement (with red circles) and TDOA only (with black circles). The root mean squared error (RMSE) of the estimated position and velocities, as shown in Fig. 5 (b) and (c). Simulation results show that x-axis is much more sensitive to disturbance than y-axis and z-axis. This is because that the three Euler angles are all relatively very large in the whole process. It reduces the mean RMSE of position by about 32.53% and 39.09% in position and velocity compared with TDOA/FDOA and the TDOA only. It is evident that both the TDOA and the FDOA measurements are effective in reducing the position/velocity error compared with TDOA only.

![Fig. 5(a) Mobile target tracking with TDOA/FDOA measurements (red) vs. TDOA only (black).](image)

![Fig. 5(b) Estimated position errors for the AEKF](image)

![Fig. 5(c) Estimated velocity errors for the AEKF](image)

VI. CONCLUSIONS

The accuracy of the location estimate is related to the frequency of the target's signal, and TDOA and FDOA are determining the location of a target from its emissions the TDOA measurements are nonlinear, target position estimation using the TDOA measurement is performed by essentially linear operations, i.e., Kalman filter update. We proposed a bias compensation algorithm based on the adaptive extended
Kalman filter (AEKF) for distributed sensor network passive localization using TDOA and FDOA measurements. It extends the EKF approach originally developed for TDOA to incorporate FDOA measurements. Simulation results reveal that the proposed TDOA/FDOA measurements in AEKF outperforms the TDOA only in localization accuracy. The use of both the TDOA and FDOA measurements was shown to be effective in the mobile target tracking. It was further proven that the AEKF deals with the nonlinear nature of the mobile target tracking problem successfully. In this paper, we focused on geolocation enabling technologies. We described TDOA/FDOA measurements embedded in AEKF tracking algorithm enabling technologies that can yield higher accurate estimate and better convergence performance than the TDOA measurement only. The proposed algorithm is expected to be widely used in such as an Internet of Things (IoT) equipment and their services that will enhance the comfort level as well as increase efficiency. Another future work is to study the more realistic scenario that the locations and velocities of the sensors are not perfectly known, but are subject to some errors. The work presented in this paper has laid the foundation for this future work.

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