A Polynomial Matrix Approach to the Descriptor Systems

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***Abstract*— In this paper, we will propose an analysis method of the descriptor systems using the regularizing polynomial matrix. The regularizing matrix compensates the singularity of the descriptor systems, like an interactor matrix. We will show that the degree of the regularizing polynomial matrix presents a structure aspect of a given descriptor system.**

Key Words: linear multi-variable systems, descriptor systems, polynomial matrix, regularizing matrix.

I. INTRODUCTION

The descriptor systems [1] are convenient and natural modeling process for the practical plants. The state space method [2] and the geometric approach [3] are used to study the structure properties and to design the controllers. Comparing these methods, there are not so many literatures using the polynomial matrix approach [4]. Since the impulsive modes in the descriptor systems cause the improper transfer function, it is natural to to discuss the treatment of the improper transfer function using the polynomial matrices.

In this paper, we will propose an analysis method of the descriptor systems using the regularizing polynomial matrix. The regularizing polynomial matrix compensates the singularity of the descriptor systems, like an interactor matrix for rational function matrices [5]. In fact, the regularizing matrix is almost equivalent to an interactor. Although some derivation methods of the interactor were proposed, almost of all were complex. Mutoh and Ortege proposed the algebraic equation, which the coefficient matrices of the interactor should be satisfied [6]. But the solution method in [6] was not adequate for computer calculations. The authors proposed a solution of the equation in [6] using Moore-Penrose pseudo-inverse [7]. Since a function to calculate the pseudo-inverse is available in some standard softwares for control engineering, the method is adequate for computer calculations.

We will show that the degree of the regularizing polynomial matrix presents a structural aspect of a given descriptor system. That is, there exists the regularizing matrix of degree one if a given system has no impulsive mode. There exists the regularizing matrix of degree two if a given system has some impulsive modes. We will also discuss a condition for the impulsive controllability of the descriptor systems using the analysis. We will also discuss the feedback controller design which removes the impulsive modes of the descriptor systems.

II. REGULARIZING POLYNOMIAL MATRIX

Consider the following $q \times m$ ($q \le m$) polynomial matrix $D(s)$:

$$\begin{aligned} D(s) &= D_0 + sD_1 + \cdots + s^\mu D_\mu \\ &= \boldsymbol{D} S^\mu_{I_m}(s) \end{aligned} \tag{1}$$

where

$$\begin{aligned} \boldsymbol{D} &= \left[\, D_0 \ D_1 \ \cdots \ D_\mu \,\right], \\ S^\mu_{I_m}(s) &= \left[\, I_m \ sI_m \ \cdots \ s^\mu I_m \,\right]^T. \end{aligned} \tag{2}$$

$D(s)$ is called $regular$ if D_μ has full rank q. The problem considered in this section is to find a $q \times q$ nonsingular polynomial matrix $L(s)$ which makes μ-th degree's coefficient matrix of $L(s)D(s)$ be full rank and the coefficient matrices which degrees are greater than μ be zeros. $L(s)$ is called a regularizing polynomial matrix of $D(s)$. The existence of such matrix is clear by considering the interactor for $D(s)/s^{\mu+1}$. In the following, we will consider the direct derivation of $L(s)$ not using the interactor.

Assume that $L(s)$ has the following structure

$$\begin{aligned} L(s) &= L_0 + sL_1 + \cdots + s^w L_w \\ &= \boldsymbol{L} S^w_{I_q}(s) \\ \boldsymbol{L} &= \left[\, L_0 \ L_1 \ \cdots \ L_w \,\right] \end{aligned} \tag{3}$$

where the integer w will be defined later. Then, $L(s)D(s)$ can be written by

$$\begin{aligned} & L(s)D(s) \\ &= \boldsymbol{L} S^w_{I_q}(s)(D_0 + sD_1 + \cdots + s^\mu D_\mu) S^\mu_{I_m}(s) \\ &= \boldsymbol{L} \begin{bmatrix} D_0 & D_1 & \cdots & D_w & \cdots & D_\mu & 0 & \cdots & 0 \\ 0 & D_0 & \cdots & D_{w-1} & \cdots & D_{\mu-1} & D_\mu & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D_0 & \cdots & D_{\mu-w} & \cdots & \cdots & D_\mu \end{bmatrix} S^{\mu+w}_{I_m}(s), \end{aligned} \tag{4}$$

where $D_{\mu-w} = 0$ if $\mu - w < 0$. Assume that the μ-th degree's coefficient matrix of $L(s)D(s)$ is $K \in \mathbf{R}^{q\times m}$. If $L(s)$ is the regularizing matrix, then the following equality must hold from the above relation:

$$\boldsymbol{L}\boldsymbol{T}_w = \boldsymbol{J} \tag{5}$$

where

$$\begin{aligned} \boldsymbol{T}_w &= \begin{bmatrix} D_\mu & 0 & \cdots & 0 \\ D_{\mu-1} & D_\mu & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ D_{\mu-w} & D_{\mu-w+1} & \cdots & D_\mu \end{bmatrix}, \\ \boldsymbol{J} &= \left[\, K \ 0 \cdots 0 \,\right]. \end{aligned} \tag{6}$$

Considering the structure of $\boldsymbol{J}$, set

$$\boldsymbol{L} = \boldsymbol{J}\boldsymbol{T}_w^\dagger = K\boldsymbol{T}_w^\dagger(1:m,:), \tag{7}$$

where $\boldsymbol{T}_w^\dagger(1:m,:)$ denote the submatrix constituted of the first m-th rows of $\boldsymbol{T}_w^\dagger$. Substituting the above equation to eqn.(5),

$$K\boldsymbol{T}_w^\dagger(1:m,:)\boldsymbol{T}_w = \boldsymbol{J}. \tag{8}$$

Define $\varLambda$ by

$$\varLambda = \boldsymbol{T}_w^\dagger(1:m,:)\begin{bmatrix} D_\mu \\ D_{\mu-1} \\ \vdots \\ D_{\mu-w} \end{bmatrix}, \tag{9}$$

the first m-th columns of eqn.(8) can be written by

$$K\varLambda = K. \tag{10}$$

That is, if eqn.(5) is solvable, its special solution is given by eqn.(7) and K must satisfy eqn.(10). Let $U\begin{bmatrix} \varGamma & 0 \\ 0 & 0 \end{bmatrix}V^T$ denote the singular value decomposition (SVD) of $\boldsymbol{T}_w$ using some nonsingular matrix $\varGamma$ and unitary matrices U and V. Then, $\boldsymbol{T}_w^\dagger$ is given by

$$\boldsymbol{T}_w^\dagger = V\begin{bmatrix} \varGamma^{-1} & 0 \\ 0 & 0 \end{bmatrix}U^T$$

and

$$\boldsymbol{T}_w^\dagger\boldsymbol{T}_w = V\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}V^T.$$

Therefore, $\varLambda$ can be written by

$$\varLambda = V(1:m,:)\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}V^T(:,1:m) \geq 0. \tag{11}$$

Eqn.(10) means that K is the left eigenvectors of $\varLambda$ which correspond to the eigenvalues at $\lambda = 1$. Since $\varLambda$ is a real symmetric matrix, the geometric multiplicity of the eigenvalue *one* in $\varLambda$ equals to the algebraic multiplicity. Thus we can find a set of linearly independent eigenvectors for the eigenvalue *one*. Therefore,

1) w is the least integer when $\varLambda$ has p multiple eigenvalue at $\lambda = 1$.
2) K is constituted of corresponding left eigenvectors.

Example 1 Consider the following polynomial matrix:

$$D(s) = \begin{bmatrix} s+1 & s+2 & s+3 \\ s+4 & s+5 & s+6 \end{bmatrix}.$$

For the above case, $q = 2$, $m = 3$ and $\mu = 1$. D_0 and D_1 are given by

$$D_0 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Setting $w = 2$, $\boldsymbol{T}_2$ is given by

$$\boldsymbol{T}_2 = \begin{bmatrix} D_1 & 0 & 0 \\ D_0 & D_1 & 0 \\ 0 & D_0 & D_1 \end{bmatrix}$$

and then $\varLambda$ is given by

$$\varLambda = \frac{1}{6}\begin{bmatrix} 5 & 2 & -1 \\ 2 & 2 & 2 \\ -1 & 2 & 5 \end{bmatrix}$$

which has the eigenvalue at $\lambda = 1$ with multiplicity $2 = p$. The left eigenvectors of $\varLambda$ corresponding to $\lambda = 1$ are given by $[1\ 0\ -1]$ and $[0\ 1\ 2]$ and thus K is given by

$$K = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}.$$

Therefore, $L(s)$ can be calculated by

$$\begin{aligned} L(s) &= \begin{bmatrix} K & 0 & 0 \end{bmatrix}\boldsymbol{T}_2^\dagger S_I^2(s) \\ &= \begin{bmatrix} .5385 & .5385 \\ -.3846 & -.3846 \end{bmatrix} + s\begin{bmatrix} -1.3077 & .3077 \\ 1.0769 & -.0769 \end{bmatrix} \\ &\quad + \frac{s^2}{3}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}. \end{aligned}$$

III. Applications to Descriptor Systems

The descriptor system is given by the following equations:

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) \end{aligned} \tag{12}$$

where $x(t) \in \mathbf{R}^n$ is the descriptor vector, $u(t) \in \mathbf{R}^m$ is a control input vector, $y(t) \in \mathbf{R}^q$ is an output vector, and E, $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times m}$ and $C \in \mathbf{R}^{q\times n}$ are constant matrices. It is assumed that rank$E = r < n$ and $(E,\ A)$ is $regular$, i.e., $\det(sE - A) \neq 0$ for almost of all s.

It is known that there are three modes for the descriptor system (12). In the followings, we will analyze the impulsive mode using the regularizing matrix.

Let $\varphi(s)$ denote the characteristic polynomial of $(E,\ A)$, i.e.,

$$\varphi(s) = \det(sE - A), \quad \deg\varphi(s) := d. \tag{13}$$

The zeros of the above polynomial are called *dynamics mode* of the system (12). Since E is singular, the system (12) has infinite mode. If $r = d$, then the infinite mode is called *static*. If $d < r$, then the system (12) has *impulsive mode*.

Lemma 1: If the regularizing polynomial matrix of $sE-A$ can be described as a first order polynomial matrix, i.e.,

$$L(s) = L_0 + sL_1, \quad L_1 \neq 0, \tag{14}$$

then the system (12) has no impulsive modes. Conversely, if the system (12) has no impulsive modes, then there exists a first order regularizing polynomial matrix.

(Proof). Consider the SVD of E as follows:

$$E = U\begin{bmatrix} E_1 & 0 \\ 0 & 0 \end{bmatrix}V^T, \quad U,\ V \in \mathbf{R}^{n\times n}, \tag{15}$$

where E_1 is nonsingular. According to the above decomposition, A and $L(s)$ are decomposed by

$$\begin{array}{l} A = U \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} V^T, \\ L(s)U = \begin{bmatrix} L_{01} & L_{02} \end{bmatrix} + s \begin{bmatrix} L_{11} & L_{12} \end{bmatrix}, \\ A_{11} \in \mathbf{R}^{r\times r}, \quad A_{12} \in \mathbf{R}^{r\times(n-r)}, \\ A_{21} \in \mathbf{R}^{(n-r)\times r}, \ A_{22} \in \mathbf{R}^{(n-r)\times(n-r)}, \\ L_{i1} \in \mathbf{R}^{n\times r}, \quad L_{i2} \in \mathbf{R}^{n\times(n-r)}, \ \ i = 0, \ 1. \end{array} \tag{16}$$

It is known that the system (12) has no impulsive modes if and only if A_{22} is nonsingular. Thus, it will be shown that nonsingularity of A_{22} if $L(s)$ is given by eqn.(14). Now,

$$\begin{aligned} & L(s)(sE - A) \\ &= \left(\begin{bmatrix} L_{01} & L_{02} \end{bmatrix} + s \begin{bmatrix} L_{11} & L_{12} \end{bmatrix} \right) \begin{bmatrix} sE_1 - A_{11} & -A_{12} \\ -A_{21} & -A_{22} \end{bmatrix} V^T \\ &= s^2 \begin{bmatrix} L_{11}E_1 & 0 \end{bmatrix} - \begin{bmatrix} L_{01} & L_{02} \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} V^T \\ & -s \left[\begin{bmatrix} L_{11} & L_{12} \end{bmatrix} \begin{bmatrix} A_{11} \\ A_{21} \end{bmatrix} - L_{01}E_1 \ \ \begin{bmatrix} L_{11} & L_{12} \end{bmatrix} \begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix} \right] V^T \end{aligned} \tag{17}$$

Since $L(s)$ is a regularizing matrix, the second degree coefficient matrix must be zero. Thus, $L_{11}E_1 = 0$. Since E_1 is nonsingular,

$$L_{11} = 0. \tag{18}$$

Then, eqn.(17) can be written by

$$\begin{aligned} L(s)(sE - A) \ = & -\begin{bmatrix} L_{01} & L_{02} \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} V^T \\ & -s \begin{bmatrix} L_{12}A_{21} - L_{01}E_1 & L_{12}A_{22} \end{bmatrix} V^T. \end{aligned}$$

Again, since $L(s)$ is a regularizing matrix, the first degree coefficient matrix must be nonsingular. Thus, $L_{12}A_{22}$ must have column full rank. Therefore, A_{22} must be nonsingular.

Conversely, if A_{22} is nonsingular, define $L(s)$ by

$$L(s) = \begin{bmatrix} I_r & 0 \\ 0 & sI_{n-r} \end{bmatrix} U^T. \tag{19}$$

Lemma 2: If the system (17) has some impulsive modes, then there exists a regularizing polynomial matrix which degree is greater than or equals to two. Conversely, if the regularizing polynomial matrix of $sE - A$ cannot be described as a first degree polynomial matrix, but there exists a regularizing polynomial matrix which degree is greater than or equals to two, then the system (17) has some impulsive modes.

(Proof). Consider the Weierstrass form of $(E, \ A)$ as follows:

$$S^{-1}ET = \begin{bmatrix} I_d & 0 \\ 0 & N \end{bmatrix}, \quad S^{-1}AT = \begin{bmatrix} A & 0 \\ 0 & I_{n-d} \end{bmatrix}, \tag{20}$$

where S and T are nonsingular, and N is given by

$$\begin{aligned} N &= diag\{N_1, \ N_2, \ \ldots, \ N_\alpha\} \in \mathbf{R}^{(n-d)\times(n-d)}, \\ N_i &= \begin{bmatrix} 0 & 1 & & \\ & 0 & \ddots & \\ & & \ddots & 1 \\ & & & 0 \end{bmatrix} \in \mathbf{R}^{k_i \times k_i}, \quad \sum_{i=1}^{\alpha} k_i = n - d. \end{aligned} \tag{21}$$

If the system has some impulsive modes, then $k_i \geq 2$ for some i. In this case $N_i \neq 0$ and thus $N \neq 0$. Then, there exists a unimodular matrix $U_2(s)$ which degree is $\max(k_i - 1)$ such that

$$U_2(s)(sN - I_{n-d}) = I_{n-d}, \quad U_2(s) = (sN - I_{n-d})^{-1} \tag{22}$$

Then, a regularizing polynomial matrix $L(s)$ is given by

$$L(s) = \begin{bmatrix} I_d & 0 \\ 0 & sU_2(s) \end{bmatrix} S^{-1}. \tag{23}$$

Since $U_2(s)$ is at least first order polynomial matrix, the order of $L(s)$ is greater than or equals to two.

Conversely, assume that the regularizing polynomial matrix of $sE - A$ cannot be described as a first degree polynomial matrix, but there exists a regularizing polynomial matrix which degree is greater than or equals to two. From the definition of the regularizing polynomial matrix, there exists an $n \times n$ matrix $\bar{A}$ such that

$$L(s)(sE - A) = sI - \bar{A}.$$

Then, $(sI - \bar{A})^{-1}$ can be written by

$$(sI - \bar{A})^{-1} = \frac{s^{n-1}I + \text{lower degree terms}}{\det(sI - \bar{A})}.$$

Since $L(s)$ is assumed to be the polynomial matrix which degree is greater than one,

$$\begin{aligned} (sE - A)^{-1} &= \{L(s)(sE - A)\}^{-1}L(s) \\ &= (sI - \bar{A})^{-1}L(s) \end{aligned}$$

is improper and thus $(E, \ A)$ has some impulsive modes.

The system (12) is said to be impulsive mode controllable if there exists a feedback gain matrix F such that $sE - A + BF$ has no impulsive mode. From the above Lemmas, we can obtain a necessary and sufficient condition for impulsive controllability.

Theorem 1: The system (12) is impulsive controllable if and only if there exists a feedback gain matrix $F \in \mathbf{R}^{m\times n}$ such that

$$\text{rank} \begin{bmatrix} E & 0 \\ A - BF & E \\ I_n & 0 \end{bmatrix} = \text{rank} \begin{bmatrix} E & 0 \\ A - BF & E \end{bmatrix}. \tag{24}$$

(Proof). From Lemma 1, the closed-loop system

$$\begin{aligned} E\dot{x}(t) &= (A - BF)x(t) + Bu(t), \\ y(t) &= Cx(t) \end{aligned}$$

has no impulsive modes if there exists a first degree regularizing polynomial matrix. In this case, $w = 1$, and then $\boldsymbol{T}_2$ and $\boldsymbol{J}$ are given by

$$\boldsymbol{T}_2 = \begin{bmatrix} E & 0 \\ -A + BF & E \end{bmatrix}, \quad \boldsymbol{J} = \begin{bmatrix} I_n & 0 \end{bmatrix}.$$

Eqn.(7) is solvable if and only if

$$\text{rank} \begin{bmatrix} \boldsymbol{T}_2 \\ \boldsymbol{J} \end{bmatrix} = \text{rank } \boldsymbol{T}_2.$$

Thus, eqn.(24) can be obtained from the above equation.

Conversely, if eqn.(24) holds, then there exists a first degree regularizing matrix. Then, from Lemma 1, the closed-loop system has no impulsive modes, i.e., the open-loop system is impulsive controllable.

Lemma 3: Define the SVD of E by eqn.(15). Corresponding decomposition of A is defined by eqn.(16) and decomposition of B and F are defined by

$$U^T BV = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad \begin{array}{l} B_1 \in \mathbf{R}^{r\times m}, \\ B_2 \in \mathbf{R}^{(n-r)\times m}, \end{array} \tag{25}$$
$$U^T FV = \begin{bmatrix} F_1 & F_2 \end{bmatrix} \quad F_1 \in \mathbf{R}^{m\times r}, \quad F_2 \in \mathbf{R}^{m\times(n-r)}.$$

Then, the system (12) is impulsive controllable if and only if there exists a gain matrix F_2 which makes $A_{22} - B_2F_2$ be nonsingular.

(Proof). Since E_1 is nonsingular,

$$\begin{aligned} &\text{rank} \begin{bmatrix} E & 0 \\ A - BF & E \\ I_n & 0 \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} E_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ A_{11} - B_1F_1 & A_{12} - B_1F_2 & E_1 & 0 \\ A_{21} - B_2F_1 & A_{22} - B_2F_2 & 0 & 0 \\ I_r & 0 & 0 & 0 \\ 0 & I_{n-r} & 0 & 0 \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} E_1 & 0 & 0 \\ 0 & 0 & E_1 \\ 0 & A_{22} - B_2F_2 & 0 \\ 0 & I_{n-r} & 0 \end{bmatrix}. \end{aligned}$$

Thus, eqn.(24) holds if and only if there exists a gain matrix K_2 which makes $A_{22} - B_2F_2$ be nonsingular.

From the view point of the transfer function matrix, $(sE-A)^{-1}B$ is proper if and only if $sE - A$ is row proper. Thus, the problem is to find the feedback gain matrix which makes $sE-A+BF$ be row proper. By an elementary row operation matrix W, sE can be decomposed by

$$sWE = \begin{bmatrix} sE_1 \\ E_2 \end{bmatrix}, \quad \begin{array}{l} E_1 \in \mathbf{R}^{\mu\times n}, \\ E_2 \in \mathbf{R}^{(n-\mu)\times n}, \end{array} \tag{26}$$
$$\text{rank } E_1 = \mu.$$

According to the decomposition, A and B are also decomposed by

$$\begin{aligned} WA &= \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}, \quad \begin{array}{l} A_1 \in \mathbf{R}^{\mu\times n}, \\ A_2 \in \mathbf{R}^{(n-\mu)\times n}, \end{array} \\ WB &= \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad \begin{array}{l} B_1 \in \mathbf{R}^{\mu\times n} \\ B_2 \in \mathbf{R}^{(n-\mu)\times n}. \end{array} \end{aligned} \tag{27}$$

Theorem 2: Let

$$\bar{A} := \begin{bmatrix} E_1 \\ A_2 \end{bmatrix}, \quad \bar{B} := \begin{bmatrix} 0_{\mu\times m} \\ B_2 \end{bmatrix}. \tag{28}$$

The descriptor system is impulsive controllable if and only if there exists a feedback gain matrix $\bar{F}$ such that $\bar{A} - \bar{B}\bar{F}$ does not have any uncontrollable eigenvalues at the origin.

Example 2 Consider the following E, A and B:

$$E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

Then, a regularizing polynomial matrix $L(s)$ of $sE - A$ is given by

$$L(s) = \begin{bmatrix} -s & -s^2 \\ 0.5 & -0.5s \end{bmatrix}$$

and thus there exist some impulsive modes for a given system by Lemma 2. In fact, $sE - A$ is a unimodular polynomial matrix and thus $d = 0$. Since rank $E = 1 > d$, the system has an impulsive mode.

Set $F = \begin{bmatrix} f_1 & f_2 \end{bmatrix}$. Then,

$$\begin{aligned} \text{rank} \begin{bmatrix} E & 0 \\ A - BF & E \end{bmatrix} &= \text{rank} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ f_1 & f_2+1 & 0 & 0 \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ f_1 & f_2+1 & 0 \end{bmatrix}. \end{aligned}$$

If we choose $f_1 \not\leq 0$ and f_2 arbitrary, eqn.(24) holds. Therefore, the system is impulsive controllable by Theorem 1.

On the other hand,

$$\bar{A} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad \bar{A}\bar{B} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}.$$

Since the pair $(\bar{A}, \bar{B})$ is controllable, we can find a feedback gain matrix $\bar{F}$ which makes $\bar{A} - \bar{B}\bar{F}$ be nonsingular.

IV. Conclusions

In this paper, a regularizing polynomial matrix was proposed. Using the matrix, an approach to the descriptor systems by polynomial matrix was proposed. A feedback controller design which removes the impulsive modes was shown.

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