A New Method for Fuzzy Ranking Based on Possibility and Necessity Measures

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***Abstract*— In this paper, a new method to rank fuzzy numbers is presented. The proposed method based on Possibility and Necessity Measures is called PNM. According to possibility and necessity measures, eight indexes are calculated to extract four rules to rank fuzzy numbers. Also a method to evaluate each rule validation especially when rules' outcomes yield conflict conclusions is presented. To test PNM performance, some controversial triangular fuzzy numbers are considered. Additionally, four extracted rules are compared with each other and fully analyzed. Furthermore, PNM is compared with other recently proposed methods. Results confirm that PNM is capable to rank a variety of fuzzy numbers and their images with any selected bandwidths, interval and any degree of closeness.**

***Index Terms*— fuzzy ranking, possibility and necessity measures, triangular fuzzy numbers**

I. Introduction

In many applications, ranking of fuzzy numbers is an important component of the decision making process [1]. In practice, many real-world problems require handling and evaluation of fuzzy data for making decision. To evaluate and compare different alternatives, it is necessary to rank fuzzy numbers. In addition, the concept of optimum or best choice is completely based on ranking or comparison [2].

Since Dubois and Prade [3] introduced the relevant concepts of fuzzy numbers and used maximizing sets to order fuzzy numbers. In addition, many other researches proposed the related methods or applications for ranking fuzzy numbers. For instance, Bortolan and Degani in 1985[4], Chen and Hwang in 1992[5], Zhu and Lee, in 1992 [6] review and comparison of these existing methods can be found. Wang and Lee extended the centroid expectation approach and proposed a preference weighting function expectation method to rank fuzzy number [7]. Furthermore, Sun and Wu proposed a new approach for ranking fuzzy numbers based on the fuzzy simulation analysis (FSA) method in which a combination method including computer and math application was developed [2]. Chu and Tsao presented a method to rank fuzzy number. They employed an area between the centroid and original points to rank fuzzy numbers; however there were some problems with the ranking method [8]. Wang and Lee revised Chu and Tsao's method which can avoid these problems [7].

Abbasbandy and Asady considered a fuzzy origin for fuzzy numbers and then according to the distance of fuzzy numbers with respect to this origin, they rank them [1]. Cheng method is based on the coefficient of variance (CV index). In this approach, the fuzzy number with smaller CV index has higher rank [9]. Li and Ma in their study proposed a novel method incorporating fuzzy preferences and range reduction techniques. The proposed model first was applied to adapt a modified Data Envelopment Analysis (DEA) model to generate reasonable upper and lower bounds of preference ratios. By referring to these ranges, a decision maker then specifies his/her fuzzy preferences partially [10].Asady and zendehnam proposed a defuzzification method using minimizer of the distance between the two fuzzy numbers [11].

Regarding to several strategies having been reviewed above, these strategies are based on methods including distance between fuzzy sets, centroid point and original point, coefficient of variation (CV index), and weighted mean value. Since each of these techniques has some problems, they are not complete. For instance, some methods, properly ranking fuzzy numbers, are not able to correctly rank fuzzy number images. In addition, when a fuzzy number is covered by another fuzzy number, most ranking methods confront several difficulties.

However, in this paper we present a novel method based on possibility and necessity measure. In the proposed method, we define AP as upper possibility distribution function and A_n

as lower possibility distribution function. Then we compare B with AP and An instead of A and vice versa.

The rest of the paper is organized as follows: in section 2, the proposed method is comprehensively described. The third section is assigned to numerical examples and further discussions on the proposed method and Validation of rules is explained in section 4. In addition, this method is compared with aforementioned techniques in section 5. Finally, section 6 concludes the paper.

II. Proposed Method

As mentioned, infrastructures of our method are based on possibility and necessity theories. Possibility theory is one of the current uncertainty theories devoted to the handling of incomplete information; more precisely, it is the mathematically simplest one. To a large extent, it is similar to probability theory because it is based on set functions. It differs from the latter by the use of a pair of dual set functions called possibility and necessity measures [12] instead of only one. Besides, it is not additive and makes sense on ordinal structures. The name "Theory of Possibility" was coined by [13]. In Zadeh's view, possibility distributions were meant to provide graded semantics to natural language statements. However, possibility and necessity measures can also be the basis of a full-fledged representation of partial belief that parallels probability. It can be seen either as a coarse, non-numerical version of probability theory, or a framework for reasoning with extreme probabilities [14], or yet a simple approach to reasoning with imprecise probabilities [15]. The theory of large deviations in probability theory also handles set functions which look like possibility measures [16]. Formally, possibility theory refers to the study of maxitive and minitive set functions, respectively, called possibility and necessity measures such that the possibility degree of a disjunction of events is the maximum of the possibility degrees of events in the disjunction, and the necessity degree of a conjunction of events is the minimum of the necessity degrees of events in the conjunction [17]. For using possibility and necessity theory, we considered bellow definitions and rules.

Definition 1: The possibility measure like the fuzzy measure is the function $\Pi : F \to [0,1]$ in which F is the σ field. In addition to boundary, uniformity and continuity conditions, the possibility measure possess the following property:

$$\Pi(A \cup B) = \Pi(A) \vee \Pi(B), \quad \forall A, B \in F \quad (1)$$

Definition 2: The necessity measure also is the function $N : F \to [0,1]$ in which in addition to boundary, uniformity and continuity conditions N holds the following feature like F:

$$N(A \cap B) = N(A) \wedge N(B), \quad \forall A, B \in F \quad (2)$$

For a fuzzy set like A, possibility and necessity measures are defined as follows:

Assume that $A \in F(U)$ and π_X is the possibility distribution function for variable X which selects its values from U; hence for the fuzzy set A, possibility and necessity measures $\Pi(A), N(A)$ are calculated according to the following formulas:

$$\Pi(A) = \sup_{u \in U} \big(A(u) \wedge \pi_X(u)\big) \quad (3)$$

$$N(A) = \inf_{u \in U} \big(A(u) \wedge (1 - \pi_X(u))\big) \quad (4)$$

In the proposed method, to compare fuzzy numbers A and B four auxiliary functions A_p, A_n, B_p and B_n defined below are needed.

$$A_P(u) = \vee_{u \le x} A(u) \quad (5)$$

$$A_n(u) = \wedge_{u \ge x} (1 - A(u)) \quad (6)$$

In fact, A_p which is a fuzzy set is possibly equal or greater than fuzzy number A. As well A_n , a fuzzy set, is necessarily greater than fuzzy number A. B_p and B_n are similarly defined.

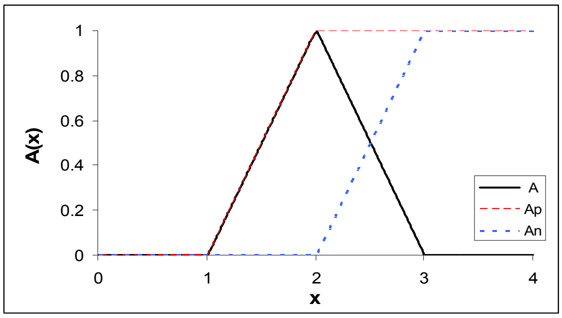
Now for a fuzzy triangular number $A(l,c,r)$, A_p and A_n are computed as:

$$A_p(u) = \begin{cases} 0 & u < l \\ A(u) & l \le u \le c, \quad u \in U \\ 1 & u > c \end{cases} \quad (7)$$

Similarly:

$$A_n(u) = \begin{cases} 0 & u \le c \\ 1 - A(u) & c \le u \le r, \quad u \in U \\ 1 & u \ge r \end{cases} \quad (8)$$

For example, A_p and A_n are exemplified in Fig 1 for A(1,2,3).



Figur1.A Fuzzy number with A_p and A_n

As it appears in figure 1, A_p and A_n respectively are upper and lower possibility distribution functions. In order to rank two fuzzy number A and B, it is more appropriate to compare B with A_p and A_n instead of A and similarly A with B_p and B_n instead of B. It can be done by developing $\Pi_B(A_p)$, $\Pi_B(A_n)$, $N_B(A_n)$ and $N_B(A_p)$ explained below:

$$\Pi_B(A_P) = \vee_v (B(v) \wedge A_P(v)) = \vee_v \left(B(v) \wedge (\vee_{u \le v} A(u)) \right) = \vee_v \left(\vee_{u \le v} (B(v) \wedge A(u)) \right) \quad (9)$$

$\Pi_B(A_p)$ indicates the possibility that the maximum value of V (the reference set of B) is greater than or equal to the minimum value of U (the reference set of A).

$$\Pi_B(A_n) = \vee_v (B(v) \wedge A_n(v)) = \vee_v \left(B(v) \wedge (\vee_{u \geq v} (1 - A(v))) \right)$$

$$= \vee_v \wedge_{u \geq v} \left((B(v) \wedge (1 - A(u)) \right) \qquad (10)$$

$\Pi_B(A_n)$ states the possibility that the maximum value of V is greater than or equal to the maximum value of U.

$$N_B(A_n) = \wedge_v ((1 - B(v)) \vee A_n(v)) = \wedge_v \left((1 - b(v)) \vee (\wedge_{u \geq v} (1 - A(u))) \right)$$
$$= 1 - \vee_{u \geq v} (B(v) \wedge A(u)) \qquad (11)$$

As well, $N_B(A_n)$ points out the necessity that the minimum value of V is greater than the maximum value of U.

$$N_B(A_P) = \wedge_v ((1 - B(v)) \vee A_P(v)$$
$$= \wedge_v \left((1 - B(v)) \vee (\vee_{v \geq u} A(u)) \right) \qquad (12)$$
$$= \wedge_v \left(\vee_{v \geq u} ((1 - B(v)) \vee A(u)) \right)$$

Finally, $N_B(A_p)$ refers to the necessity that the minimum value of V is greater than or equal to the minimum value of U.

We correspondingly define $\Pi_A(B_p)$, $\Pi_A(B_n)$, $N_A(B_n)$ and $N_A(B_p)$ as mentioned above. In fact based on the above-mentioned discussion, eight different indexes are consequently calculated. Hence, although different combinations of these indexes-chosen two by two- exist; all indexes cannot be measured through pair comparison because most of these comparisons are not significant. For example, $\Pi_A(B_p)$ cannot be evaluated by $N_B(A_n)$, $\Pi_B(A_n)$ or $N_B(A_p)$ due to having different dimensions; hence, $\Pi_A(B_p)$ can merely be compared with $\Pi_B(A_p)$. Therefore, according to these comparison indexes, only four rules, not three or five rules, can be extracted.

Rule1. If $\Pi_B(A_p)$ is greater than $\Pi_A(B_p)$ then B is greater than A.

In fact, rule1 compares two available possibilities in the "if part". $\Pi_B(A_p)$ as stated indicates the possibility showing that the maximum value of B at least is equal to the minimum value of A. In contrast with $\Pi_B(A_p)$, $\Pi_A(B_p)$ depicts the possibility uttering that the maximum value of A at least is equal to the minimum value of B; hence in if-part of rule 1, these two possibility are compared. If $\Pi_B(A_p) > \Pi_A(B_p)$ then it can be inferred that B is greater than A.

Rule2. If $\Pi_B(A_n)$ is greater than $\Pi_A(B_n)$ then B is greater than A.

As well, while in the rule1, the minimum value of A is compared with Maximum value of B and vice versa, the second rule compares maximum values of each fuzzy number.

Rule3. If $N_B(A_n)$ is smaller than $N_A(B_n)$ then B is greater than A.

Despite rule1, indicating the possibility, rule3 states necessity showing that the minimum value of B is bigger than the maximum value of A.

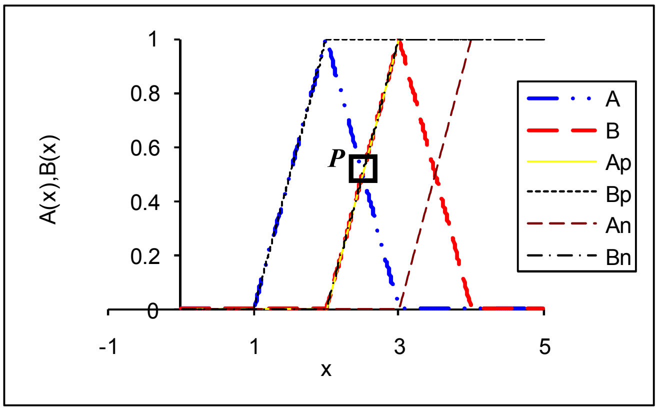
Rule4. If $N_B(A_p)$ is smaller than $N_A(B_p)$ then B is greater than A.

This rule evaluates the necessity indicating that the minimum value of B at least is equal to the minimum value of A with the necessity indicating that the minimum value of A at least is equal to the minimum value of B. in fact this rule compares the necessity of these two fuzzy number minima.

In order to demonstrate the effectiveness of each rule, these rules are evaluated by several numerical examples in the following section.

III. Numerical Examples

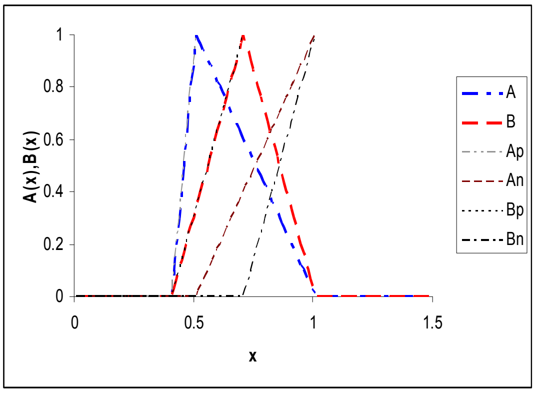
A. Example 1. Consider two fuzzy numbers A(1,2,3) and B(2,3,4), table 1 lists the results of simulation.



Figur2. A and B Fuzzy number and their index for example 1

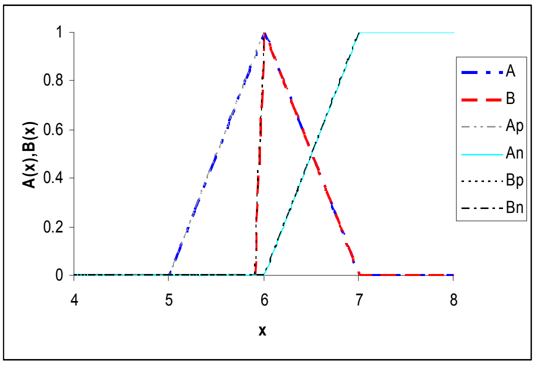
In addition to fuzzy numbers A and B, figure 2 illustrates A_p, A_n, B_p and B_n. In fact, each of the aforementioned indexes like $\Pi_A(B_p)$ can be extracted from figure 2. For example, $\Pi_A(B_p)$ is corresponding to intersection point of the right hand side of A with the left hand side of B (Point P in figure 2).

B. Example 2. In this example, A(0.4,0.5,1) and B(0.4,0.7,1), which are very close to each other, are compared. The closeness of these numbers makes trouble for the appropriate ranking. Figure 3 shows their indexes.



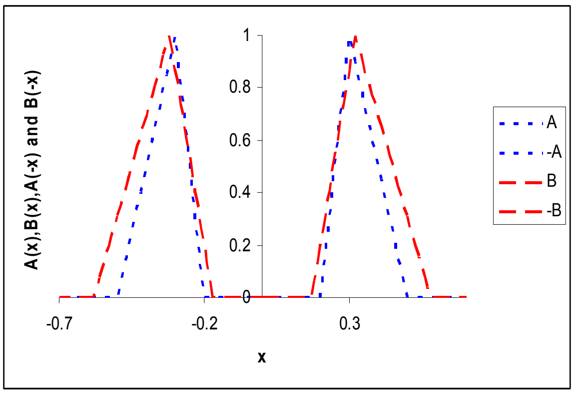
Figur3. A and B Fuzzy number and their index for example 2

C. Example 3. A (5, 6, 7) and B(5.9,6,7) are selected to test PNM method. According to the classical set theory B is subset of A because it is completely covered by A. in such cases how to rank these numbers is somewhat crucial.



Figur4. A and B Fuzzy number and their index for example 3

D. Example 4. A (-0.5,-0.3,-0.2) and B (-0.58,-0.32,-17) are negative fuzzy numbers. As stated, some techniques performing well in the fuzzy number ranking, cannot properly rank images of them.



Figur5. A and B Fuzzy number and their index for example 4

In the examples of 2, 3 and 4 given above, since different fuzzy ranking methods have various results on these examples; we selected them to test our methods. For example, some fuzzy numbers have equal band widths and their cores are very close so that decision making becomes complicated (see example 2). In addition, when a fuzzy number is covered by another one, there is no consensus how to rank those numbers (see example 3). Furthermore, if A<B, we logically infer that -B<-A while most of proposed methods are unable to correctly rank inverse fuzzy numbers (see example 4). The proposed method ranks fuzzy numbers and their images acceptably.

IV. Rules Validation Assessment

In this section, a method to evaluate each rule validation is presented. This method is used when rules' outcomes yield conflict conclusions. Since the afore-mentioned four rules may have conflicting results, the decision maker should finally come up with a concrete solution. To do so, we are required to measure somehow the strength of each rule in order to reach to a final conclusion. The following procedure is proposed.

Let $\eta_{ij}\ ; i=1,2\ \ j=1,2,3,4$ denote the ith parameter value of the jth rule and define:

$$\phi_j = \max_i \{\eta_{ij}\}; j = 1,2,3,4 \qquad (13)$$

$$\psi_j = \min_i \{\eta_{ij}\}; j = 1,2,3,4 \qquad (14)$$

The Validation Degree (VD) of each rule is calculated as:

$$VD_j = \frac{\phi_j - \psi_j}{\psi_j + \varepsilon}\ ; \qquad \forall\ j = 1,2,3,4 \qquad (15)$$

where, ε is a small positive real number . Accordingly, two sets are defined, Ω_1 which is the set of those rules saying that A<B and Ω_2 which is the set of those rules saying that A>B. Thereafter, we calculate

$$\lambda_1 = \sum_{k\in\Omega_1} VD_k \qquad (16)$$

$$\lambda_2 = \sum_{l\in\Omega_2} VD_l \qquad (17)$$

$\lambda_1 \succ (\prec) \lambda_2$ implies $A \prec (\succ) B$. Furthermore, for $\lambda_1 = \lambda_2$, we say that A is almost equal to B.

To demonstrate the efficiency of the proposed method, some examples in which rules' outcomes still hold disagreement outputs are used. The results are listed in table 2.

The results of the aforementioned examples and commutated indexes are illustrated in table 1. As mentioned, four rules exploited from possibility and necessity measures are applied to rank fuzzy numbers. These rules do not agree with each other in all cases. On the other hands, in some cases rule 2 and rule 4 have different results while rule 1 and rule 3 comply to the same consequence in all numbers which we tested.

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In this table, four pair of fuzzy numbers in which the aforementioned rules disagree whether A is greater then B or not are selected. In these examples, some rules express that A is greater than B while some others state that B is greater than A. Column 4 presents the result of each rule. By following the procedure described above, λ_1 and λ_2 are consequently calculated and used to determine the final result as given in the last column of this table.

V. More Comparison Studies

In this section, we compare PNM with other recently proposed methods reviewed in section 1. Table 3 depicts the comparative results.

Set 1 : A(.2,.5,.9) B(.1,.6,.8)

Set 2 : A(.3,.7,1.1) B(.5,.7,.9)

Set 3 : A(.4,.5,1) B(.4,.7,1) C(.4,.9,1)

To compare fuzzy ranking methods, 3 different set of fuzzy numbers in which the proposed methods do not agree on how to rank them, are selected. As easily observed from table 3, these methods have different results for same fuzzy numbers. Results of ranking demonstrate that the proposed method performs as well as most of the recently proposed techniques (like Yager's method) to rank fuzzy numbers.

TABLE 1. INDEXES CALCULATED IN PNM FOR DIFFIRENT EXAMPLES

Example	FUZZY NUMBER	Rules	Results	INDEX							
				$\Pi_B(A_p)$	$\Pi_B(A_n)$	$N_B(A_n)$	$N_B(A_p)$	$\Pi_A(B_p)$	$\Pi_A(B_n)$	$N_A(B_n)$	$N_A(B_p)$
1	A(1,2,3) B(2,3,4)	1 2 3 4	$A<B$ $A<B$ $A<B$ $A<B$	1	1	.5	1	.5	0	0	0
2	A(.4,.5,1) B(.4,.7,1)	1 2 3 4	$A<B$ $A<B$ $A<B$ $A<B$	1	.62	.26	.7667	.74	.3667	0	.2667
3	A(5,6,7) B(5.9,6,7)	1 2 3 4	$A<B$ $A<B$ $A<B$ $A<B$	1	.5	0	.91	1	.5	0	1
4	A(-.5,-.3,-.2) B(-.58,-.32,-.17)	1 2 3 4	$A>B$ $A<B$ $A>B$ $A>B$	.9333	.5	2.2× 10^-16	.4	1	.4667	.0667	.6154
5	A(.2,.3,.5) B(.17,.32,.58)	1 2 3 4	$A<B$ $A<B$ $A<B$ $A>B$	1	.6	.0667	.5	.9333	.3846	0	.5333
6	A(-1,-.5,-.4) B(-1,-.7,-.4)	1 2 3 4	$A>B$ $A>B$ $A>B$ $A>B$	.74	.2333	3.33× 10^-16	.38	1	.7333	.26	.6333
7	A(-7,-6,-5) B(-7,-6,-5.9)	1 2 3 4	$A<B$ $A>B$ $A>B$ $A>B$	1	.09	0	.5	1	.9	8.88× 10^-16	.5
8	A(0,2,3) B(1,2,3)	1 2 3 4	$A<B$ $A<B$ $A<B$ $A<B$	1	.5	0	.67	1	.5	0	.335
9	A(0,1,2) B(0,1,3)	1 2 3 4	$A<B$ $A<B$ $A<B$ $A<B$	1	.6650	0	.5	1	.33	0	.5
10	A(-2,-1,0) B(-3,-1,0)	1 2 3 4	$A<B$ $A<B$ $A<B$ $A>B$	1	.5	0	.3350	1	.5	0	.67

TABLE 2. RULES VALIDATION ASSESMENT FOR EXAMPLES

Example	Fuzzy Number	No. Rule	Conflict Rules	ϕ_j	ψ_j	VD_j	λ	Final Result
4	A(-.5,-.3,-.2) B(-.58,-.32,-.17)	1 2 3 4	A>B A<B A>B A>B	$\phi_1 = 1$ $\phi_2 = .5$ $\phi_3 = .06$ $\phi 4 = .61$	$\psi_1 = .93$ $\psi_2 = .46$ $\psi_3 = .06$ $\psi_4 = .4$	$VD_1 = .07$ $VD_2 = .46$ $VD_3 = 0$ $VD_4 = .53$	λ_1=.07 λ_2=.60	A>B
5	A(.2,.3,.5) B(.17,.32,.58)	1 2 3 4	A<B A<B A<B A>B	$\phi_1 = 1$ $\phi_2 = .6$ $\phi_3 = .06$ $\phi 4 = .53$	$\psi_1 = .93$ $\psi_2 = .38$ $\psi_3 = 0$ $\psi_4 = .5$	$VD_1 = .07$ $VD_2 = .55$ $VD_3 = .66$ $VD_4 = .06$	λ_1=1.2 λ_2=.09	A<B
7	A(-7,-6,-5) B(-7,-6,-5.9)	1 2 3 4	A<B A>B A>B A>B	$\phi_1 = 1$ $\phi_2 = .9$ $\phi_3 = 0$ $\phi 4 = .5$	$\psi_1 = 1$ $\psi_2 = .09$ $\psi_3 = 0$ $\psi_4 = .5$	$VD_1 = 0$ $VD_2 = 8.9$ $VD_3 = 0$ $VD_4 = 0$	λ_1= 0 λ_2= 8.9	A>B
10	A(-2,-1,0) B(-3,-1,0)	1 2 3 4	A<B A<B A<B A>B	$\phi_1 = 1$ $\phi_2 = .5$ $\phi_3 = 0$ $\phi 4 = .67$	$\psi_1 = 1$ $\psi_2 = .5$ $\psi_3 = 0$ $\psi_4 = .33$	$VD_1 = 0$ $VD_2 = 0$ $VD_3 = 0$ $VD_4 = .99$	λ_1= 0 λ_2=.99	A>B

TABLE3. COMPARISON RESULTS OF PNM WITH OTHER METHODS

Methods	SET 1	SET 2	SET 3
Choobineh	*A<B*	*A<B*	*A<B<C*
Yager	*A<B*	*A<B*	*A<B<C*
Chen	*A<B*	*A<B*	*A<B<C*
Bakwin	*A≈B*	*A<B*	*A<B<C*
Chu & Tsao	*A>B*	*A<B*	*A<B<C*
Cheng distance	*A>B*	*A<B*	*A<B<C*
Cheng CV	*A<B*	*A>B*	*B<C<A*
Wang central	*A>B*	----	*A<B<C*
Wang distance	*A>B*	----	*A<B<C*
Asady	*A≈B*	*A<B*	*A<B<C*
PNM	*A<B*	*A<B*	*A<B<C*

VI. Conclusion

In this paper, a new method called PNM to rank fuzzy numbers was presented. To develop PNM, possibility and necessity measures were employed. According to these measures, eight indexes were calculated by which four rules were extracted. These rules were tested for different types of fuzzy triangular numbers which are controversial to rank. In addition, we proposed a method to evaluate each rule validation. It can be used when the four mentioned rules have contradictory results. Results demonstrated that PNM can remarkably rank these fuzzy numbers. Finally, PNM was compared with 10 different methods, some of them recently proposed. Results proved that PNM has ability to rank a variety of fuzzy numbers and their images with any selected bandwidths, interval and any degree of closeness.

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