Lanczos Model Reduction for Switched Linear systems

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***Abstract*—In today the methods reduction of large-scale linear time invariant and complexe systems are very many, the best choices today is the used of the krylov subspace methods based on moment matching. As hybrid dynamical systems are of rising spread and complexity, for these reasons, we present in this paper two model reduction methods applied to linear switched system. Which is an important class of hybrid and non linear system. Tow methods for reduction systems are present. In first part we present the *modified non symmetric Lanczos algorithm*, which is numerically efficient and applicable of any order. In second part we present the *modified global lanczos algorithm*, it is also numerically efficient, applicable of any order and having a best numerical stability. The effectivity and suitability of these new methods is illustrated by one simulation example.**

Keywords*—Model-order reduction, Krylov subspace, Multiple points moment matching, Lanczos, Hybrid systems, Switched systems.

I. INTRODUCTION

Hybrid dynamical systems are frequently encountered in some fields such as Electrical circuit, Power electronics system, Thermal-fluid systems and Mechanical system,...,many modeling and control methods are developed of large scale system [11, 12, 13]. However, these high order models are difficult to manipulate, the resolution of such models is indeed very demanding in computational resources, especially when applying a control strategy which become very difficult to determine. Switched system, representing an important class of hybrid system, which the latter is a general way an interconnection of continuous and discreet dynamics [1, 14]. However, in the switched system the discreet dynamics are reducing to switching events. Definitely, these systems consists of a finite amount $q \in IN$ of continuous dynamical linear time invariant (LTI) subsystems, with q is a function piecewise constant over time called a switching signal, for simplicity we write q [14].
The states representation of switched systems is as follows [1, 9, 10, 18]:

$$\Sigma_q = \begin{cases} x(t+1) = A_q x(t) + B_q u(t) \\ y(t) = C_q x(t) + D_q u(t) \end{cases} \quad (1)$$

In which $A_q \in IR^{n\times n}$, $B_q \in IR^{n\times p}$, $C_q \in IR^{p\times n}$, $D_q \in IR^{p\times p}$, $u(t) \in IR^{n\times p}$, $y(t) \in IR^{p\times n}$ and q is a switching signal.
Reduction of these systems is an important task of treatment and analysis of high order systems, especially, in the case of determination of a controller parameters. Several approaches exit in the literature for calculation of these parameters but they are easy to apply on the second order system. The problem is to obtain a reduced order model of second order, guaranteeing stability and minimizing the error between the original system and reduced one by the use of the Lanczos approaches [4, 8, 9, 10].
The states representation of reduction hybrid dynamic systems is as follows [1, 9, 12, 18]:

$$\hat{\Sigma}_q = \begin{cases} \hat{x}(t+1) = \hat{A}_q x(t) + \hat{B}_q u(t) \\ \hat{y}(t) = \hat{C}_q x(t) + \hat{D}_q u(t) \end{cases} \quad (2)$$

In which $\hat{A}_q \in IR^{k\times k}$, $\hat{B}_q \in IR^{k\times p}$, $\hat{C}_q \in IR^{p\times k}$, $\hat{D}_q \in IR^{k\times k}$ and $\hat{y}(t) \in IR^{p\times k}$ with $k << n$.
For hybrid dynamical system we can not get the bode diagram of the entire system, thus we presents the error $e(t)$ between the output of two systems, which defined by [1, 11]:

$$e(t) = y(t) - \hat{y}(t) \quad (3)$$

This paper is organized as follows: in section 2, the some preliminaries are given. section 3, the Modified Non Symmetric Lanczos method, will be presented with application on the numerical example. In section 4, we detailed the Modified Global Lanczos method and evaluate by the use of the numerical example. Section 5, we give a comparison between these proposed methods and the others methods of the literature. The last section is dedicated to conclude this paper.

II. SOME PRELIMINARIES

In this part we will take $q = 0$ and treating the LTI system in a general way, then the state space of system is as form [5, 6, 7]:

$$\Sigma = \begin{cases} x(t+1) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \quad (4)$$

A. Principle of The Moment Matching

The principle of the moment matching are as follows,given a linear system in state space form equ.1 and equ.2, with the transfer function as in this form $G(s) = C(sI - A)^{-1}B + D$ [5, 6, 7], for simplicity we assume that $D = 0$. if $G(s)$ is expanded in Laurent series around a given point $s_0 \in \mathcal{C}$ in the complex plane [5, 6, 7, 8, 9]:

$$G(s_0 + \sigma) = \eta_0 + \eta_1\sigma + \eta_2\sigma^2 + \eta_3\sigma^3 + ... \quad (5)$$

The η_t are called the moments of LTI system at s_0. We are interested in determining a reduced system, which matches the $2k$ coefficients, such that the transfer function as in this form $G(\hat{s}) = \hat{C}(sI - \hat{A})^{-1}\hat{B} + \hat{D}$ and the Laurent expansion of the reduced transfer function at s_0 has the form :

$$\hat{G}(s_0 + \sigma) = \hat{\eta_0} + \hat{\eta_1}\sigma + \hat{\eta_2}\sigma^2 + \hat{\eta_3}\sigma^3 + ... \quad (6)$$

With, $\eta_j = \hat{\eta_j} forj = 1, 2, ..., 2k$.

B. Moment matching through Lanczos Methods

Take is a linear dynamical system in a state space form equ.1 and equ.2. Let us define a initial vectors r_0, q_0 and a matrix ψ. The Lanczos process is based to compute two rectangular matrices $W_k, V_k \in IR^{n*k}$ which satisfy the biorthogonality condition $W_T^k V_K = I$ and the Krylov subspace conditions $colspV_k = K_k(\psi, r_0)$ and $colspW_k = K_k(\psi^T, q_0)$, where the Krylov subspace are as follows [2, 3]:

$$K_k(\psi, r_0) = span\{r_0, \psi r_0, ..., \psi^{k-1} r_0\} \quad (7)$$

and

$$K_k(\psi^T, q_0) = span\{q_0, \psi^T q_0, ..., \psi^{k-1^T} q_0\} \quad (8)$$

Where, in the general case $\psi = A$, $r_0 = B$ and $q_0 = C$.
After K steps, the Lanczos Algorithm can iteratively generate two orthonormal basis V_k and $W_k \in IR^{n*k}$ from the successive Krylov subspace [1, 2, 3]:

$$K_k(\psi, r_0) = span\{v_1, v_2, ..., v_k\} \quad (9)$$

and

$$K_k(\psi^T, q_0) = span\{w_1, w_2, ..., w_k\} \quad (10)$$

Where $v_i \in V_k$ and $w_i \in W_k$, for $i = 1, ..., k$.
During the iteration process, a tridiagonal Matrix $T_k \in IR^{k*k}$ is generate that satisfies the following relationships:

$$AV_k = V_k T_k + \delta_{k+1} v_{k+1} e_T^T \quad (11)$$

and

$$A^T W_k = W_k T_k^T + \beta_{k+1} w_{k+1} e_T^T \quad (12)$$

Where e_q is the qth unit vector in IR^k.

$$T_q = \begin{pmatrix} \alpha_1 & \beta_3 & . & . & . & . & . & . & . \\ \delta_2 & \alpha_2 & \beta_2 & . & . & . & . & . & . \\ . & \delta_3 & \alpha_3 & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & \beta_k \\ . & . & . & . & . & . & . & \delta_k & \alpha_k \end{pmatrix} \quad (13)$$

C. BIBO Stability of Linear Switching Systems

Theorem.[16, 17] We say that the system in equation (2) is BIBO stable, we can proved by the using of the exponentially stability over the switching signal set q, if there exist two positive constants $0 < \epsilon < 1$ and $0 < \mu < \infty$, such that for any switching signal q and for the identically zero-input $u(t) = 0$, $t \geq 1$, the norm of the reduced output sequence $\hat{x}(t)$, $t \geq 0$ can be bounded above as follows:

$$\|x(t)\| \leq \mu\epsilon^t \|x(0)\| \quad (14)$$

Proof. The proof can be found in [16].

III. Modified non Symmetric Lanczos for Switched Linear System

Take a linear switched system as the form:

$$x(t+1) = A_q x(t) + B_q u(t) \quad (15)$$

$$y(t) = C_q x(t) + D_q u(t) \quad (16)$$

In our case, we take $m = p = 1$, we seek to find the reduced model as this form:

$$\hat{x}(t+1) = \hat{A}_q x(t) + \hat{B}_q u(t) \quad (17)$$

$$\hat{y}(t) = \hat{C}_q x(t) + \hat{D}_q u(t) \quad (18)$$

The order of reduced model is equal of $k << n$, such that the first $2k$ Markov parameters $\eta_{i_{(q)}} := C_q A_q^{i-1} B_q$ and $\hat{\eta}_{i_{(q)}} := \hat{C}_q \hat{A}_q^{i-1} \hat{B}_q$,of each original sub-system and reduced sub-system respectively are matched:

$$\eta_{i_{(q)}} = \hat{\eta}_{i_{(q)}}, \quad for \quad i = 1, ..., 2k \quad (19)$$

The parameters of the reduced order model are obtained by using the following biorthogonal projection $\hat{x}(t) = W_{k_{(q)}}^T x(t) V_{k_{(q)}}$.
The reduced sub-system parameters in equ.3 and equ.4 can be obtained by the congruence transformation [4, 9]:
$\hat{A}_q = W_{k_{(q)}}^T A_q V_{k_{(q)}}$, $\hat{B}_q = W_{k_{(q)}}^T B_q$, $\hat{C}_q = V_{k_{(q)}}^T C_q$, $\hat{D}_q = D_q$.
The detail of the Modified Lanczos algorithm can be found in Table1 [7, 2, 3]:
Table1:Lanczos

Modified Lanczos Algortihm:*(Input:*A_q,B_q,C_q,D_q,r_0,q_0,k,q;
Output:W_{k_q},V_{k_q}*)*
Switch q{
*(1):/*Initialize*/*
$\beta_{1_q} := \sqrt{C_q B_q}$,
$\gamma_{1_q} := sgn(C_q B_q)\beta_{1_q}$,
$v_{1_q} := B_q/\beta_{1_q}$,
$w_{1_q} := C_q^*/\gamma_{1_q}$
*(2):/*Generate the new orthonormal vector*/*
for j=1,...,k do
$\alpha_{j_q} := w_{j_q}^* A_q v_{j_q}$
$r_{j_q} := A_q v_{j_q} - \alpha_{j_q} v_{j_q} - \gamma_{j_q} v_{j-1_q}$
$q_{j_q} := A_q^* w_{j_q} - \alpha_{j_q} w_{j_q} - \beta_{j_q} w_{j-1_q}$
$\beta_{j+1_q} = \sqrt{|r_{j_q}^*|}$
$\gamma j + 1_q = sgn(r_{j_q}^* q_{j_q})\beta_{j+1_q}$
$v_{j+1_q} = r_{j_q}/\beta_{j+1_q}$
$w_{j+1_q} = q_{j_q}/\gamma_{j+1_q}$
end for
}

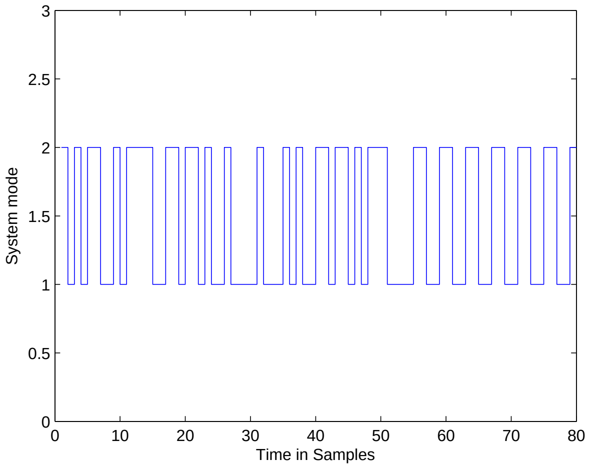

Fig. 1. Switching Signal

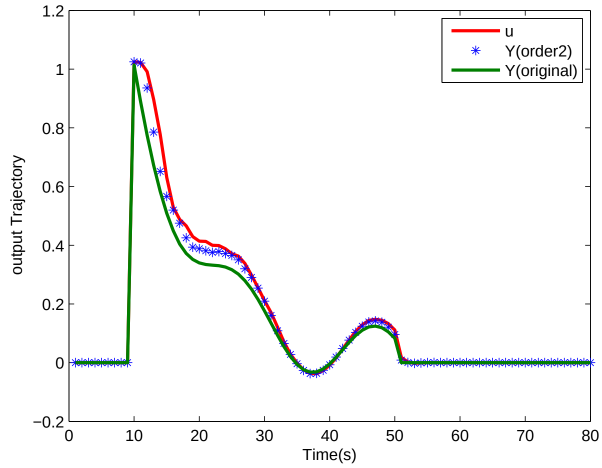

Fig. 2. Output trajectories of order reduction 2 by Modified Non Symmetric Lanczos method

A. Numerical example

To evaluate this approach we take the model used by [Gao Huijun] in the paper [5] and a switched signal where q=1,2 [5],which parameters of States representation are as follows:

$$A_1 = \begin{pmatrix} 0.1612 & 0.0574 & -0.0144 & 0.1846 \\ 0.0434 & -0.3638 & 0.5258 & -0.0357 \\ -0.0747 & -0.3146 & -0.0487 & -0.1043 \\ -0.1664 & 0.4031 & 0.0347 & 0.2864 \end{pmatrix},$$

$$B_1 = B_2 = \begin{pmatrix} 0.2023 \\ -0.2313 \\ -0.1137 \\ 0.1279 \end{pmatrix},$$

$$C_1 = C_2 = \begin{pmatrix} 1.4419 & 0.672 & 0.1387 & -0.8595 \end{pmatrix},$$

$D_1 = D_2 = 1.$

The input signal u(t) is:

$$u(t) = \begin{cases} exp(0.1(-t+10)) + 0.1sin(0.3t) & \text{if } 10 \le t \le 50 \\ 0 & \text{otherwise} \end{cases}$$

The switching signal is generate randomly as: {2, 1, 2, 1, 2, 2, 1, 1, 2, 1, 2, 2, 2, 2, 1, 1, 2, 2, 1, 2, 1, 1, 2, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 2, 1, 1, 2, 2, 1, 2, 2, 1, 2, 1, 2, 2, 2, 1, 1, 1}.

The figure 1 present the arbitrary switching signal generate by Matlab with a possible case.

The output trajectories of the original system and reduced one of second order and the input signal are show in the figure 2, we see that a good correlation between the output trajectories of original and reduced system. The output error between the original system and reduced one is depicts in figure 3, we note a slight variation of error, the maximum value of error is equal to 0.12.

IV. Modified Global Lanczos for Switched Linear System

The Global Lanczos Algorithm is an overall improvement of the standard Lanczos algorithm applied to the matrix pairs

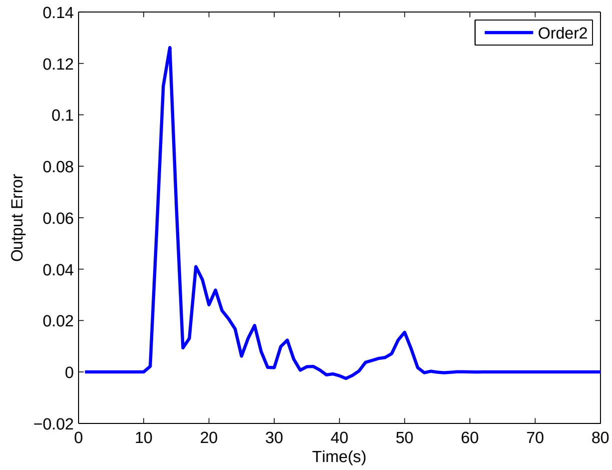

Fig. 3. Output errors of order reduction 2 by Modified Non Symmetric Lanczos method

$(\psi_q,\ \xi_q)$and $(\psi_q^T,\ C_q^T)$ where $\psi = (s_{1_q}E - A_q)^{-1}E$ and $\xi = (s_{1_q}E - A_q)^{-1}B_q$.

This method can be generate recursively two Frobenuis orthonormal bases for two Krylov subspaces [15]:

$$K_{k_q}(\psi_q, \xi_q) = span\{\xi_q, \psi_q\xi_q, ..., \psi_q^{k-1}\xi\} \tag{20}$$

$$L_{k_q}(\psi_q^T, C_q^T) = span\{C_q^T, \psi_q C_q^T, ..., \psi_q^{k-1}C_q^T\} \tag{21}$$

The detail of the Modified Lanczos algorithm can be found in Table2 [15]:

Table2:Global Lanczos

Modified Global Lanczos Algortihm:*(Input:A_q,B_q,C_q,D_q,ψ_q, ξ_q,k,q; Output:W_{g,k_q},V_{g,k_q})*

Switch q*{ (1):/*initialize*/ Set $\psi_q = -(s_qE - A_q)^{-1}E$,*

Set $\xi_q = (s_qE - A_q)^{-1}E$,

Set $\beta_{1_q} = sqrt(trace(abs(\xi_q C_q)))$,

Set $\delta_{1_q} = \beta_{1_q} sgn(trace(C_q\xi_q))$,

Define $V_{1_q} = \xi_q/\delta_{1_q}$,

Define $W_{1_q} = C_q/\beta_{1_q}$,

Let $V_{g,k_q} = [V_{1_q}]$,
Let $W_{g,k_q} = [W_{1_q}]$.
*(2):/*Generate the new orthonormal vector*/*
for i=1,2,...,k do
$\alpha_{i_q} = trace((W_{i_q}^T)\psi_q V_{i_q})$,
$\hat{V}_{(i+1)_q} = \psi_q V_{i_q} - \alpha_{i_q} V_{i_q} - \beta_{i_q} V_{(i-1)_q}$
(When i_q*=1, take* $\beta_{1_q} V_0 = 1$*),*
$\hat{W}_{(i+1)_q} = \psi_q^T W_{i_q} - \alpha_{i_q} W_{i_q} - \delta_{i_q} W_{(i-1)_q}$
(When i_q*=1, take* $\delta_{1_q} W_0 = 1$*),*
$\beta_{(i+1)_q} = \|\hat{W}_{(i+1)_q}, \hat{V}_{(i+1)_q}\|_F$,
$\delta_{(i+1)_q} = \beta_{(i+1)_q}.sgn[trace(\hat{W}_{(i+1)_q}^T \hat{V}_{(i+1)_q})]$,
$V_{(i+1)_q} = \hat{V}_{(i+1)_q} / \delta_{(i+1)_q}$,
$W_{(i+1)_q} = \hat{W}_{(i+1)_q} / \beta_{(i+1)_q}$,
$V_{g,k_q} = [V_{g,k_q} V_{(i+1)_q}]$,
$W_{g,k_q} = [W_{g,k_q} W_{(i+1)_q}]$.
end for }

During the iteration process, a tridiagonal Matrix $T_{(g,k)_q} \in IR^{k*k}$ and two Frobenius orthonormal bases $V_{g,k_q} = [V_{1_q} V_{2_q} ... V_{k_q}] \in K_{k_q}(\psi_q, \xi_q)$ and $W_{g,k_q} = [W_{1_q} W_{2_q} ... W_{k_q}] \in L_{k_q}(\psi_q^T, C_q^T)$ are generate that satisfies the following recursively relations:

$$\psi_q V_{g,k_q} = V_{g,k_q} \widetilde{T}_{g,k_q} + \delta_{(k+1)_q} V_{(k+1)_q} E_q^T \tag{22}$$

$$\psi_q^T W_{g,k_q} = W_{g,k_q} \widetilde{T}_{g,k_q}^T + \beta_{(k+1)_q} W_{(k+1)_q} E_q^T \tag{23}$$

Where $\widetilde{T}_{(g,k)_q}^T = T_{(g,k)_q} \otimes I_k$.
The parameters of the reduced order model are obtained by using the following biorthogonal projection $\hat{x}(t) = \widetilde{W}_{(g,k)_{(q)}}^T x(t) V_{(g,k)_{(q)}}$.
Where $\widetilde{W}_{(g,k)_{(q)}}^T = W_{(g,k)_{(q)}} (W_{(g,k)_{(q)}}^T V_{(g,k)_q})^{-T}$.
The reduced sub-system parameters in equ.3 and equ.4 can be obtained by the congruence transformation:
$\hat{A}_q = \widetilde{W}_{(g,k)_{(q)}}^T A_q V_{(g,k)_{(q)}}$, $\hat{B}_q = \widetilde{W}_{(g,k)_{(q)}}^T B_q$, $\hat{C}_q = V_{(g,k)_{(q)}}^T C_q$, $\hat{D}_q = D_q$.
Since that $\widetilde{W}_{(g,k)_{(q)}}^T V_{(g,k)_q} = I_k$ is an identity matrix.

A. Numerical example

To evaluate this approach we take the same model used previously, with the same switching signal. In the first, we make various s, taken s around zero $s1 = 0$ for each subsystem and takes $s2 \simeq \infty$.

The figure 4 and 6 show the output trajectories of the original system 15, and reduced one of second order around two expansion point (s1 and s2) respectively ,due to the above input signal, we see a good correlation between the output of the original system and reduced one.
The figure 5 and 7 present the output error between the original system and reduced one,we note that the choice of expansion point influences in the variation of error, for s1 we see that the maximum of value of error is equal to 0.23, but by the use of s2 is equal to 0.02.
The figure 8 show the output trajectories of the original system and reduced one by the use of two methods.
The variation of error is given in figure 9. We can see from these figures the results obtained by the Modified Global

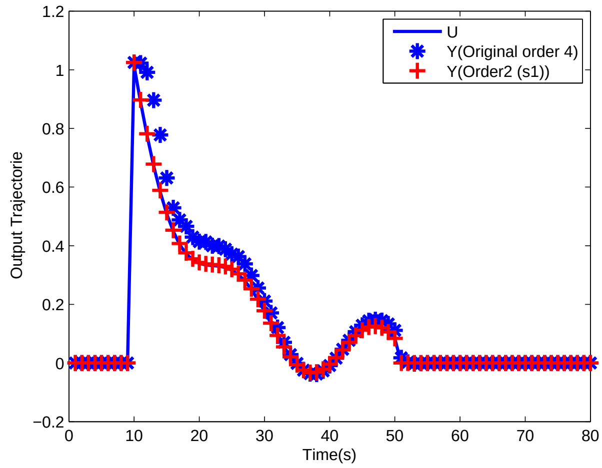

Fig. 4. Output trajectories of order reduction 2 (s1) by Modified Global Lanczos method

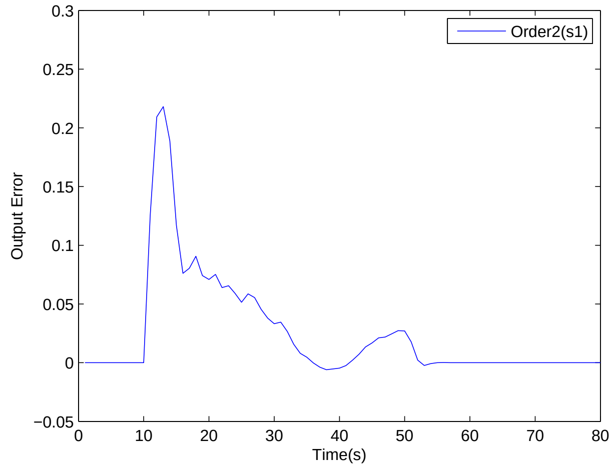

Fig. 5. Output errors of order reduction 2 (s1) by Modified Global Lanczos method

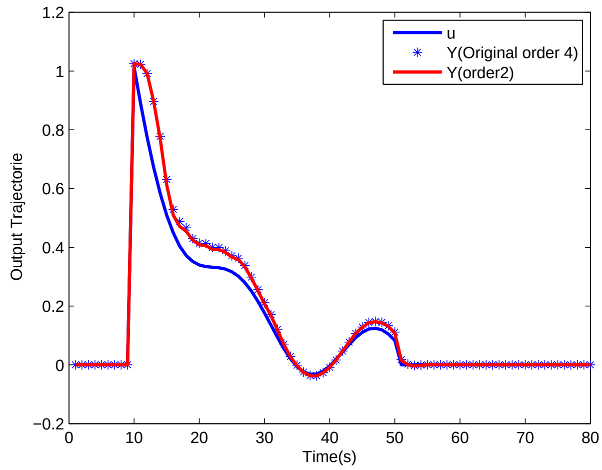

Fig. 6. Output trajectories of order reduction 2 (s2) by Modified Global Lanczos method

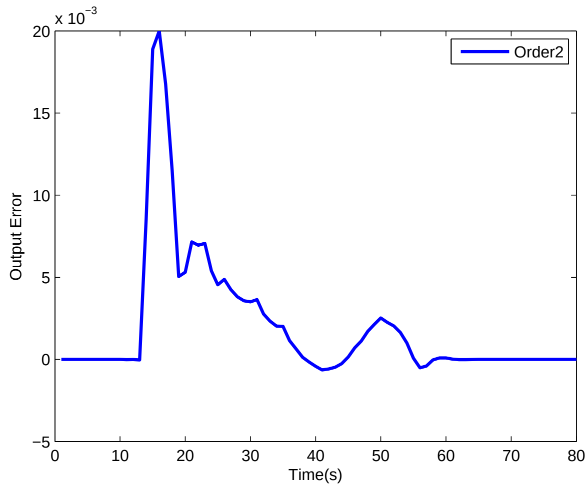

Fig. 7. Output Error of order reduction 2 (s2) by Modified Global Lanczos method

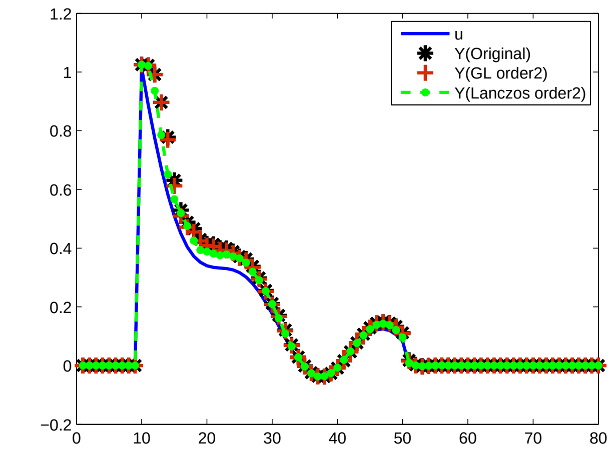

Fig. 8. Output trajectories of order reduction 2 by Modified Non symmetric Lanczos and Modified Global Lanczos methods

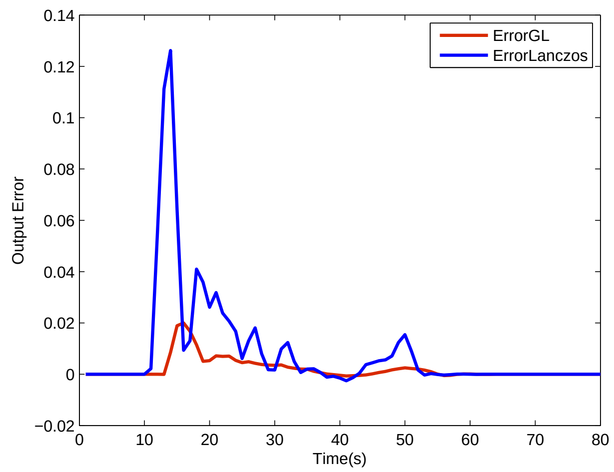

Fig. 9. Output errors of order reduction 2 by Modified Non symmetric Lanczos and Modified Global Lanczos method

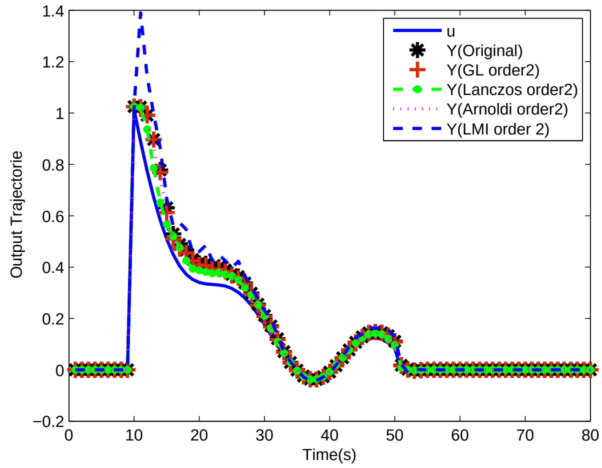

Fig. 10. Output errors of order reduction 2 by some methods

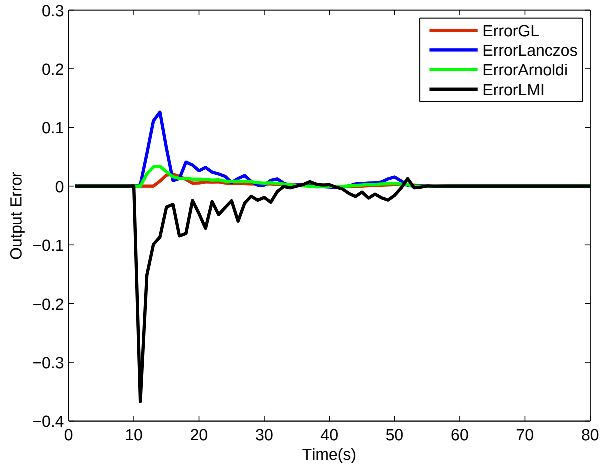

Fig. 11. Output errors of order reduction 2 by some methods

Lanczos method are better that those obtained by the Modified Non Symmetric Lanczos.

V. Comparison Study

In this section we compare the results obtained by the Lanczos methods with other methods of the literature (Arnoldi, linearization approach (LMI))[1, 9].
We present tow figures,the figure 10 present the output trajectory by for methods (Non symmetric Lanczos, Global Lanczos, Arnoldi and Linearization approach) we see that the good result is obtained by the Global Lanczos of order 2 if compare with the input U; Figure 11 shows the variation of error trajectory, we note the best result is obtained by Global Lanczos.

VI. Conclusion

In this paper we have proposed a news methods for reduction of linear switched systems based on generation of Krylov subspace for each sub-systems. We present the modified

Non symmetric Lanczos and Modified Global Lanczos. This methods are numerically efficient, guarantee the stability of subsystems, gives good results and easy to study compared to other methods (Arnoldi,LMI,...). To evaluate and demonstrate the accuracy and efficient of these methods, we present also a comparative study with the other methods. From simulation results we noted that the best results is obtained by Modified Global Lanczos Algorithm around a large expansion point.

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