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An Invariant Three-polar Representation for R^3 Surfaces: Robustness and Accuracy for 3D Faces Description

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***Abstract*—In this paper, we intend to introduce a new curved surface representation that we qualify by three-polar. It is constructed by the superposition of the three geodesic potentials generated from three reference points of the surface. By considering a pre-selected levels set of this superposition, invariant points are obtained. The accuracy of the three-polar representation for 3D human faces description is performed in the mean of the Hausdorff distance. A comparison between this representation and the one based on the level curves around the nose tip is established in the sense of the robustness under errors on the nose tip positions.**

***Keywords*—Three-polar, geodesic, 3D, potential, superposition, level set, face, curve, Hausdorff, robustness.**

I. Introduction

3D shape analysis and description has received a great deal of attention over the last few years due to the increasing development of 3D sensors, their low cost and the good 3D data quality they provide. Actually, R^3 surfaces description becomes a necessary task for pattern recognition, computer vision and 3D movement analysis. In practice, the data obtained from 3D sensors is not organized or partially organized like the 3D triangular mesh known as the conventional 3D discrete surface representation. Therefore, one of the major issues faced today in the three dimensional imaging field is the construction of a surface representation that ensures several properties. The invariance under some transformations and different parametrisations, the independence from the point of view and the robustness to some local variations in shape consist the most important ones. In the litterature, the three dimensional surface description methods can be classified into four major categories: the transform based approaches, the 2D views, the graph ones and those based on statistical features.

The transform based approaches consist in the application of specific transformations on the surface after its conversion onto 3D voxels or a spherical grid. The most known transformations are 3D Fourier [1], the 3D Radon [2], the angular radial transform [3], the rotation-invariant spherical harmonics [4], the uniformization [5] and the spherical wavelet descriptors [6].

For the two dimensional view based methods, a collection of 2D projections from canonical viewpoints is realized. Then, planar image descriptors are computed as Zernike moments [7] and Fourier descriptors [8].

The graph based approaches have the potential to code geometrical and topological shape properties in an intuitive manner. However, they are complex, in general harder to be constructed and not applicable easily to all 3D objects. The usually used descriptors are Reeb graphs [9] and the skeletal ones [10].

For the fourth description category, numerical attributes of the 3D object are collected. Several past works adopted this approach for invariant features extraction like the works of high curvature area determination [11], the generalized shape distribution [12], the extend Gaussian image [13], the CIRCON representation [28] and the canonical 3D Hough transform descriptor [14]. Bannour et al. [15] proposed a new surface pseudo-reparametrisation by the extraction of a curves network determined by iso-curvature features computation. Other methods impose the use of local coordinates by the exponential map around a point belonging to the two dimensional manifold obtained by wrapping a neighborhood of this point by the polar coordinates of the tangent plane at the same point or by constructing a set of geodesic circles relative to a given reference point [16], [17], [18]. The stability of these last methods remains dependent on the robustness of the reference point detection. In recent works, Jribi et al. [27] and Ghorbel et al. [26] proposed a new representation that they called a bipolar one. It consists on the superposition of the two geodesic potentials generated from two reference points instead of one reference point. The goal was to provide a more stability to the representations based on only one reference point [16], [17], [18].

We propose in this paper a novel curved surface representation that we qualify by three-polar. It is an attempt to generalize what is known by local coordinates around a reference point. It is constructed from the superposition of the three geodesic potentials generated from three reference points of the surface. The proposed representation is obtained by sampling the sum of these three geodesic potentials. The accuracy of such representation for 3D human faces' description is proved. A comparison between the proposed representation and the one based on only one reference point (the nose tip) is established in the sense of the robustness under errors on the reference points positions.

Thus, this paper will be structured as follows: We present in the second section the mathematical formulation of the three-polar representation. The used similarity metric to compare between shapes is illustrated in the third section. We show in the fourth section the performance of the three-polar representation for 3D faces' description. In the last section, a comparison between the three-polar representation and the unipolar one in the sense of the robustness under errors on the nose tip positions is established.

II. The three-polar representation: Mathematical formulation

We consider here a two dimensional differential manifold S and we denote by U_r the geodesic potential generated from a reference point r. For each point p of S, $U_r(p)$ is the length of the geodesic curve joining p to r. This function is well defined since a geodesic curve between any two points of the surface always exists. For a given real value λ, the points with geodesic potential values equal to λ form a curve that we denote by C_r^{λ}. It belongs to the surface and is called the geodesic of level λ.

We describe here a representation that we call a three-polar one. It is constructed from the superposition of the three geodesic potentials generated from the three reference points. Thus, let consider $\{r_i, i = 1..3\}$ three points of the two differential manifold S. Let $\{U_{r_i}, i = 1..3\}$ be their corresponding potentials functions. We denote by $U_{r3} = \sum_{i=1}^{3} U_{r_i}$ the geodesic potential constructed by the sum of the three geodesic potentials generated from $\{r_i, i = 1..3\}$.

Let p^* be a point of S. Therefore there exist $\{\lambda_i^*, i = 1..3\}$ such that p^* belongs to the three level curves $\{C_r^{\lambda_i^*}, i = 1..3\}$. Let $U_{r3}^* = min\{U_{r3}\}$. The points of the surface with the same geodesic sum are invariant. We construct a system of invariant points under the rotations' group $SO(3)$ by considering a levels set of this sum. The representation that we propose is constructed by varying these levels from 0 to the integer K. This integer represents the maximum value determined by the interest region extend which lies between the three reference points and its neighborhood on the surface. Therefore, the descriptor can be written as following:

$$M_{r3}^{k}(S) = \{p^* \in S; U_{r3}(p^*) = U_{r3}^* + \frac{k}{K}(\alpha_K - U_{r3}^*), k = 0..K\} \quad (1)$$

Where α_K is the maximum of geodesic sum.

III. Simlarity metric

It is important to define the used similarity metric to compare between different shapes. The well known Hausdorff shape distance introduced by Ghorbel in [20], [21] is chosen. Following the same process, we denote by G the group representing all possible normalized parametrisations of surfaces which can be the real plane R^2 or the unit sphere S^2. we consider the space of all surface pieces as the set of all 3D objects assumed diffeomorphic to G which can be assimilated to a subspace of $L^2_{R^3}(G)$ formed by all square integrated maps from G to R^3. The direct product of the Euler rotations group $SO(3)$ by the group G , acts on such space in the following sense:

$$SO(3) \times G \times L^2_{R^3}(G) \rightarrow L^2_{R^3}(G) \quad (2)$$
$$\{A, (u_0, v_0), S(u, v)\} \rightarrow AS(u + u_0, v + v_0)$$

The 3D Hausdorff distance Δ can be written for every S_1 and S_2 belonging to $L^2_{R^3}(G)$ and g_1 and g_2 to $SO(3)$ as follows:

$$\Delta(S_1, S_2) = max(\rho(S_1, S_2), \rho(S_2, S_1)) \quad (3)$$

Where:

$$\rho(S_1, S_2) = \sup_{g_1 \in SO(3)} \inf_{g_2 \in SO(3)} \parallel g_1 S_1 - g_2 S_2 \parallel_{L^2} \quad (4)$$

$\parallel S \parallel_{L^2}$ denotes the norm of the functional banach space $L^2_{R^3}(G)$.
Due to the fact that the euclidean rotations preserve this norm, it is easy to show that this distance is reduced to the following quantity:

$$\Delta(S_1, S_2) = \inf_{h \in SO(3)} \parallel S_1 - hS_2 \parallel_{L^2} \quad (5)$$

After that, we consider a normalized version of Δ so that the variations of this normalized distance are confined to the interval [0,1]. We try to achieve the real value of this distance with an adaptative version of the well known Itertive Closest Point (ICP) algorithm [19].

IV. Human face description with the three-polar representation

The description and the analysis of three dimensionnal shapes have become more and more attracting especially with the availability of 3D shape scanners. The 3D face description has received a great deal of attention over the last few years because of its various application domains. The biometrics are one of the most important applications. We test here the accuracy of the three-polar representation on the 3D meshes of the database Bosphorus [22] in the sense of the Hausdorff distance. We use a total of ten faces that can be grouped into two classes. A first class contains five faces of the same person with different expressions and a second one contains five faces of different persons.

A. The choice and the automatic extraction of the reference points

The first step of the three-polar representation construction consists on the choice of the reference points from the face. There is a general agreement that eyes are the most important facial features [25]. Indeed, they are a crucial source of information about the state of human being. Moreover, their appearance is less variant to certain face changes. Therefore, we choose to use the two outer corners of the eyes as reference points for the proposed three-polar representation. Since the nose tip is commonly used for many facial surfaces' description [16], [17], [18], it will be also chosen as a reference point in the three-polar representation. For the automatic extraction of these points, we refer to the work of Szeptycki et al. [24]. This method is based on a curvatures analysis with the use of a generic face model generated from a set of faces.

B. Geodesic potentials computation

Having extracted the reference points, the geodesic distances between any reference points and each vertex of the surface should be computed. Several past methods have been proposed in the litterature to compute distances on discrete meshes. This computation must take in consideration the trade off between the the computational cost and the accuracy. We use in the present work a simple method in which the geodesic distance is approximated by Dijkstra's algorithm [23] based on the edges length. This algorithm has a computational cost of order $O(n \log n)$. n is the number of vertices in the mesh.

C. Accuracy of the three-polar representations for human face description

After determining the reference points and computing the geodesic distances from these points, the next step consists on the extraction of the levels set of the geodesic sum for the construction of the three-polar representation.

The figure 1 shows the proposed representation with different resolutions which are linked to the number of levels in the representation construction.

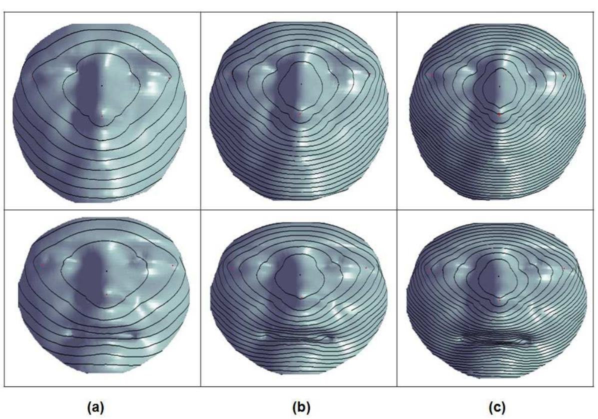

Fig. 1. Row 1: A neutral face. Row 2: A face with a surprise expression. (a) The three-polar representation with 9 levels. (b) The three-polar representation with 19 levels. (c) The three-polar representation with 29 levels.

To illustrate the effectiveness of the joint introduction in this context of the two notions: the three-polar representation and the Hausdorff distance, the matrix representing the pairwise normalized distances between the ten faces is computed. The first five faces correspond to the first class. The rest belongs to the second class. The figure 2 illustrates this matrix.

This matrix shows that the distances between the faces of the same persons are smaller compared with the ones computed between faces of different individuals.

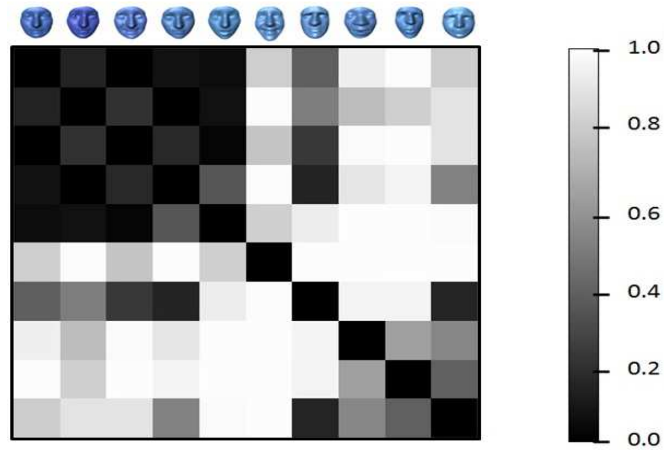

Fig. 2. Matrix of pairwise normalized Hausdorff distances between the ten facial surfaces. The first five faces correspond to the same person while others belong to different individuals

V. ROBUSTNESS UNDER ERRORS ON REFERENCE POINTS POSITIONS

We propose to make here a comparison between the three-polar representation and the unipolar one based on only one reference point which corresponds to the nose tip. This comparison will be performed in the sense of the robustness under error on their common reference point (the nose tip) positions.

To evaluate this robustness, we assume that the nose tip is moved by some geodesic distance around the same point without errors of extraction. The figure 3 illustrates the unipolar representation in two cases: a good extraction of the nose tip (a), and with error of extraction (b).

The figure 4 shows the three-polar representation for the same two cases with the same errors of the nose tip extraction.

The superposition of the two representations (with and without errors of the nose tip extraction) is illustrated in the figure 5 for the three-polar representation and the unipolar one.

This figure tends to prove that the three-polar representation is more robust than the unipolar one under errors on the nose tip positions. Indeed, for the three-polar representation, there is a better superposition of the level curves (with and without errors of extraction of the nose tip) than the unipolar one.

In order to more illustrate this robustness under the small distortions of the nose tip positions, this point is chosen randomly in a small neighborhood around the same point without errors. The matrices representing the pairwise normalized distance are computed for the two representations. The figure 6 illustrates these matrices.

From the comparison of these two matrices, we can note that the three-polar representation is more robust than the unipolar representation under errors of the nose tip positions.

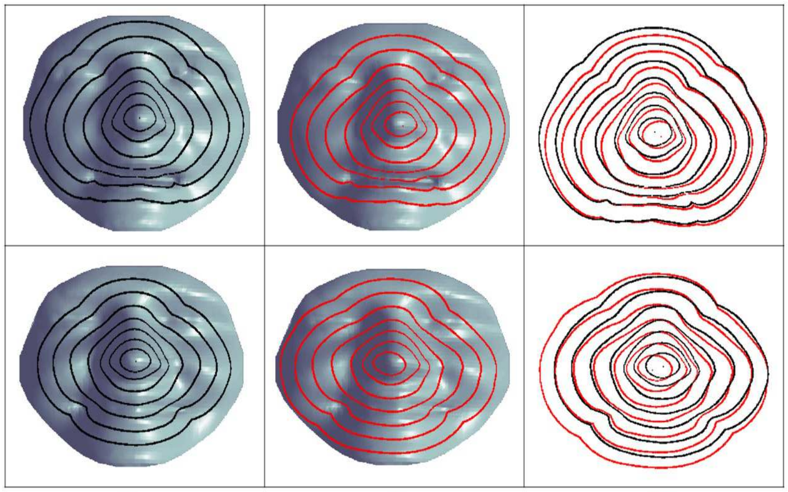

Fig. 3. Row 1: A face with a surprise expression. Row 2: A face with a happiness expression (a) The unipolar representation with a good extraction of the nose tip. (b) The unipolar representation with errors on the nose tip positions. (c): The superposition of the two representations (with and without errors).

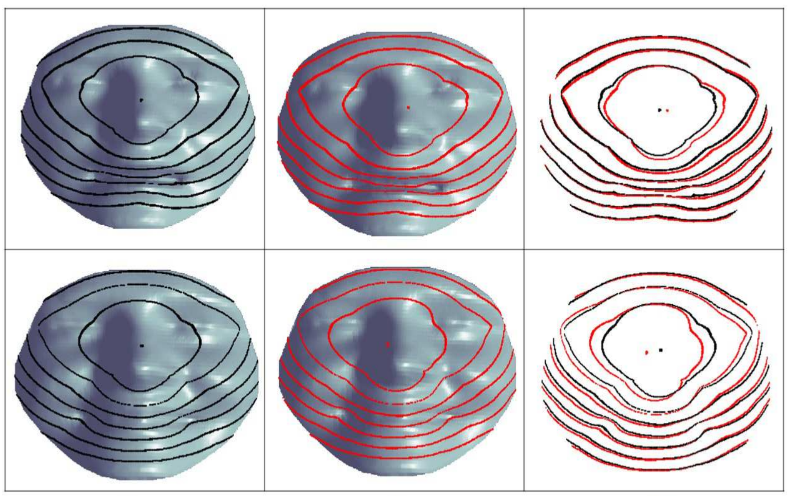

Fig. 4. Row 1: A face with a surprise expression. Row 2: A face with a happiness expression. (a) The three-polar representation with a good extraction of the nose tip. (b) The three-polar representation with errors on the nose tip positions. (c): The superposition of the two representations (with and without errors).

VI. Conclusion

We have proposed in this paper a novel curved surface representation. It is called a three-polar one since it consists on the superposition of the three geodesic potentials generated from three reference points of the surface. By sampling this continuous representation, a levels set of this superposition is computed. The accuracy of this representation for human face description is proved. Its robustness under errors on one reference point positions is established.

We intend in future works to perform the experimentation on a larger number of faces. We propose also to apply the representation for other types of surfaces (medical, archeological,...). Another track of future works can be the determination of the optimal number of the level curves of the proposed representation.

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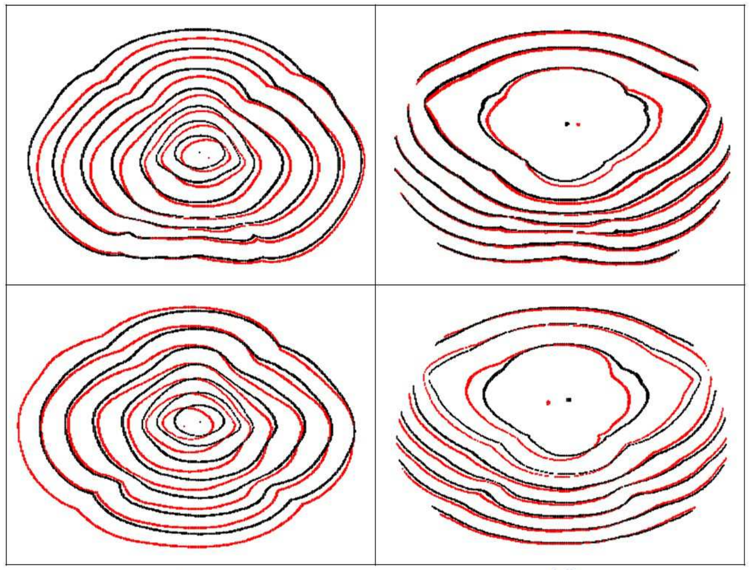

Fig. 5. Row 1: A face with a surprise expression. Row 2: A face with a happiness expression. (a) The superposition of the unipolar representations (with and without errors of the nose tip extraction). (b) The superposition of the three-polar representations (with and without errors of the nose tip extraction).

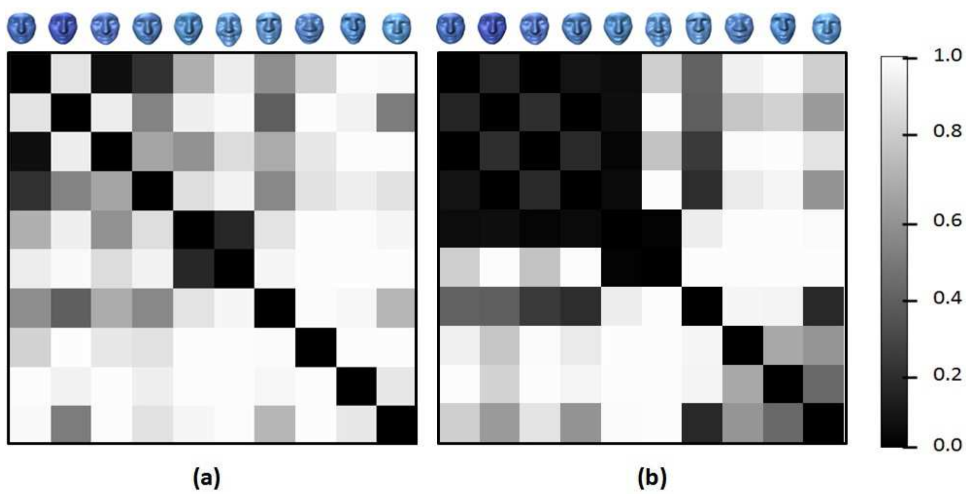

Fig. 6. Matrices of pairwises normalized distances betwen the ten faces with errors on the nose tip position. (a): The unipolar representation. (b): The three-polar representation.

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