Bank Closure Policies and Capital Requirements: a Mathematical Model

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Abstract—A bank closure policy problem is analysed in a mathematical model within a Black-Scholes framework where an appropriate notion of capital adequacy is introduced. The value of the deposit insurance liabilities and bank equity are derived. The effects of capital requirements on risk-shifting and bank reorganization are discussed, with a comparison of the impact of the Basel I and II Accords on banks' behaviour.

Keywords— bank closure policies, Black-Scholes framework, deposit insurance.

I. Introduction

THE objective of this paper is to analyze a major question of bank regulatory policy, which has renewed its urgency in view of the recent Basel II Accord on capital ratios: that is, should regulatory agencies close (reorganize) a near insolvent but insured bank? If so, when? What are the effects of the new regulatory capital levels on bank's closure policies?

Although there has been a sizeable literature on deposit insurance pricing since the pioneering work by Merton (1977, 1978), who first suggested to model deposit insurance as a put option on bank assets that is written by the regulator/insurer and held by the bank equity holders with a strike price equal to the face value of the insured deposits - and since then there have been a few contributions on the impact of deposit insurance on risk-shifting and on bank equity capital (see Pennacchi 1987; Ronn and Verma 1986; Marcus and Shaked 1984; Pyle 1984; Duan, Moreau and Sealy 1992) - little guidance has been offered about the timing of bank reorganizations and the impact of regulatory capital requirements.

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Acharya and Dreyfus (1989) first derived an endogenous bank closure rule, while previous work assumed an exogenously specified minimum level of asset-to-deposits ratio below which to close a bank. In most literature there is no divergence between the bank's and the regulator's incentives to exercise their options and close the bank. A few exceptions are Acharya (1996), because a stochastic charter value is included, Bhattacharya, Plank, Strobl and Zechner (2002), where the regulatory authority chooses the closure rule such that equity holders become indifferent with respect to the risk the bank takes, and Allen and Saunders (1993), because regulators are allowed to exercise the deposit insurance put prematurely, which has the effect of lowering the value of the deposit insurance put, hence increasing the critical closure threshold. Other papers have examined forbearance, that is, the regulator's intentional delay in forcing a bank closure, following a policy that would grant the bank the time to return to solvency before the closure rule is enforced (see, f.e. Duan and Yu 1994). An argument against forbearance is that banks have a greater incentive to gamble the greater is the probability of forbearance if deposits are insured. In fact, banks seem to be willing to transfer wealth from the regulator either by decreasing bank capital (Calomiris and Kahn 1991) or by holding assets with high variability of returns (McCulloch 1986; Duan, Moreau and Sealey 1992). As a consequence, policies are called for in order to mitigate risk-shifting and to induce banks to improve the quality of their assets.

Such concern is fundamental in the Basel accords on capital standards. The 1988 Basel accord explicitly considered only credit risk and imposed an 8% capital requirement on assets belonging to the same asset category or asset risk bucket. For example, all commercial loans belonged to the same 100% category, irrespective of the differences in credit quality, obligor or industry. The more recent Basel II Accord has been fine-tuned to accommodate some risks not initially considered and, in particular, to allow for different risk weights within the same asset category.

In this paper we examine the effects of capital adequacy rules on banks' behaviour using a dynamic framework. Most papers on capital requirements (Kahane 1977, Furlong and Keeley 1989, Rochet 1992, Kim and Santomero 1988, Kohen and Santomero 1980) develop static models on the asset-substitution problem. Kahane (1977) and Koehn and Santomero (1980) use a mean-variance model, while Furlong and Keeley (1989) utilize both a state-preference framework and an option pricing model. They show that a bank that is sufficiently risk-lover will choose a riskier mix of assets in response to a higher capital requirement. Kim and Santomero (1988) extend the portfolio-selection approach to the case of an asset-risk weighted system. Basically, most results are explained by the different underlying assumptions about the curvature of bank managers risk-return preference, as Rochet (1992) and Gennotte and Pyle (1991) have stressed. In most circumstances, an increase in bank capital is unambiguously associated with a reduction in the bank investments in risky assets, so that capital requirements are seen as useful instruments for reducing the incentive to increase risk. Other papers (Gennotte and Pyle 1991, Blum 1999, Pelizzon and Schaefer 2003) show that capital requirements may sometimes have a perverse incentive, that is, banks take on more risk. In particular, when a dynamic model is taken into account (Blum 1999, Pelizzon and Schaefer 2003) a tighter capital requirement may lower the expected profits of the bank, hence the bank has less to lose in the event of bankruptcy, and, as a consequence, the risk-taking incentive may increase. Indeed, as Gennotte and Pyle (1991) have stressed, there are two effects of higher capital requirements on the probability of bankruptcy: on one side, they reduce leverage, hence the probability of bankruptcy reduces; on the other hand, asset risk might increase, which increases the probability of bankruptcy. Which effect prevails depends on a ratio of the elasticities to the net present value of investment with respect to the mean and variance of the present value.

Other papers have emphasized that capital requirements alone will not necessarily improve the overall safety and soundness of banks. Hellman, Murdock and Stiglitz (2000) suggest that combining a deposit rate ceiling with capital regulation can unambiguously induce all banks to decrease riskiness in their portfolios. Morrison and White (2005) suggest a mixing of an audit policy and capital regulation to improve efficiency and to limit the possibility of adverse selection. Further recent studies of bank capital regulation are reviewed in VanHoose (2007).

Almost all the above-mentioned papers are mainly concerned with the impact of risk-based capital requirements on the bank portfolio strategies and asset risk, but do not address bank closure policies specifically.

The objective of this paper is twofold. First, we study the timing of bank reorganization, formulating a bank closure policy and the corresponding pricing of deposit insurance in a setting that is more complex than the one studied in the literature, because we introduce a more appropriate notion of capital adequacy. Our notion is in keeping with the basic standardized model of the Basel II Accord on capital ratios and allows us to make a comparison of the impact of the two Basel I and Basel II accords on banks' behaviour.

Second, we build on a model which extends Merton (1978) in two directions. We introduce two risky assets instead of one in order to have different risk weights and a more appropriate notion of capital requirements; moreover, we allow for dividends payouts and consider the cost of reorganization when bank capital proves to be inadequate. We examine the effects of capital requirements on risk-shifting, bank reorganization and bankruptcy. In common with much of the literature we study the present value of the deposit insurance liabilities as a metric for riskiness.

Our paper is more closely related with Bhattacharya, Plank, Strobl and Zechner (2002), Blum (1999), Pelizzon and Schaefer (2003). However, Blum (1999) does not deal with bank closure policies, considers a two-period model with a single risky asset and uses a very special definition of risk. Bhattacharya, Plank, Strobl and Zechner (2002) consider a model of optimal bank closure rules but have a single risky asset, do not take into account the possibility of bank reorganization and their measure of capital requirements does not conform to the definition stated in the capital adequacy directives. Pelizzon and Schaefer (2003) deal mainly with risk management strategies and their main results are obtained with numerical simulations only.

The paper is organized as follows. Section 2 presents the model. Section 3 describes the regulator's problem, deriving the deposit insurance premium and the closure policy. Section 3 contains the evaluation of bank equity and discusses the main results about the effects of capital requirements on risk-shifting and bank reorganization. Finally, Section 4 concludes and indicates further avenues of research.

II. THE MODEL

We consider a bank which will remain in operation unless the regulator/insurer intervenes to close (or reorganize) the bank. The bank holds some specialized assets, notably loans, and is financed by equity and a variety of other liabilities, collectively referred to as deposits, which are assumed to be all insured. The insurer charges a premium for the deposit insurance, which is paid by the bank's equity holders. Bank managers make decisions in the interest of the equity holders. Bank assets are classified into two categories, according to their riskiness and the credit quality of the obligor. We denote by V_i, $i = 1, 2$, the two categories of the same type of financial asset which enter the total asset value with weights θ_i and assume that $dV_i = \mu_i V_i dt + \sigma_i V_i dZ_i$, where μ_i and σ_i are constant, dZ_i denotes a Wiener process and $E(dZ_1 dZ_2) = \rho dt$. Denote by D the value of the bank's aggregate deposits. Let g be the rate of growth in deposits, r_D the rate of interest paid by the bank on deposits and suppose that depositors withdraw a constant fraction γ of interest, accrued over the preceding period, so that the remaining fraction $(1-\gamma)$ is added to the value of the bank deposits. The dynamics for aggregate deposits are non-stochastic and described by:

$dD = (g + r_D(1-\gamma))Ddt = nDdt$ with $r \geq r_D$ and $\delta = g - r_D\gamma \leq 0$ (in order to avoid that the bank may run a "Ponzi game"). Thus, the dynamics of the value of total bank assets follow:

$$\delta Ddt + \theta_1(\mu_1 V_1 d + \sigma_1 V_1 dZ_1) + \theta_2(\mu_2 V_2 dt + \sigma_2 V_2 dZ_2)$$

The regulator charges the bank a premium to insure all the deposits of the bank in perpetuity, provided that the bank is solvent, that is if $\theta_1 V_1 + \theta_2 V_2 > D$. Following Merton (1978) we suppose that solvency of the bank is ascertained by audit. The regulator may appraise the economic value of the bank assets and liabilities on an appointed date. The residual capital position is then compared to the capital adequacy standard, which is computed as follows, in keeping with the basic standardized model of the Basel II Accord on capital ratios. The book value of each asset category is multiplied by a risk weight y_i according to the different risk bucket into which the loan is classified and then by 8% (which is the coefficient required by the Basel Accords on capital ratios to generate the minimum capital requirement). Table 1 shows the different capital requirements under Basel I (third line from above) and Basel II (fifth line from above).

Table 1 (**Basel I** vs **Basel II**)

Rating	AAA/ AA-	A+/A-	B+/B-	below BB-	unrated
Weights y_i	100%	100%	100%	100%	100%
Capital Requirements	8%	8%	8%	8%	8%
Weights y_i	20%	50%	100%	150%	100%
Capital Requirements	1.6%	4%	8%	12%	8%

Source: Basel Committee on Banking Supervision, 2001

Let:

$$\theta_1 V_1 + \theta_2 V_2 - D \geq 0.08(y_1\theta_1 V_1 + y_2\theta_2 V_2) \quad (1)$$

Suppose $\sigma_2 > \sigma_1$. Then $y_2 \geq y_1$, in keeping with most standard default prediction models. If (1) is satisfied, then bank capital is judged to be adequate and there is no regulatory interference. Notice that if $y_1 = y_2 = 1,$ then expression (1) states the notion of capital adequacy for the same type of financial instrument as from the Basel I Accord on capital ratios. Under the foundation approach of the new Basel II Accord the same type of financial instrument is assigned different risk weights, that is $y_i = 20\%, 50\%, 100\%, 150\%$, depending on the credit quality of the obligor. If (1) is not satisfied, then bank capital proves to be inadequate and its classification varies with the extent of the deficiency. As a condition for continued insurance, we assume that bank managers are expected to make up some of the deficiency by restricting current and subsequent dividends. Denote by α the proportional dividend that is assumed to be distributed to equity holders in case of solvency. We suppose that if (1) is not satisfied, then $\alpha = 0$ as a condition for continued deposit insurance. Finally, when the book value of equity is assessed to zero, the regulator declares the bank "technically insolvent". We can consider also the case where it can force a bank to technical insolvency only when the market value of assets falls seriously below that of its deposit liabilities, so that forbearance is allowed. In this case, an insolvency resolution occurs if the asset value falls below βD where $\beta \leq 1$ (that is, if $\theta_1 V_1 + \theta_2 V_2 - \beta D \leq 0$). If $\beta = 1$ the liabilities facing the insurer reduce to the familiar put option: then the regulator liquidates the bank and exercises the put option to pay the depositors off. In any case there is a cost of audit, which is assumed to be $c(D) = kD$, borne by the insurer and taken into account when the insurance premium is computed.

III. THE REGULATOR'S POLICY

The regulator chooses the insurance premium P and the closure policy. Given the audit report, it may either liquidate the bank or keep it in operation, deciding what $P = P(V_1, V_2, D)$ to charge the bank. We suppose the event of audit to be Poisson distributed, with the probability of an audit over the next instant equal to λdt the probability of no audit is equal to $1-\lambda dt$ and the probability of more than one audit of order $O(dt)$. It is assumed that the Poisson process and dZ_i are independent. Following Merton (1978), we put $\delta = 0$ for simplicity and indicate cash outflows as positive inflows, so that the derived values are positive instead of negative. In the absence of costs and if there are no dividends,

$$dP = (\mu_1\theta_1 V_1 \tfrac{\partial P}{\partial V_1} + \mu_2\theta_2 V_2 \tfrac{\partial P}{\partial V_2} + \tfrac{1}{2}\sigma_1^2\theta_1^2 V_1^2 \tfrac{\partial^2 P}{\partial V_1^2}$$
$$+ \tfrac{1}{2}\sigma_2^2\theta_2^2 V_2^2 \tfrac{\partial^2 P}{\partial V_2^2} + \rho\sigma_1\sigma_2\theta_1\theta_2 V_1 V_2 \tfrac{\partial^2 P}{\partial V_1 \partial V_2})dt$$
$$+ \sigma_1\theta_1 V_1 \tfrac{\partial P}{\partial V_1} dZ_1 + \sigma_2\theta_2 V_2 \tfrac{\partial P}{\partial V_2} dZ_2 + \tfrac{\partial P}{\partial D} nDdt \quad (2)$$

Adopting the standard Dixit-Pindyck framework, we get $dP + \Psi_1\, dV_1 + \Psi_2\, dV_2 = r(P + \Psi_1 V_1 + \Psi_2 V_2)dt$, so that for i = 1,2, $\Psi_i = -\theta_i \frac{\partial P}{\partial V_i}$.

Thus, we get the partial differential equation of the Black-Scholes type:

$$\frac{1}{2}\sigma_1^2\theta_1^2 V_1^2 \frac{\partial^2 P}{\partial V_1^2} + \frac{1}{2}\sigma_2^2\theta_2^2 V_2^2 \frac{\partial^2 P}{\partial V_2^2} + \\ + \rho\sigma_1\sigma_2\theta_1\theta_2 V_1 V_2 \frac{\partial^2 P}{\partial V_1 \partial V_2} + \frac{\partial P}{\partial D} nD + \\ + r(\theta_1 V_1 \frac{\partial P}{\partial V_1} + \theta_2 V_2 \frac{\partial P}{\partial V_2}) - rP = 0 \tag{3}$$

Suppose now there are dividends, so that $dV_i = (\mu_i - \alpha)V_i dt + \sigma_i V_i dZ_i$. Then:

$$\frac{1}{2}\sigma_1^2\theta_1^2 V_1^2 \frac{\partial^2 P}{\partial V_1^2} + \frac{1}{2}\sigma_2^2\theta_2^2 V_2^2 \frac{\partial^2 P}{\partial V_2^2} + \\ + \rho\sigma_1\sigma_2\theta_1\theta_2 V_1 V_2 \frac{\partial^2 P}{\partial V_1 \partial V_2} + \frac{\partial P}{\partial D} nD + \\ + (r - \alpha)(\theta_1 V_1 \frac{\partial P}{\partial V_1} + \theta_2 V_2 \frac{\partial P}{\partial V_2}) - rP = 0 \tag{4}$$

To simplify the notation let:

$$\mathrm{G}(V_1, V_2, D) = \frac{1}{2}\sigma_1^2\theta_1^2 V_1^2 \frac{\partial^2 P}{\partial V_1^2} + \frac{1}{2}\sigma_2^2\theta_2^2 V_2^2 \frac{\partial^2 P}{\partial V_2^2} + \\ \rho\sigma_1\sigma_2\theta_1\theta_2 V_1 V_2 \frac{\partial^2 P}{\partial V_1 \partial V_2} + \frac{\partial P}{\partial D} nD .$$

Therefore, P must satisfy:

$$G(V_1, V_2, D) + (r - \alpha)(\theta_1 V_1 \frac{\partial P}{\partial V_1} + \theta_2 V_2 \frac{\partial P}{\partial V_2}) \\ - rP + \lambda kD = 0, \tag{5}$$

if $\theta_1 V_1 + \theta_2 V_2 - D \geq 0.08(y_1\theta_1 V_1 + y_2\theta_2 V_2)$,

$$\mathrm{G}(V_1, V_2, D) + r(\theta_1 V_1 \frac{\partial P}{\partial V_1} + \theta_2 V_2 \frac{\partial P}{\partial V_2}) \\ - rP + \lambda kD = 0, \tag{6}$$

if $0 < \theta_1 V_1 + \theta_2 V_2 - \beta D$
and $\theta_1 V_1 + \theta_2 V_2 - D < 0.08(y_1\theta_1 V_1 + y_2\theta_2 V_2)$,

$$\mathrm{G}(V_1, V_2, D) + r(\theta_1 V_1 \frac{\partial P}{\partial V_1} + \theta_2 V_2 \frac{\partial P}{\partial V_2}) - rP \\ + \lambda(kD + D - \theta_1 V_1 - \theta_2 V_2 - P) = 0, \tag{7}$$

if $\theta_1 V_1 + \theta_2 V_2 - \beta D \leq 0$.

They have the following interpretation. If the bank is solvent and (1) is satisfied, then bank capital is judged to be adequate and there is no regulatory interference, as from (5); otherwise, as from (6), the bank cannot pay any dividends ($\alpha = 0$) and reorganization is required. In any case, if an audit takes place there is a cash flow of $c(D) = kD$. Finally, if the market value of assets falls seriously below that of its deposit liabilities, like in (7), there is a second cash flow of $D - \theta_1 V_1 - \theta_2 V_2$ and the liability of the insurer ceases. To simplify the notation let us define $\sigma = (\sigma_1\theta_1 V_1 + \sigma_2\theta_2 V_2)/V$ where $V = \theta_1 V_1 + \theta_2 V_2$. Under a suitable change in variables and choice of parameters, with $x = V/D$ and $p = P/D$, the equation system (5)-(6)-(7) becomes:

$$\frac{1}{2}p''\Sigma x^2 + p'(r - \alpha - n)x - p(r - n) + \lambda k = 0 \tag{8}$$

if $x > 1/(1-\xi)$

$$\frac{1}{2}\ p''\Sigma x^2 + p'(r - n)x - p(r - n) + \lambda k = 0 \tag{9}$$

if $\beta < x \leq 1/(1-\xi)$

$$\frac{1}{2}p''\Sigma x^2 + p'(r - n)x - p(r - n + \lambda) + \\ \lambda\,(k + 1 - x) = 0 \tag{10}$$

if $x \leq \beta$.

Here $\Sigma = \sigma_1^2\theta_1^2 + \sigma_2^2\theta_2^2 + 2\rho\sigma_1\sigma_2\theta_1\theta_2$ and $\xi = 0.08(y_1(\sigma_2 - \sigma) + y_2(\sigma - \sigma_1))/(\sigma_2 - \sigma_1)$. We will consider parameter values such that $\xi < 1$ for any $\sigma_1, \sigma_2, \sigma$ and y_1, y_2 as requested in the foundation approach of the Basel II Accord. Observe that if $y_1 = y_2 = 1$ then $\xi = 0.08$, as from the Basel I Accord. By solving equation system (8)-(9)-(10) we get Proposition 1, where $q^{\pm}$ are the solutions to the algebraic equation related to (8), $1, a^{-}$ the solutions to the algebraic equation related to (9) and $b^{\pm}$ the solutions to the algebraic equation related to (10):

Proposition 1. *The present value of the deposit insurance liability (p) chosen by the regulator has the following expression:*

$$\overline{p}(x) = (x/\tilde{x})^{q^-}\ (a^- - 1)\ (x^*(1-b^+) - \Gamma b^+)\Psi^{-1} + \frac{\lambda k}{r-n}, \quad \text{if } x > \tilde{x}, \qquad (11)$$

$$\underline{p}(x) = ((x/\tilde{x})\ (a^- - q^-) + (x/\tilde{x})^{a^-}\ (q^- - 1)).$$
$$(x^*(1-b^+) - \Gamma b^+)\Psi^{-1} + \frac{\lambda k}{r-n}, \quad \text{if } x^* < x \le \tilde{x}, \qquad (12)$$

$$p_0(x) = (x/x^*)^{b^+}\ ((x^*/\tilde{x})^{a^-}\ (q^- - 1)\ (x^*(1-a^-) - \Gamma a^-) + (x/x^*)\ (q^- - a^-)\Gamma)\Psi^{-1} + \frac{\lambda(k+1)}{r+\lambda-n} - x,$$
$$\text{if } x \le x^*, \qquad (13)$$

with $\tilde{x} = 1/(1-\xi)$, $x^* = \beta$, $\Gamma = \frac{\lambda k}{r-n} - \frac{\lambda(k+1)}{r-n+\lambda}$,

$\Psi = (x^*/\tilde{x})^{a^-}\ (q^- - 1)(b^+ - a^-) + (x^*/\tilde{x})(a^- - q^-)(b^+ - 1)$.

Proof. Expressions (11), (12), (13) can be obtained applying the smooth-pasting and high-order conditions on equations (8), (9), (10) that is, $\overline{p}(\tilde{x}) = \underline{p}(\tilde{x})$, $\underline{p}(x^*) = p_0(x^*)$, $\overline{p}'(\tilde{x}) = \underline{p}'(\tilde{x})$, $\underline{p}'(x^*) = p_0'(x^*)$ and the no-bubble condition on p(x).

Observe that if $\beta = 1$ and $\alpha = 0$ then expressions (11)-(12)-(13) collapse into Merton's expressions for the regulator's liabilities (see Merton 1978). We obtain the following results from a comparative statics analysis.

REMARK 1. p *is not a monotonically decreasing function of* x.

Such result follows from the property of the audit cost, that is, a monotonically increasing function of x. For x sufficiently large, the expected number of audits prior to an audit where the bank is found to be insolvent increases: thus, the cost increases with x, which completely offset the "put option part" which is decreasing in *x*.

REMARK 2. *If capital forbearance is in place, cash payments resulting from the deposit insurance guarantee are higher, other things being equal.*

Straightforward computation shows that for $x > x^*$, the derivative of p with respect to β is negative, given the other parameter values. Therefore, the future liability increases as a result of continuing to provide insurance when the market value of assets falls seriously below that of its deposit liabilities and the regulator allows for forbearance. If $\beta = 1$ the liabilities facing the insurer reduce to the familiar put option.

REMARK 3. *The value of deposit insurance increases as capital requirements increase, other things being equal.*

For $x > x^*$, the derivative of p with respect to ξ is positive, given the other parameter values. Thus, increasing capital requirements may limit the bank's ability to exploit its rents in the future, so that it can lead to an increase in the value of deposit insurance liability. Such result has to be compared with the usual asset-substitution effect which has been emphasized in the literature.

REMARK 4. *Under Basel II Accord, the range of values of* x *where bank's reorganization is required may reduce relative to Basel I.*

If the bank chooses assets so that y_1 (y_2) < 1 as a consequence of Basel II Accord (for example, 20%, 50%), then ξ is lower than under Basel I. Since $\frac{\partial \tilde{x}}{\partial \xi} > 0$, then $\tilde{x}$ decreases as ξ decreases. Actually, the Basel Committee on Banking Supervision**,** Results of the QIS 5 (2006) show that minimum required capital under Basel II Accord would decrease relative to Basel I: such result holds both for the standardized approach and even more for the internal ratings-based approaches. For all group of countries examined by the Basel Committee on Banking Supervision**,** Results of the QIS 5 (2006), retail portfolios drive the reduction in minimum required capital. Also the corporate and SME corporate portfolios and equity exposures produce a decrease in minimum required capital. Therefore, we can conjecture that under Basel II relative to Basel I ξ is expected to decrease, reducing in turn the range of values of x where reorganization is required to make up some of the deficiency through a restriction of current and subsequent dividends.

IV. THE BANK EQUITY

Let us consider now the bank that has paid its premium to the regulator. Following the same procedure as above, we can derive the value of equity per units of deposits, denoted by $e = E/D$, which satisfy the following equations:

$$\frac{1}{2}e''\Sigma x^2 + e'(r-\alpha-n)x - e(r-n) = 0 \qquad (14)$$

if $x > 1/(1-\xi)$

$$\frac{1}{2}e''\Sigma x^2 + e'(r-n)x - e(r-n) = 0 \qquad (15)$$

if $\beta < x \leq 1/(1-\xi)$

$$\frac{1}{2}e''\Sigma x^2 + e'(r-n)x - e(r-n-\lambda) = 0 \qquad (16)$$

if $x \leq \beta$

By solving equation system (14)-(15)-(16) we get Proposition 2:

Proposition 2. *The equity per units of deposits (e) has the following expression:*

$$\overline{e}(x) = \frac{x*}{(q^- - q^+)(b^+ - a^-)} .$$

$$\{ \ (x/\tilde{x})^{q^+} ((\tilde{x}/x*)^{a^-} .(1-b^+)(q^- - a^-) - (\tilde{x}/x*)\ (b^+ - a^-)(1-q^-))+$$

$$-(x/\tilde{x})^{q-} .((\tilde{x}/x*)^{a^-} (1-b^+)(a^- - q^+) + (\tilde{x}/x*)\ (b^+ - a^-)(1-q^+)) \ \}$$

if $x > \tilde{x}$, *(17)*

$$\underline{e}(x) = x + (x/x*)^{a^-} x * \tfrac{1-b^+}{b^+-a^-} , \qquad (18)$$

if $x* < x \leq \tilde{x}$,

$$e_o(x) = (x/x*)^{b^+} x * \tfrac{1-a^-}{b^+-a^-} , \qquad (19)$$

if $x \leq x*$,

where $\tilde{x} = 1/(1-\xi)$ *and* $x* = \beta$.

Proof. Expressions (17), (18), (19) can be obtained applying the smooth-pasting and high-order conditions on equations (14), (15), (16) that is, $\overline{e}(\tilde{x}) = \underline{e}(\tilde{x})$, $\underline{e}(x*) = e_0(x*)$, $\overline{e}'(\tilde{x}) = \underline{e}'(\tilde{x})$, $\underline{e}'(x*) = e_0'(x*)$ and the no-bubble condition on *e(x).*

If $\beta = 1$ and $\alpha = 0$ then expressions (17)-(18)-(19) collapse into Merton's expressions for the bank equity evaluation (see Merton 1978). From (17), (18), (19) the equity per units of deposits is a monotonically increasing function of x for $x \leq \tilde{x}$. It is strictly convex for $x \leq x*$, as is usually the case for limited liability levered equity, and strictly concave for $x > x*$. Since the equity position can be viewed as ownership of the assets levered by a riskless debt issue (the rate paid on which is *n*) combined with an implicit put option on the value of the assets (Merton 1978), in the case of the bank equity it is the positive spread $r - n$ that induces the concavity. The spread becomes lower if dividends are paid out.

REMARK 5. *The value of equity does not increase as capital requirements increase, other things being equal.*

For $x > \tilde{x}$, the derivative of e with respect to ξ is negative, given the other parameter values, while it is equal to zero for $x < \tilde{x}$ Capital requirements limit the bank's ability to invest in risky assets. An effect of regulation is the reduction of bank's equity: capital requirements do lower "bank's profits", and may lower bank's incentive to preserve future rents.

REMARK 6. *Insolvent banks increase value by increasing portfolio variance; sufficiently capitalized banks maximize value by minimizing variance.*

It is straightforward to compute $\frac{\partial e_o}{\partial \Sigma} = (\frac{x}{x*})^{b^+} x * (\frac{1-a^-}{b^+-a^-} \frac{\partial b^+}{\partial \Sigma} \ln(\frac{x}{x*}) + \frac{\partial}{\partial \Sigma}(\frac{1-a^-}{b^+-a^-}))$ which is positive for $x < x*$, since $\frac{\partial b^+}{\partial \Sigma}$ and $\ln(\frac{x}{x*})$ are negative and the first term in parenthesis dominates the second for small enough x. On the contrary, $\frac{\partial \underline{e}}{\partial \Sigma} = (\frac{x}{x*})^{a^-} x * (\frac{1-a^-}{b^+-a^-} \frac{\partial a^-}{\partial \Sigma} \ln(\frac{x}{x*}) + \frac{\partial}{\partial \Sigma}(\frac{1-a^-}{b^+-a^-}))$ is negative for $x > x*$, since $\frac{\partial a^-}{\partial \Sigma}$ and $\ln(\frac{x}{x*})$ are positive, hence $\frac{1-a^-}{b^+-a^-} \frac{\partial b^+}{\partial \Sigma} \ln(\frac{x}{x*})$ is negative, and the first term in parenthesis dominates the second for large enough x . It is in keeping with the observation that as long as it is solvent, the bank pays less than the riskless rate on its deposits. Therefore, as long as $x > x*$ an increase in portfolio volatility would increase the probability of becoming insolvent, and thus of losing this rent. Thus, sufficiently capitalized banks would like to minimize variance. On the contrary, if the bank is insolvent, that is $x < x*$, the bank may find it more convenient to increase portfolio volatility, gambling for resurrection by the time of the next audit. Actually, we can obtain the following:

REMARK 7. *If $x < x*$, the bank would choose the portfolio with the highest possible risk level. If $x > x*$, such strategy is no longer optimal .*

Here we have to look for the optimal θ_1 such that equity is maximized. Suppose $\sigma_1 < \sigma_2$. If we compute $\partial e_0 / \partial \theta_1$ an interior solution for a maximum does not exist; indeed, the only optimal solution is $\theta_1 = 0$ for $x < x*$. It is no longer true for $x > x*$ since $\partial \underline{e} / \partial \Sigma < 0$. Indeed, sufficiently capitalized banks would prefer to invest a strictly positive fraction of their total assets in the less risky asset class too.

REMARK 8. *Under Basel II riskiness reduces relative to Basel I, other things being equal.*

Straightforward computation shows that $\frac{\partial \tilde{x}}{\partial \theta_1} < 0$ if $y_1 < y_2$, while $\frac{\partial \tilde{x}}{\partial \theta_1} = 0$ if $y_1 = y_2$. Moreover, $\frac{\partial \overline{e}}{\partial \theta_1} = \frac{\partial \overline{e}}{\partial \tilde{x}} \cdot \frac{\partial \tilde{x}}{\partial \theta_1} > 0$ if $y_1 < y_2$ while $\frac{\partial \overline{e}}{\partial \theta_1} = \frac{\partial \overline{e}}{\partial \tilde{x}} \cdot \frac{\partial \tilde{x}}{\partial \theta_1} = 0$ if $y_1 = y_2$. If different risk weights are assigned, that is $y_1 \neq y_2$, the optimal portfolio choice shifts to the less risky asset. Therefore, under Basel II with $y_1 \neq y_2$ riskiness is reduced relative to the case $y_1 = y_2$.

REMARK 9. *Under Basel II equity does not decrease relative to Basel I.*

If we can conjecture that under Basel II relative to Basel I ξ is expected to decrease, as it is suggested by the Basel Committee on Banking Supervision, Results of the QIS 5 (2006) , then for $x > \tilde{x}$ an increase in equity is expected $(\partial \overline{e} / \partial \xi < 0)$ while for $x < \tilde{x}$ equity is unchanged $(\partial \underline{e} / \partial \xi = 0)$.

REMARK 10. *The sensitivity of the optimal portfolio choices can be checked numerically.*

One can implement the findings of Remarks 6,7,8 with a *Mathematica* programme in order to compute the optimal portfolio choice for different values of the relevant parameters. For this purpose we can use the numerical solver FindRoot.

After specifying the three branches of the equity function (which will be written as LOWEQ, MIDDLEQ, HIGHEQ, for $e_0(.)$, $\underline{e}(.)$ and $\overline{e}(.)$ respectively), we need to write the first order conditions to have a maximum (ADIFF [.]=0, BDIFF[.]=0, CDIFF[.]=0, respectively) and then apply the numerical solver FindRoot. Denote mu = r-n. For simplicity and without loss of generality, let us take β=1. Let:

```
LOWEQ[σ1_,σ2_,θ1_,θ2_,ρ_,α_,λ_,mu_,y1_,y2_,w1_,w2_,k_,x_]:=
x^BP[σ1,σ2,θ1,θ2,ρ,α,λ,mu]*(1-
AM[σ1,σ2,θ1,θ2,ρ,α,λ,mu])/(BP[σ1,σ2,θ1,θ2,ρ,α,λ,mu]-
AM[σ1,σ2,θ1,θ2,ρ,α,λ,mu]);

MIDDLEEQ[σ1_,σ2_,θ1_,θ2_,ρ_,α_,λ_,mu_,y1_,y2_,w1_,w2_,k_,x
_]:=x+x^AM[σ1,σ2,θ1,θ2,ρ,α,λ,mu]*(1-
BP[σ1,σ2,θ1,θ2,ρ,α,λ,mu])/(BP[σ1,σ2,θ1,θ2,ρ,α,λ,mu]-
AM[σ1,σ2,θ1,θ2,ρ,α,λ,mu]);

HIGHEQ[σ1_,σ2_,θ1_,θ2_,ρ_,α_,λ_,mu_,y1_,y2_,w1_,w2_,k_,x_]:=
(1/(QM[σ1,σ2,θ1,θ2,ρ,α,λ,mu]-
QP[σ1,σ2,θ1,θ2,ρ,α,λ,mu])*(BP[σ1,σ2,θ1,θ2,ρ,α,λ,mu]-
AM[σ1,σ2,θ1,θ2,ρ,α,λ,mu]))*(x^QP[σ1,σ2,θ1,θ2,ρ,α,λ,mu]*Xtilde[σ
1,σ2,θ1,θ2,ρ,α,λ,mu,y1,y2,w1,w2]^(AM[σ1,σ2,θ1,θ2,ρ,α,λ,mu]-
QP[σ1,σ2,θ1,θ2,ρ,α,λ,mu])*
(1-BP[σ1,σ2,θ1,θ2,ρ,α,λ,mu])*(QM[σ1,σ2,θ1,θ2,ρ,α,λ,mu]-
AM[σ1,σ2,θ1,θ2,ρ,α,λ,mu])-
x^QP[σ1,σ2,θ1,θ2,ρ,α,λ,mu]*Xtilde[σ1,σ2,θ1,θ2,ρ,α,λ,mu,y1,y2,w1,
w2]^(1-QP[σ1,σ2,θ1,θ2,ρ,α,λ,mu])*(BP[σ1,σ2,θ1,θ2,ρ,α,λ,mu]-
AM[σ1,σ2,θ1,θ2,ρ,α,λ,mu])*(1-
QM[σ1,σ2,θ1,θ2,ρ,α,λ,mu])+(x^QM[σ1,σ2,θ1,θ2,ρ,α,λ,mu]*
Xtilde[σ1,σ2,θ1,θ2,ρ,α,λ,mu,y1,y2,w1,w2]^(AM[σ1,σ2,θ1,θ2,ρ,α,λ,
mu]-QM[σ1,σ2,θ1,θ2,ρ,α,λ,mu])*(1-
BP[σ1,σ2,θ1,θ2,ρ,α,λ,mu])*(QP[σ1,σ2,θ1,θ2,ρ,α,λ,mu]-
AM[σ1,σ2,θ1,θ2,ρ,α,λ,mu])-
x^QM[σ1,σ2,θ1,θ2,ρ,α,λ,mu]*Xtilde[σ1,σ2,θ1,θ2,ρ,α,λ,mu,y1,y2,w1
,w2]^(1-QM[σ1,σ2,θ1,θ2,ρ,α,λ,mu])*(BP[σ1,σ2,θ1,θ2,ρ,α,λ,mu]-
AM[σ1,σ2,θ1,θ2,ρ,α,λ,mu])*(1-QP[σ1,σ2,θ1,θ2,ρ,α,λ,mu]));

LDIFF[σ1_,σ2_,θ1_,θ2_,ρ_,α_,λ_,mu_,y1_,y2_,w1_,w2_,k_,x_]:=N
D[LOWEQ[σ1,σ2,θ1,θ2,ρ,α,λ,mu,y1,y2,w1,w2,k,x],θ1];
FindRoot[LDIFF[σ1,σ2,θ1,θ2,ρ,α,λ,mu,y1,y2,w1,w2,k,x]==0,{θ1,{0,
1}];

MDIFF[σ1_,σ2_,θ1_,θ2_,ρ_,α_,λ_,mu_,y1_,y2_,w1_,w2_,k_,x_]:=N
D[MIDDLEEQ[σ1,σ2,θ1,θ2,ρ,α,λ,mu,y1,y2,w1,w2,k,x],θ1];
FindRoot[MDIFF[σ1,σ2,θ1,θ2,ρ,α,λ,mu,y1,y2,w1,w2,k,x]==0,{θ1,{0
,1}];
HDIFF[σ1_,σ2_,θ1_,θ2_,ρ_,α_,λ_,mu_,y1_,y2_,w1_,w2_,k_,x_]:=N
D[HIGHEQ[σ1,σ2,θ1,θ2,ρ,α,λ,mu,y1,y2,w1,w2,k,x],θ1];
FindRoot[HDIFF[σ1,σ2,θ1,θ2,ρ,α,λ,mu,y1,y2,w1,w2,k,x]==0,{θ1,{0
,1}]
```

REMARK 11. *Although data are not available yet to perform a calibration of our model, existing empirical evidence seems not to disagree with our predictions.*

The effects of the new capital regulation are likely to depend on the extent to which individual banks adopt the new Basel regime. Peura and Jokivuolle (2004) provide an analysis of bank capital ratios under the current Basel regime and a simulation of banks' regulatory capital adequacy. Heid (2007) surveys empirical evidence about the effects of Basel I and II and provides a simulation study which is mainly devoted to assessing the cyclical patterns of capital charges. It is estimated that capital relief for high (average) quality portfolios is 50% (18%) going from Basel I to Basel II. For Italian banks (see Cannata 2006) capital relief for high (average) quality portfolios is 5.4% (1.3%) under the standardized method, 9.8% (24.5%) under IRB method for credit risk going from Basel I to Basel II. Most of their results are in line with our predictions. Other works (see, Mehran and Thakor 2006) have stressed that banks hold capital beyond their regulatory requirements. It would imply that although capital requirements decrease going from Basel I to Basel II, capital requirements are not binding. Whether this fact determines nonetheless a change in banks' behaviour or not has still to be examined empirically. We believe this issue becomes of central importance in the debate about Basel II when data will be available.

V. FINAL REMARKS

This paper makes two main contributions to the literature on inter-temporal bank regulation. First, we introduce a more appropriate notion of capital requirements than is usually adopted in the literature. We allow for different risk weights in the measure of capital adequacy, in a way that is in keeping with the basic standardized model of the Basel II Accord on capital ratios so that we are able to make a comparison of the impact of the two Basel I and Basel II accords on banks' behaviour. Second, we allow for dividends payouts and consider the cost of reorganization when bank capital proves to be inadequate and examine the effects of capital requirements on risk-shifting, bank reorganization and bankruptcy in this more complex setting.

Our analysis reveals how the model parameters influence the regulatory policy, the value of deposit insurance liabilities, bank equity and riskiness. Our main results, summarized in Remarks 1-9, provide a further contribution to the debate about the role and effectiveness of minimum required capital rules. In Remark 10 we show how numerical methods play a role in the design of bank strategies.

A few avenues for future research can be indicated. In this paper we have assumed that the audit frequency is independent of the previously observed level of asset values, in keeping with all the literature following Merton (1978). However, this is clearly an artifact and affects both the closure threshold, the expressions of p and e and some of their properties (see Remark 1). A more plausible assumption, such that the audit frequency is inversely related to the asset value, would cancel out the property that the cost of audit is a monotonically increasing function of the asset value, which implies, for example, that p would have the usual put option behaviour. As suggested by Bhattacharya, Plank, Strobl and Zechner (2002), one could also let λ change after an audit, because the regulator might be willing to increase λ after an audit reveals that the bank is close to bankruptcy.

Another possible extension refers to the possibility of alternative forms of reorganization, for example, through new equity issue as well. Finally, since the Basel II Accord has started its implementation only a few months ago and is applied by a negligible number of banks and countries by now, market data are not yet available to perform a check of the results of this paper. Further calibration of our results will be interesting when Basel II data will be available.

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