On a modal approach for oscillations damping in affine and piecewise affine systems

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***Abstract*—**In the case of continuous affine systems, an oscillating behavior, due to the presence of pairs of complex eigenvalues in the spectrum of the system matrix, is considered. The control problem consists of damping the oscillations and tracking a piecewise constant reference signal. A control solution is proposed, based on pole placement combined with the internal model principle. Sufficient conditions for the controller existence are deduced and some issues concerning fixed and variable step simulations approaches are discussed. The results are extended to piecewise affine hybrid systems, composed of a set of piecewise affine systems and a switching strategy based on a state space partition.

***Keywords*—**affine systems, controllability, discrete systems, internal model, hybrid systems, pole placement, tracking.

I. INTRODUCTION

THERE is an increasing interest, in recent years, in the study of different parts of an automobile: suspension system [1], brake system [2], power train [3] etc. Hybrid systems, describing the interaction between time-driven and even-based dynamics are often used [4]-[10]. Piecewise affine systems represent a class of hybrid systems frequently encountered as modeling approximations of a large class of nonlinearities [4] arising both in control engineering [5], [6] and biological modeling [7], [8]. The control of oscillations is an important objective for hybrid piecewise affine systems, generally associated with driving quality and comfort in the evolution of automotive systems [9].

In order to analyze and control complex piecewise affine nonlinear dynamics, a prior study of affine systems is relevant for the description of the local behavior of the hybrid system [3]. This paper proposes a control approach for an affine system, ensuring oscillations damping and the tracking of a piecewise constant reference input. The two objectives are reached, under certain assumptions, using a classic pole placement procedure combined with the internal model principle. For simplicity, in order to analyze strictly the impact of the closed loop pole allocation, it is assumed that the process state is known; however, a state estimator can be synthesized in a straightforward manner and embedded in the controller. The control procedure is then extended, through an example, to a piecewise affine system composed of two oscillating affine systems and a switching strategy based on a state space partition.

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The paper is structured as follows. Section II presents a modal approach for oscillations damping in a class of affine systems, illustrated by a simulation example. Section III extends, through a simulation example, the modal control approach. Concluding remarks are finally discussed.

II. OSCILLATIONS CONTROL FOR AN AFFINE SYSTEM

This section introduces a control problem for a single continuous affine system, which represents a starting point for developing a control approach for a piecewise affine hybrid system.

A. The affine model and the control problem

Consider the continuous-time affine system

$$\dot{x} = Ax + bu + f \ , \ y = c^T x \ , \tag{1}$$

with $x \in \mathbf{R}^n$ the continuous state, u a scalar-valued input signal, y the scalar output and A, $b(\neq 0)$ and $f(\neq 0)$ real matrices of appropriate dimensions, respectively. Denote $\Lambda(A)$ the set of eigenvalues of A.

Assumption 1

(a) The matrix A has at least one pair of complex eigenvalues, i.e. $\exists(\alpha \pm j\beta) \in \Lambda(A)$.

(b) The pair (A, b) in (1) is controllable [11].

(c) The pair (c^T, A) in (1) is observable.

(d) Consider also a class of piecewise-constant exogenous signals, $r:[0,T] \to \mathbf{R}$,

$$r(t) = \begin{cases} r_1, & 0 \le t < t_1 \\ \vdots & \\ r_i, & t_{i-1} \le t < t_i \\ \vdots & \\ r_s, & t_{s-1} \le t \le T \end{cases} \tag{2}$$

where $r_i \in \mathbf{R}$, $i = 1:s$, and the time intervals between any two consecutive switching moments t_i, $i = 1 : s-1$, may not be equal, respectively.

Problem 1

Given the affine system (1) satisfying the Assumption 1 and an exogenous signal r (2), find a control law $u(x,r)$ s.t. in closed loop:

(i) the system is stable and the oscillations [12] of the output y are damped and

(ii) the output y tracks the reference signal (2).

B. The control system: pole assignment, internal model and extended system

The first part of the control objective can be realized by a pole assignment strategy, implemented as a static feedback control law, while the second part of the control objective requires - for zero asymptotic tracking error - the presence of an internal model of the exogenous signal. Since the reference signal (2) is a combination of delayed step signals, the tracking is ensured by an integrator as internal model. This solution is implemented in the control structure depicted in Fig.1, with the state equations

$$\begin{aligned} \dot{x} &= Ax + bu + f, \\ \dot{z} &= -c^T x + r, \end{aligned} \tag{3}$$

output signal $y = c^T x$, and tracking error $\varepsilon = r - y$.

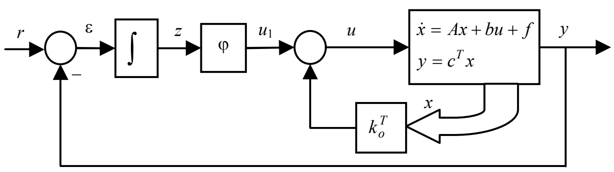

Fig.1 control structure for the affine system in Problem 2

Remark 1. Note that condition (3) is stronger than condition (i) in Problem 1. In order to achieve the first part of the control objective, Assumption 2 can be relaxed: the oscillations can still be damped in closed loop if not all eigenvalues of (A,b) are controllable, but the uncontrollable eigenvalues are real and stable. In this case, the uncontrollable "part" produces neither oscillations nor instability and, after a structural decomposition, the feedback matrix k_o^T - with appropriate reduced dimensions – can be applied only to the controllable part, assumed to generate output oscillations.

Define the extended affine system of order $(n+1)$

$$\dot{x}_e = A_e x_e + b_e u + f_e + g_e r, \tag{4}$$

where $x_e = [x^T \quad z]^T$ is the extended state, u is the control input, r is the reference signal to be tracked and the system matrices are

$$A_e = \left[\begin{array}{c|c} A & 0_{n\times 1} \\ \hline -c^T & 0 \end{array}\right], \; b_e = \left[\begin{array}{c} b \\ \hline 0 \end{array}\right], \qquad f_e = \left[\begin{array}{c} f \\ \hline 0 \end{array}\right], \; g_e = \left[\begin{array}{c} 0_{n\times 1} \\ \hline 1 \end{array}\right], \tag{5}$$

and consider the following problem associated to Problem 1.

Problem 2

Given:

(i) the system (4) - (5), with (A,b) satisfying Assumption 1 and

$$\det\begin{bmatrix} A & b \\ -c^T & 0 \end{bmatrix} \neq 0, \tag{6}$$

and

(ii) a set Λ_e of $(n+1)$ strictly negative real numbers, find a control law

$$u = k_e^T x_e = [k_o^T \quad \varphi] \cdot \begin{bmatrix} x \\ z \end{bmatrix} \tag{7}$$

so that

$$\Lambda(A_e + b_e k_e^T) = \Lambda_e. \tag{8}$$

In order to show that solving Problem 2 drives to a solution to Problem 1, two aspects have to be proved: firstly, the fact that the pole placement for (A_e, b_e) is possible, provided that Assumption (b) is true and, secondly, the fact that the control system (4),(5) with the control law (7),(8), can track a reference signal from the class (2), i.e. that no steady state error occurs.

The solution to the first part of the control objective is based on the next result.

Lemma. If the pair (A,b), with $A \in \mathbf{R}^{n\times n}$ and $b \in \mathbf{R}^n$, is controllable and (A,b,c^T) satisfies (9), then the pair (A_e, b_e), defined in (5), is also controllable.

Proof. Controllability is equivalent, for single input - single output systems, to the non-singularity of the controllability matrix. Hence, the controllability matrix of the pair (A,b) is defined by

$$R_e = \left[\begin{array}{c|c|c|c|c} b & Ab & A^2b & \cdots & A^n b \\ 0 & -c^T b & -c^T Ab & \cdots & -c^T A^{n-1} b \end{array}\right]. \quad (9)$$

The above matrix can be written in the form

$$R_e = \begin{bmatrix} b & AR \\ 0 & -c^T R \end{bmatrix} = \begin{bmatrix} b & A \\ 0 & -c^T \end{bmatrix} \cdot \begin{bmatrix} 1 & 0_{1\times n} \\ 0_{n\times 1} & R \end{bmatrix}. \quad (10)$$

which, in view of (6) and of $\det R \neq 0$, results also non-singular.

The controllability of the extended pair (A_e, b_e) is necessary and sufficient for *pole assignment*, but, obviously, if (A,b) in (1) is not controllable, but the uncontrollable eigenvalues of A are *real and stable* then the uncontrollable "part" produces neither oscillations nor instability and, after a structural decomposition, the pole assignment procedure can be applied for the controllable "part". In other words, a solution to Problem 2 is also a solution to Problem 1; however, a solution to Problem 1 can still be obtained in more relaxed conditions, as specified above.

The second part of the control objective concerns the asymptotic error behavior, for a reference signal from the class (2). Denote

$$y(s) = L\{y(t)\},\ u_1(s) = L\{u_1(t)\},\ \varepsilon(s) = L\{\varepsilon(t)\}, \\ r(s) = L\{r(t)\} \quad (11)$$

the Laplace transformed of the corresponding time signals in Fig.1, respectively. Also, note that for the closed loop matrix

$$A_o = A + bk_o^T, \quad (12)$$

the spectrum $\Lambda(A_o) \subset \Lambda_e$ is stable. Then, for zero initial conditions $x_e(0) = 0$, the Laplace transformed of the affine system state in Fig. 1 is

$$x(s) = (sI - A_o)^{-1}\left[bu_1(s) + f \cdot \frac{1}{s}\right], \quad (13)$$

With

$$u_1(s) = L\{u_1(t)\} = \frac{\varphi}{s} \cdot \varepsilon(s) = \frac{\varphi}{s} \cdot [r(s) - y(s)]. \quad (14)$$

After some computation

$$y(s) = \frac{c^T (sI - A_o)^{-1}[b \cdot \varphi r(s) + f]}{s + c^T (sI - A_o)^{-1} b \cdot \varphi}, \quad (15)$$

Then, for

$$r(t) = r_i \cdot 1(t), \quad (16)$$

with the Laplace transformed

$$r(s) = r_i / s,\ r_i > 0, \quad (17)$$

the stationary output value is

$$y(\infty) = \lim_{s\to 0} sy(s) = r_i \quad (18)$$

and the resulting the steady state error is $\varepsilon(\infty) = 0$.

The control procedure solving Problem 2 can be summarized as follows.

Modal control algorithm for affine systems

Input: the n order model (1) with (A, b, c^T) in observable canonical form, the reference signal (2), and desired closed loop spectrum

$$\Lambda_e = \{\lambda_i \mid \lambda_i < 0\}_{i=1:n+1}. \quad (19)$$

Output: the feedback matrix

$$k_e^T = [k_o^T \quad \varphi] \quad (20)$$

in (7).

1. If the pair (A,b) is controllable, then go to step 2. Else, stop.
2. If the condition (6) is satisfied, then go to step 3. Else, stop.
3. Build the matrices (5) of the extended system (4).
4. For the pair (A_e, b_e) build the feedback matrix (20).
5. Given

$$\Lambda_{est} = \{\lambda_{est\,j} \mid \lambda_{est\,j} < 0\}_{j=1:n-1}, \quad (21)$$

the control law (7) is implemented using a minimal observer for the process state x in (3).

6. Stop.

Remark 2 If (A,b) in (1) is not controllable, but the uncontrollable eigenvalues of A are real and stable then, when solving Problem 2, Remark 1 can be taken into account. Also, if the condition (6) is not satisfied, for example due to the sensors placement (modelled by the output matrix c^T), then a reconfiguration of the sensors system can be tried.

C. A simulation example – an affine system

Consider the second order affine system (1) in observable canonical form, with the matrices

$$A = \begin{bmatrix} -2\zeta & -1 \\ 1 & 0 \end{bmatrix},\ b = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ f = \begin{bmatrix} 10 \\ 0 \end{bmatrix} \tag{22}$$

$$c^T = [0 \quad 1],\ 0 < \zeta < 1 .$$

The output of the system defined by (1) and (14) has to become stable, with no oscillations and to track the signal $r : [0, T] \to \mathbf{R}$,

$$r(t) = \begin{cases} r_1, & 0 \le t < t_1 \\ r_2, & t_1 \le t < t_2 \\ r_3, & t_2 \le t \le T \end{cases} . \tag{23}$$

The matrices defined in (22) satisfy condition (6). The matrices of the extended system (4) are

$$A_e = \begin{bmatrix} -2\zeta & -1 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}, b_e = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, f_e = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}, g_e = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \tag{24}$$

and the desired closed loop spectrum is

$$\Lambda_e = \{-1, -2, -10\} . \tag{25}$$

The resulting extended feedback matrix is, for $\zeta = 0.1$,

$$k_e^T = [k_o^T \mid \varphi] = [-12.8 \quad -31 \mid 20] . \tag{26}$$

In order to avoid possible Zeno behavior [13], in the presence of switching, a discrete time simulation model is defined by

$$x_d(+1) = A_d x_d + b_d u_d + f_d ,$$

$$y_d = c_d^T x_d , \tag{27}$$

where, denoting $h > 0$ the sampling step and introducing the matrices in (22),

$$A_d = \exp(Ah) ,\ b_d = \int_0^h \exp(A\theta) d\theta \cdot b ,$$

$$f_d = f \cdot h ,\ c_d^T = c^T , \tag{28}$$

and the discrete variables are

$$x_d(k) = x(kh) ,\ u_d(k) = u(kh) , \text{with } k = 0, 1, \dots . \tag{29}$$

In this regard, an already classic example in the literature of automotive control is the mixed logical dynamical approach discussed in [14].

Denote

$$A_{oe} = A_e + b_e k_e^T . \tag{30}$$

The simulation model of the closed loop system without observer is

$$x_{ed}(+1) = A_{oed} x_{ed} + f_{ed} + g_{ed} r_d ,$$

$$y_d = c_{ed}^T x_{ed} , \tag{31}$$

where,

$$A_{oed} = \exp(A_{oe} h) ,\ f_{ed} = f_e \cdot h ,\ g_{ed} = g_e \cdot h ,\ c_{ed}^T = c_e^T , \tag{32}$$

and, for any $k > 0$, the sampled signals are

$$x_{ed}(k) = x_e(kh) ,\ r_d(k) = r(kh) . \tag{33}$$

With the simulation values

$$T = 30 ,\ h = 0.001 ,\ r_1 = 0 ,\ r_2 = 2 ,\ r_3 = 1 ,\ t_1 = 5 ,$$
$$t_2 = 15 ,\ \zeta = 0.1 , \tag{34}$$

the open loop evolution of the affine system (27) with the discrete input signal associated to (23) is depicted in Fig.2. The evolution of the closed loop system (27) with the same discrete input signal associated to (23) is depicted in Fig.3.

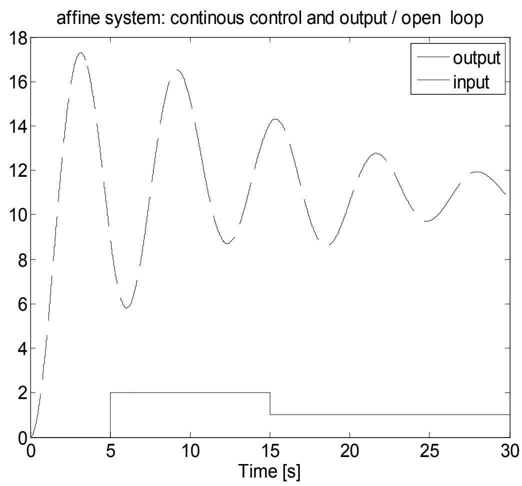

Fig. 2 simulation of the discrete affine system (27) with sampled piecewise constant input (23) and parameters (34)

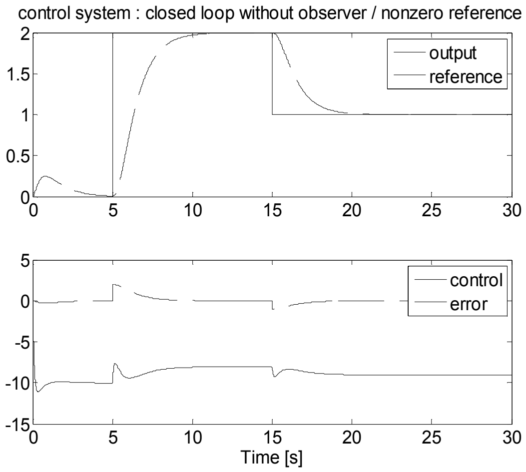

Fig. 3 simulation of the discrete control system (27) with sampled piecewise constant input (23) and parameters (34); the process state is assumed to be known

In what concerns the simulation, for arbitrary initial state conditions, of the *free* continuous affine systems, variable step methods can be successfully applied. Fig. 4 (up) represents the phase portrait of the free discrete affine system (31), and the trajectory evolves towards the stationary point of the continuous affine stable system (1), given by

$$x(\infty)=\lim_{s\to 0} sx(s)=(-A)^{-1}f=\begin{bmatrix}0.2 & 1\\ -1 & 0\end{bmatrix}\cdot\begin{bmatrix}0\\ 10\end{bmatrix}=\begin{bmatrix}10\\ 0\end{bmatrix}.$$

Remark 3 In the simulation experiment, it was assumed, for simplicity, that the entire process state is available for building the control law (7). However, in practice, the feedback matrix (26) has to be applied to an estimation of the unmeasured process state. In order to build a minimal observer for the system, with eigenvalue $\lambda_{est}<0$, rewrite the matrices (22) in the partitioned form

$$A=\left[\begin{array}{c:c} A_1 & A_3\\ \hdashline A_2 & A_4\end{array}\right],\ b=\left[\begin{array}{c} b_1\\ \hdashline b_2\end{array}\right],\ f=\left[\begin{array}{c} f_1\\ \hdashline f_2\end{array}\right]$$

$$c^T=[0\ \vdots\ 1], \tag{35}$$

where $A_1=-2\zeta=-0.2$, $b_1=1$, $f_1=10$ and the rest results by identification.
The continuous time minimal observer is

$$\dot{w}=Jw+Hy+Mu+f_L,\ \hat{x}_1=w-Ly, \tag{36}$$

with

$$J=A_1+LA_2,\ M=b_1+Lb_2,\ f_L=f_1+Lf_2, \tag{37}$$

$$H=-A_1L+A_3-LA_2L+LA_4,$$

and L chosen s.t.

$$\sigma(J)=\{\lambda_{est}\}. \tag{38}$$

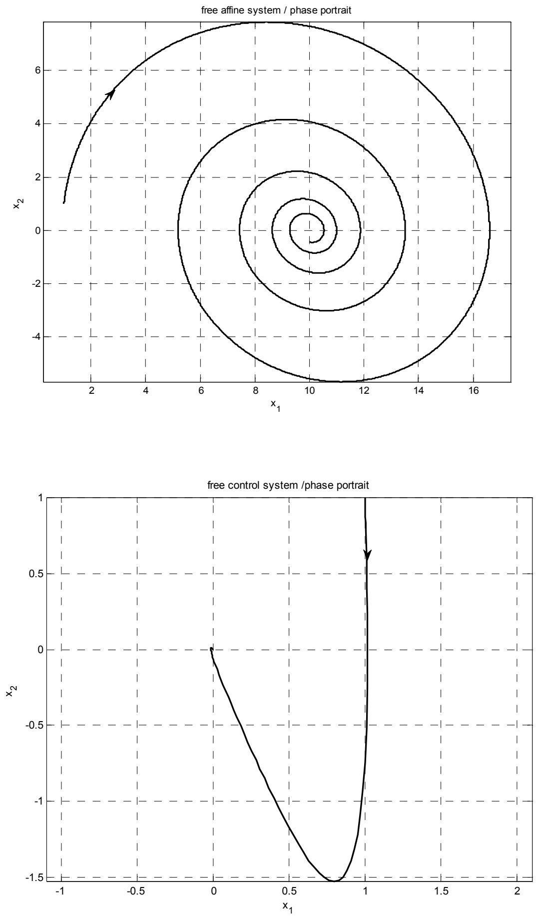

Fig. 4 phase portraits: the free discrete affine system (23) with $u_d=0$, $x_d(0)=[1\quad 1]^T$ (up) and the continous extended system (26) with $r=0$, $x_e(0)=[1\quad 1\quad 0]^T$, simulated with ode45 (down).

On the other side, the phase portrait depicted in Fig. 4 (down), associated to the first two components of the free extended state vector in (31) - the controlled affine process -, with matrices given in (22) and (26), is obtained by using the MATLAB routine ode45, based on the Runge-Kutta method. However, the simulation results of the discrete affine systems and of the corresponding continuous time model, simulated with variable step integration methods, may slightly differ, due to the approximation introduced by the sampling procedure.

The feedback law (7) is implemented, in continuous time, in the form

$$u=-12.8\hat{x}_1-31y+20z, \tag{39}$$

and the simulation model comprises the discrete dynamics of the extended system (4) and of the observer (36), respectively.

Evidently, a difference in the simulated behavior, compared to Fig. 3, is obtained only if there is a disturbance in the process dynamics, in order to alter the initial condition or the measured signal y.

III. OSCILLATIONS CONTROL FOR A PIECEWISE AFFINE SYSTEM

This section presents, through a simulation example, an extension of the oscillations control procedure for affine systems to piecewise affine systems, hence to a class of hybrid systems.

A. *The piecewise affine system as a hybrid automaton*

Consider the hybrid automaton with the structure depicted in Fig. 5(a), with the local continuous dynamics given, in each location, by the affine model (1) with the matrices

$$A_i = \begin{bmatrix} -2\zeta_i & -1 \\ 1 & 0 \end{bmatrix},\ b = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ f = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{40}$$

$$c^T = [0 \quad 1],\ i = 1,2,\ \zeta_1 = 0.1,\ \zeta_2 = 0.5\,.$$

and the partition based discrete dynamics

$$q = \begin{cases} q_1, & x_1 x_2 \geq 0 \\ q_2, & x_1 x_2 \leq 0 \end{cases} \tag{41}$$

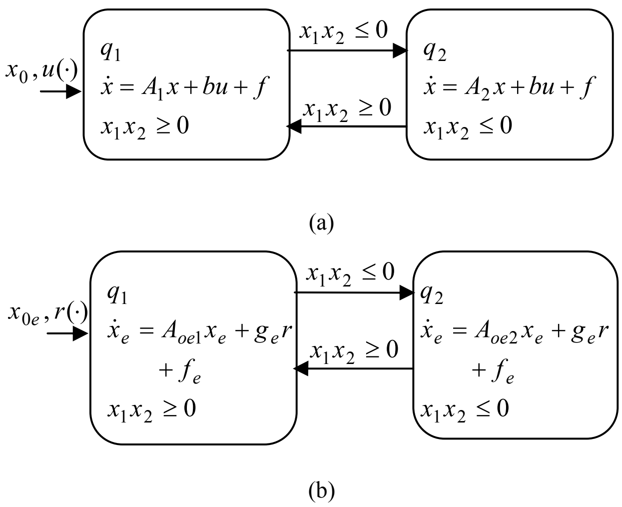

Fig. 5 hybrid automaton in open loop (a) and closed loop (b)

The simulated behavior of the open loop hybrid automaton with control input

$$u(t) = \begin{cases} u_1 = 0, & 0 \leq t < t_1 = 5 \\ u_2 = 2, & t_1 \leq t < t_2 = 15 \\ u_3 = 1, & t_2 \leq t \end{cases} \tag{42}$$

is given in Fig. 6.

The control objective is the oscillations damping of the system measured output

$$y = x_2\,, \tag{43}$$

while tracking the reference signal $r(\cdot) = u(\cdot)$, with $u(\cdot)$ specified in (42). The desired closed loop spectrum is Λ_e (25).

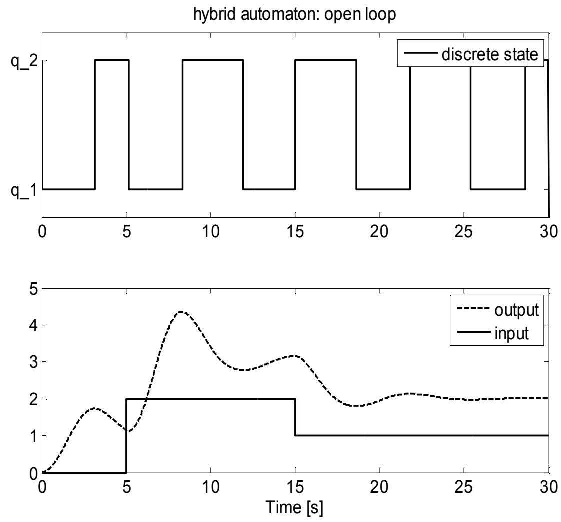

Fig. 6 simulated dynamics of the hybrid automaton in Fig. 4(a)

By applying, in each location, the steps 1-4 of the Control algorithm described in Section II, and assuming that the entire process state is available –i.e., for simplicity, no state observer is build and the feedback matrices are applied directly to the process state x– one obtains the closed loop hybrid automaton in Fig. 5(b), where matrices of the extended system (4) are

$$A_{ei} = \begin{bmatrix} -2\zeta_i & -1 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix},\ b_e = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \tag{44}$$

$$f_e = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix},\ g_e = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ i = 1,2\,,$$

and the closed loop matrices are:

$$A_{oei} = A_{ei} + b_e k_{ei}\,,\; i = 1,2\,, \tag{45}$$

with the corresponding feedback matrices

$$k_{e1}^T = [k_{o1}^T \;\vdots\; \varphi_1] = [-12.8 \quad -31 \;\vdots\; 20]\,, \tag{46}$$

and

$$k_{e2}^T = [k_{o1}^T \;\vdots\; \varphi_1] = [-12 \quad -31 \;\vdots\; 20]\,, \tag{47}$$

respectively. The simulated behavior of the closed loop hybrid automaton is presented in Fig. 7.

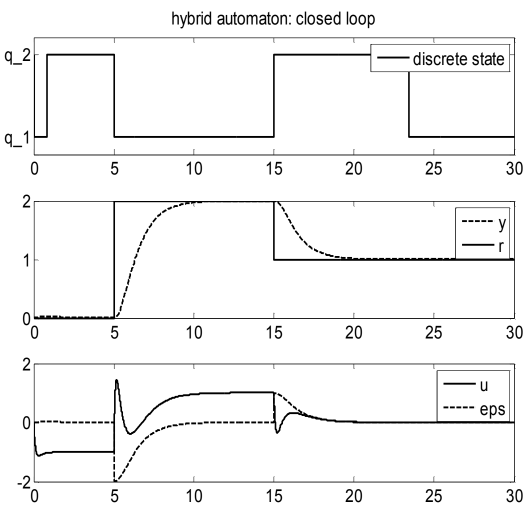

Fig. 7 simulated dynamics of the hybrid automaton in Fig. 4(b)

IV. Conclusions

The study of affine systems, as subsystems of piecewise affine hybrid systems, is important for understanding the overall complex system behavior. Also, the presence of the affine term imposes specific control approaches, which slightly differ from the classic linear control strategies. The control problem considered in this paper has an objective frequently encountered in the automotive systems literature: oscillations damping and reference tracking.

The proposed modal control approach is based on the controllability of the classic linear system associated to the affine system and implies a combination of a classic pole placement procedure – driving to a closed loop stable and aperiodic behavior – and of the internal model principle.

The two strategies are unified by introducing the extended affine system model, which aggregates the state equations of the affine process and of the internal model, respectively. Then, provided that an additional rank condition is fulfilled – related to the controllability of the extended system – it is proved that an extended pole placement procedure can be applied and implemented into the control system.

A control problem implying sufficient conditions for the solution of the original control problem is introduced and solved by the design of a feedback pole placement matrix. The implementation of the control solution has to incorporate also a minimal state estimator adapted to the affine model.

The simulation experiments are based on fixed step discretized models associated to the continuous time original affine systems. However, special care has to be taken when interpreting the results, due to the approximation introduced by the sampling procedure, despite the fact that this aspect is not generally detailed in the automotive systems literature.

The modal control approach can be successfully extended from affine to piecewise affine systems, as illustrated by the simulation example in Section III. It is important to emphasize that, in this case, a single desired closed loop pole assignment is chosen for each local continuous dynamics. Also, despite the fact that, naturally, the local feedback matrices may differ – depending on the distinct local process dynamics – the internal model in the controller is common, because it is designed according to the exogenous signals.

In the proposed simulation examples, a state observer can easily be incorporated to the controller.

The proposed modal approach for oscillations damping can be tested, as an alternate control procedure [9], for automotive systems, such as driveline oscillations damping.

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