Products reliability assessment using Monte-Carlo simulation

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Abstract**—** Product reliability is a critical part of total product quality. Reliability is a measure of a product's performance that affects both product function and operating and repair costs. Too often performance is thought of only in terms of speed, capacity, range, and other "normal" measures. This paper presents an approach to evaluate the reliability and unreliability of the industrial products taking into consideration the parameters deviation. For this reason, a variety of measurement methods, including test methods and specialized analytical techniques, it have been developed. Based on a Monte Carlo simulation, it was elaborated a program which we allows to calculate the normal distribution parameters. The methods used for estimation of products reliability should be those that meet the customer's needs in accordance with the strategy of the organization making the measurements. The case study consists of measuring process of an industrial product dimension and estimation of reliability and unreliability functions.

Keywords**—**improvement, quality, normal distribution, Monte Carlo simulation, reliability estimation, parameters deviation.

I. Introduction

THE everyday usage term quality of a product is loosely taken to mean its inherent degree of excellence. In industry, this is made more precise by defining quality to be "conformance to requirements at the start of use" [1]. Assuming the product specifications adequately capture customer requirements, the quality level can now be precisely measured by the fraction of units shipped that meet specifications.

The objective of reliability prediction is to support decisions related to the operation and maintenance of the product including to [2]:

- Reduce output penalties including labor costs and outage repair;
- Help in the design of future products, by improved safety margins and reduced failures;
- Increase profitability;
- Optimize maintenance cycles and spares holdings;
- Maintain the effectiveness of equipment through optimized repair actions.

"Quality" is the fundamental enterprises stay in business, training of all staff, the quality of the ceremony and build a reliable system of quality control is consistent two aspects. In the supplier's selection process and finished goods, all aspects of processing, checking, "substandard materials warehousing", "orderly transfer does not fail", the letter, the real operation to ensure the validity of the ISO 9001 quality assurance system [3], [5], [6]. The first step to product quality control is to reflect in the development of products the users' requirements for product quality. After setting the target, we aim at producing the products that are consistently high in quality and can meet the standards that we set. The basic concept is that each process of production is equally important as it plays an important role in product quality control.

One phase from the next is generally a decision milestone, sometimes referred to as a gate. For many products, the phases may be abbreviated or combined. For example, the Concept/Planning and Design/Development phases may be combined under a compressed schedule for a new product that is simply an update or slightly modified version of an older, proven product. Reliability measurement tasks would concentrate only on the differences between the old and the modified product. As a result, the number of engineering tasks would be reduced. It is important to understand that tasks performed in one phase are often the result of the analysis, trade-offs and planning performed in an earlier phase. For example, analytical measurement techniques addressing alternative approaches to the reliability of products would be performed during design/development, with reliability testing to measure the results of the process decision following during the production/manufacturing phase.

II. Reliability Concepts

Reliability is considered to be a performance attribute that is concerned with the probability of success and frequency of failures, and is defined as: "The probability that an item will perform its intended function understated conditions, for either a specified interval or over its useful life", [3].

Product reliability is a critical part of total product quality. Product reliability refers to the ability of a product to continue to perform its intended functions over time in the hands of the customer [14]. Product reliability is important for the following reasons presented in figure 1.

Reliability is a traditional measure of quality. A reliable product rarely fails. This aspect, which is sometimes reformulated as "being free of deficiencies", it is a very important dimension of quality.

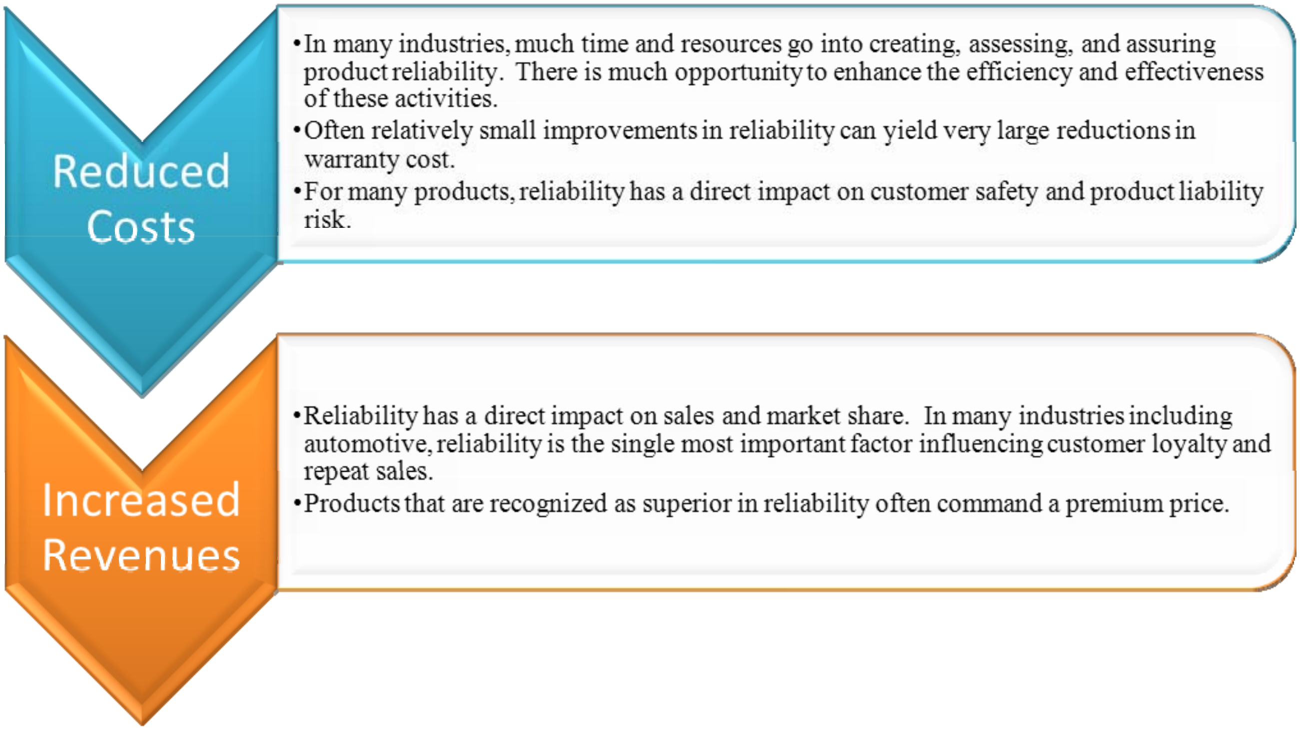

Fig. 1 Importance of product reliability study

If a product has such poor reliability that it is seldom available for use, these other performance measures become meaningless. Reliability is also critical to safety and liability issues.

Superficially, measuring reliability is a simple matter. One merely counts failures and divides by operating time to come up with a mean time between failures (MTBF), the most common reliability measure.

A. Statistical Process Control

While the need to set targets has been widely discussed in the TQM, benchmarking and re-engineering literature, guidance aimed at helping management report and interpret performance against targets has been fragmented [7]. The three forms are first, the target is a single lower limit; second, the target is a single upper limit; and third, the target is a zone between an upper and a lower limit. The four methods to calculate the level of improvement needed to reach target are the counting, distance, histogram and capability index methods.

For every measured parameter, upper control limit has to be determined. It is used to recognize and react to significant deviations from statistically acceptable limits in product or process reliability.

Upper control level (UCL) or alert value is statistical value which shows the limit below which deviations are considered statistically acceptable.

In case of exceeded alert limit value, system reliability is considered to be unstable.

Repeated exceeding of the alert value represents a negative trend which has to be stopped by application of appropriate corrective action.

Upper control limit is based on statistical calculation of standard deviation covering recent twelve month period. It should not be placed too high because in that case negative trends would not be shown and opposite, if it is placed too low, because even small deviations from mean values will trigger exceeding.

Upper control limit is established by multiplying standard deviation above mean value with deviation factor (normally between 2 and 3). Deviation factor is defined by operator and it is depending on dispersion of data – smaller factor is more appropriate for large fleets and greater factor is appropriate for small fleets.

Procedure of UCL establishing is [8], [9]:

- calculation of standard deviation:

$$\sigma = \sqrt{\frac{\Sigma\left(x^2\right) - \frac{(\Sigma x)^2}{N}}{N-1}}\,, \qquad (1)$$

where: x – monthly value of parameter in observed months; σ – standard deviation; N – number of observed months for which standard deviation is calculated.

- calculation of upper control limit – U:

$$UCL = \bar{x} + k\sigma\,, \qquad (2)$$

where: $\bar{x} = \Sigma x / N$, k – deviation factor (normally between 2 and 3).

- calculation of lower control limit – L:

$$LCL = \bar{x} - k\sigma \ . \tag{3}$$

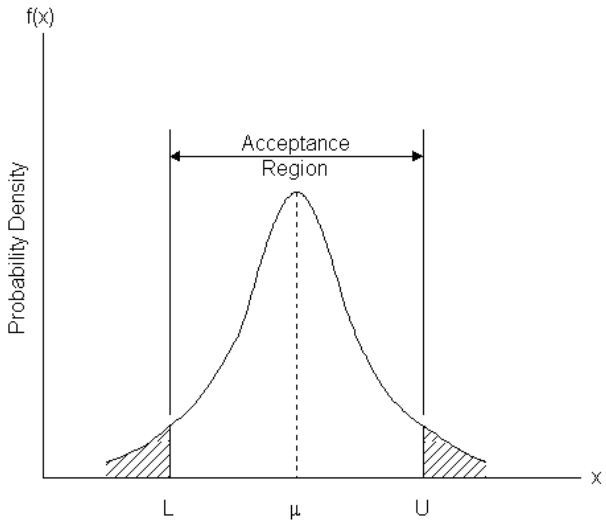

Fig. 2 Probability distributions and acceptance regions

This calculation of UCL should be repeated every 12 months. Upper control limit can be increased or decreased by maximum 10 % compared to the previous UCL.

This methodology enables analysts to determine whether there is any margin left in the product after calculating its worst-case maximum and minimum performance. It also allows these values to be compared to the product's specified upper and lower tolerance limits, which cannot be exceeded per its design specifications.

B. Monte-Carlo Simulation

Where appropriate statistical model known for technological system considered, based on knowledge of deviation function of the parameters can be predicted when processing errors exceed a certain percentage of the tolerance range allowed. This requires an adjustment operation of analyzed technological equipment.

Figure 3 illustrates the parameters deviation for normal distribution where both parameters mean μ and standard deviation σ are variables.

A random variable X is said to be normally distributed if it density function is specified by [5]:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \ . \tag{4}$$

Suppose that:

$$\mu_i = \mu_0 + a \cdot t \ , \tag{5}$$

$$\sigma_i = \sigma_0 + b \cdot t \ , \tag{6}$$

where σ and μ are two parameters that denote the mean and the standard deviation.

Let

$$z = \frac{\Delta - \mu_i}{\sigma_i} \ . \tag{7}$$

Substituting equation 7 in linear variation function, we obtain:

$$z(\sigma_0 + b \cdot t) = \Delta - \mu_0 - a \cdot t \ . \tag{8}$$

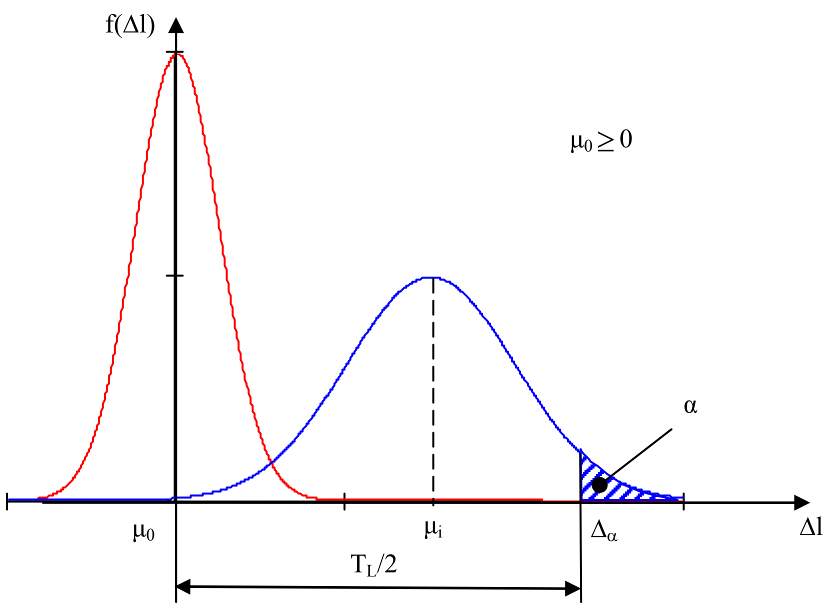

Fig. 3 Parameters deviation for normal distribution

Based on notations in figure 3, it can write:

$$\Delta_\alpha = \mu_0 + T_L/2 \,, \tag{9}$$

where T_L is tolerance interval.

If it is eliminating Δ_α between the two equations given by relations 8 and 9, it obtain an expression for calculating the adjustment time:

$$t \le \frac{T_L/2 - z \cdot \sigma_0}{b \cdot z + a} \,. \tag{10}$$

If α is the level of significance, then z can be obtained from the relationship:

$$\alpha = 0.5 - \int_0^z f(z)dz \,. \tag{11}$$

III. Six Sigma Methodology

Six Sigma has evolved over the last two decades and so has its definition. The term "Sigma" is often used as a scale for levels of "goodness" or quality. Using this scale, "Six Sigma" equates to 3.4 defects per one million opportunities. Therefore, Six Sigma started as a defect reduction effort in manufacturing and it was then applied to other business processes for the same purpose.

Six Sigma is a business improvement methodology that focuses an organization on:

- Understanding and managing customer requirements;
- Aligning key business processes to achieve those requirements;
- Utilizing rigorous data analysis to minimize variation in those processes;
- Driving rapid and sustainable improvement to business processes.

When practiced as a management system, Six Sigma is a high performance system for executing business strategy. Six Sigma is a top-down solution to help organizations:

- Align their business strategy to critical improvement efforts;
- Mobilize teams to attack high impact projects;
- Accelerate improved business results;
- Govern efforts to ensure improvements are sustained.

The Six Sigma Management System drives clarity around the business strategy and the metrics that most reflect success with that strategy. It provides the framework to prioritize resources for projects that will improve the metrics, and it leverages leaders who will manage the efforts for rapid, sustainable, and improved business results.

IV. Case Studies

The methods used for estimation of products reliability should be those that meet the customer's needs in accordance with the strategy of the organization making the measurements. For this reason, a variety of measurement methods, including test methods and specialized analytical techniques, it have been developed.

The case study consists of measuring process of an industrial product dimension. The lower specified limit (LSL) is 12.50 and the upper specified limit (USL) is 21.9.

The link between parameters deviation and reliability or unreliability of the technological processes, with two specified limits for m = constant and s $\neq$ constant. Considering normal distribution, the simulation results are illustrated in figure 4. Also, the Monte-Carlo simulation program is detailed in figure 5.

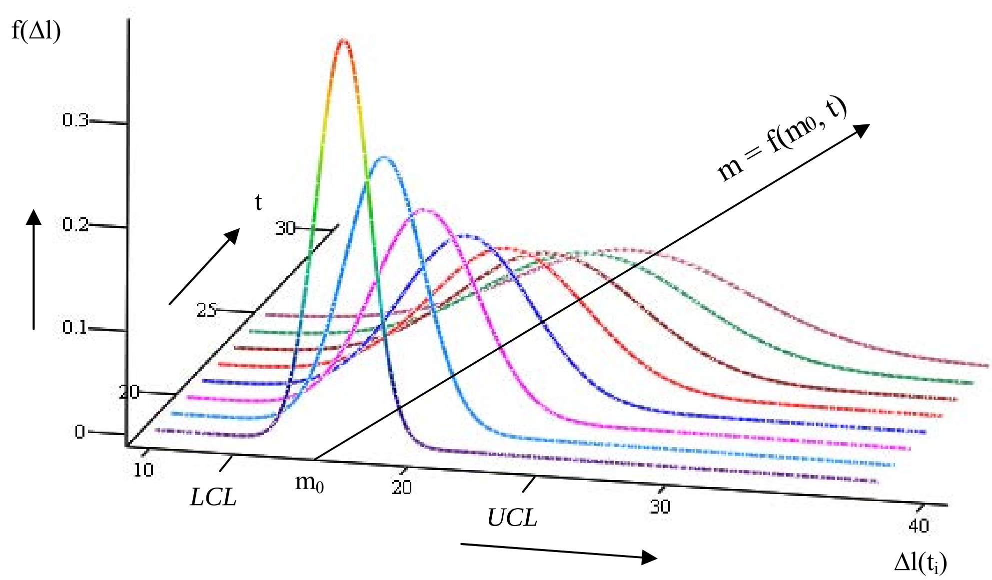

Fig. 4 Simulation results of parameters deviation for m $\neq$ constant and s $\neq$ constant, t > 0

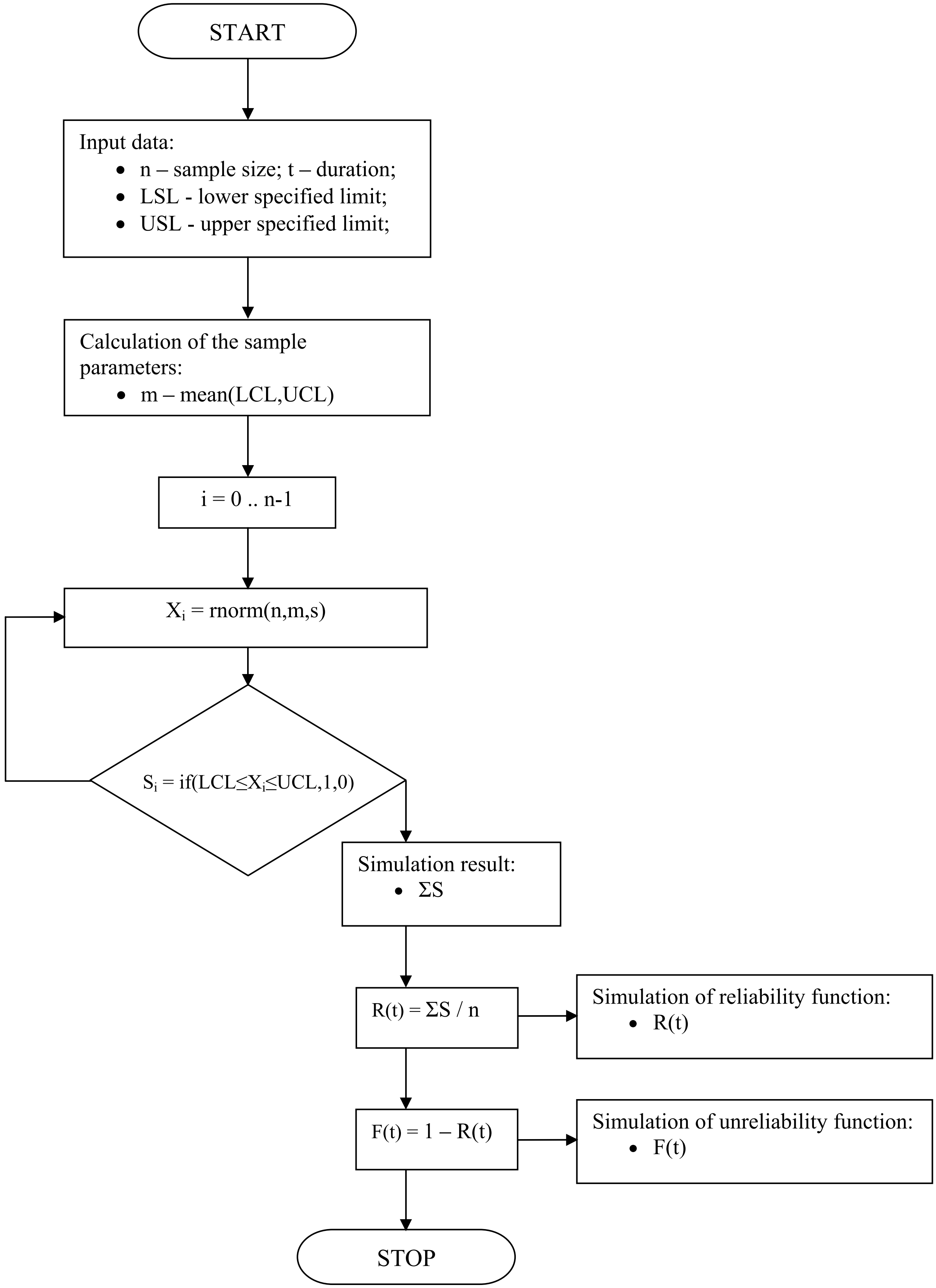

Fig. 5 Monte-Carlo simulation program

The design simulation program was made using the specialized functions from Mathcad soft.

Based on a Monte Carlo simulation, the program allows calculating the normal distribution parameters using the following function:

- mean: returns the sample average;
- Stdev: returns the sample standard deviation;
- rnorm: returns a vector X of n random numbers having the normal distribution with mean m and standard deviation s.

TABLE I

m	*s*	*R(t)*	*F(t)*
17.2	1.024	1	0
17.2	1.524	0.99804	1.96×10^{-3}
17.2	2.024	0.97935	0.02065
17.2	2.524	0.93693	0.06307
17.2	3.024	0.88058	0.11942
17.2	3.524	0.81865	0.18135
17.2	4.024	0.75833	0.24167
17.2	4.524	0.70270	0.2973
17.2	5.024	0.64854	0.3514
17.2	5.524	0.60365	0.3964
17.2	6.024	0.56755	0.4325
17.2	6.524	0.52927	0.4707
17.2	7.024	0.49739	0.5026
17.2	7.524	0.46803	0.5320
17.2	8.024	0.44158	0.5584
17.2	8.524	0.41835	0.5816
17.2	9.024	0.39631	0.6037
17.2	9.524	0.37860	0.6214
17.2	10	0.36058	0.6394

Monte Carlo simulation consists of random variables generations. Assuming a uniform distribution, we can use the specific functions.

The simulation results of reliability and unreliability functions for m = constant and s ≠ constant can be easily plotted to produce. These are presented in table I, figure 6 and figure 7.

The reliability and unreliability functions simulated for m ≠ constant and s ≠ constant are presented in table II.

TABLE II

m	*s*	*R(t)*	*F(t)*
17.2	1.024	1	0
18.2	1.524	0.99259	0.00741
19.2	2.024	0.90989	0.09011
20.2	2.524	0.74842	0.25158
21.2	3.024	0.59075	0.40925
22.2	3.524	0.46483	0.53517
23.2	4.024	0.37027	0.62973
24.2	4.524	0.29947	0.70053
25.2	5.024	0.24780	0.75220
26.2	5.524	0.21229	0.78771
27.2	6.024	0.18292	0.81708
28.2	6.5024	0.15952	0.84048
29.2	7.024	0.13851	0.86149
30.2	7.5024	0.12616	0.87384
31.2	8.024	0.11197	0.88803
32.2	8.5024	0.10364	0.89636

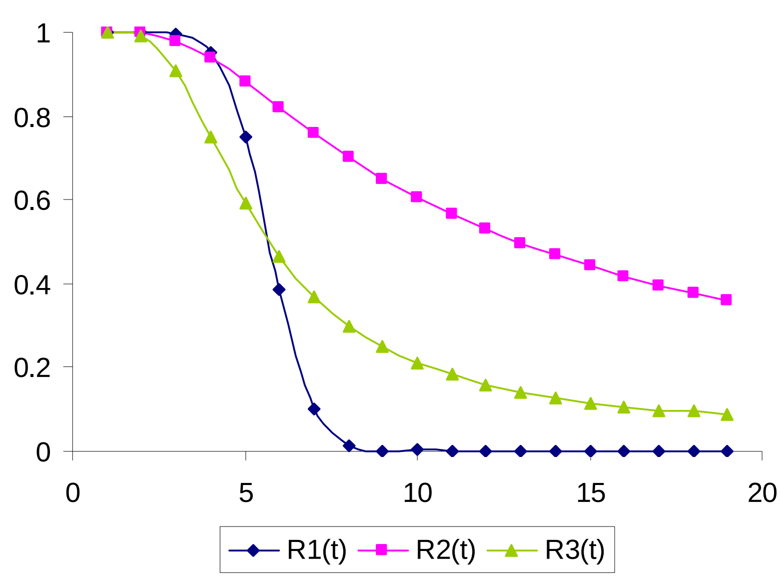

Fig. 6 The simulation of reliability functions for m = constant and s ≠ constant

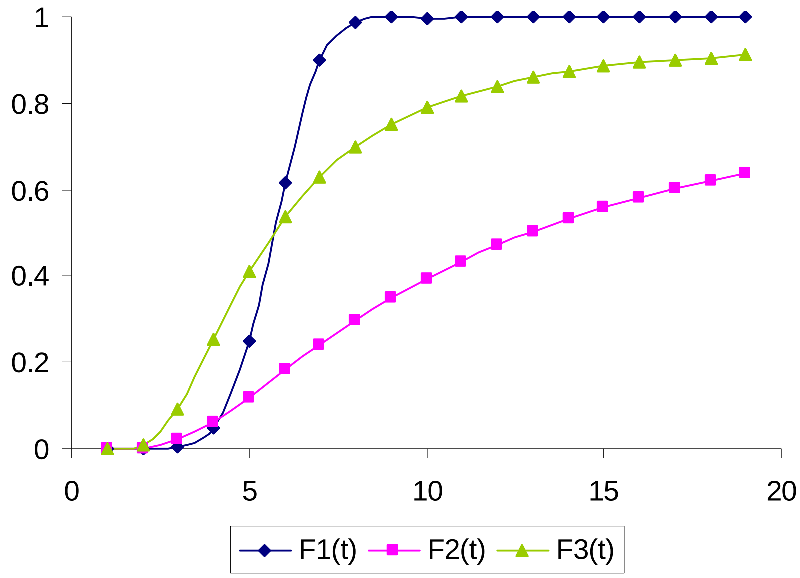

Fig. 7 The simulation of unreliability functions for m = constant and s ≠ constant

Figure 8 shows the estimation of the products reliability function based on the experimental data. The unreliability function estimation for analyzed products in the case of m ≠ constant and s ≠ constant it is shown in figure 9.

In a worst-case scenario, survival of all parts simultaneously at their maximum drift values ensures survival to any degree of part variation in any combination.

Calculating product performance under the worst-case scenario it is not exceeding the specified upper and lower performance tolerance limits, ensures a solid process against part variation.

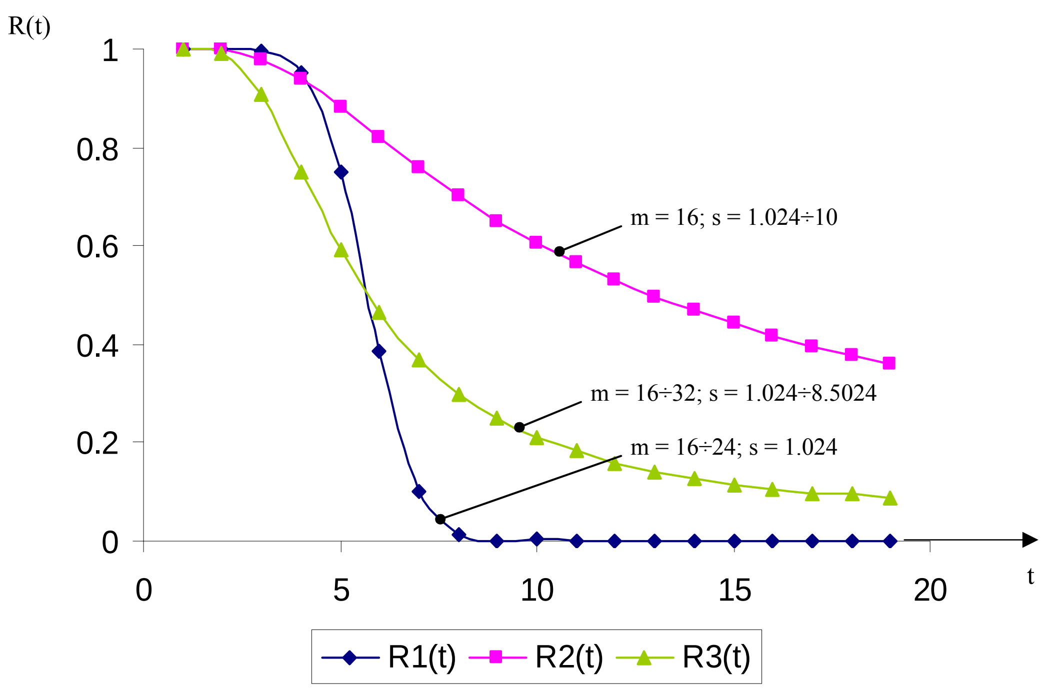

Fig. 8 Estimation of products reliability function

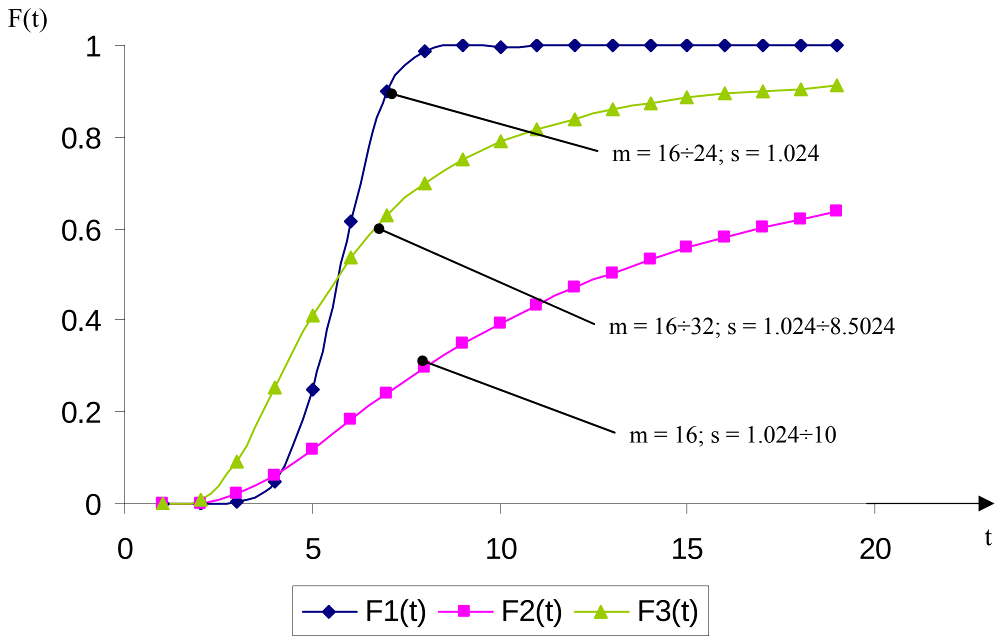

Fig. 9 Estimation of products unreliability function

V. Conclusions

Functions and performance of equipment, the social impact and damages produced by failures are increasing, and high reliability has come to be demanded of products, processes or systems.

The importance of reliability is revealed from the following:

An unreliable product will negatively affect customer satisfaction severely:

- High reliability is a mandatory requirement for customer satisfaction;
- If a product fails to perform its function within the warranty period, the replacement and repair costs will negatively affect profits, as well as gain unwanted negative attention;
- A concentrated effort towards improved reliability shows existing customers that a manufacturer is serious about their product, and committed to customer satisfaction. This type of attitude has a positive impact on future business;
- Resellers may take reliability data and combine it with other cost information to illustrate the cost effectiveness of their products. This life cycle cost analysis can prove that although the initial cost of their product might be higher, the overall lifetime cost is lower than a competitor's because their product requires fewer repairs or less maintenance.

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