# Kinematics of the seven gear automatic gearboxes and vehicle dynamics

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***Abstract*—** Kinematic analysis for the automatic gearboxes with four and seven gears reveal both the evolution and state of the elements of these types of automatic gearboxes in operation. Kinematics and dynamics are both dependent thus the analysis is very rigorous and complex. Dynamic performances and fuel consumption of an automobile depend on the engine that is used on it and the performance of its transmission. The dependency of the energy consumption and the value of the gearboxes final ratios it is also highlighted.

***Keywords*—**automatic gearbox, fuel, kinematics, transmission, energy.

## I. Introduction

THE acceleration, fuel consumption and reliability are important factors for the development and optimization of the gearboxes. In sporty vehicles the gearboxes are adapted to a dynamic driving style, which allows faster accelerations in each gear. It is known that the dynamic performance and the fuel consumption of an automobile are influenced by the engine and gearbox ratios.

The optimization of the gearbox is made based on a single criterion-acceleration or fuel consumption and this two criterions cannot be satisfied simultaneously.

It is well known that there are two criterions of calculating gear ratios.

Dynamic performances and fuel consumption of an automobile depend on the engine that is used on it. Also, a considerable effect on the vehicle performance is created by the gear ratios.

According to the construction and functioning of the gearbox, the main elements of a gearbox are the gears that form the transmission. The variation of the gear ratios from one gear to another one represents the gearing of the gearbox.

The gear ratios in the gearbox allow the automobile to meet the following criterions:

-drive in difficult condition (road with a very steep incline)

-reach the maximum speed

-to function in the minimum consumption domain of the internal combustion engine

## II. Kinematic Analysis of the 4 Speed Automatic Gearbox

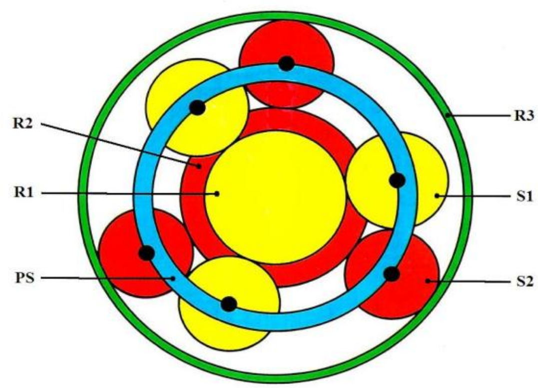


Figure 1. „Ravigneaux" planetary gear system

In figure 1 we can identify the following elements: two sun gears $R_1$ and $R_2$ , the $R_3$ crown gear, the carrier PS and two planet gears $S_1$ and $S_2$.

The "Ravigneaux" type planetary system has 4 forward gear ratios,and one reverse gear ratio. The input elements are different type of combinations of two by two pairs of the elements $R_1$, $R_2$ and PS,and the output element is the crown gear, $R_3$ with inner teeth [1; 2; 7; 8; 9; 10].

The main advantages of the „Ravigneaux" planetary gear system are:

- compact and space-saving, because the energy transfer is performed in parallel in many ways divided

-it can realize wide gear ratios

-high reliability, ensured by the good lubrication conditions

For the kinematic analysis of a automatic 4 gear transmission, we choose a Renault A R4 automatic transmission with 4 gear plus one reverse gear.

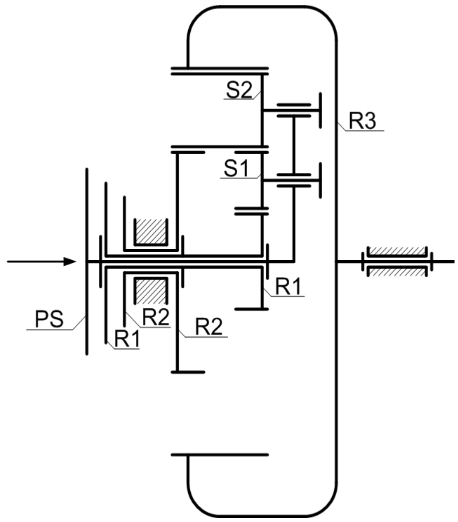


Figure 2. Kinematic scheme of a „Ravigneaux" planetary gear system

There are three known methods to determine the ratio of a planetary gear system :

-the reverse motion method (Willis method) and the transformation of the planetary transmission system into an ordinary transmission with fixed axis

-decomposition of composed movements into simple movements using the decomposition method (Swamp method);

-graphic-analytical method (method of instantaneous centers or method Kutzbach). The most utilized method is "the Willis method"

First gear: PS-fixed, $R_1$-input, $R_2$-output

Second gear: PS-free, $R_1$-input, $R_2$- fixed

Third gear: PS-free, $R_1$-input, $R_2$- input

Fourth gear: PS-input, $R_1$-free, $R_2$- fixed

The reverse gear: PS-fixed, $R_1$-free, $R_2$- input

For the following steps of the kinematic calculus it is used only the Willis method.

Next, we calculate the gear ratio on each gear, using the teeth numbers

First gear kinematic calculation:

The transmission ratio between R1 gear and R2 gear is the ratio between the angular speed $\omega_{R_1}$ and $\overline{\omega_{R_3}}$

$\overline{\omega_{R_3}}$

$$\frac{\overline{\omega_{R_1}}}{\overline{\omega_{R_3}}} = \left(+\frac{z_{R_3}}{z_{S_2}}\right)\cdot\left(-\frac{z_{S_2}}{z_{S_1}}\right)\cdot\left(-\frac{z_{S_1}}{z_{R_1}}\right) = \frac{z_{R_3}}{z_{R_1}} = i_1 \quad (1)$$

For the second gear we will use "The Willis method" (inversation movement method)

Table 1. Willis Method table

| MOTION | $R_1$ | $R_2R_2$ (FIXED ELEMENT) | PS | $R_2$ |
|---|---|---|---|---|
| Real | $\overline{\omega_{R_1}}$ | $\overline{0}$ | $\overline{\omega_{PS}}$ | $\overline{\omega_{R_3}}$ |
| With – $\omega_{PS}\overline{\omega_{PS}}$ overlapped | $\overline{\omega_{R_1}} - \overline{\omega_{PS}}$ | $-\overline{\omega_{PS}}$ | $\overline{0}$ | $\overline{\omega_{R_3}} - \overline{\omega_{PS}}$ |

From the planetary cycloidal gear train the transfer is made into an ordinary gear train by immobilizing the carrier (equation 2):

$$\frac{\overline{\omega_{R_1}} - \overline{\omega_{PS}}}{-\overline{\omega_{PS}}} = \left(-\frac{z_{R_2}}{z_{S_2}}\right)\cdot\left(-\frac{z_{S_2}}{z_{S_1}}\right)\cdot\left(-\frac{z_{S_1}}{z_{R_1}}\right) = -\frac{z_{R_2}}{z_{R_1}} \quad (2)$$

$$\frac{\omega_{R_1}}{-\omega_{PS}} + 1 = -\frac{z_{R_2}}{z_{R_1}} \quad (3)$$

$$\frac{\omega_{R_1}}{\omega_{PS}} = 1 + \frac{z_{R_2}}{z_{R_1}} \quad (4)$$

Using "The Willis method", we have the same calculation, ,but this time the input is on the R3 gear

$$\frac{\overline{\omega_{R_3}} - \overline{\omega_{PS}}}{-\overline{\omega_{PS}}} = \left(-\frac{z_{R_2}}{z_{S_2}}\right)\cdot\left(+\frac{z_{S_2}}{z_{R_3}}\right) = -\frac{z_{R_2}}{z_{R_3}} \quad (5)$$

$$\frac{\omega_{R_3}}{\omega_{PS}} - 1 = \frac{z_{R_2}}{z_{R_3}} \quad (6)$$

$$\frac{\omega_{R_3}}{\omega_{PS}} = 1 + \frac{z_{R_2}}{z_{R_3}} \quad (7)$$

Returning to the second gear transmission ratio we have:

$$i_2 = \frac{\overline{\omega_{R_1}}}{\overline{\omega_{R_3}}} = \frac{1+\frac{z_{R_2}}{z_{R_1}}}{1+\frac{z_{R_2}}{z_{R_3}}} = \frac{\frac{z_{R_1}+z_{R_2}}{z_{R_1}}}{\frac{z_{R_2}+z_{R_3}}{z_{R_3}}} = \frac{z_{R_1}+z_{R_2}}{z_{R_2}+z_{R_3}}\cdot\frac{z_{R_3}}{z_{R_1}} \quad (8)$$

Third gear calculation:

$$\overline{\omega_{R_1}} = \overline{\omega_{R_3}}\ ; \quad \frac{\omega_{R_1}}{\omega_{R_3}} = 1 = i_3 \quad (9)$$

Using "The Willis method" ,but this time the input is element PS, the relation for the fourth gear is the following:

$$\frac{-\overline{\omega_{PS}}}{\overline{\omega_{R_3}} - \omega_{PS}} = \left(+\frac{z_{R_3}}{z_{S_2}}\right)\cdot\left(-\frac{z_{S_2}}{z_{R_2}}\right) = -\frac{z_{R_3}}{z_{R_2}} \quad (10)$$

$$\frac{\omega_{R_3} - \omega_{PS}}{-\overline{\omega_{PS}}} = -\frac{z_{R_2}}{z_{R_3}} \quad (11)$$

$$-\frac{\omega_{R_3}}{\omega_{PS}} = -\frac{z_{R_2}}{z_{R_3}} - 1 \quad (12)$$

$$\frac{\overline{\omega_{PS}}}{\overline{\omega_{R_3}}} = \frac{z_{R_3}}{z_{R_3}+z_{R_2}} = i_4 \quad (13)$$

The reverse gear calculation:

$$\frac{\overline{\omega_{R_2}}}{\overline{\omega_{R_3}}}=\left(+\frac{z_{R_3}}{z_{S_2}}\right)\cdot\left(-\frac{z_{S_2}}{z_{R_2}}\right)=-\frac{z_{R_3}}{z_{R_2}}=i_R \quad (14)$$

By having the teeth number of all gears for the „Ravigneaux" system for the Renault AR4 automatic gear box we can easily verify the transmission ratios [14].

Given: $z_{R_1}=21,\ z_{R_2}=27,\ z_{R_3}=57,\ z_{S_1}=15,\ z_{S_2}=14$

We have the next calculus relation for the transmission ratios:

$$i_1=\frac{z_{R_3}}{z_{R_1}}=2{,}71 \quad (15)$$

$$i_2=\frac{z_{R_1}+z_{R_2}}{z_{R_2}+z_{R_3}}\cdot\frac{z_{R_3}}{z_{R_1}}=1{,}55 \quad (16)$$

$$i_3=\frac{\omega_{R_1}=\omega_{R_2}}{\overline{\omega_{R_3}}}=1 \quad (17)$$

$$i_4=\frac{z_{R_3}}{z_{R_3}+z_{R_2}}=0{,}68 \quad (18)$$

$$i_R=-\frac{z_{R_3}}{z_{R_2}}=-2{,}11 \quad (19)$$

Table 2.Gear speed and transmission ratios for four speed automatic gear box

| Gear | | Transmission ratio | | State of motion of the input elements | | |
|---|---|---|---|---|---|---|
| | | Relation | Value | R1 | R2 | PS |
| Forward movement | 1 | $i_1=\frac{z_{R3}}{z_{R1}}$ | 2,71 | 1 | - | 0 |
| | 2 | $i_2=\frac{z_{R_1}+z_{R_2}}{z_{R_2}+z_{R_3}}\cdot\frac{z_{R_3}}{z_{R_1}}$ | 1,55 | 1 | 0 | - |
| | 3 | $i_3=1$ | 1 | 1 | 1 | - |
| | 4 | $i_4=\frac{z_{R_3}}{z_{R_3}+z_{R_2}}$ | 0,68 | - | 0 | 1 |
| Reverse gear | | $i_R=-\frac{z_{R3}}{z_{R2}}$ | -2,11 | - | 1 | 0 |

Analyzing the upward table we can see that the automatic gearbox in four gears has one amplifier transmission ratio, which means that in the fourth gear it has a lower energetic consumption.

## III. 7G TRONIC AUTOMATIC GEARBOX KINEMATIC ANALYSIS

As stated in the technical literature, the automatic gearbox 7G Tronic with 7 gears of forward movement and 2 reverse gears is divided in 3 planetary systems: the „Ravigneaux" type planetary system of gears and two simple planetary systems, named specifically to their positioning: Simple front planetary system – SPSF and simple rear planetary system – SPSS (Figure 3) [2; 3;10; 15].

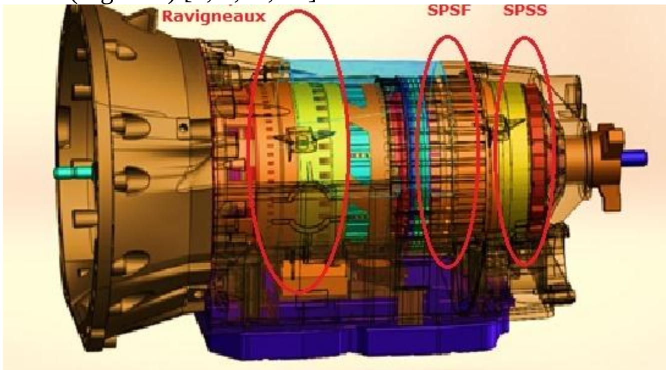


Figure 3. 7G Tronic automatic gearbox

There are three distinct cases of reduction transmission ratios and three amplifier transmission ratios for a simple planetary transmission. Two of these cases have an inversion role, for the reverse gear.

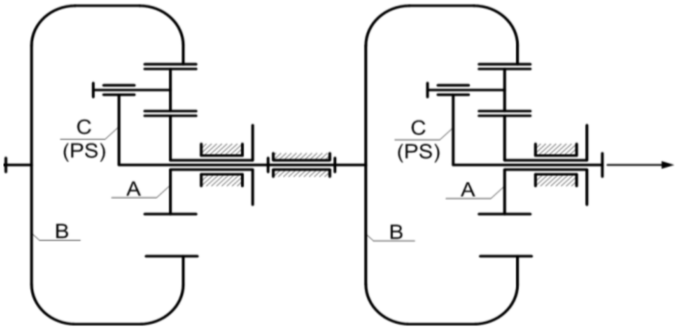


Figure 4 simple front planetary system SPSF and simple rear planetary system SPSS (A-central sun gear, B-crown gear, C<PS>-carrier)

Knowing all technical data for the 7G Tronic transmission, means that we also have all the transmission ratios. We can extract the final drive ratio, by dividing the transmission ratios like below [7]:

$$i_{Final}=\frac{i_{1\,7G}}{i_{14R}}=\frac{4{,}377}{2{,}71}\cong 1{,}617$$

$$i_{Final}=\frac{i_{R1}}{i_{R\,4R}}=\frac{-3{,}416}{-2{,}11}\cong 1{,}617$$

These values are given by the transmission ratios of the two simple planetary system.

In the case of a simple reduction planetary system, the calculation formula for that kind of system is:

$$i=1+\frac{z_A}{z_B} \quad (20)$$

For a simple reductive planetary system with the ratio between 1,25 and 1,67, and in the case of 7G Tronic gear box, having the final ratio $i_{Final}$ = 1,617 on both simple planetary system. That means that for an individual simple planetary system we have:

$$i_{SPSF}=i_{SPSS}=\sqrt{i_{Final}} \quad (21)$$

From where we have: $i_{SPSF}=i_{SPSS}=\sqrt{1{,}617}=1{,}272$$i_{SPSF}=i_{SPSS}=\sqrt{1{,}617}=1{,}272$

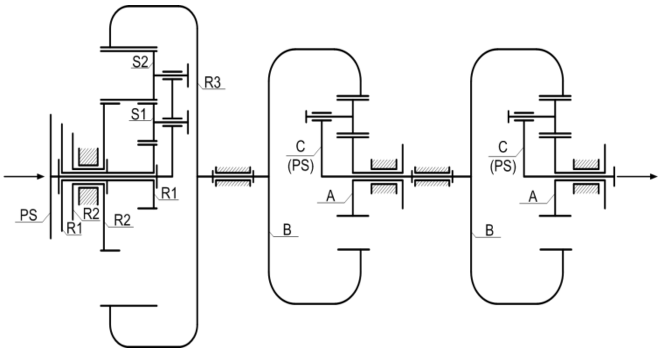


Figure 5. Kinematic scheme of the 3 planetary systems of the 7G Tronic automatic gearbox

According to these data and figure 4 and 5, we can notice that in both cases(SPSF and SPSS) the input is in the crown gear B, the output is on carrier C, and the blocked element, the central sun gear A, using the same notation for both simple planetary system [3; 5; 10;13].

Comparing the data from table 3 we can notice that the composition of the transmission ratios of the 7G Tronic gearbox by the “Ravigneaux” planetary system, of which kinematic calculation was determined before for the Renault A R4 automatic gearbox and combination of one, two or none simple planetary system.

Table 3. Comparison beetwen the gear ratios of a „Ravigneaux” planetary system and the ones of the 7G Tronic automatic gearbox

| Relations for calculating the ratios of 7G Tronic | Calculated values | 7G Tronic Ratios |
|---|---|---|
| $i_{17G}=i_{Final}\cdot i_{14R}$ | $i_{17G}=4{,}377\cong 1{,}617\cdot 2{,}71$ | $i_2$= 4,377 |
| $i_{2\ 7G}=i_{SPSS}\cdot i_{14R}$ | $i_{2\ 7G}= 2{,}859\cong 1{,}272\cdot 2{,}71$ | $i_2$= 2,859 |
| $i_{3\ 7G}=i_{SPSS}\cdot i_{2\ 4R}$ | $i_{3\ 7G}= 1{,}921\cong 1{,}272\cdot 1{,}55$ | $i_3$= 1,921 |
| $i_{4\ 7G}=i_{2\ 4R}$ | $i_{4\ 7G}= 1{,}368\cong 1{,}55$ | $i_4$= 1,368 |
| $i_{5\ 7G}=i_{3\ 4R}$ | $i_5= 1$ | $i_5$= 1 |
| $i_6=i_{SPSF}\cdot i_{4\ 4R}$ | $i_6= 0{,}821\cong 1{,}272\cdot 0{,}68$ | $i_6$= 0,821 |
| $i_7=i_{4\ 4R}$ | $i_7= 0{,}728\cong 0{,}68$ | $i_7$= 0,728 |
| $i_{R1}=i_{Final}\cdot i_{R4R}$ | $i_{R1}= -3{,}416\cong 1{,}617\cdot(-2{,}11)$ | $i_{R1}$= -3,416 |
| $i_{R2}=i_{R4R}$ | $i_{R2}= -2{,}23\cong -2{,}11$ | $i_{R2}$= -2,23 |

According to www.gearsmagazine.com in Gear Ratios section there is a calculating algorithm, which defines the number of teeth for the gears of the planetary system, according to the transmission ratios.

Using figure 5 we can optimize the planetary gear sets SPSF and SPSS with the following teeth numbers: 71, 14, 28, 60.

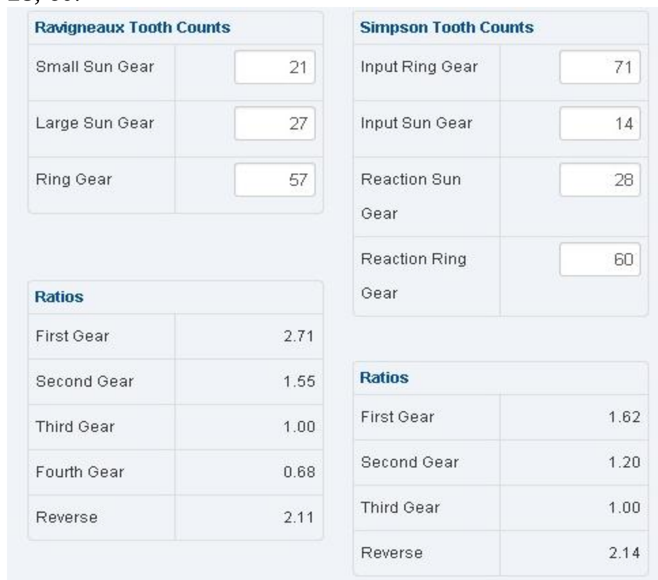


Figure 6. Teeth number using the given transmission ratios [16]

## IV. 7 Gear Automatic Gearbox Traction Analysis

The car's traction power is given by its external feature which is the dependence function of the torque developed by the engine, on the rotation angular rate of the bent axle. When deciding the external characteristic, the speed variation can be obtained only by decreasing or amplifying the engine torque. For a value of the engine torque M and for the angular rate of the bent axle, regular motion is possible to determine the engine's power.

$$P=M\cdot\omega \tag{22}$$

In this way we can easily notice the external characteristic using graphic method.

The evaluation of car's traction performance is made for a single type specific to Mercedes-Benz WDC 164.063 equipped with 7G Tronic having the maximum torque , but also on the other two types Mercedes-Benz WDC 204.984 and WDC 251.022

Characteristics of the first type of Mercedes-Benz WDC 164.063:

Cylinder volume: $C_c$=2987;

Maximum power: $P_{max}$=150 kW;

Max power speed: $n_P$= 4000 rot/min;

Maximum torque: $M_{max}$=500 Nm;

Maximum torque speed: $n_{Mmax}$=1600 rot/min;

Gear box type: automatic, 7 gears

Gear ratios: $i_1$= 4,377, $i_2$= 2,859, $i_3$= 1,921, $i_4$= 1,368, $i_5$=1, $i_6$= 0,821, $i_7$= 0,728, $i_{R1}$= -3,416, $i_{R2}$= -2,23

Overall dimensions: L= 4,781 m, B= 1,91 m, H=1,815 m;

Wheelbase: Amp= 2,915 m;

Front & back track: Ec= 1,67 m;

Tire type: 235/65 HR17;

Weight: $m_0$= 2185 kg;
Maximum speed: $V_{amax}$= 210 km/h;
Acceleration time: $t_{acc}$= 8,2 s;
Medium fuel consumption: $C_m$= 6,3 l;

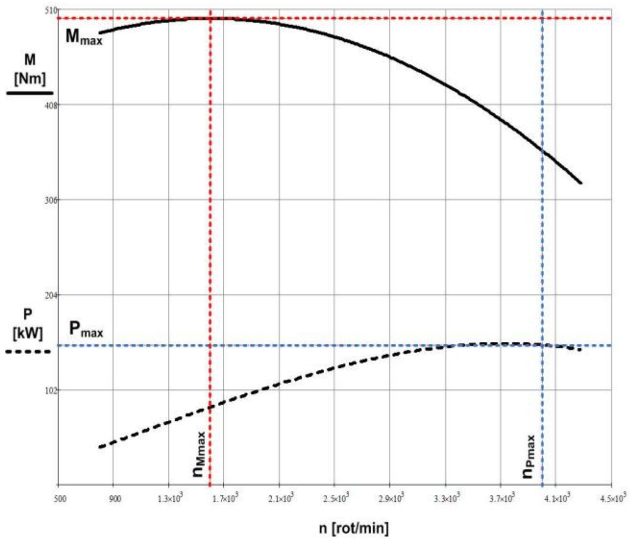


Figure 7. Engine characteristics

Fig. 7 presents the external characteristic of WDC 164.063. The power curve, P (kW) increases till it touches the maximum speed $n_{Pmax}$. This image also exposes the engine torque curve M (Nm), which increases depending on the engine rate before maximum torque, when it goes down.

The place where the engine is working is called constant area, because when increasing the task and speeding down, the engine torque appeared is increasing and balances redundant torques.

The bigger the constant area is, the better for the driving force. The measure of this area is the elasticity factor $C_e$.

For spark ignition internal combustion engines, $C_e$=0.45…0.65 and for compression internal combustion engines, $C_e$=0,55…0,75.

For every point of external characteristic of the engine in action $M=M(\omega)$, it is possible to estimate, for each gear of the gearbox, both speed $F_R$ and the driving speed of the car.

$$F_R=\frac{\eta_t \cdot M \cdot i_{cv} \cdot i_0}{r_d} \tag{23}$$

Where $\eta_t$ is the transmission efficiency, approximately $\eta_t = 0{,}89$; M-maximum engine torque; $i_{cv}$-central transmission gear ratio; $i_0$-first gear ratio and also starting gear ratio, $r_d$-dynamic wheel radius.

For the aerodynamic force the calculus is at follows:

$$F_a=\frac{1}{2} \cdot \rho \cdot C_x \cdot A \cdot \frac{v^2}{13} \tag{24}$$

Where $C_x$ -aerodynamic coefficient, $\rho$-represents air density in normal temperature conditions at 20°C, $\rho=1{,}205\ \frac{kg}{m^3}$ ; A-transversal section area.

Image 8 presents speed and force characteristics of a vehicle using 7G Tronic gear box.

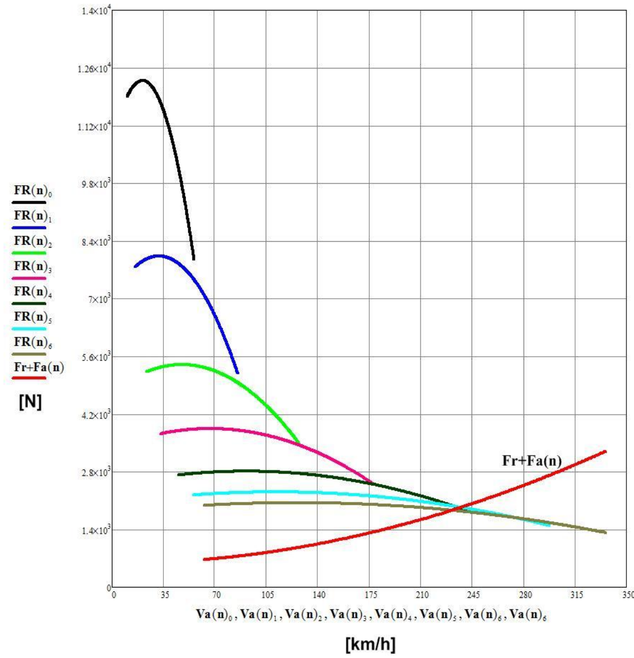


Figure 8. Traction and speed characteristics on all seven gears

The cross point of the curve resistance force along with aerodynamic force $F_R$ + Fa..with drag, only for 5th gear, 6th and 7th of the gearbox, determines maximum speed, the start-up is no longer possible. Worth mentioning is that in the maximum speed point its fluxion is null in relation to time.

In figure 8 there are captured speeds for each gear of the gearbox, starting with the first, Fr0, where the torque is maximum and the speed is low, ending with speed Fr7, associated with the seventh gearbox gear where it is reached the maximum speed of the vehicle.

## V. CONCLUSION

If we realize the ratio between two consecutive gears, we obtain the jump from one gear to the other. This coefficient lets us evaluate the quality of the process of changing between gears for a specifically gearbox. The jump between gears is calculated for each gear change by dividing the values of the current gear to the values of the neighbor gear. For example for the automobile equipped with the gasoline engine the jump between the first and second gear is i2/i1=4,377/2,859.

The more the jump has lower values between the gears, the more the gear changes are made more easily and more comfortably. Also the dissipated energy of the process of the synchronization depends on the value of the jump between gears. If the jump between gears has high values, then synchromesh must eliminate a bigger difference of revolutions between shafts, and that results in more intense friction in the gear change process.

From the example presented above, we observe that the jump between gears drops as we approach higher gears, so that the process of changing the higher gears is more comfortable and easier.

Usually manufacturers opt for a compromise, respectively fast accelerations in the first gears and good fuel consumption in the higher gears.

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