Stabilizing time delay systems with prespecified gain and phase margins by lead-lag controllers

N. Ben Hassen, K. Saadaoui and M. Benrejeb

***Abstract* —In this paper, the problem of stabilizing a class of time delay systems by phase lead-lag controllers is investigated. One of the controller's parameter is first determined by a necessary condition based on Kharitonov's lemma. Then, the stabilizing regions in the space of the remaining parameters, for a fixed value of the first parameter, are determined using the D-decomposition method. By sweeping over the first parameter the complete set of stability regions in the space of the controller can be identified. Further, gain margin and phase margin specifications are added in the design and robust phase lead-lag controllers are sought. Illustrative examples are given to show the effectiveness of the proposed procedure.**

***Keywords*—** Second order controller; time delay; D-decomposition; stability; stabilization; phase margin; gain margin.

I. Introduction

Recently, the problem of determining all stabilizing fixed order, fixed structure, low order controllers for linear time invariant systems was addressed by several authors, see [1], [2] and [3] and the references therein. The problem is worthwhile as determining this set of all stabilizing controllers is a first and an essential step in calculating optimal fixed order controllers. This line of research was later extended to include time delay systems [4] and [5]. In fact, Studying stability of dynamical systems with time delay has received the attention of many researchers from the control community in the past decades; see [6] and the references therein. One of the main reasons for this continuing interest in this class of systems comes from the fact that many physical systems are inherently associated with time delays, this includes population dynamics, communication systems, nuclear reactors and power systems with loss-less transmission lines, see [6] and [7]. It is also known that delay is one of the main sources of poor performance and even instability [7] and [8]. This is another reason for the extensive literature on stability and stabilization of time delay processes. The above mentioned parameterization methods were successfully applied to get stabilizing classical low order controllers, such as PI controller [9], PID controller [10] and [11], and first order controllers [12] and [13]. In industrial processes, conventional controllers such as PI, PID, Phase-lead, phase-lead and phase-lead-lag controllers are widely used. Traditionally phase-lead, phase-lag and phase lead-lag controllers are tuned using trial and error methods. An analytic method for designing these controllers has been around for decades [6]. In this paper, we propose a method to calculate stabilizing lead-lag controllers for delay systems. It consists of determining the admissible values of one of the controller's parameters. Then, this parameter is fixed within the admissible range and the D-decomposition method is used to determine the stabilizing regions in the space of the remaining two parameters. By sweeping over the first parameter the complete set of stabilizing gains can be determined. This can be considered as a first step in the direction of designing optimal lead-lag controllers for delay systems. The obtained results are then extended to obtain robust lead-lag controllers by imposing gain and phase margin specifications ensuring stability margins, see [14] and [15], for the closed-loop system.

The paper is organized as follows. In section II, the set of all stabilizing lead-lag controllers for time delay systems is calculated. In section III, the problem of stabilizing with a pre-specified gain and phase margins is solved. Then an extended form for this controller to a PID controller is given. An illustrative example is given in section V. Finally, the last section gives some concluding remarks.

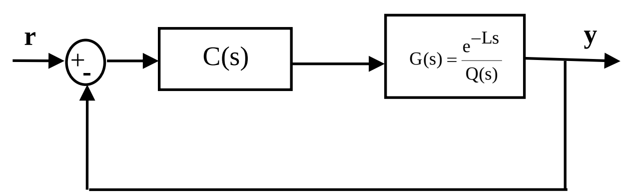

Fig. 1. Classical feedback system

II. Stabilizing second order controllers for time delay systems

In this section, the stabilizing regions in the parameter space of a second order controller are determined. We consider the classical feedback system of Fig. 1, where the system's transfer function is given by

$$G(s) = \frac{e^{-Ls}}{Q(s)} \quad (1)$$

with $L > 0$ the time delay. Many practical systems can be represented by (1).
In [6], the dynamic behavior of temperature control in a mix process is represented by (1) and in [7] it is used to model a ship positioning an underwater vehicle through a long cable, to name just few examples. Our objective is to determine the set of all second order controllers given by

$$C(s) = \frac{s^2 + \alpha_3 s + \alpha_1}{s^2 + \alpha_2 s + \alpha_1} \quad (2)$$

that stabilizes the feedback system of Fig 1.

In fact, the controller given by (2) is a lead-lag controller which combines the effects of phase lead and phase lag in certain frequency ranges and can realize the behavior of a PID controller [16]. Let

$$C(s) = \frac{(1+\tau_2 s)(1+\tau_3 s)}{(1+\tau_1 s)(1+\tau_4 s)} \quad (3)$$

with $\tau_1 > \tau_2 > \tau_3 > \tau_4$. In order to get the same gain for high frequencies and low frequencies, we impose $\tau_2\tau_3 = \tau_1\tau_4$ which leads to the following expression of C(s)

$$C(s) = \frac{\beta_1 s^2 + \beta_3 s + 1}{\beta_1 s^2 + \beta_2 s + 1} \quad (4)$$

or equivalently the controller given by (2). The closed loop characteristic equation is given by

$$\Delta^*(s) = (s^2 + \alpha_2 s + \alpha_1)Q(s) + (s^2 + \alpha_3 s + \alpha_1)e^{-Ls} \quad (5)$$

Our aim in this section is to determine the set of all stabilizing regions in the parameter space of the controller. As there are only three parameters $(\alpha_1, \alpha_2, \alpha_3)$, the problem will be solved in two steps. First, we calculate the admissible ranges for one of the controller's parameters. Next, this parameter is fixed within the admissible range and the stabilizing region in the space of the remaining two parameters, if it exists, is determined. By sweeping over the admissible values of the first parameter, the complete set of stabilizing controllers can be obtained.

A. *Admissible values of* (α_1, α_2)

Let us start by determining the admissible values in the parameter space of (α_1, α_2). In order to reduce the number of parameters $(\alpha_1, \alpha_2, \alpha_3)$ in the original stability problem from three into a simpler sub-problem with only two parameters, the following lemma will be used.

Lemma [17]. *Consider the quasi-polynomial*

$$\Delta(s) = \sum_{i=0}^{n} \sum_{l=1}^{r} h_{il} s^{n-1} e^{\tau_l s}$$

such that $\tau_1 < \tau_1 < \cdots \tau_r$, *with main term* $h_{0r} \neq 0$, *and* $\tau_1 + \tau_r > 0$. *If* $\Delta(s)$ *is stable, then* $\Delta^{'}(s)$ *is also a stable quasi-polynomial, where* $\Delta^{'}(s)$ *is the derivative of* $\Delta(s)$.

Now, the closed loop characteristic equation of the closed loop system of Fig. 1 is given by (5). Since the term e^{Ls} has no finite roots, the quasi-polynomial $\Delta^*(s)$ and $\Delta(s) = e^{Ls}\Delta^*(s)$ have the same roots, therefore stability of $\Delta(s)$ is equivalent to stability of $\Delta^*(s)$. In the sequel, the quasi-polynomial $\Delta(s)$ will be used to study stability of the closed-loop system of Fig 1, where $\Delta(s)$ is given by

$$\Delta(s, \alpha_1, \alpha_2, \alpha_3) = (s^2 + \alpha_2 s + \alpha_1)e^{Ls}Q(s) + (s^2 + \alpha_3 s + \alpha_1) \quad (6)$$

Using the condition of Lemma, if $\Delta(s)$ is stable then $\Delta^{'}(s)$ is also a stable quasi-polynomial, where $\Delta^{'}(s)$ is given by

$$\begin{aligned}\Delta'(s, \alpha_1, \alpha_2, \alpha_3) &= (2s + \alpha_2)P(s) + (s^2 + \alpha_2 s + \alpha_1)P'(s) \\ &\quad + 2s + \alpha_3\end{aligned} \quad (7)$$

where $P(s) = Q(s)e^{Ls}$. Repeating the same reasoning once again, If $\Delta^{'}(s)$ is stable then $\Delta^{''}(s)$ is also stable, where $\Delta^{''}(s)$ is given by

$$\begin{aligned}\Delta''(s, \alpha_1, \alpha_2) &= (s^2 P''(s) + 4sP'(s) + 2P(s) + 2) \\ &\quad + \alpha_1 P''(s) + \alpha_2 (sP''(s) + 2P'(s))\end{aligned} \quad (8)$$

At this step, note that the number of parameters is reduced and only two parameters appear in (8). It is possible now to apply the D-decomposition method [18], [19] and [20], and calculate the stabilizing regions in the parameter space of (α_1, α_2). To this end, we evaluate the characteristic polynomials on the imaginary axis by substituting s by jw and equating the real and imaginary parts of (8) to zero. Let

$$P(j\omega) = R(\omega) + jI(\omega)$$
$$P'(j\omega) = R'(\omega) + jI'(\omega)$$

and

$$P''(j\omega) = R''(\omega) + jI''(\omega)$$

then we get the following set of equations represented in matrix form

$$\begin{bmatrix} R'' & -\omega I'' + 2R' \\ I'' & \omega R'' + 2I' \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = -\begin{bmatrix} -\omega^2 R'' - 4\omega I' + 2R + 2 \\ -\omega^2 I'' + 4\omega R' + 2I \end{bmatrix} \quad (9)$$

Three cases will be considered:

Case1: Setting $\omega = 0$ corresponds to the case of a root crossing the imaginary axis through the real line. This leads to the following equation

$$\alpha_1 = -\frac{2P'(0)}{P''(0)}\alpha_2 - \frac{2(P(0)+1)}{P''(0)} \quad (10)$$

Case2: By sweeping over all $\omega > 0$, we consider the case of a pair of conjugate complex roots crossing the imaginary axis. Setting the real and imaginary parts of (9) to zero we get the following solution

$$\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix} = -\frac{1}{B_1}\begin{bmatrix}\omega R''+2I' & \omega I''-2R'\\ -I'' & R''\end{bmatrix}\begin{bmatrix}-\omega^2 R''-4\omega I'+2R+2\\ -\omega^2 I''+4\omega R'+2I\end{bmatrix} \quad (11)$$

where

$$B_1 = \omega R''^2(\omega) + \omega I''^2(\omega) + 2(I'(\omega)R''(\omega) - I''(\omega)R'(\omega)) \quad (12)$$

Case 3. When $\omega \to \infty$ corresponds to a root leaving the left-half plane (alternatively the right half-plane) at infinity. Since e^{Ls} does not have any finite roots, we consider the quasi-polynomial

$$\Delta(s) = \Delta^*(s)e^{Ls} = (s^2+\alpha_2 s+\alpha_1)Q(s)e^{Ls} + (s^2+\alpha_3 s+\alpha_1)$$

which has the principal term [21]. Clearly the quasi-polynomial $\Delta(s)$ possesses a root chain of retarded type that goes deep in the left-half plane and does not affect stability properties [22]. In the rest of the paper this case will not be considered as it does not affect the stability regions, only the first two cases will be studied.

The (α_1, α_2) plane can be partitioned using equations (10) and (11) into several regions and stability of (8) can be checked by choosing a point inside a region and applying classical methods for testing stability such as Nyquist criterion or Bode method.

B. Stability regions in (α_1, α_3) plane

Once the admissible values of (α_1, α_2) are determined by the procedure described in the previous sub-section, one parameter is fixed within the admissible range and we determine the stability regions in the space of the remaining two parameters. We choose to fix α_2 and calculate stability regions in (α_1, α_3) plane. Using (6) and replacing $Q(s)e^{Ls}$ by $P(s)$ we get

$$\Delta(s,\alpha_1,\alpha_2,\alpha_3) = (s^2+\alpha_2 s+\alpha_1)P(s) + (s^2+\alpha_3 s+\alpha_1) \quad (13)$$

substituting s by $j\omega$ and equating the real and imaginary parts of (13) to zero, we get

Case1: For $\omega = 0$

$$\alpha_1 = 0 \quad (14)$$

Case2 : For $\omega > 0$

$$\alpha_1 = \frac{\omega^2 R(\omega) + \omega^2 + \alpha_2 \omega I(\omega)}{R(\omega)+1} \quad (15)$$

$$\alpha_3 = \frac{(\omega^2 - \alpha_1)I(\omega) - \alpha_2 \omega R(\omega)}{\omega} \quad (16)$$

where $P(j\omega) = R(\omega) + jI(\omega)$. By the D-decomposition method, using (14), (15) and (16) for $\omega \geq 0$ the (α_1, α_3) plane can be partitioned into several regions and the stability region, if any, can be determined by employing classical methods. By sweeping over admissible values of α_2 the complete set of stabilizing lead lag controller for the linear time delay system given by (1) can be calculated.

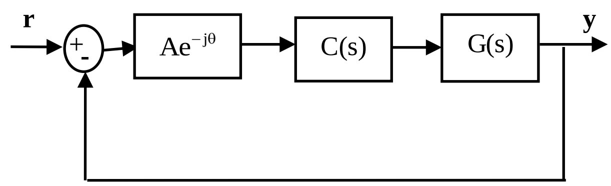

Fig. 2. Closed-loop feedback system with margin tester.

III. STABILIZING WITH PRESPECIFIED GAIN AND PHASE MARGIN

In this section, controllers are designed to meet gain and phase margin specifications. Indeed, to ensure the stability of the closed loop system, we can modify the procedure described in the previous section, by integrating in chain as shown in Fig.2, a block of action $Ae^{-j\theta}$ that introduces a gain margin A and phase margin θ [5], [23]. The characteristic function of the system in Fig. 2 is given by

$$\Delta^*(s) = (s^2+\alpha_2 s+\alpha_1)Q(s) + Ae^{-j\theta}(s^2+\alpha_3 s+\alpha_1)e^{-Ls} \quad (17)$$

multiplying this equation by $e^{(Ls+j\theta)}$ does not change the study of stability. Then the quasi-polynomial $\Delta(s)$ will be used to study stability of the closed-loop system of Fig.2 is given by

$$\Delta(s,\alpha_1,\alpha_2,\alpha_3) = (s^2+\alpha_2 s+\alpha_1)Q(s)e^{(Ls+j\theta)} + A(s^2+\alpha_3 s+\alpha_1) \quad (18)$$

the purpose of the proposed study is to determine the stabilizing values α_1, α_2 and α_3 giving the looped system a gain margin and phase margin imposed supplementary. As before, we will determine the set of the allowable values of (α_1, α_2) by the same principle, using the D-decomposition method described earlier. Thereafter for fixed values of α_2, we go in search of allowable values of couples (α_1, α_3), considering the gain margin and phase margin block.

The second derivative of the characteristic function of the system is given by

$$\Delta''(s,\alpha_1,\alpha_2) = \left[t_1(s)\alpha_1 + t_2(s)\alpha_2 + t_3(s)\right]e^{(j\theta+Ls)} + 2A \qquad (19)$$

with

$$t_1(s) = L^2Q(s) + 2LQ'(s) + Q''(s)$$
$$t_2(s) = 2LQ(s) + L^2Q(s) + 2Q'(s) + 2LsQ'(s) + sQ''(s)$$
$$t_3(s) = 2Q(s) + L^2s^2Q(s) + 4LsQ(s) + 4sQ'(s) + 2Ls^2Q'(s) + s^2Q''(s)$$

When A=1 then the phase margin of the system is θ, and when θ=0, then the gain margin of the system is A.
Let

$$Q(j\omega) = R(\omega) + jI(\omega)$$
$$Q'(j\omega) = R'(\omega) + jI'(\omega)$$

and

$$Q''(j\omega) = R''(\omega) + jI''(\omega)$$

A. Gain Margin equal to A

We start by determining the admissible values of (α_1, α_2). Once they are determined, we choose to fix α_2 and calculate stability regions in (α_1, α_3) plane. Two cases are considered:

Case1: For $\omega = 0$

$$\alpha_1 = 0 \qquad (20)$$

Case2 : For $\omega > 0$

$$\alpha_1 = -\frac{\begin{bmatrix} -\alpha_2(I(w)w\cos(Lw) + R(w)w\sin(Lw)) \\ -w^2\cos(Lw)R(w) + w^2\sin(Lw)I(w) - Aw^2 \end{bmatrix}}{-I(w)\sin(Lw) + R(w)\cos(Lw) + A} \qquad (21)$$

$$\alpha_3 = -\frac{1}{A*\omega}\begin{bmatrix} \alpha_1(I(w)\cos(Lw)+R(w)\sin(Lw))+ \\ \alpha_2(wR(w)\cos(Lw)-wI(w)\sin(Lw))- \\ w^2I(w)\cos(Lw)-w^2R(w)\sin(Lw) \end{bmatrix} \qquad (22)$$

B. Phase Margin equal to θ

To impose a margin phase θ, we just choose A=1. We proceed the same way as before. Two cases are considered:

Case1: For $\omega = 0$

$$\alpha_1 = 0 \qquad (23)$$

Case2 : For $\omega > 0$

$$\alpha_1 = -\frac{\begin{bmatrix} -\alpha_2(I(w)w\cos(Lw+\theta) + R(w)w \\ \sin(Lw+\theta)) - w^2R(w)\cos(Lw+\theta) \\ +w^2I(w)\sin(Lw+\theta) - w^2 \end{bmatrix}}{-I(w)\sin(Lw+\theta) + R(w)\cos(Lw+\theta) + 1} \qquad (24)$$

$$\alpha_3 = -\frac{1}{\omega}\begin{bmatrix} \alpha_1(I(w)\cos(Lw+\theta)+R(w)\sin(Lw+\theta))+ \\ \alpha_2(wR(w)\cos(Lw+\theta)-wI(w)\sin(Lw+\theta))- \\ w^2I(w)\cos(Lw+\theta)-w^2R(w)\sin(Lw+\theta) \end{bmatrix} \qquad (25)$$

The intersection of the two regions, gives us the desired controllers that achieve the gain and phase margin specifications.

IV. Design of a PID controller

In this section, the stabilizing regions in the parameter space of a PID controller are determined. The proposed approach in this paper can be slightly modified and applied to PID controllers, which are a special case of second order controllers.
Let

$$C(s) = \frac{k_d s^2 + k_p s + k_i}{s} \qquad (26)$$

applied to a plant transfer function

$$G(s) = \frac{e^{-Ls}}{Q(s)}$$

As in precedent section, our aim is to determine the set of all stabilizing regions in the parameter space of the controller. We first determine the stability regions of (k_p, k_d) by applying the D-decomposition method. Then for a fixed k_d, we determine the stabilizing regions in the plane of (k_p, k_i) as will shown in the next sub-section.

a. admissible values of (k_p, k_d)

We start by determining the admissible values of (k_p, k_d). Two cases are considered:

Case1: For $\omega = 0$

$$k_p = -(R(0) + j\,I(0)) \qquad (27)$$

Case2 : For $\omega > 0$

$$k_p = -[R(\omega)-L\omega I(\omega)-\omega\dot{I}(\omega)]\cos(L\omega) + [\omega LR(\omega)+\omega\dot{R}(\omega)]\sin(L\omega) \quad (28)$$

$$k_d = \frac{\sin(L\omega)}{2\omega}[R(\omega)-L\omega I(\omega)-\omega\dot{I}(\omega)] + \frac{\cos(L\omega)}{2\omega}[\omega LR(\omega)+\dot{R}(\omega)] \quad (29)$$

b. admissible values of (k_p, k_i)

Once they are determined, we choose to fix k_d and calculate stability regions in (k_p, k_i) plane.

Case1: For $\omega = 0$

$$k_i = 0 \quad (30)$$

Case2 : For $\omega > 0$

$$k_p = -R(\omega)\cos(L\omega)+I(\omega)\sin(L\omega) \quad (31)$$

$$k_i = \omega R(\omega)\sin(L\omega)+\omega I(\omega)\cos(L\omega) \quad (32)$$

V. Illustrative Example

Example 1. Consider stabilizing the third-order plant given by

$$G(s) = \frac{e^{-0.25s}}{s^3+2s^2+3s+5}$$

by a second order controller

$$C(s) = \frac{s^2+\alpha_3 s+\alpha_1}{s^2+\alpha_2 s+\alpha_1}$$

as described in the previous section, we start by calculating the admissible values of (α_1,α_2). After deriving twice the characteristic polynomial of the closed loop system, the sub-problem to be solved at this step is stabilizing the quasi-polynomial given by

$$\begin{aligned}\Delta''(s,\alpha_1,\alpha_2) &= (0.0625s^5+2.625s^4+24.1875s^3+28.8125s^2 \\ &+23s+10)e^{0.25s}+\alpha_1(0.0625s^3+1.625s^2+8.1875s \\ &+5.8125)e^{0.25s}+\alpha_2(0.0625s^4+2.125s^3+15.1875s^2 \\ &+15.3125s+8.5)e^{0.25s}+2\end{aligned}$$

Using (10) and (11), the stability region is found as shown in Fig. 3. Fixing a stabilizing value of controller parameter α_2 within the stability region, for instance $\alpha_2 = 2$, and applying (14), (15) and (16) we get the stability region in the (α_1,α_3) plane as shown in Fig. 4. By sweeping over admissible values of α_2 the complete stabilizing regions in the parameter space $(\alpha_1,\alpha_2,\alpha_3)$ of the controller can be determined. In Fig. 5, a 3D plot of the stabilizing regions is given for values of α_2 between 2 and 10. The figure shows that for smaller values of α_2, the stability region is getting smaller and eventually important while the parameter gets even bigger. The gain and phase margin are given by A=3 and θ=45°, respectively. Fixing $\alpha_2 = 2$, we obtain the stability region with specified phase and gain margins as shown in Fig. 6. The shaded regions shows the intersection between stability region with a gain margin equals 3 and stability region with a phase margin equals 45°.

Example 2. Consider stabilizing the third-order plant given by

$$G(s) = \frac{e^{-0.25s}}{s^3+2s^2+3s+5}$$

by a PID controller

$$C(s) = \frac{k_d s^2+k_p s+k_i}{s}$$

we start by calculating the admissible values of (k_p, k_d). After deriving twice the characteristic polynomial of the closed loop system, the sub-problem to be solved at this step is stabilizing the quasi-polynomial given by

$$\begin{aligned}\Delta''(s,k_p,k_d) &= (0.0625s^4+2.125s^3+12.1875s^2+11.3125s \\ &+6.5)e^{0.25s}+2k_d\end{aligned}$$

The solution of this problem can be achieved by using the D-decomposition method. Applying (27), (28) and (29), Fig. 7 shows the stability region in the (k_p, k_d) plane. Now , let us fix $k_d = -10$ within the admissible set and go back to (30), (31) and (32), we get the stabilizing region in (k_p, k_i) plane as shown in Fig. 8.

VI. Conclusion

In this paper, the D-decomposition method is used to compute the stability regions of a second order controller applied to an n-th order all poles linear time delay system. The second order controller used to stabilize the feedback system is a lead lag controller. The proposed method is based on determining first the set of one of the controller's parameter, α_2 in our case, and then determining stability regions in the parameter space of the remaining two parameters. These results are extended to include gain and phase margin specifications. The proposed lead-lag controller, can be extended to a PID controller for some frequence range. First, The parameter k_d is fixed a priori within an admissible range determined using the D-decomposition method.

Then the stabilizing region in the plane of (k_p, k_i) of a PID controller is determined.

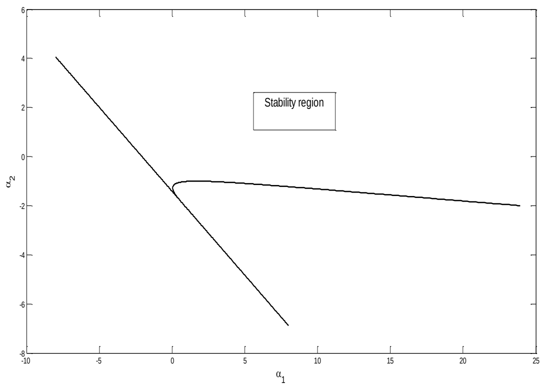

Fig. 3. Stabilizing region in the (α_1, α_2) plane

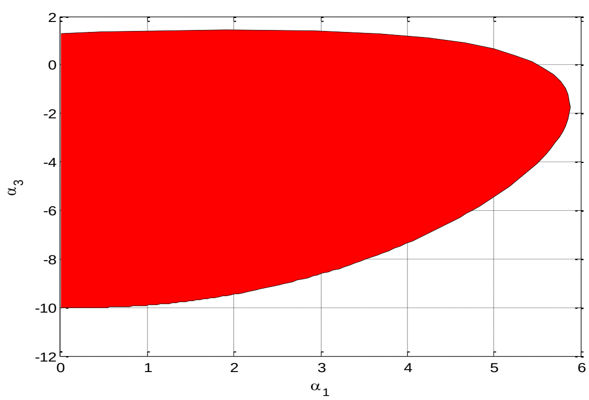

Fig. 4. Stabilizing region in the (α_1, α_3) plane for $\alpha_2 = 2$

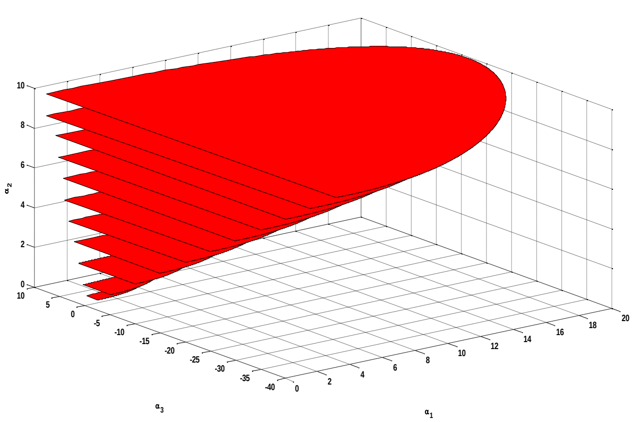

Fig. 5. Stabilizing region in the $(\alpha_1, \alpha_2, \alpha_3)$ plane for $\alpha_2 \in \lfloor 2\ 10 \rfloor$

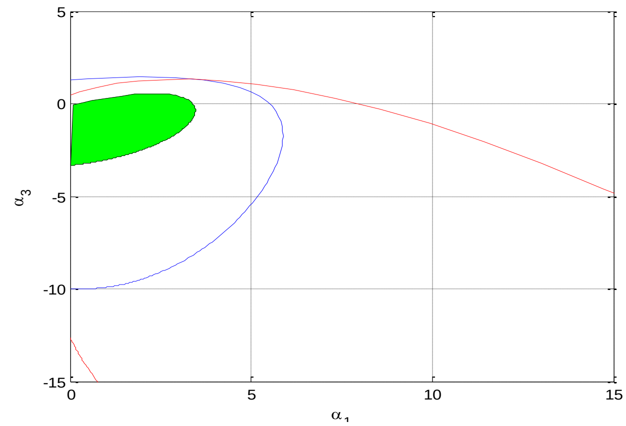

Fig. 6. Stabilizing regions with gain margin A=3 and phase margin θ =45°

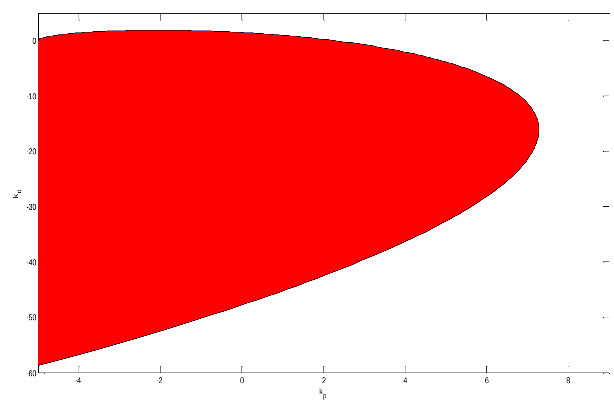

Fig. 7. Stabilizing region in the (k_p, k_d) plane

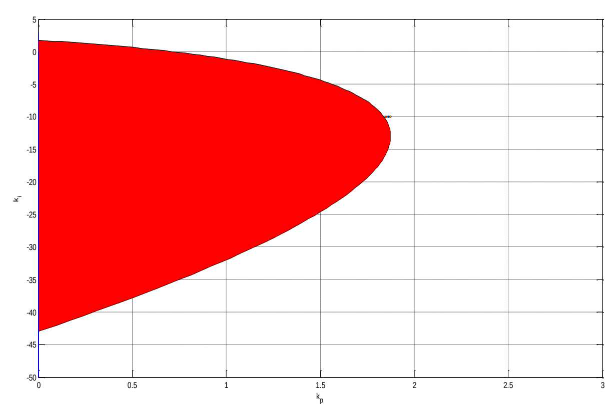

Fig. 8. Stabilizing region in the (k_p, k_i) plane for $k_d = -10$

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