Modal Parameter Identification Using Particle Swarm Optimization

M. El-Kafafy, A. Elsawaf, B. Peeters, T. Vampola, P. Guillaume

Abstract**—**In this paper, the applicability of Particle Swarm Optimization (PSO) to identify the modal parameters will be tested. PSO is a heuristic optimization method which does not require the calculation of the error derivatives with respect to the model parameters hence the Jacobian matrix formulation is not required. The modal parameters will estimated by optimizing the modal model using PSO in order to decrease the error between the modal model and the measured frequency response functions (FRFs) of the structure under test. The applicability of PSO to optimize the modal model is evaluated by means of real-life measurement example.

Keywords**—**Complex structures, Frequency response functions, Modal parameter identification, Particle swarm optimization.

I. INTRODUCTION

MODAL analysis is currently one of the key technologies used for analysing the dynamic behaviour of complex structures such as cars, trucks, aircrafts, bridges, offshore platforms and industrial machinery A number of textbooks give a good overview of the theory and practice in the domain of modal analysis [1-4]. Modal analysis is a process whereby a structure is described in terms of its natural characteristics, which are the resonance frequencies, damping ratios and mode shapes - its dynamic properties. These modal parameters are the basic characteristics of any vibration (resonance) mode. A small force exciting the structure at one of these resonance frequencies causes large vibration responses resulting in possible structural damage. The damping ratios of the different vibration modes control the vibration level at the corresponding resonance frequency. The mode shape that is not global but local property of the structure describes how the structure will vibrate when it is excited at a certain resonance frequency. For the mode shape, local property means that it depends on the number of measured degrees of freedom (DOFs) and the locations of these DOFs with respect to the structure under test. Indeed, we can also say it is global property in the sense that it depends on the mass and stiffness distribution of the structure under test. Indeed, getting accurate estimates for these parameters helps to better understanding, modelling and controlling the dynamics of the vibratory structures.

In this contribution, based on successfully experiences obtained with the application of Particle Swarm Optimization (PSO) in other areas [5-8], the authors decided to investigate the applicability of PSO technique in the field of modal parameter identification. PSO is a heuristic optimization method which is different from the optimization algorithms that are being used in the modal parameter estimation community. PSO was originally contributed by Kennedy and Eberhart [9] and was first introduced for simulating social behaviour as a stylized representation of the movement of organisms in a bird flock or fish school. PSO algorithm is a population based algorithm that makes few or no assumptions about the optimized problem and can search very large spaces of candidate solutions.

The validation of applying PSO to modal parameter estimation in this paper will be done using a real-life measurement from aerospace application. The outline of the paper is as follows: in section II, a review over the modal parameter estimation techniques will be given. Then, the problem statement will be described in section III. In section IV, a theoretical background about PSO will be given. Some validation results will be shown in section V. Then, some concluding remarks are given in section VI.

II. MODAL PARAMETER ESTIMATION: A REVIEW

Over the last decades, a number of algorithms have been developed to estimate modal parameters from measured frequency or impulse response function data. The algorithms have evolved from very simple single degree of freedom (SDOF) techniques to algorithms that analyse data from multiple-input excitation and multiple-output responses simultaneously in a multiple degree of freedom (MDOF) approaches. In the time domain modal parameter identification techniques, the Complex Exponential (CE) algorithm is one of the earliest modal analysis methods. The CE was improved using least square solution by Brown et al. [10], and it was called Least Square Complex Exponential (LSCE) method. The LSCE method was developed for MIMO systems as the Polyreference Least Square Complex Exponential (pLSCE) by Vold et al.[11]. Even though the pLSCE method uses FRFs as an input, it essentially operates in the time domain. This is

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achieved by computing the impulse response functions (IRFs) from the FRFs by inverse Fourier transformation. In the frequency-domain modal parameter identification side, a very popular implementation of the frequency-domain linear least squares estimator optimized for the modal parameter estimation is called Least Squares Complex Frequency-domain (LSCF) estimator [12]. That method was first introduced to find initial values for the iterative maximum likelihood method [13]. The LSCF estimator uses a discrete-time common denominator transfer function parameterization. In [14], the LSCF estimator is extended to a poly-reference case (pLSCF). The pLSCF estimator uses a right matrix fraction description (RMFD) model. Both of those estimators have been developed for handling modal data sets that are typically characterized by a large number of response DOFs, high modal density and a high dynamic range. LSCF and pLSCF estimators were optimized both for the memory requirements and for the computation speed. The main advantages of those estimators are their speed and the very clear stabilization charts they yield even in the case of highly noise-contaminated frequency response functions (FRFs).

LSCF and pLSCF estimators are curve fitting algorithms in which the estimation process is achieved without using information on the statistical distribution of the data. By taking knowledge about the noise on the measured data into account, the modal parameters can be derived using the so-called frequency-domain maximum likelihood estimator (MLE) with significant higher accuracy compared to the ones developed in the deterministic framework. MLE for linear time invariant systems was introduced in [15] and it is extended to multivariable systems in [16]. A multivariable frequency-domain maximum likelihood estimator was proposed in [13] to identify the modal parameters together with their confidence intervals where it was used to improve the estimates that are initially estimated by LSCF estimator. In [17], the poly-reference implementation for MLE was introduced to improve the starting values provided by pLSCF estimator. Both of the ML estimators introduced in [13, 17] are based on a rational fraction polynomial model, in which the coefficients are identified. The modal parameters are then estimated from the coefficients in a second step. In these estimators, the uncertainties on the modal parameters are calculated from the uncertainties on the estimated polynomial coefficients by using some linearization formulas. These linearization formulas are straightforward when the relation between the modal parameter and the estimated coefficients is explicitly known but can be quite involved for the implicit case. Moreover, they may fail when the signal-to-noise ratio is not sufficiently large [18].

A combined deterministic–stochastic modal parameter estimation approach called Polymax Plus has been introduced and successfully validated in [19-24]. This estimator combines the best features of both the pLSCF estimator [14] of having a clear stabilization chart in a fast way and the MLE –based estimation [13] of having consistent estimates of the modal parameters together with their confidence bounds. A recent maximum likelihood modal parameter identification method (ML-MM), which identify directly the modal model instead of rational fraction polynomial model, is introduced and validated with simulated datasets and several real industrial applications in [25-31]. Basically, the design requirements to be met in the ML-MM estimator were to have accurate estimate for both of the modal parameters and their confidence limits without using the linearization formulas which have to be used in case of identifying a rational fraction polynomial models. And, meanwhile, to have a clear stabilization chart which enables the user to easily select the physical modes within the selected frequency band. Another advantage of the ML-MM estimator lies in its potential to overcome the difficulties that the classical modal parameter estimation methods face when fitting an FRF matrix that consists of many (i.e. 4 or more) columns, i.e. in cases where many input excitation locations have to be used in the modal testing. For instance, the high damping level in acoustic modal analysis requires many excitation locations to get sufficient excitation of the modes.

All the previously mentioned modal parameter identification methods are based on fitting a mathematical model (e.g. polynomial-based models or modal model) to the measured data (i.e. FRFs). This fitting can be done either in a linear-least squares sense (e.g. pLSCF) or in a non-linear least squares sense (e.g. MLE or ML-MM). In case of non-linear least squares-based estimators [13, 28, 32, 33], a non-linear optimization algorithm is required since the cost function to be minimized is nonlinear in the parameters of the model. In system identification community Levenberg- Marquardt method [33, 34], which combines Gauss-Newton and Gradient descent methods, is commonly used to minimize the cost function of these estimators.

III. Problem Statement

The modal model is considered as one of the important objectives of any modal estimation process, by which we are characterizing the system dynamics in terms of its modal parameters (i.e. poles, mode shapes and participation factors). This model proposes that the frequency response function matrix of the system can be formulated in its modal form as follows [3]:

$$\left[H\left(\Omega_k\right)\right]_{N_o\times N_i} = \sum_{r=1}^{N_m}\left(\frac{\psi_r L_r^T}{\Omega_k-\lambda_r}+\frac{\psi_r^* L_r^H}{\Omega_k-\lambda_r^*}\right) + \frac{LR}{\Omega_k^{\ 2}}+UR \tag{1}$$

with $\mathrm{H}\left(\Omega_k\right)\in\mathbb{C}^{N_o\times N_i}$ the frequency response function matrix with N_o the number of the measured outputs and N_i the number of the measured inputs, $\Omega_k = j\omega_k$ the polynomial basis function in case of using continuous-time formulation (s-domain), ω_k the circular frequency in rad/sec., $\psi_r \in \mathbb{C}^{N_o \times 1}$, $L_r^T \in \mathbb{C}^{1 \times N_i}$, λ_r the mode shape, the participation factor and the pole corresponding to the rth mode. $LR \in \mathbb{C}^{N_o \times N_i}$ and $UR \in \mathbb{C}^{N_o \times N_i}$ are the lower and upper residual terms. Since the modal model formulation uses a limited number of modes to model the FRF matrix within the analysis frequency band, the lower and upper residual terms are used to compensate for the residual effects that come from the out-of-band modes. Equation 1 considers a displacement FRF, while often the acceleration FRF is measured in modal analysis tests or sometime velocity FRFs like in case of using the Laser Doppler Vibrometer. In such cases, equation 1 should be corrected as follows:

$$\begin{aligned} H_{Vel}(\Omega_k) &= \Omega_k H_{Dis}(\Omega_k) \\ H_{Accel}(\Omega_k) &= \Omega_k{}^2 H_{Dis}(\Omega_k) \end{aligned} \tag{2}$$

with H_{Dis}, H_{Vel} and H_{Accel} the displacement, velocity and acceleration FRFs. In case of operational modal analysis (OMA), the upper and lower residual terms (operational residuals) are different from the residuals in case of experimental modal analysis (EMA) used in equation 1. They were determined by verifying the asymptotic behaviour of the output spectra of a single-degree of freedom system excited by a white noise in [35]. Once the modal model is derived for a certain structure, a number of applications of modal analysis can be instigated using this modal model. In the following, some of the applications in which the modal model can be used

- Correlation of FEM
- Damage detection
- Structural modification
- Sensitivity analysis
- Forced response prediction
- Substructure coupling
- Active and semi-active control

It is obvious that all the above-mentioned applications heavily depend on the extracted modal parameters. In other word, a successful applicability of all those applications depends on the quality of the modal model. The quality of the estimated modal model mainly depends on the quality of the measured data and on the asymptotic properties of the modal parameter estimator used to extract the modal model parameters from the measured data. As mentioned in section II, in the literature there are several modal parameter estimators are successfully introduced and validated to obtain accurate modal model. In this contribution, the objective now is to try PSO in optimizing the modal model presented by equation 1 with the aim to have accurate modal parameters (i.e. $\psi_r, L_r^T, \lambda_r, LR,\ UR$) for the structure under test within the analysis frequency band. Therefore, to optimize the modal model in equation 1 the following cost function (equation 3) will be minimized using PSO:

$$\ell_{PSO}(\omega_k, \theta) = \sum_{l=1}^{N_o N_i} \sum_{k=1}^{N_f} \left| E_l(\omega_k, \theta) \right|^2 \tag{3}$$

with N_f the number of the frequency lines and $E_l(\omega_k, \theta) = \hat{H}_l(\Omega_k, \theta) - H_l(\omega_k)$ is the error between the modal model $\hat{H}_l(\Omega_k, \theta)$ represented by equation 1 and the measured FRFs $H_l(\omega_k)$ at frequency line k. So, the cost function is simply the sum of the squares of the absolute value of the error over all the inputs, outputs and frequency lines. So, the modal parameters of the modal model in equation 1 will be tuned by the PSO algorithm in the way which minimizes the cost function described above by equation 3. To reduce the computational time taken by PSO, the poles λ_r and the participation factors L_r are taken as the parameters to be optimized by PSO, and then the mode shapes ψ_r and the residual terms ($LR\ \&UR$) are calculated in a linear least squares sense as implicit functions of the optimized poles and participation factors using equation 1.

IV. Particle Swarm Optimization

Recently, particle swarm optimization (PSO) has attracted a lot of attention because it's easy to implement, robust, fast convergence, and for its ability to solve many optimization problems. PSO algorithm optimizes a problem using a population (swarm) of candidate solutions (particles). Particles have their own positions, and fly around in the problem solution space looking for best fitness value. Those particles are initially scattered in the solution space with initial positions. The position $\alpha_\beta^{(\tau)}$ and velocity $v_\beta^{(\tau)}$ of a particle β at the generation τ, are iteratively enhanced in the solution space towards the optimum solution. Each movement of a particle is influenced by its local best position $b_\beta^{(\tau)}$ and the global best position $g^{(\tau)}$ obtained from all candidates in the solution space. When the process repeated for sufficient number, the best solutions eventually will be found. Equation (4), shows the mathematical formula used for updating the positions and the velocities of the particles [36];

$$\begin{aligned} \alpha_\beta^{(\tau+1)} &= \alpha_\beta^{(\tau)} + v_\beta^{(\tau+1)} \\ v_\beta^{(\tau+1)} &= \chi \times \left(\begin{aligned} & v_\beta^{(\tau)} + acc_1 \times rand_1 \times \left[b_\beta^{(\tau)} - \alpha_\beta^{(\tau)} \right] \\ & + acc_2 \times rand_2 \times \left[g^{(\tau)} - \alpha_\beta^{(\tau)} \right] \end{aligned} \right) \end{aligned} \tag{4}$$

χ is the constriction coefficient, acc_1 and acc_2 are acceleration coefficients, $rand_1$ and $rand_2$ are random numbers between 0 and 1. The values used in this study for χ, acc_1 and acc_2 are 0.729, 2.05 and 2.05 respectively [36]. Figure 1 shows a flow chart for the optimization procedures.

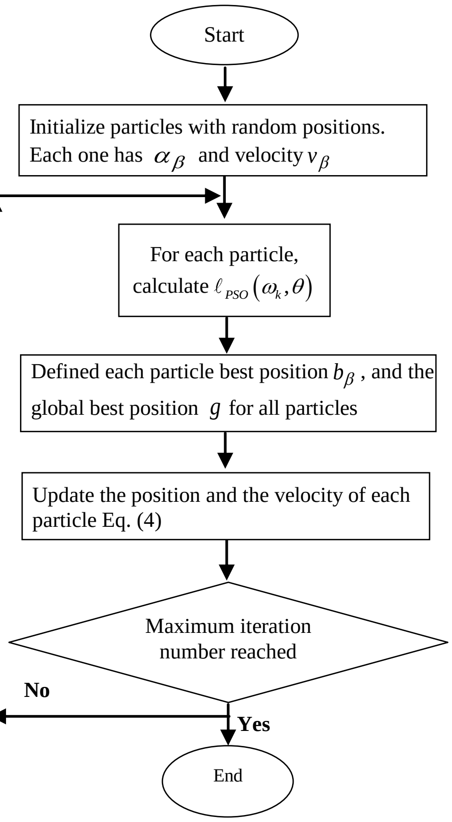

Figure 1: Optimization procedures flow chart

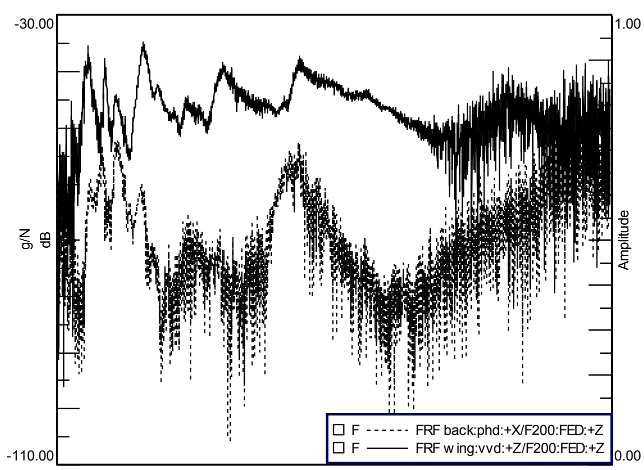

Figure 2: Typical FRFs for the tested business jet (the frequency axis is hidden for confidentiality)

V. Validation Results

A. Inflight Dataset Example

In this section, the proposed PSO for modal parameter estimation will be validated using experimentally measured FRFs that were measured during a business jet in-flight testing. These types of FRFs are known to be highly contaminated by noise. During this test, both the wing tips of the aircraft are excited during the flight with a sine sweep excitation through the frequency range of interest by using rotating fans. The forces are measured by strain gauges. Next to these measurable forces, turbulences are also exciting the plane resulting in rather noisy FRFs. Figure 2 shows some measured frequency response functions (FRFs), which clearly show the noisy character of the data. During the flight, the accelerations were measured at nine locations while both the wing tips were excited (two inputs).

The PSO needs an initial guess about the number of the modes within the analysis band. The pLSCF estimator is applied to the measured FRFs to have a clue about how many modes are expected in the desired band. It was found that there are 13 physical vibration modes within the analysis band. The modal model (1) is then optimized by minimizing the cost function (3) using the PSO technique. To start the PSO, lower and upper bounds have to be defined for all the parameters that have to be optimized (i.e. the poles and participation factors). The pole λ_r consists of real and imaginary parts. For each mode, the undamped resonance frequency is $\omega_{n_r} = |\lambda_r| / 2\pi$ and the damping ratio is $\xi_r = -Re(\lambda_r) / |\lambda_r|$. Assuming lower and upper bounds for the poles can be made easier if the pole for each mode is written as a function of the resonance frequency and damping ratio. The pole can be written as a function of ω_{n_r} and ξ_r as $\lambda_r = -\xi_r \omega_{n_r} + j\omega_{n_r}\sqrt{1-\xi_r^{\,2}}$.

So, instead of optimizing the poles, the frequencies ω_{n_r} and damping ratios ξ_r will be optimized. The resonance frequency for all the modes is bounded by the minimum and maximum frequency of the analysis band. The damping ratio is allowed to vary between 0 and 10% since the stable systems have to have a positive damping and 10 % is a logical value for the damping ratio for the mechanical structures. The real and imaginary parts of participation factors are allowed to vary between -1 and 1.

It should be said that modal parameters estimation using PSO in such complex cases will depend on the user's experience and, in most of the cases, the procedure must be repeated a few times modifying the lower and upper bounds for the optimized parameters until the results converge to optimum values.

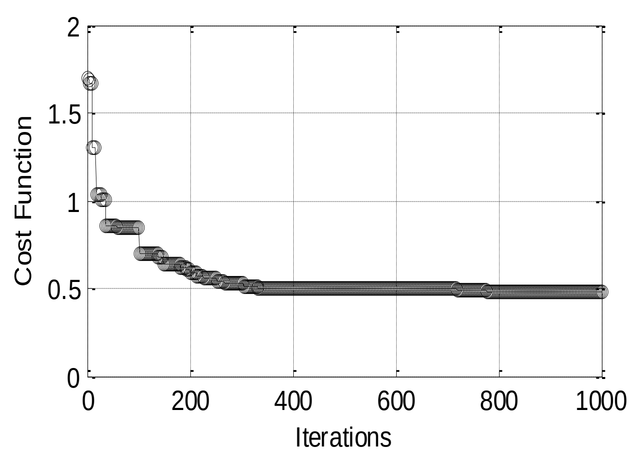

Figure 3: Decreasing of the error at different iteration

Figure 3 shows the decreasing of the cost function as a function of the number of iterations. In this case study, there are 9 outputs and 2 inputs and 13 modes. For this data set, the number of parameters to be optimized by PSO is 78 parameters: the frequency and the damping ratio for each mode ($2N_m = 2\times 13$) plus the real and the complex parts of the participation factors for each mode ($2N_m N_i = 2\times 13\times 2$). The Maximum number of iterations taken was 1000 iterations, and the PSO takes about 5 minutes to achieve those iterations.

To check the accuracy of the estimated (optimized) modal model, a simple but very popular way to validate the model is to compare the obtained model to the measurements. Figure 4 shows the quality of the fit between the measured and synthesized FRFs calculated based on the obtained modal model.

It can be seen from this figure that the PSO is able to converge to a modal model that closely fits the measured data, which indicates that the model represents well the dynamic of the system under test in the analysis band. In Figure 5, the auto Modal Assurance Criterion (auto MAC) of the estimated mode shapes is shown. It shows that the identified modes are not correlated except for some higher frequency modes. This correlation of the higher frequency modes is due to the spatial aliasing since the number of the measured outputs is only 9. Figure 6 shows some of the identified mode shapes.

VI. Conclusion

The estimation of modal parameters using Particle Swarm Optimization (PSO) was tried and its application to real measured data showed that it can be used with a dependency on the user's experience and the quality of the defined lower and upper bounds of the optimized parameters. In some cases, the procedure has to be repeated few times modifying the defined lower and upper bounds of the parameters to reach an optimum solution. The PSO does not require the calculation of error derivatives and Jacobian matrix which might be taken as an advantage for the method. On the other hand, the quality of the solution and the calculation time of the PSO in such application have been found to be highly dependent on the quality of the defined bounds for the parameters. For the future work, the PSO will be investigated for the modal parameter estimation using more industrial applications.

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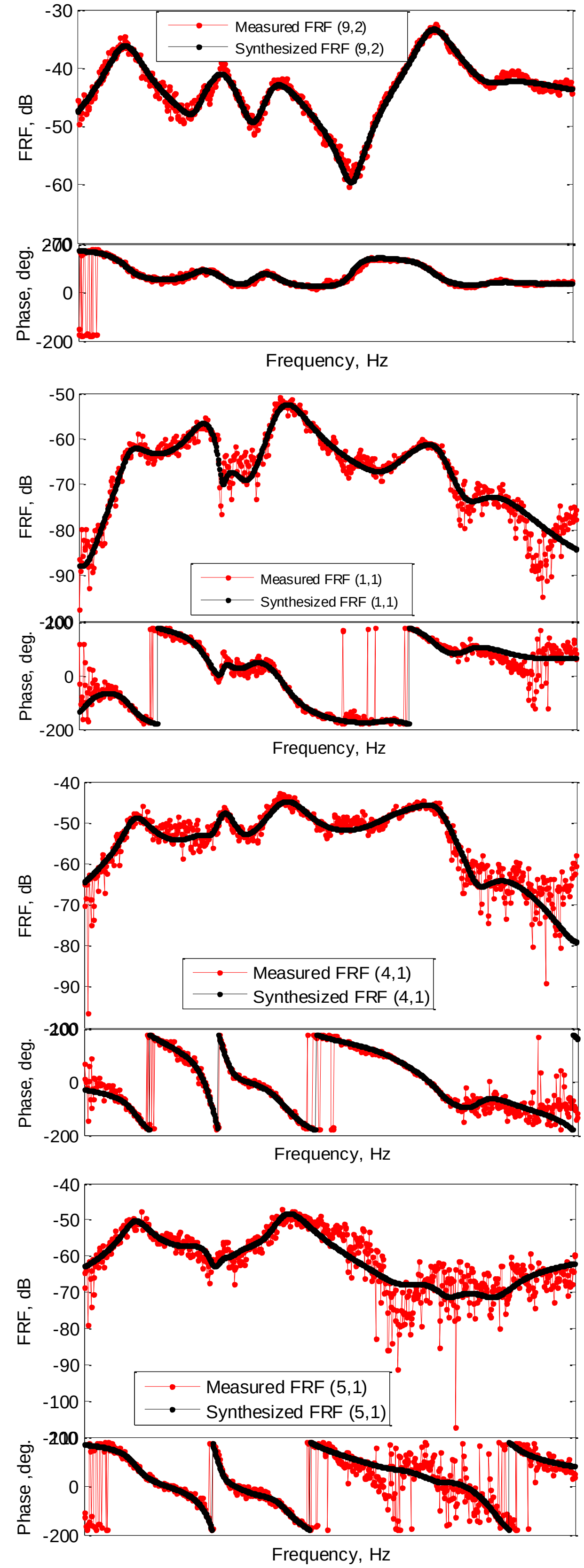

Figure 4: Some typical synthesized FRF compared with the measured ones

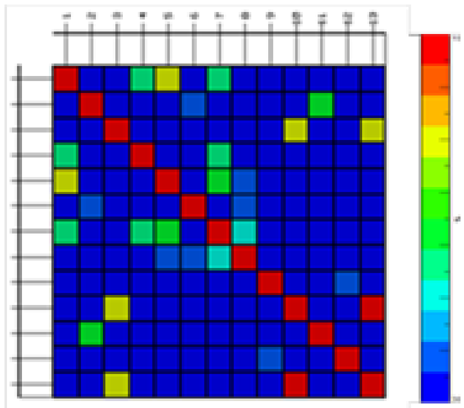

Figure 5: Auto modal assurance criterion (MAC) of the identified mode shapes

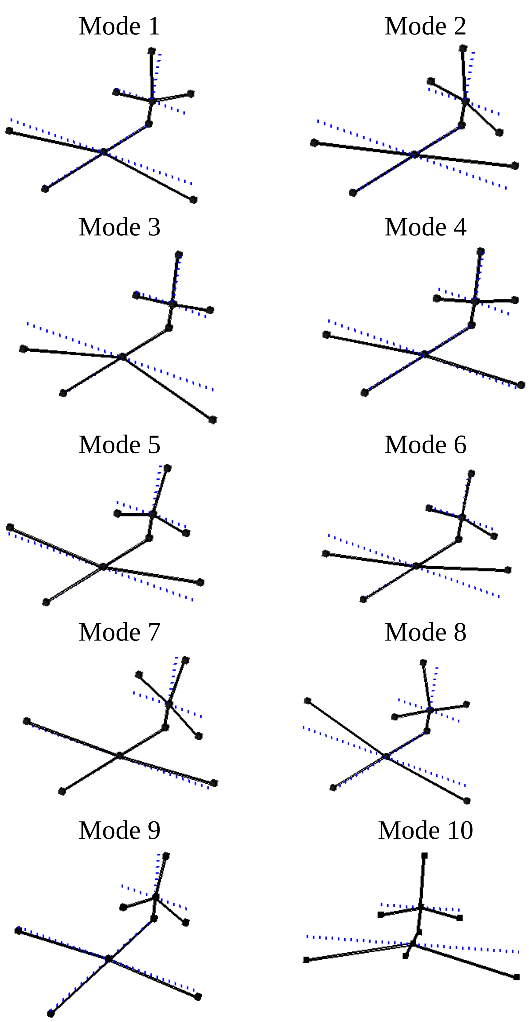

Figure 6: Graphical representation of some typical estimated mode shapes

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