# An Efficient Large Eddy Simulation (LES) Methodology for Jet Aeroacoustics – Recent Developments

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Abstract—Jet noise is an important issue due to the concerns of people living or working in the vicinity of airports. Increased commercial air traffic, penalty fees for noisier aircraft, stringent noise regulations, and military operational requirements are issues that need to be addressed. Processing speeds and memory limitations of existing supercomputers limit the faithfulness of these simulations. Thus the simulations are not accurate enough to allow design and testing of noise reduction strategies. In order to simulate realistic situations very fine grids (e.g. on the order of tens of billions of points) are sometimes needed, requiring significant computational resources. Thus very efficient algorithms are needed. An efficient, petascalable code has been developed based on the large eddy simulation (LES) technique. It is a multi-block structured solver capable of using cylindrical grids and simulating both subsonic jets and supersonic jets with shock waves. Recent advancements have targeted improved prediction accuracy by enabling inclusion of nozzle geometries in simulations. A digital filter-based approximate turbulent inflow boundary condition is used and can be coupled with several noslip wall boundary conditions for the nozzle walls. A wall model is employed in the nozzle walls to save computational time. Finally, a ghost-point-based immersed boundary method is implemented to allow simulation of complex nozzle shapes that show promise of noise reduction, e. g. chevrons, lobed mixers, beveling, and corrugations. We will show validation efforts highlighting recent efforts with beveled nozzles and summarize future research directions.

*Keywords*—computational fluid dynamics, aeroacoustics, air-craft noise.

### I. INTRODUCTION

Aviation is playing a significant role in supporting global economic growth. Aircraft noise, a byproduct of aviation, however, is proving to have an adverse impact on the overall benefit of aviation, ranging from hearing damage to financial penalties imposed on its originators and costs of noise mitigation measures. In recent years, designing quieter aircraft has become a new arena of competition for major aircraft and engine manufacturers. Also, a low-noise in-flight experience is gaining emphasis in advertising campaigns targeting airline

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A. Lyrintzis is with the Department of Aerospace Engineering at Embry-Riddle Aeronautical University, Daytona Beach, FL 32114, USA. customers. In military operations, injury-incurring sound levels near vertical and/or short takeoff and landing (VSTOL) aircraft also corroborates the necessity of noise reduction in aircraft design.

High costs are associated with the trial-and-error based experimental examination of novel low-noise design ideas. Computational prediction tools can complement the experimental research to reduce this cost. Computational aeroacoustic (CAA) simulation of sound levels generated by aircraft airframes and engines. CAA, is a relatively new discipline of computational fluid dynamics (CFD), intended as a robust and accurate technique that complements traditional theoretical and empirical approaches in aircraft noise prediction. Different from practices in general CFD, CAA strongly relies on accurate prediction of small-amplitude acoustic fluctuations and their correct propagation to the far field. This demands that the underlying numerical methods provide high accuracy and good spectral resolution, while keeping diffusion and dispersion errors low.

The state-of-the-art CAA prediction of far-field noise is based on time-dependent CFD simulation of turbulent flows followed by integral postprocessing [1] that propagates the noise to the far field. This two-step approach first appeared in the 1980s and has continued to mature, partly owing to the emergence of large-scale computing platforms, which have turned computationally intensive techniques including direct numerical simulations (DNS) and large eddy simulation (LES) into reality. For reference, DNS seeks to resolve all of the relevant turbulent length and time scales, thereby being the most expensive, but also the most realistic. LES compromises by spatially filtering the governing equations to generate solutions that resolve directly the large turbulent structures (i.e. the most energetic ones), modeling the impact of the small ones. It is less costly than DNS, but requires a suitable choice for the subgrid scale (SGS) model and still consumes relatively large computer resources. Reference [2] is a recent review of the application of LES to the prediction of jet noise. We summarize some recent high-fidelity LES jet noise studies below.

For subsonic jets, references [3]–[5] utilize a multi-block solver with overset grid blocks to simulate nozzles with and without chevrons on grids up to 500 million points. They have also performed studies of inflow conditions upstream of the nozzle. In general, their results predict the acoustic high frequencies well but underpredict the lower ones. The specific cause of the underprediction is unknown. References [6], [7]

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have studied the effect of important parameters on the noise. Using high-order methods on meshes with 252 million points, they have found that as the Reynolds number and boundary layer turbulence intensities increase, the jet potential core is lengthened, peak turbulence intensities are reduced, and far-field sound pressure levels decrease. This indicates that it is crucial to predict accurate turbulent quantities at the nozzle exit and perform simulations at realistic Reynolds numbers.

There are comparatively fewer high-fidelity supersonic jet simulations. Many high-quality simulations have been completed without the nozzles in the LES, however. See, for example, Lo et al. [8] who use characteristic filters to extend a high-order structured solver for supersonic jets with shock waves. Another example is Shur et al. [9], [9], [10], who have studied many subsonic and supersonic noise-reduction concepts even though they do not actually include the nozzle in the LES. They use a coupled Reynolds averaged Navier– Stokes equations (RANS)/LES approach where a RANS nozzle simulation is used for the inflow conditions to the LES, thereby removing the cost of including the nozzle in the LES computation. They have examined chevrons [9] and microjets [10], and both static and forward flight conditions [11].

While structured solvers for CAA with LES have been dominant, there is also a push towards unstructured solvers, which allow for more flexibility in meshing and modeling complex geometries, but also reduce the order of accuracy. Utilizing this approach, Mendez et al. [12] include round converging-diverging nozzles to analyze heated and unheated jets at near-perfect expansion. Other applications include large simulations of hot overexpanded round jets issuing from realistic converging-diverging nozzles with chevrons [13] and underexpanded isothermal rectangular jets from nozzles with and without chevrons [14]. These simulations utilize hundreds of millions of grid points and have been completed using as many as 163,840 processors on the Intrepid cluster at the Argonne National Laboratory. More recently, they have tested their  $CharLES^{x}$  code on 1.57 million cores [15] using the IBM Blue Gene/Q Sequoia cluster at Lawrence Livermore National Laboratory. Additional details on their numerical methods can be found in [16].

The above discussion highlights some of the various LES techniques in use for jet noise studies. A survey of the literature finds that fine grids, high-order numerical methods, and accurate turbulent nozzle boundary layers are required for accurate quantitative sound level predictions. That said, there is much room for improvement and many challenges remain. Examples include simulation of flows at realistically high Reynolds numbers and inclusion of realistic nozzle geometries in high-order structured solvers.

We have recently developed an efficient highly-parallel LES application [17], [18] which aims to address these concerns and more accurately predict noise levels at the far field. Our LES application is designed to feature scalability across a wide range of processor core counts and has successfully carried out performance experiments utilizing between 2,744 and 91,125 cores. It is a multi-block structured solver which has capabilities for simulating jets using cylindrical grids and has shock-capturing routines for supersonic jets with shock waves

[8]. Fulfilling the requirements for accurate CAA simulations, it uses high-order numerical methods including compact finite difference schemes for spatial derivatives. Furthermore, it is capable of including walls in simulations and has an approximate turbulent inflow boundary condition for producing realistic conditions inside of jet nozzles.

Previously, the LES application capabilities were limited to wall-resolved simulations including simple nozzle geometries [18]. Such simulations, however, are often prohibitively expensive. Recently, we have implemented a more efficient approximate wall model boundary condition and validated it for a turbulent flat plate boundary layer [19]. The wall model was validated for jet flows [20] and extended to compressible flows; an analysis of the noise reduction concept of beveling the nozzle exit plane was presented [21], using a body-conforming mesh. Currently, the solver is being further developed to include a sharp immersed boundary method (IBM) [22]. An immersed boundary method enables the use of grids that do not conform to the nozzle shape under study. This facilitates easier and efficient grid generation, and use of identical grid resolution for different nozzles, thus providing a fairer comparison between their computed noise signatures. The proposed method has been implemented and validated for wall-resolved simulations [23], and efforts are ongoing to extend it to include a wall model. This will allow efficient simulations of more complex nozzle shapes, such as the ones including the chevrons. As an example of the code capabilities, we present an analysis of the noise reduction concept of beveling the nozzle exit plane, performed using our wall model approach [19] with a body-conforming mesh.

The paper is organized as follows. Section II outlines the simulation methods utilized in our methodology. Section III presents the analysis of an example noise reduction concept, i.e. beveling the nozzle exit plane. Finally, concluding remarks and future directions are provided in section IV.

## **II. LES METHODOLOGY**

In LES, the large turbulent eddies are simulated directly. The effect of small unresolved turbulent features on the dynamics of the large scales is modeled by using an (SGS) model. A variety of SGS models have been employed in LES of jets, such as the Smagorinsky [24] and dynamic Smagorinsky [25] models. Alternatively, one can use a spatial filter [26], [27] or rely on dissipation inherent in upwind-biased numerical schemes [9], [28] as an implicit SGS model. While unstructured solvers are frequently used for CFD because of their ability to easily handle complex geometries, the accuracy requirements for CAA often call for higher-order numerical schemes. This is due to the small amplitude of the acoustic fluctuations and the wide range of turbulent length scales that must be accurately resolved for many CAA problems. Structured solvers are ideal then, due to their available highorder methodologies and efficiency.

In our LES application, the Favre–filtered Navier–Stokes equations are solved in conservation form in generalized curvilinear coordinates on a uniform grid after spatial transformation. The classical fourth-order Runge–Kutta method is used for time integration and a surface integral method based on the porous surface Ffowcs Williams–Hawkings (FWH) equation [1] is used for evaluating farfield acoustic signals. Spatial partial derivatives appearing in the governing equations are discretized using a sixth-order compact finite difference scheme [29]

$$\frac{1}{3}f'_{i-1} + f'_i + \frac{1}{3}f'_{i+1} = \frac{7}{9h}(f_{i+1} - f_{i-1}) + \frac{1}{36h}(f_{i+2} - f_{i-2}),$$
(1)

where  $f_i$  and  $f'_i$  are the values of a function f and its approximate first derivative at the *i*th point, and h is the grid spacing. For points in the vicinity of the boundaries, lower-order and one-sided implicit difference formulae are used to ensure an overall tridiagonal formulation [29]. For nonlinear problems with non-periodic boundary conditions and non-uniform grids, the above compact difference scheme is unstable. As a result, we use a sixth-order compact spatial filter from Visbal and Gaitonde [30] to attenuate the highwavenumber modes. Furthermore, we do not use an explicit SGS model but use the spatial filter for implicit LES instead. The filter function is given by

$$\alpha_f \bar{f}_{i-1} + \bar{f}_i + \alpha_f \bar{f}_{i+1} = \sum_{n=0}^3 \frac{a_n}{2} (f_{i-n} + f_{i+n}), \quad (2)$$

where  $f_i$  is the filtered value of f at the *i*th point,  $\alpha_f$  is a filter parameter satisfying  $|\alpha_f| < 0.5$ , and  $a_0, a_1, a_2, a_3$ are constants depending on  $\alpha_f$ . Partially one-sided implicit formulae are used for points in the vicinity of boundaries [31].

A fast and accurate solver for the diagonally dominant tridiagonal linear systems induced by equations 1 and 2 is crucial to our LES application. To this end, the transposition [32] and Schur complement methods [33] are known to have severe scalability limitations [17]. Better in this regard are some schemes that have overlapping blocks [34], [35] and constrain the linear systems to individual blocks. This is trivial to parallelize, but inherently less accurate. For both high accuracy and scalability, our LES application uses the truncated SPIKE algorithm [36], [37] on non-overlapping blocks to solve these tridiagonal linear systems with theoretically optimal weakscaling scalability [38]. In references [17], [39] we report a strong scaling test case with a  $1,260 \times 1,260 \times 1,260$  grid where our LES application achieves a speedup of 24.6 for an efficiency of 74% when the number of processor cores increases from 2,744 to 91,125.

Our present solver has many other advantages over legacy versions of the code. These include support for cylindrical grids, extensions for supersonic jet flows, and a wide variety of boundary conditions. All of these improve the range of simulations that can be performed as well as the physical accuracy of the simulations. Below, each is discussed briefly.

First, for better simulations of jets, advanced support for cylindrical grids has been included in our LES application. Various approaches [40]–[42] exist to handle the centerline singularity and to avoid the time step penalty associated with the inherent concentrated grid point distribution in the azimuthal direction of cylindrical grids. We have implemented three approaches: a point skipping method [42], the spectral

method, and the windowed sinc filter [18]. The point skipping method produces good results [18], but our implementation uses the transposition method and thus inherits its scalability weaknesses. For our present simulations, however, this is acceptable since our production runs do not use tens of thousands of cores.

Second, to extend the implementation to enable simulations of off-design supersonic jets, it is necessary to add shock capturing routines. Based on the success of characteristic filters [43], [44] in our legacy code [8], they are ported to our current LES application. Characteristic filters are based on the idea that discontinuous flow predictions with baseline methodologies can be corrected by adding the dissipative portion of a nonlinear scheme (e.g., total variation diminishing (TVD) or weighted essentially non-oscillatory (WENO) schemes) after a full time step. In our application, they are made even more efficient and less dissipative by applying them locally to shock wave regions only [8] and using the filter formulation from Kim and Kwon [45]. The shock capturing modules have been validated by using jet simulations without nozzles and comparing to past results. Additionally, a centerline treatment method has been developed to reduce time step restrictions and allow simulations with cylindrical grids [46].

Lastly, many new boundary conditions have been added to our LES application that enable inclusion of the nozzle geometry. For these simulations, it is crucial to achieve a fully turbulent boundary layer on the nozzle wall to better match standard jet operating conditions. A realistic turbulent boundary layer is required to accurately feed the turbulent shear layer downstream of the nozzle exit which directly influences the noise produced. Thus, a digital filter-based turbulent inflow boundary condition [47]–[49] has been implemented and extended to non-uniform curvilinear coordinates in a novel way [50].

Furthermore, boundary conditions have been implemented for including the nozzle walls themselves. Options include walls based on characteristic analysis [51], [52] and more economical extrapolation-based no-slip conditions [53]. The combination of these methods with the approximate turbulent inflow has since been successfully validated and used for jet noise simulations.

The methods presented so far are capable of performing high-quality simulations for jet noise predictions. The problem, however, is that these simulations are often prohibitively expensive due to the high cost of accurately resolving the near-wall flow (i.e., the boundary layer) inside of the nozzle [54], [55]. As a result, we have implemented an approximate wall model boundary condition based on the standard log-law velocity profile. The log-law allows us to estimate the wall shear stress using the flowfield at some point off of the wall. The wall shear stress is then used to specify the flux at the wall as a boundary condition. This enables us to use much coarser grids than would otherwise be possible. This approach has been validated for turbulent flat plate boundary layer flow [19] and for jet flows [20], [21]. Many more implementation details for the wall model are provided in [46].

# III. ANALYSIS OF CONVERGING-DIVERGING BEVELED NOZZLE JETS

A beveled nozzle case is chosen as an example to show the capabilities of our methodology. More details about this case can be found in references [21], [46] The beveled nozzle is depicted in figure 1 using the design of Powers et al. [56]. Beginning with an axisymmetric nozzle and maintaining a constant diverging angle, one side of the nozzle is lengthened in the streamwise direction while the other is shortened. Compared to other noise mitigation techniques (see, for example, chevrons [57]–[60], corrugated nozzles [61]–[63], fluidic corrugations [64], [65], etc.), beveling the nozzle exit has received less attention. Beveled nozzles have been found to decrease noise for some observers with less than a 1% decrease in thrust [56], or possibly a slight increase [66], depending on how the beveling is completed. Furthermore, converging-diverging (CD) beveled nozzles do not show large undesirable deflections of the jet plume [56], [62], [63], [66] unlike purely converging beveled nozzles [67]-[70]. Powers et al. [56] examined CD nozzles with bevel angles of  $24^{\circ}$  and 35°. For the latter, they found a reduction of 3 dB overall sound pressure level (OASPL) in the peak direction with little penalty at other angles,  $\Theta$ , relative to the jet axis. This reduction was only for observers positioned along the long side of the nozzle,  $\Phi = 0^{\circ}$ , however. The levels were comparable to the baseline case at  $\Phi = 45^{\circ}$  and  $180^{\circ}$  from the long side of the nozzle (see figure 1). At  $\Phi = 90^{\circ}$ , however, the levels were 1 to 2 dB higher for observers close to the jet axis. These results for heated jets are comparable to those found by Viswanathan and Czech [66] for unheated jets. While the azimuthal variation of the sound field may not be ideal for all situations, it could be used advantageously. For example, in dual-nozzle configurations the long sides of each nozzle could



Fig. 1: Definition of the nozzle bevel angle, the long and short sides of the beveled nozzle, and the azimuthal angle,  $\Phi$ .

For the presented analysis, the baseline axisymmetric nozzle is modeled after a GE F400-series engine. A heated and overexpanded operating point is chosen to mimic takeoff conditions and simulations with and without bevel are performed. A wall model is used to limit the grids to realistic turnaround times. Grids with approximately 100 million points are utilized to ensure accurate simulation of the jet flowfield and to capture a wide range of acoustic frequencies. This provides more insight into beveling as a noise-reduction technique and gives additional flowfield and acoustic validation for the computational methodology including the wall model.

In this work, two simulations are performed: one of a baseline axisymmetric nozzle and one with the nozzle exit plane beveled by an angle of 35°. This provides a good test of the prediction accuracies that can be generated with the wallmodeling approach. Furthermore, beveled nozzles are ideal for the described high-order structured-grid methods because their geometry is continuous and smooth. The utilized nozzles and operating condition are chosen for their practical importance. The Mach 1.65 nozzles are modeled after the GE F404 family which are used on the F-18 aircraft [64]. The operating condition is for a nozzle total pressure ratio (NPR) of 3.5 and a total temperature ratio (TTR) of 3. This mimics critical takeoff conditions for fighter aircraft: an overexpanded and heated jet. This condition and nozzle has also been examined experimentally at Pennsylvania State University (PSU) by Kuo et al. [58] for the baseline geometry, Powers et al. [56] for both the beveled and baseline geometries, and at NASA by Bridges et al. [71] for the baseline jet. For a more detailed discussion of the operating conditions of the simulations, the boundary conditions used, the grid resolution details, and the statistics accumulation, please refer to [21].

Figure 2 shows 2-D contour plots comparing the mean axial velocity between the baseline simulation and the experiment. First, notice that both show a Mach disk but they are at different locations. This also affects the rest of the shock cell structure downstream. These discrepancies are suspected to result due to the several differences between the simulations and the experiments. The biggest known difference between the experiment and simulation is that the experiment [71] uses a dual-stream nozzle with a cold bypass flow, which is absent in the simulations. We have noticed that the peak turbulence levels in the shear layer are approximately two times higher for the simulation [21]. This is likely impacted by the bypass flow. Bridges et al. [71] show that the peak turbulence levels are higher for single-stream flows like that simulated here. The wall model, however, is also likely playing a role. It has previously been found to produce higher turbulence levels than those found in a wall-resolved simulation for a subsonic jet [72]. One of the other differences is that the military-style experimental geometries have a sharp throat that is difficult to simulate using a structured grid with high-order numerical methods. The throat impacts the flowfield downstream because shock waves form there [73]. Another difference is that the converging portion of the nozzles is shortened considerably from that used in the experiments, to minimize the length of the simulated nozzles and the amount by which the boundary layer thins within the converging portion. Currently, we are working on quantifying the effect of changes to the nozzle geometry, the boundary layer thickness within the nozzle, and the mean flow profiles imposed at the nozzle inlet on the average shock cell structure. These efforts will help bridge the gap between the experimental and the simulation results in our future studies.

To gain an appreciation for the differences in the flowfield between the baseline and beveled jets, instantaneous images of the velocity dilatation are shown in figure 3 for each.



Fig. 2: Comparisons between mean simulation and particle image velocimetry (PIV) experimental data  $(u/U_j)$  for the baseline jet.

Velocity dilatation allows one to clearly see the sound waves propagating outwards and the shock cell structure within the jet. The regularly-spaced shock cells of the baseline case are altered significantly for the beveled jet. The Mach disk is deflected towards the long lip by approximately  $0.5R_j$  and the entire jet plume is deflected towards the long side of the nozzle. A change in the acoustic wave radiation pattern is also apparent.

Figure 4 presents the OASPL results for the present baseline simulation and the mentioned experimental data. For detailed information on the locations of the FWH surfaces used for aeroacoustic predictions and the end-cap treatments used, please see [21]. The angle  $\Theta$  is measured from the downstream jet axis (see figure 1). Simulation results are shown for the different end-cap methods using surface S1. Note that the chosen method does not make a substantial impact on the results. The data without an end-cap are quieter for low observer angles and using a regular end-cap is slightly louder at high observers. Comparing with the experimental data, all share a peak OASPL location of 50° but the magnitudes are different. The simulation results are always quieter than the Kuo et al. [58] smooth-wall data and the Powers et al. [56] rough-wall results. The NASA experiment, however, is matched reasonably well for  $55^{\circ} < \Theta < 100^{\circ}$ . The trend of underpredicting the levels at low and high observer angles has been noted in other predictions also [46]. Notice, however, that there is up to 3 dB of scatter even between the various experimental results.

It is easier to see the changes to the OASPL due to beveling by defining

$$\Delta OASPL = OASPL \Big|_{\text{beveled iet}} - OASPL \Big|_{\text{baseline iet}}.$$
 (3)

This quantity is plotted as a function of observer angle,  $\Theta$ , and azimuthal angle,  $\Phi$ , in figure 5. Results for both the simulation and experiment are shown. At the long side of the nozzle ( $\Phi = 0^{\circ}$ ), the peak noise is reduced by almost 4 dB in the simulation and closer to 3 dB in the experiment. The trends are followed



Fig. 3: x-y plane showing instantaneous contours of velocity dilatation near the nozzle for both nozzle simulations.

well, however, especially for  $\Theta > 70^{\circ}$ . It is not clear why the simulation  $\Delta OASPL$  approaches zero as  $\Theta$  approaches  $20^{\circ}$ . This is not supported in the experiment. A similar trend is noted at  $\Phi = 45^{\circ}$  for small  $\Theta$ . The trend of reduced noise for lower angles and slightly increased noise for higher angles is maintained, however. At  $\Phi = 90^{\circ}$  the levels are higher for all but  $\Theta > 115^{\circ}$ . The experiment even finds higher levels in this region, but this is not seen in the simulations. The  $\Delta OASPL$  changes most significantly with  $\Theta$  at the short side of the nozzle ( $\Phi = 180^{\circ}$ ). At  $\Theta = 20^{\circ}$ , the simulation finds a 2 dB increase in OASPL which drops nearly linearly to -2 dB by  $\Theta = 60^{\circ}$ . The levels then increase as a function of  $\Theta$ . Overall, the experimental trends are captured well by the simulation although the quantitative predictions show errors as high as 1-2 dB.

Lastly, OASPL and  $\Delta$ OASPL contour plots for the present simulations are shown as functions of  $\Theta$  and  $\Phi$  in figure 6. This provides a more complete picture of the effect of beveling the nozzle on the resultant noise. It is clear from the OASPL



Fig. 4: Baseline nozzle OASPL results at  $r = 200R_j$  for FWH surface S1 and various end-cap treatments.



Fig. 5: Change in OASPL between baseline and  $35^{\circ}$  beveled nozzles using FWH surface S2 and the Ikeda et al. [74] end-cap method. Lines are used for the simulation data and the symbols are for the experiment.

contour plot that the maximum noise is near an azimuthal angle of 90°. The lower levels for the long and short sides of the nozzle, however, are plainly seen. Examining the  $\Delta$ OASPL plot, the reduction in the peak noise for the long and short sides of the beveled nozzle is clear. An interesting feature is that there is a large  $\Delta$ OASPL gradient for  $50^{\circ} < \Theta < 60^{\circ}$  at nearly all azimuthal angles. The exception to this rule is near  $\Phi = 90^{\circ}$  – the azimuthal angle for which the noise is the

loudest with the beveled nozzle. It is also interesting that the noise tends to be reduced for large observer angles and, again, for most azimuthal angles. This is related to the broadband shock-associated noise (BBSAN) peak in the spectra being larger for the baseline jet compared to the beveled case.



(a) OASPL at  $r = 200R_j$ . Contour lines are spaced every 1 dB.



(b)  $\triangle OASPL$ . Contour lines are spaced every 0.5 dB and dashed lines negative.

Fig. 6: OASPL  $\Theta$ - $\Phi$  maps for 35° beveled jet using FWH surface S2 and the Ikeda et al. [74] end-cap method.

## **IV. CONCLUSIONS AND FUTURE DIRECTIONS**

The accurate prediction and ultimate reduction of jet noise are problems that require significant effort and resources. We have developed an efficient highly-parallel LES methodology to accurately predict noise levels at the far field. Our LES code is designed to feature scalability across a wide range of processor core counts. It is a multi-block structured solver which has capabilities for simulating jets using cylindrical grids and has shock-capturing routines for supersonic jets with shock waves Fulfilling the requirements for accurate CAA simulations, it uses high-order numerical methods including compact finite difference schemes for spatial derivatives. Furthermore, it is capable of including walls in simulations and utilizing a wall model to reduce turnaround time. The solver includes a sharp immersed boundary method (IBM) to allow the use of grids that do not conform to the nozzle shape under study. This facilitates easier and efficient grid generation, This will allow efficient simulations of more complex nozzle shapes, such as the ones including the chevrons. As an example of current code capabilities, we present an analysis of the noise reduction concept of beveling the nozzle exit plane, performed using our wall model approach with a body-conforming mesh.

A practically important bevelled nozzle geometry and operating condition is simulated. The geometry is a Mach 1.65 nozzle modeled after the GE F404 family which is used on the F-18 aircraft. Additionally, the operating condition is overexpanded (nozzle pressure ratio of 3.5) and heated (total temperature ratio of 3) to approximate typical takeoff conditions. This is a critical condition for ground crews. In addition to an axisymmetric baseline simulation, the concept of beveling the nozzle exit [56], [62], [63], [66] is also studied. Experimentally, this modification has been found to reduce noise at some azimuthal observer locations. The flowfield for the baseline simulation is compared with available PIV data [71]. Reasonable trends are obtained. Although the acoustics levels are not always well-predicted for the individual simulations, the change between the results with the two geometries shows good agreement with the experimental trends. This further validates the presented methodology and highlights the capabilities of LES for evaluating noise-mitigation techniques. It also suggests that beveling the nozzle exit should be considered for further analysis based on the promising experimental and computational results. A reduction of 3 to 4 dB in the peak noise direction is shown in the simulations along the long side of the nozzle. This is competitive with other noise-reduction techniques, and therefore beveled nozzles are suggested for future research.

To allow simulations of more complex nozzle geometries, a sharp immersed boundary method [22] has been recently implemented in our LES solver [23]. Certain nozzle geometries involve sharp corners (such as chevrons) that cannot be perfectly modeled with a body-conforming grid due to the limitations of the structured nature of the solver. The ability to immerse a nozzle surface geometry in a non-bodyconforming background grid facilitates efficient and more realistic modeling of complicated nozzles. The use of a wall model is imperative to allow simulations at high Reynolds numbers at a practical computational cost. Therefore, the current IBM implementation is being extended to include wall modeling of the immersed boundaries. Using these techniques we plan to examine promising noise reduction concepts such as chevrons or fluidic inserts.

We have also developed a time-domain equivalent source method [75] for predicting acoustic scattering from hard surfaces such as the fuselage or deck near an exhausting jet aircraft or rotorcraft. The method is important to predicting the effects of various engine and nozzle configurations. The method can be linked to an LES prediction code [76] and account for these refraction effects.

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#### REFERENCES

- A. S. Lyrintzis, "Integral acoustics methods: From the (CFD) near-field to the (acoustic) far-field," *I.J. Aeroacoustics*, vol. 2, no. 2, pp. 95–128, 2003.
- [2] D. J. Bodony and S. K. Lele, "Current status of jet noise predictions using large-eddy simulation," *AIAA Journal*, vol. 46, no. 2, pp. 364–380, 2008.
- [3] A. Uzun and M. Y. Hussaini, "High-fidelity numerical simulations of a round nozzle jet flow," AIAA Paper No. 2010-4016, June 2010.
- [4] A. Uzun and M. Y. Hussaini, "On some issues in large-eddy simulations for chevron nozzle jet flows (invited paper)," AIAA Paper No. 2011-0019, January 2011.
- [5] A. Uzun and M. Y. Hussaini, "Some issues in large-eddy simulations for chevron nozzle jet flows," *Journal of Propulsion and Power*, vol. 28, no. 2, pp. 246–258, 2012.
- [6] C. Bogey, O. Marsden, and C. Bailly, "Influence of initial turbulence level on the flow and sound fields of a subsonic jet at a diameter-based Reynolds number of 10<sup>5</sup>," J. Fluid Mech., vol. 701, pp. 352–385, 2012.
- [7] C. Bogey, O. Marsden, and C. Bailly, "Effects of moderate Reynolds numbers on subsonic round jets with highly disturbed nozzle-exit boundary layers," *Physics of Fluids*, vol. 24, no. 10, p. 105107, 2012.
- [8] S.-C. Lo, K. Aikens, G. Blaisdell, and A. Lyrintzis, "Numerical investigation of 3-D supersonic jet flows using large-eddy simulation," *International Journal of Aeroacoustics*, vol. 11, no. 7, pp. 783–812, 2012.
- [9] M. L. Shur, P. R. Spalart, and M. K. Strelets, "Noise prediction for increasingly complex jets. part II: Applications," *International Journal* of Aeroacoustics, vol. 4, no. 3–4, pp. 247–266, 2005.
- [10] M. L. Shur, P. R. Spalart, and M. K. Strelets, "LES-based evaluation of a microjet noise reduction concept in static and flight conditions," *Journal of Sound and Vibration*, vol. 330, pp. 4083–4097, 2011.
- [11] M. L. Shur, P. R. Spalart, and M. K. Strelets, "Noise prediction for underexpanded jets in static and flight conditions," *AIAA Journal*, vol. 49, no. 9, pp. 2000–2017, 2011.
- [12] S. Mendez, M. Shoeybi, A. Sharma, F. E. Ham, S. K. Lele, and P. Moin, "Large-eddy simulations of perfectly expanded supersonic jets using an unstructured solver," *AIAA Journal*, vol. 50, no. 5, pp. 1103–1118, 2012.
- [13] G. A. Brès, J. W. Nichols, S. K. Lele, and F. E. Ham, "Towards best practices for jet noise predictions with unstructured large eddy simulations," AIAA Paper No. 2012-2965, June 2012.
- [14] J. W. Nichols, S. K. Lele, F. E. Ham, S. Martens, and J. T. Spyropoulos, "Crackle noise in heated supersonic jets," ASME Paper GT2012-69624, ASME Turbo Expo 2012, 2012.
- [15] I. Bermejo-Moreno, J. Bodart, and J. Larsson, "Scaling compressible flow solvers on the IBM Blue Gene/Q platform on up to 1.97 million cores," Center for Turb. Res. Annual Research Briefs, pp. 343–358, 2013.
- [16] Y. Khalighi, J. W. Nichols, S. Lele, F. Ham, and P. Moin, "Unstructured large eddy simulations for prediction of noise issued from turbulent jets in various configurations," AIAA Paper No. 2011-2886, June 2011.

- [17] C. S. Martha, Y. Situ, M. E. Louis, G. A. Blaisdell, A. S. Lyrintzis, and Z. Li, "Development and application of an efficient, multiblock 3-D large eddy simulation tool for jet noise," 50th Aerospace Sciences Meeting, Nashville, Tennessee, AIAA Paper No. 2012-0833, January 2012.
- [18] C. S. Martha, "Toward high-fidelity subsonic jet noise prediction using petascale supercomputers," Ph.D. dissertation, School of Aeronautics and Astronautics, Purdue University, West Lafayette, IN, May 2013.
- [19] K. M. Aikens, N. S. Dhamankar, C. S. Martha, Y. Situ, G. A. Blaisdell, A. S. Lyrintzis, and Z. Li, "Equilibrium wall model for large eddy simulations of jets for aeroacoustics," AIAA Paper No. 2014-0180, Aerospace Sciences Meeting, National Harbor, MD, January 2014.
- [20] K. Aikens, D. N., G. Blaisdell, A. Lyrintzis, and Z. Li, "High fidelity wall-modeled large eddy simulation for jet noise predictions," in 6<sup>th</sup> International Conference from Scientific Computing to Computational Engineering (IC-SCCE), Athens, Greece, July 2014.
- [21] K. M. Aikens, G. A. Blaisdell, A. S. Lyrintzis, and Z. Li, "Analysis of converging-diverging beveled nozzle jets using large eddy simulation with a wall model," AIAA Paper No. 2015-0509, Aerospace Sciences Meeting, Kissimmee, FL, January 2015.
- [22] R. Mittal and G. Iaccarino, "Immersed boundary methods," Annual Review of Fluid Mechanics, vol. 37, pp. 239–261, 2005.
- [23] N. S. Dhamankar, G. A. Blaisdell, A. S. Lyrintzis, and Z. Li, "Implementation of a sharp immersed boundary method in a 3-D multi-block large eddy simulation tool for jet aeroacoustics," AIAA Paper No. 2015-0504, January 2015.
- [24] J. Smagorinsky, "General circulation experiments with the primitive equations," *Monthly Weather Review*, vol. 91, no. 3, pp. 99–164, 1963.
- [25] M. Germano, U. Piomelli, P. Moin, and W. H. Cabot, "A dynamic subgrid-scale eddy viscosity model," *Physics of Fluids A*, vol. 3, no. 7, pp. 1760–1765, 1991.
- [26] A. Uzun, G. A. Blaisdell, and A. S. Lyrintzis, "Impact of subgrid-scale models on jet turbulence and noise," *AIAA Journal*, vol. 44, no. 6, pp. 1365–1368, 2006.
- [27] C. Bogey, C. Bailly, and D. Juve, "Noise investigation of a high subsonic, moderate Reynolds number jet using a compressible LES," *Theoretical* and Computational Fluid Dynamics, vol. 16, no. 4, pp. 273–297, 2003.
- [28] M. L. Shur, P. R. Spalart, and M. K. Strelets, "Noise prediction for increasingly complex jets. part I: Methods and tests," *International Journal of Aeroacoustics*, vol. 4, no. 3–4, pp. 213–246, 2005.
- [29] S. K. Lele, "Compact finite difference schemes with spectral-like resolution," J. Comp. Phys., vol. 103, pp. 16–42, 1992.
- [30] M. R. Visbal and D. V. Gaitonde, "Very high-order spatially implicit schemes for computational acoustics on curvilinear meshes," *Journal of Computational Acoustics*, vol. 9, no. 4, pp. 1259–1286, 2001.
- [31] D. Gaitonde and M. Visbal, "Pade-type higher-order boundary filters for the Navier-Stokes equations," *AIAA Journal*, vol. 38, no. 11, pp. 2103–2112, 2000.
- [32] A. Uzun, A. S. Lyrintzis, and G. A. Blaisdell, "Coupling of integral acoustics methods with LES for jet noise prediction," *International Journal of Aeroacoustics*, vol. 3, no. 4, pp. 297–346, 2004.
- [33] P.-T. Lew, "A study of sound generation from turbulent heated round jets using 3-D large eddy simulation," Ph.D. dissertation, School of Aeronautics and Astronautics, Purdue University, West Lafayette, IN, 2009.
- [34] X. Zhang, G. A. Blaisdell, and A. S. Lyrintzis, "High-order compact schemes with filters on multi-block domains," *Journal of Scientific Computing*, vol. 21, pp. 321–339, 2004.
- [35] A. Uzun and M. Y. Hussaini, "High-fidelity numerical simulations of a chevron nozzle jet flow," AIAA Paper No. 2009-3194, May 2009.
- [36] E. Polizzi and A. Sameh, "A parallel hybrid banded system solver: The SPIKE algorithm," *Parallel Computing*, vol. 32, no. 2, pp. 177– 194, 2006. [Online]. Available: http://linkinghub.elsevier.com/retrieve/ pii/S0167819105001353
- [37] Y. Situ, L. Liu, C. S. Martha, M. E. Louis, Z. Li, A. H. Sameh, G. A. Blaisdell, and A. S. Lyrintzis, "Reducing communication overhead in large eddy simulation of jet engine noise," in 2010 IEEE International Conference on Cluster Computing, 2010, pp. 255–264.
- [38] Y. Situ, L. Liu, C. S. Martha, M. E. Louis, Z. Li, A. H. Sameh, G. A. Blasidell, and A. S. Lyrintzis, "A communication-efficient linear system solver for large eddy simulation of jet engine noise," *Cluster Computing*, vol. 16, no. 1, pp. 157–170, 2013.
- [39] Y. Situ, C. S. Martha, M. E. Louis, Z. Li, A. H. Sameh, G. A. Blaisdell, and A. S. Lyrintzis, "Petascale large eddy simulation of jet engine noise based on the truncated SPIKE algorithm," *Parallel Computing*, vol. 40, no. 9, pp. 496–511, 2014.

- [40] K. Mohseni and T. Colonius, "Numerical treatment of polar coordinate singularities," J. Comp. Phys., vol. 157, pp. 787–795, 2000.
- [41] A. Uzun and M. Y. Hussaini, "Simulation of noise generation in nearnozzle region of a chevron nozzle jet," *AIAA Journal*, vol. 47, pp. 1793– 1810, 2009.
- [42] C. Bogey, N. de Cacqueray, and C. Bailly, "Finite differences for coarse azimuthal discretization and for reduction of effective resolution near origin of cylindrical flow equations," *J. Comp. Phys.*, vol. 230, pp. 1134– 1146, 2011.
- [43] H. C. Yee, N. D. Sandham, and M. J. Djomehri, "Low-dissipative high-order shock-capturing methods using characteristic-based filters," *J. Comp. Phys.*, vol. 150, pp. 199–238, 1999.
- [44] E. Garnier, P. Sagaut, and M. Deville, "A class of explicit ENO filters with applications to unsteady flows," *J. Comp. Phys.*, vol. 170, no. 1, pp. 184–204, 2001.
- [45] D. Kim and J. H. Kwon, "A low dissipative and dispersive scheme with a high order WENO dissipation for unsteady flow analyses," AIAA Paper No. 2004-2705, June 2004.
- [46] K. M. Aikens, "High-fidelity large eddy simulation for supersonic jet noise prediction," Ph.D. dissertation, School of Aeronautics and Astronautics, Purdue University, West Lafayette, IN, August 2014.
- [47] M. Klein, A. Sadiki, and J. Janicka, "A digital filter based generation of inflow data for spatially developing direct numerical or large eddy simulations," *J. Comp. Phys.*, vol. 186, no. 2, pp. 652–665, 2003.
- [48] Z.-T. Xie and I. P. Castro, "Efficient generation of inflow conditions for large eddy simulation of street-scale flows," *Flow, Turbulence and Combustion*, vol. 81, no. 3, pp. 449–470, 2008.
- [49] E. Touber and N. D. Sandham, "Large-eddy simulation of low-frequency unsteadiness in a turbulent shock-induced separation bubble," *Theoretical and Computational Fluid Dynamics*, vol. 23, no. 2, pp. 79–107, 2009.
- [50] N. S. Dhamankar, C. S. Martha, Y. Situ, K. M. Aikens, G. A. Blaisdell, A. S. Lyrintzis, and Z. Li, "Digital filter-based turbulent inflow generation for jet aeroacoustics on non-uniform structured grids," AIAA Paper No. 2014-1401, Aerospace Sciences Meeting, National Harbor, MD, January 2014.
- [51] J. W. Kim and D. J. Lee, "Generalized characteristic boundary conditions for computational aeroacoustics," *AIAA Journal*, vol. 38, no. 11, pp. 2040–2049, 2000.
- [52] J. W. Kim and D. J. Lee, "Generalized characteristic boundary conditions for computational aeroacoustics, part 2," *AIAA Journal*, vol. 42, no. 1, pp. 47–55, 2004.
- [53] J. Liu, C. Kaplan, and E. Oran, "A brief note on implementing boundary conditions at a solid wall using the FCT algorithm," Memorandum report NRL/MR/6410-06-8943, Naval Research Laboratory, Tech. Rep., April 2006.
- [54] D. R. Chapman, "Computational aerodynamics development and outlook," AIAA Journal, vol. 17, no. 12, pp. 1293–1313, 1979.
- [55] H. Choi and P. Moin, "Grid-point requirements for large eddy simulation: Chapman's estimates revisited," *Physics of Fluids*, vol. 24, no. 1, p. 011702, 2012.
- [56] R. W. Powers, M. Senft, and D. K. McLaughlin, "Acoustics measurements of scale models of military style supersonic beveled nozzle jets," AIAA Paper No. 2011-2702, June 2011.
- [57] B. Henderson and J. Bridges, "An MDOE investigation of chevrons for supersonic jet noise reduction," AIAA Paper No. 2010-3926, June 2010.
- [58] C.-W. Kuo, J. Veltin, and D. K. McLaughlin, "Advanced acoustic assessment of small-scale military-style nozzles with chevrons," AIAA Paper No. 2010-3923, June 2010.
- [59] S. Martens and J. T. Spyropoulos, "Practical jet noise reduction for tactical aircraft," ASME Paper GT2010-23699, ASME Turbo Expo 2010, Glasgow, UK, 2010.
- [60] D. Munday, N. Heeb, E. Gutmark, J. Liu, and K. Kailasanath, "Acoustic effect of chevrons on supersonic jets exiting conical convergentdivergent nozzles," *AIAA Journal*, vol. 50, no. 11, pp. 2336–2350, 2012.
- [61] J. M. Seiner, L. S. Ukeiley, and B. J. Jansen, "Aero-performance efficient noise reduction for the F404-400 engine," AIAA Paper No. 2005-3048, May 2005.
- [62] K. Viswanathan, A. Krothapalli, J. Seiner, M. Czech, B. Greska, and B. Jansen, "Assessment of low-noise nozzle designs for fighter aircraft applications," *Journal of Aircraft*, vol. 48, no. 2, pp. 412–423, 2011.
- [63] R. W. Powers and D. K. McLaughlin, "Acoustics measurements of scale models of military style supersonic beveled nozzle jets with interior corrugations," AIAA Paper No. 2012-2116, June 2012.
- [64] R. W. Powers, C.-W. Kuo, and D. K. McLaughlin, "Experimental comparison of supersonic jets exhausting from military style nozzles

with interior corrugations and fluidic inserts," AIAA Paper No. 2013-2186, May 2013.

- [65] P. J. Morris, D. K. McLaughlin, and C.-W. Kuo, "Noise reduction in supersonic jets by nozzle fluidic inserts," *Journal of Sound and Vibration*, vol. 332, no. 17, pp. 3992–4003, 2013.
- [66] K. Viswanathan and M. J. Czech, "Adaptation of the beveled nozzle for high-speed jet noise reduction," *AIAA Journal*, vol. 49, no. 5, pp. 932–944, 2011.
- [67] K. Viswanathan, "Nozzle shaping for reduction of jet noise from single jets," AIAA Journal, vol. 43, no. 5, pp. 1008–1022, 2005.
- [68] K. Viswanathan, M. Shur, P. R. Spalart, and M. K. Strelets, "Flow and noise predictions for single and dual-stream beveled nozzles," *AIAA Journal*, vol. 46, no. 3, pp. 601–626, 2008.
- [69] K. Viswanathan, P. R. Spalart, M. J. Czech, A. Garbaruk, and M. Shur, "Tailored nozzles for jet plume control and noise reduction," *AIAA Journal*, vol. 50, no. 10, pp. 2115–2134, 2012.
- [70] M. L. Shur, P. R. Spalart, M. K. Strelets, and A. V. Garbaruk, "Analysis of jet-noise-reduction concepts by large-eddy simulation," *International Journal of Aeroacoustics*, vol. 6, no. 3, pp. 243–285, 2007.
- [71] J. Bridges, M. P. Wernet, and F. C. Frate, "PIV measurements of chevrons on F400-series tactical aircraft nozzle model," AIAA Paper No. 2011-1157, January 2011.
- [72] K. M. Aikens, N. S. Dhamankar, G. A. Blaisdell, A. S. Lyrintzis, and Z. Li, "High-fidelity wall-modeled large eddy simulation for jet noise predictions," 6th Int. Conf. from Scientific Computing to Computational Engineering, Athens, Greece, July 9-12, 2014.
- [73] D. Munday, E. Gutmark, J. Liu, and K. Kailasanath, "Flow structure and acoustics of supersonic jets from conical convergent-divergent nozzles," *Physics of Fluids*, vol. 23, no. 11, p. 116102, 2011.
- [74] T. Ikeda, S. Enomoto, and K. Yamamoto, "On the modification of the Ffowcs Williams–Hawkings integration for jet noise prediction," AIAA Paper No. 2013-2277, May 2013.
- [75] S. H. Swift, G. Blaisdell, and A. Lyrintzis, "An efficient time-domain equivalent method for acoustic scattering," *International Journal of Aeroacoustics*, vol. 14, no. 1–2, pp. 133–160, 2015.
- [76] S. H. Swift, G. Blaisdell, and A. Lyrintzis, "Enhancements of a timedomain equivalent source method for acoustic scattering," in AIAA 21<sup>st</sup> Aeroacoustics Meeting, Dallas, TX, June 2015.